21-120: Differential and Integral Calculus
Lecture #34 Outline

Read: Section 3.1 of the OpenStax Calculus 2 Textbook

Objectives and Concepts:

- Integration by Parts is a technique derived from the Product Rule for derivatives that allows us to rewrite an integral of the product of two functions in a possibly more simplified way that is easier to integrate.
- The appropriate choice of functions u and dv in integration by parts depends on how easy they are to differentiate and integrate, respectively. We can use the LIATE approach to help us choose which function would be the u and the dv.

Suggested Textbook Exercises:

- 3.1: 9-35 odd, 39-45 odd, 53-57 odd

Integration By Parts, Continued

Here we will get some more practice applying the Integration by Parts formula

$$\int u\,dv = uv - \int v\,du.$$

Recall that the choice of u can be guided by LIATE:

- **L - logarithmic:** Functions whose outermost operation is a logarithm usually convert to algebraic functions when you differentiate them.
- **I - inverse trigonometric:** Inverse trig functions usually have algebraic derivatives.
- **A - algebraic:** Algebraic functions include polynomials, rational functions, and functions with roots of polynomials. These may simplify when you differentiate them.
- **T - trigonometric:** Trig functions have trig function derivatives, so choosing u to be a trig function may not be of much benefit.
- **E - exponential:** Exponential functions have exponential function derivatives so the problem is usually not simplified any by choosing u to be an exponential function.

Example 1: Evaluate $\int_1^e x^3 \ln x \, dx$.

Example 2: Evaluate $\int_0^1 x \arctan x \, dx$.

In some cases, we could end up with the same integral twice. Once we see two occurrences of the same integral in a single equation, we can solve for the integral.

Example 3: Evaluate $\int e^{2x}\cos 3x\,dx$.

Sometimes making an appropriate substitution prior to attempting integration by parts can be helpful
Example 4: Evaluate the following integrals.

(a) $\int e^{\sqrt{2x+4}}\,dx$

(b) $\int \sin(\ln x)\,dx$