
21-120: Differential and Integral Calculus
Recitation #6 Outline: 09/12/24

1. Use the definition of derivative to directly compute $f'(0)$ for the following functions.

(a) $f(x) = x^2 + 3x - 2$

(b) $f(x) = x \cos(x)$

2. Show that the following function is continuous but not differentiable at 0.

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

3. Find an equation of the tangent line to the curve at the given point.

(a) $y = x^3 - 3x + 1$, $(2, 3)$

(b) $y = \frac{3x+1}{x+3}$, $(1, 1)$

4. Suppose that the positions (in meters) of two runners, A and B, running a 100-meter race are respectively given by

$$f(t) = \frac{100}{14}t \text{ and } g(t) = \frac{100}{196}t^2$$

where t is measured in seconds.

- (a) Sketch the graphs of the two runners positions.
 - (b) Describe and compare how the runners run the race. Who won?
 - (c) At what time are they running at the same velocity? What is that velocity?
 - (d) What was the fastest velocity runner B ran at? If runner B ran at this pace for the entire race, how many seconds would they have beaten runner A by?
5. The number of bacteria after t hours in a controlled laboratory experiment is $n = f(t)$.
- (a) What is the meaning of the derivative $f'(5)$? What are its units?
 - (b) Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger, $f'(5)$ or $f'(10)$? If the supply of nutrients is limited, would that affect your conclusion? Explain.
 - (c) Suppose that $f'(t) = e^{-t}$. Which of the two situations above would this correspond to?