21-120: Differential and Integral Calculus
Recitation #19 Outline: 11/7/24

1. Find the antiderivatives of the following functions.

   (a) $4x^5 + 2x^3 - 7x^2$ (b) $\cos\left(\frac{x}{2}\right) - \frac{3}{x^2+1}$ (c) $3^e - \frac{1}{x^{4/3}}$

2. Evaluate the following sums.

   (a) $\sum_{i=4}^{8} \left((i+2)^2 - 1\right)$ (b) $\sum_{k=3}^{12} \left(k^3 + k^2\right)$

3. Given $a > 0$, evaluate the following limits.

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i a^2}{n^2}$ (b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2 a^3}{n^3}$

   What areas do these limits correspond to?

4. (a) Estimate the area under the graph of $f(x) = \sin(x)$ from $x = 0$ to $x = \pi/2$ using three approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

   (b) Repeat part (a) using left endpoints.

5. Oil leaked from a tank at a rate of $r(t)$ liters per hour for 10 hours before the leak was discovered. The rate decreased as time passed and values of the rate at two-hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

t (h)	0	2	4	6	8	10
$r(t)$ (L/h)	8.7	7.6	6.8	6.2	5.7	5.3