21-120: Differential and Integral Calculus
Recitation #16 Outline: 10/24/24

1. Find the following limits. You may use the fact that $\lim_{t\to 0}\frac{\sin t}{t} = 1$ and $\lim_{t\to 0}\frac{\cos t - 1}{t} = 0$. You may NOT use L'Hôpital's rule.

   (a) $\lim_{x\to 0}(\cot(2x)\cdot\sin x)$;

   (b) $\lim_{x\to 0}\frac{\sec x - 1}{x}$.

2. Determine if the Mean Value Theorem can be applied to the function $f(x) = x^2 - x^{2/3}$ on the the interval $[-1,8]$. If so, find all possible values of c that satisfy the conclusion of the Mean Value Theorem.

3. Without graphing anything, explain why the equation $2^x - 2x + 1 = 0$ cannot have more than two real solutions.

4. Using an appropriate linear approximation, estimate $(1.01)^{-3}$.

5. Find $\frac{dy}{dx}$ for each of the following:

   (a) $\cos(x^2 + 2y) + xe^{y^2} = 1$;

   (b) $y = (2x - e^{8x})^{\sin(2x)}$.

6. Let $f(x) = \frac{e^{-3x}}{x^2+1}$. Find the equation of the tangent line to f^{-1} at $(1,0)$.

7. Let $f(x) = x^{1/3}(x-2)$. Without doing any calculations, explain why this function must have both absolute maximum and minimum on $[-1,3]$. Then find the critical numbers as well as the absolute maximum and minimum values of f on the interval $[-1,3]$.

8. Let $f(x) = x^2 - x - \ln x$.

   (a) Find the intervals on which f is increasing and decreasing.

   (b) Find the local minimum and maximum values of f (if any).

   (c) Find the inflection points of f (if any), and the intervals of concavity.