21-120: Differential and Integral Calculus Recitation #22 Outline: 11/19/24

1. Evaluate each indefinite integral.

(a)
$$\int \frac{(\ln x)^2}{x} \, dx$$

(b)
$$\int \sin x \sin(\cos x) dx$$
 (c) $\int \frac{x^2}{x^3 + 1} dx$

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$$\int \frac{x^2}{x^3 + 1} \, dx$$

2. Evaluate each definite integral.

(a)
$$\int_0^1 \frac{e^x + 1}{e^x + x} dx$$

(b)
$$\int_0^3 \frac{dx}{5x+1}$$

$$\text{(c)} \int_0^4 \frac{x}{\sqrt{2x+1}} \, dx$$

3. Let f be an everywhere-continuous function.

(a) Find
$$\int_0^2 f(2x) dx$$
 knowing that $\int_0^4 f(x) dx = 10$.

(b) Find
$$\int_0^3 x f(x^2) dx$$
 knowing that $\int_0^9 f(x) dx = 4$.

4. Let *f* be an everywhere-continuous function. Show that

$$\int_0^a x^3 f(x^2) \, dx = \frac{1}{2} \int_0^{a^2} x f(x) \, dx$$

for any a > 0.