21-120: Differential and Integral Calculus
Lecture #35 Outline

More Integration Practice

Example 1: Evaluate the following integrals.

(a) $\displaystyle\int \frac{\ln x}{x^2}\,dx$

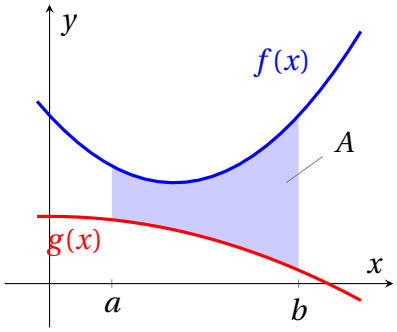
(b) $\displaystyle\int x^5 e^{x^3}\,dx$

(c) $\displaystyle\int_0^{1/\sqrt{2}} 2x \arcsin(x^2)\, dx$

(d) $\displaystyle\int_{-\sqrt{7}}^{0} t(t^2+1)^{1/3}\, dt$

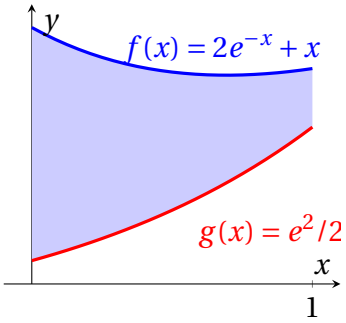
Application: The Area Between Two Curves

In some instances, we wish to find the area of a region bounded by two different curves, $f(x)$ and $g(x)$, between $x = a$ and $x = b$. For example, in the figure to the right, the area A is the area bounded above by the curve $y = f(x)$ and below by the curve $y = g(x)$. In this situation, it's not too hard to see that the area between the two curves is the area under $f(x)$ minus the area under $g(x)$:

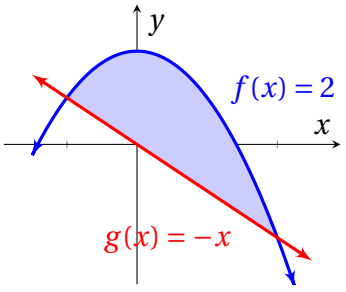


$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b \big(f(x) - g(x)\big)\,dx.$$

Example 2: Find the area bounded above by the curve $f(x) = 2e^{-x} + x$ and bounded below by the curve $g(x) = \frac{1}{2}e^x$ from $x = 0$ to $x = 1$.

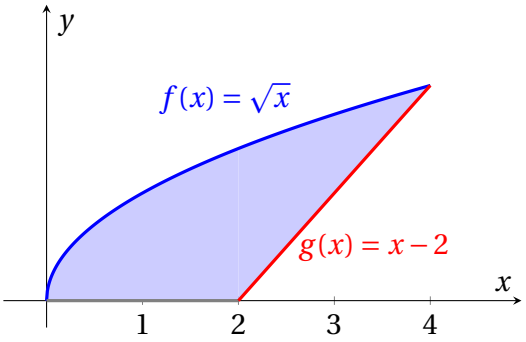


Example 3: Find the area bounded by the curves $f(x) = 2 - x^2$ and $g(x) = -x$.



You can also set up this kind of integral with respect to y if it makes more sense to do so. You may have to solve for $x = f^{-1}(y)$ given $y = f(x)$ in order to set the problem up.

Example 4: Set up the integral(s) that would calculate the area in the first quadrant bounded above by the curve $f(x) = \sqrt{x}$ and bounded below by the x-axis and the line $g(x) = x - 2$, first by integrating with respect to x, then by integrating with respect to y.



Example 5: Set up integrals that would find the area in the first quadrant bounded by the curves $y = 1$, $y = x$, and $y = x^2/4$, first by integrating with respect to x, then by integrating with respect to y.