21-120: Differential and Integral Calculus
Recitation #9 Outline: 09/24/24

1. Here we will derive the Chain Rule in the special case that $g(x) \neq g(a)$ for $x \neq a$.

   (a) Suppose that $g(x)$ is continuous at a, $f(x)$ is differentiable at $g(a)$, and $g(x) \neq g(a)$ for all $x \neq a$. Then evaluate the following limit
   $$\lim_{x \to a} \frac{f\big(g(x)\big) - f\big(g(a)\big)}{g(x) - g(a)}.$$

   (b) Use (a) to show that if $h(x) = f\big(g(x)\big)$, f is differentiable at $g(a)$, g is differentiable at a, and $g(x) \neq g(a)$ when $x \neq a$ then
   $$h'(a) = \lim_{x \to a} \frac{f\big(g(x)\big) - f\big(g(a)\big)}{x - a} = f'\big(g(a)\big)g'(a)$$
   (Hint: multiply and divide by $g(x) - g(a)$).

2. Use the chain rule to evaluate the following derivatives.

   (a) $f(x) = (5x^6 + 2x^3)^4$

   (b) $y = \sqrt{\dfrac{x}{x+1}}$

   (c) $k(h) = \cos\left(\sqrt{h^2 - 2}\right)$

3. Find y' and y''.

   (a) $y = \sin(3x)$

   (b) $y = \sqrt{1 - \sec(t)}$

4. Find an equation of the tangent line to the curve at the given point.

   (a) $y = \sqrt{1 + x^3}$, $(2, 3)$

   (b) $y = \sin\big(\sin(x)\big)$, $(\pi, 0)$

5. Use the table of values for $f(x)$, $g(x)$, $h(x)$, $f'(x)$, $g'(x)$, and $h'(x)$ to evaluate the following derivatives.

x	f(x)	g(x)	h(x)	f'(x)	g'(x)	h'(x)
1	3	2	4	3	2	2
2	1	1	0	1	3	2
3	4	3	2	2	2	4

   (a) $F(x) = g\big(f(x)\big)$, find $F'(2)$

   (b) $G(x) = g\Big(h\big(f(x)\big)\Big)$, find $G'(1)$

   (c) $H(x) = g\Big(g\Big(g\big(g(x)\big)\Big)\Big)$, find $H'(2)$