21-120: Differential and Integral Calculus
Lecture #13 Outline

Read: Section 3.8 of the textbook

Objectives and Concepts:

- When y is implicitly defined as a function of x, we can find dy/dx through implicit differentiation, which is accomplished by differentiating every term in the equation with respect to x, and then solving for dy/dx.

Suggested Textbook Exercises:

- 3.8: 301-321 odd.

Implicit Differentiation

The functions that we have met so far can be described by expressing one variable explicitly in terms of another variable. Example: $y = x \sin x$. However, some functions are defined *implicitly*, like

$$x^2 + y^2 = 25 \qquad \text{or} \qquad x = y \sin y$$

Sometimes we can solve such equations for y and get an explicit function (or several) of x. For example, the first equation becomes $y = \pm\sqrt{25 - x^2}$ which we can write as the two functions

$$f(x) = \sqrt{25 - x^2} \qquad \text{and} \qquad g(x) = -\sqrt{25 - x^2},$$

and it is straightforward to find $f'(x)$ and $g'(x)$. Unfortunately, the second equation isn't easy to solve (in fact no closed-form solution of y in terms of x exists for $x = y \sin y$). Thus it is impossible to apply our standard method for finding the derivative dy/dx. However, we can find $\dfrac{dy}{dx}$ without first having to solve for y. The method we used is called Implicit Differentiation.

Implicit Differentiation: Given an equation involving x and y, **first** take the derivative of each side of the equation. Keep in mind that y is a function of x, so we must apply the Chain Rule. This produces a factor of y' or $\dfrac{dy}{dx}$ whenever we must take the derivative of y. **Second,** solve the equation for y' or dy/dx.

Example 1: Use implicit differentiation to find $\dfrac{dy}{dx}$ when $x^2 + y^2 = 25$.

Example 2: Use implicit differentiation to find $\frac{dy}{dx}$:

(a) $\sqrt{x+y} = 5y - 4x$

(b) $y^5 + x^2 y^3 = 1 + y \arctan(x^2)$

(c) $y^2 = \cos(xy) - y^4$

Example 3: Find y' and y'' when $x^2 + 6xy - 2y^2 = 3$.

Example 4: If $\left[f(x)\right]^4 = \left[x + f(x)\right]^3$ with $f(1) = 2$, find $f'(1)$.

Example 5: Find the equation of the line tangent to the graph of the equation

$$\arcsin(x) + \arcsin(y) = \frac{\pi}{6}$$

at the point $\left(0, \frac{1}{2}\right)$.

Example 6: Find the equation of the tangent line to the curve below at the point $(1, 0)$.

$$x^2 - \sin(xy) + y^3 = 1$$