21-120: Differential and Integral Calculus
Lecture #23 Outline

Read: Section 4.7 of the textbook

Objectives and Concepts:

- Optimization problems involve situations where there is a function that you wish to maximize or minimize.
- A procedure for solving optimization problems involves first understanding the problem, drawing a diagram, finding the quantity to be optimized in terms of an independent variable, and then finding the absolute max/min of that function on its domain.
- When the quantity to be optimized is initially expressed in terms of more than one independent variable, look for information (perhaps a constraint) that will allow you to write the quantity in terms of a single variable.

Suggested Textbook Exercises:

- 4.7: 311-314 all, 315-355 odd.

Applied Optimization Problems

One of the most important applications of calculus is finding optimal solutions to problems in the sciences, engineering, and business. Usually, the most challenging aspect of optimization is coming up with the correct mathematical model that represents the quantity to be optimized. For the problems we study in this course, it is often the case that the quantity to be optimized depends on more than just one independent variable. In these situations, we usually must also incorporate one or more **constraints** that can be used to eliminate some independent variables, reducing the scenario to a single-variable optimization problem.

Steps involved in solving an optimization problem:

1. Read and understand the problem (identify the question).
2. Draw a diagram that represents the situation.
3. Find the quantity to be optimized and express it in terms of independent variable(s).
4. Identify any constraints on the independent variables.
5. Write objective function with just one independent variable, using the constraint.
6. Find the *absolute* min or max, depending on the situation.

We start with some simple problems that could arise in some geometric situations.

Example 1: Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.

Example 2: Find two positive integers such that their sum is 10, and the sum of their squares is as large as possible. Then find two positive integers such that their sum is 10, and the sum of their squares is as small as possible.

Example 3: A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 meters of wire at your disposal, what is the largest area you can enclose?

Example 4: A printed poster is to have a margin of 1 inch on the sides and 1.5 inches at the top and bottom. What dimensions of the poster board will minimize the total area of the poster if the printed area of the poster must be 96 square inches?

Example 5: A standard can in the shape of a right circular cylinder with a top and a bottom contains a volume of 2000 cubic centimeters. Find the dimensions of the can that will minimize the metal needed to manufacture the can.