21-120: Differential and Integral Calculus
Recitation #4

1. (a) Using the Squeeze Theorem, evaluate the following limit:

$$\lim_{x\to 0} x^4 \cos\left(\frac{2}{x}\right)$$

(b) Using the Squeeze Theorem, evaluate the following limit:

$$\lim_{x\to 1} (x-1)^2 \cos\left(\frac{1}{x-1}\right)$$

(c) Let f be a function. If $4x-9 \le f(x) \le x^2-4x+7$ for $x \ge 0$, using the Squeeze Theorem, evaluate the following limit

$$\lim_{x\to 4} f(x)$$

2. We consider the function f defined as follows:

$$f(x) = \begin{cases} 6x+8 & \text{if } x \le -1 \\ -3x+7 & \text{if } -1 < x < 2 \\ x-1 & \text{if } x \ge 2. \end{cases}$$

Is the function f continuous at -1 ? Is the function f continuous at 2?

3. Let a and b be two real numbers. We consider the function

$$f(x) = \begin{cases} ax^2+bx+1 & \text{if } x < 2 \\ x^2+ax+b & \text{if } x \ge 2. \end{cases}$$

Give a condition on the real numbers a and b for the function f to be continuous everywhere.

4. For all real numbers x, let f be the following function

$$f(x) = x^5 - 2x - 4.$$

Calculate $f(1)$ and $f(2)$. Explain why the equation $x^5 = 2x+4$ has at least one solution in the interval $[1,2]$.

5. Let f be a function continuous on $(0,1)$ such that, for every real number x in this interval, $0 \le f(x) \le 1$. Show that there exists a real number $x \in [0,1]$ such that $f(x) = x$.

Hint: Consider for all $0 \le x \le 1$, the function $g(x) = f(x) - x$.