21-120: Differential and Integral Calculus
Lecture #29 Outline

Read: Section 5.3, 5.4 of the textbook

Objectives and Concepts:

- The integral of a rate of change f' from a to b is the net change in the function f from a to b.

Suggested Textbook Exercises:

- 5.3: 149-163 odd, 171-197 odd.
- 5.4: 207-211 odd, 219-225 odd.

Evaluating Definite Integrals

Here we will use Part II of the Fundamental Theorem of Calculus,

$$\int_a^b f'(x)\,dx = f(b) - f(a),$$

to evaluate several definite integrals. This amounts to finding an antiderivative of $f(x)$, evaluating the antiderivative at both limits of integration, and then subtracting.

Example 1: Evaluate the definite integrals.

(a) $\displaystyle\int_{-1}^{2} \left(4x - 6x^2 + 1\right)\,dx$

(b) $\displaystyle\int_1^8 \frac{x+x^2}{x^{2/3}}\,dx$

(c) $\displaystyle\int_{-\pi/6}^{\pi/3} (2\sec(x)\tan(x)+\cos x)\,dx$

(d) $\displaystyle\int_0^1 \frac{4}{t^2+1}\,dt$

(e) $\displaystyle\int_{-e^2}^{-e} \frac{3}{x}\,dx$

(f) $\displaystyle\int_{-4}^{4} \sin x\,dx$

If f is an odd function ($f(-x) = -f(x)$ for all x), then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

If f is an even function ($f(-x) = f(x)$ for all x), then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

Example 2: Can you evaluate $\int_{-2}^{3} \frac{6}{x^2}\,dx$? Why or why not?

The Net Change Theorem: The new value of a changing quantity equals the initial value plus the integral of the rate of change:

$$F(b) = F(a) + \int_a^b F'(x)\,dx.$$

Here are a few instances of this idea:

- If $V(t)$ is the volume of water in a pool at time t, then its derivative $V'(t)$ is the rate at which the pool loses or gains water at time t. So

  $$\int_{t_1}^{t_2} V'(t)\,dt = V(t_2) - V(t_1)$$

  is the change in the amount of water in the pool between time t_1 and t_2.

- If $C(x)$ is the cost of producing x units of a product, then the marginal cost is the derivative $C'(x)$. So

  $$\int_{x_1}^{x_2} C'(x)\,dx = C(x_2) - C(x_1)$$

  is the increase in cost when production is increased from x_1 to x_2 units.

- If a particle moves along a straight line with position function $s(t)$, then its velocity is $s'(t) = v(t)$, so

  $$\int_a^b v(t)\,dt = s(b) - s(a)$$

  represents the *displacement* of the particle from time $t = a$ to time $t = b$. Meanwhile,

$$\int_a^b |v(t)|\,dt = \text{ total distance traveled from } t = a \text{ to } t = b.$$

Example 3: The velocity function for a particle moving along a line is given by $v(t) = t^2 - 2t$ meters per second, where $0 \le t \le 5$. Find the displacement and the total distance traveled by the particle during this time interval.