21-120: Differential and Integral Calculus
Recitation #5

1. Show that the equation $-x^3+x^2-x+2=0$ has at least one solution in the interval $(1,2)$.

2. Show that the equation
$$\cos(x)=\frac{1}{x}$$
has infinitely many solutions in $(0,+\infty)$.
Hint: Think about what happens between $2k\pi$ and $2k\pi+\pi$ when $k\geq 1$ is an integer.

3. Evaluate the following limits:

   (a) $\lim\limits_{x\to+\infty}\frac{\sqrt{x^2-7}}{3x+5}$

   (b) $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2-7}}{3x+5}$

   (c) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+6x+1}-x\right)$

   (d) $\lim\limits_{x\to+\infty}\frac{2e^x+1}{e^x-2}$.

4. For the function f defined for every $x\in\mathbb{R}$ as follows:
$$f(x)=\frac{3x}{x^2-x-6},$$
determine the equations of all horizontal or vertical asymptotes.

5. (a) Show, using the (ϵ,δ) definition that :
$$\lim_{x\to 0}x^2=0.$$
(b) Translate the statement into a mathematical formula (with quantifiers):
$$\lim_{x\to 0}\ln(1+x)=0.$$