21-120: Differential and Integral Calculus
Recitation #15 Outline: 10/22/24

1. Given that

$$\lim_{x\to a} f(x) = 0 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 1 \qquad \lim_{x\to a} p(x) = \infty \qquad \lim_{x\to a} q(x) = \infty$$

which of the following limits are indeterminate forms? For those that are not, evaluate the limit where possible.

(a) $\lim_{x\to a} \dfrac{f(x)}{g(x)}$

(b) $\lim_{x\to a} \dfrac{f(x)}{p(x)}$

(c) $\lim_{x\to a} \dfrac{p(x)}{q(x)}$

(d) $\lim_{x\to a} \dfrac{p(x)}{f(x)}$

(e) $\lim_{x\to a} \dfrac{h(x)}{p(x)}$

(f) $\lim_{x\to a} f(x)p(x)$

(g) $\lim_{x\to a} p(x)q(x)$

(h) $\lim_{x\to a} h(x)p(x)$

2. Find the limit using l'Hospital's rule.

(a) $\lim_{x\to\infty} \dfrac{\ln(x)}{\sqrt{x}}$

(b) $\lim_{t\to 0} \dfrac{8^t - 5^t}{t}$

(c) $\lim_{x\to-\infty} x\ln\left(1 - \dfrac{1}{x}\right)$

(d) $\lim_{x\to 0} \dfrac{x - \sin(x)}{x - \tan(x)}$

(e) $\lim_{x\to\infty} \sqrt{x}e^{-x/2}$

(f) $\lim_{x\to 0} \dfrac{x3^x}{3^x - 1}$

3. If f' is continuous, $f(2) = 0$, and $f'(2) = 7$, evaluate

$$\lim_{x\to 0} \frac{f(2+3x) + f(2+5x)}{x}$$

4. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is

$$v = \frac{mg}{c}(1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant that governs the strength of the air resistance.

(a) Calculate $\lim_{t\to\infty} v$. What is the meaning of this limit?

(b) For fixed t, use l'Hospital's Rule to calculate $\lim_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?