

21-120: Differential and Integral Calculus
Recitation #8 Outline: 09/19/24

1. Find all vertical and horizontal asymptotes (if any) of the following functions and justify all your work:

(a) $f(x) = \frac{x^2 + 7x + 6}{x + 1}$

(c) $f(x) = x - \frac{1}{x^2}$

(b) $f(x) = \frac{x + 1}{x^2 + 7x + 6}$

(d) $f(x) = \frac{x \sin x}{x^2 - 1}$

2. Is the following function differentiable at $x = 1$? Prove your answer using the limit definition of the derivative.

$$f(x) = \begin{cases} -x^2 + 2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases}$$

3. Using the Squeeze Theorem, prove that

$$\lim_{x \rightarrow \infty} \frac{2 - \cos x}{x + 3} = 0.$$

4. Using the Intermediate Value Theorem, prove that the equation

$$x^3 \cos x = 4$$

has at least one solution in $(-\infty, +\infty)$.¹

5. Let

$$f(x) = \begin{cases} 3x & \text{if } x > 1 \\ x^3 & \text{if } x < 1 \end{cases}$$

Is it possible to find a value a such that $f(1) = a$ which makes the modified f continuous at all points?

6. At which points (if any) are the functions below discontinuous? Classify each discontinuity.

(a) $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3 - x & \text{if } x > 1 \end{cases}$

(c) $f(x) = \begin{cases} \frac{x^2 + 7x + 6}{x + 1} & \text{if } x \neq -1 \\ 5 & \text{otherwise} \end{cases}$

(b) $f(x) = \frac{x + 1}{x^2 + 7x + 6}$

7. For which values of a is the function

$$f(x) = \begin{cases} e^x & \text{if } x < 0 \\ a + x & \text{if } x \geq 0 \end{cases}$$

continuous on $(-\infty, +\infty)$?

¹Note that you can solve this problem without introducing the function $x^3 \cos x - 4$.

8. Give an example of a function f such that

$$\lim_{x \rightarrow \infty} f(x) = -\infty$$

and prove that your answer works using the ϵ - δ definition. (The proof should be very easy.)

9. Let

$$f(x) = \frac{1}{\sqrt{1+x}}$$

- (a) Using the *definition* of the derivative only, find a formula for $f'(x)$.
- (b) Find an equation of the tangent line to f at $x = 1$.

10. Using differentiation rules, find the derivative of the function

$$f(x) = 3x \left(18x^4 + \frac{13}{x+1} \right).$$

11. Evaluate the following limits (if they exist):

(a) $\lim_{x \rightarrow 4} \frac{1}{\sqrt{x} - 2}$

(c) $\lim_{x \rightarrow 2^+} \left(3\sqrt{x-2} + 5 \cdot \frac{x^2 + x - 6}{x-2} \right)$

(b) $\lim_{x \rightarrow -\infty} \frac{4x}{\sqrt{x^2 - 1}}$