21-120: Differential and Integral Calculus
Lecture #31 Outline

Read: Sections 5.5, 5.6, 5.7 of the textbook

Objectives and Concepts:

- Integration by substitution (or u-substitution) is a technique for integrating functions whose derivatives are also present in the integrand. Substitution can be thought of as the chain rule for derivatives in reverse.
- To use u-substitution, we must substitute the entire integrand and the variable of integration completely.
- When performing u-substitution in a definite integral, you should change the limits of integration to reflect the new variable. This often simplifies the problem and makes the antiderivative easier to evaluate at the limits of integration.

Suggested Textbook Exercises:

- 5.5: 257-287 odd, 293-297 odd, 307-311 odd.
- 5.6: 321-365 odd, 373-381 odd.
- 5.7: 391-405 odd, 411-415 odd, 423-431 odd.

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u-Substitution, Continued

As a reminder, our techniques for finding indefinite and definite integrals now include a method for handling more complicated integrands which either might be the result of a chain rule derivative calculation or can be simplified via a replacement of the variable of integration with another variable. Here are some important takeaways from the previous lecture to emphasize:

- Identifying a good choice for the variable u usually amounts to finding some component of the integrand (call it $g(x)$) such that a multiple of its derivative ($g'(x)$) is also present in the integrand. Setting this component to u should reduce the integrand to some function of u whose antiderivative is fairly straightforward to find.
- Once you determine $u = g(x)$, you must calculate the differential $du = g'(x)\,dx$. You may need to manipulate this equation before making the substitutions for $g(x)$ and whatever multiple of $g'(x)\,dx$ is present in the integrand. Be sure that your new integral is written completely in terms of the new variable - there should be no instances of the old variable (x) in the integrand.
- When performing u-substitution on a definite integral (given in the variable x, for example), we need to convert the limits of integration from the variable x to the new variable u. Do NOT forget to do this.
- Sometimes our motivation for making a substitution is to simply rewrite the integrand in an alternate way that has an easier antiderivative. These types of problems often involve fractions, radicals, or both.
- Finally, there are integrals that require not just one, but two or even three successive substitutions in order to evaluate completely. Just be patient and treat each newly-substituted integral as its own problem.

Example 1: Evaluate each integral.

(a) $\displaystyle\int \frac{e^{\sin^{-1}(x)}}{\sqrt{1-x^2}}\,dx$

(b) $\displaystyle\int (x+5)(x-5)^{1/3}\,dx$

(c) $\displaystyle\int \frac{5}{9+4x^2}\,dx$

(d) $\displaystyle\int_0^{\sqrt[3]{\pi^2}} \sqrt{\theta}\cos^2\left(\theta^{3/2}\right)\sin\left(\theta^{3/2}\right)\,d\theta$

(e) $\displaystyle\int_1^9 t\sqrt{4+5t}\,dt$

(f) $\displaystyle\int_{\pi/4}^{\pi/3} \left(1 + e^{\cot\theta}\right) \csc^2\theta \, d\theta$

(g) $\displaystyle\int_{2}^{16} \frac{dx}{2x\sqrt{\ln x}}$

(h) $\displaystyle\int_1^{e^{\sqrt{3}}} \frac{4\,dx}{x\left[1+(\ln x)^2\right]}$

(i) $\displaystyle\int_0^{\pi/2} \sqrt{\sin x}\cos^3 x\,dx$