21-120: Differential and Integral Calculus
Recitation #23 Outline: 11/21/24

1. Calculate the following limits:

$$\lim_{x\to 0}\frac{e^x-1-x}{x^2},\quad \lim_{x\to 1}\frac{\ln(x)-1+x}{\arctan(x)-\frac{\pi}{4}},\quad \lim_{x\to+\infty}\ln(1+e^x)\sin\left(\frac{1}{x}\right)$$

2. Calculate the following integrals:

$$\int_0^3 |1-x|\,dx$$

and

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2\sin(x)}{1+x^6}\,dx$$

3. Calculate the following antiderivatives by substitution:

$$\int(\cos(x))^{1234}\sin(x)\,dx$$

$$\int\frac{1}{x\ln(x)}\,dx$$

4. Calculate the derivative of the following function:

$$f(x)=\int_0^{x^3}\cos(t^2)\,dt$$

5. Determine the antiderivative F of the function $f(x)=x^3-3x^2+7$ on $\mathbb{R}$ such that $F(1)=2$.

6. The height, in meters, of an electrical line of 160 meters can be modeled by the function h defined on $[-80,80]$ as

$$h(x)=10\left(e^{x/40}+e^{-x/40}\right).$$

What is the average height of this electrical line?

7. Suppose we want to construct a chocolate box for a friend's anniversary. The open box will be made from a 24-inch by 36-inch cardboard by cutting a square from each corner of the box and folding up the flaps on each side. What is the size of the square that should be cut from each corner in order to obtain a box with the maximum volume?