21-120: Differential and Integral Calculus
Recitation #17 Outline: 10/29/24

1. Given that

$$\lim_{x\to a} f(x) = 0 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 1 \qquad \lim_{x\to a} p(x) = \infty \qquad \lim_{x\to a} q(x) = \infty$$

which of the following limits are indeterminate forms? For those that are not, evaluate the limit where possible.

(a) $\lim_{x\to a}(f(x) - p(x))$
(b) $\lim_{x\to a}(p(x) - q(x))$
(c) $\lim_{x\to a}(p(x) + q(x))$
(d) $\lim_{x\to a}(f(x))^{g(x)}$
(e) $\lim_{x\to a}(f(x))^{p(x)}$
(f) $\lim_{x\to a}(h(x))^{p(x)}$
(g) $\lim_{x\to a}(p(x))^{f(x)}$
(h) $\lim_{x\to a}(p(x))^{q(x)}$
(i) $\lim_{x\to a} \sqrt[q(x)]{p(x)}$

2. Find the limit using l'Hospital's rule.

(a) $\lim_{x\to 0}(\csc x - \cot x)$
(b) $\lim_{t\to\infty}(x - \ln x)$
(c) $\lim_{x\to 0^+} x^{\sqrt{x}}$
(d) $\lim_{x\to\infty} x^{e^{-x}}$
(e) $\lim_{x\to 1}(2-x)^{\tan(\pi x/2)}$
(f) $\lim_{x\to 0^+}(1+\sin(3x))^{1/x}$

3. Suppose f is a positive function. If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \infty$, show that

$$\lim_{x\to a}(f(x))^{g(x)} = 0.$$

This shows that 0^∞ is not an indeterminate form.