21-120: Differential and Integral Calculus
Recitation #3 Outline: 09/03/24

1. (a) The equation below states that two functions are equal. Explain what is wrong (if anything) with this equation.
$$\frac{x^2+x-6}{x-2} = x+3$$
(b) Explain why the equation
$$\lim_{x\to 2}\frac{x^2+x-6}{x-2} = \lim_{x\to 2}(x+3)$$
is nevertheless correct.

2. Evaluate the following limits, if possible. For any limits that do not exist, provide an explanation for why they do not exist.

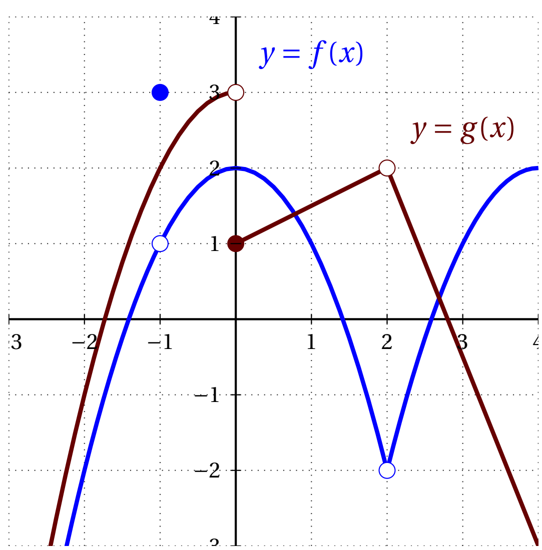
(a) $\displaystyle\lim_{x\to\pi^+}\csc(x)$
(b) $\displaystyle\lim_{x\to\pi}\csc(x)$
(c) $\displaystyle\lim_{x\to 0^+}\sin\left(\frac{1}{x}\right)$
(d) $\displaystyle\lim_{x\to 7^-}\frac{3}{(x-7)(x-1)}$
(e) $\displaystyle\lim_{x\to 7^+}\sqrt{x-7}$
(f) $\displaystyle\lim_{x\to 7}\sqrt{x-7}$

3. Suppose $\displaystyle\lim_{t\to 0}\frac{f(t)}{t^2} = 3$. Find the following limits:

(a) $\displaystyle\lim_{x\to 0}f(x)$
(b) $\displaystyle\lim_{x\to 0}\frac{f(x)}{x}$

4. Using the graphs of functions f and g, find the limits below if they exist. For each limit that does not exist, provide an explanation of why it does not exist.

(a) $\displaystyle\lim_{x\to -1}\big(f(x)\cdot g(x)\big)$
(b) $\displaystyle\lim_{x\to -1}\big(x^3\cdot g(x)\big)$
(c) $\displaystyle\lim_{x\to -1}\big(f(x)\cdot g(0)\big)$
(d) $\displaystyle\lim_{x\to -1}\big(f(x)\cdot g(2)\big)$
(e) $\displaystyle\lim_{x\to 2}\frac{x^2+x}{g(x)}$
(f) $\displaystyle\lim_{x\to 0}\big(f(x)+g(x)\big)$
(g) $\displaystyle\lim_{x\to 0}\big(f(x)+g(0)\big)$
(h) $\displaystyle\lim_{x\to 0}\big(f(0)+g(x)\big)$
(i) $\displaystyle\lim_{x\to 0}\big(10560\cdot f(x)\big)$



5. For each of the limits below, state whether it exists, and if so, evaluate it.

(a) $\lim_{t\to 3}\left(\sqrt{t}+\frac{t}{2t-5}\right)$

(b) $\lim_{t\to 1} 2$

(c) $\lim_{x\to -4}\frac{\sqrt{x^2+9}-5}{x+4}$

(d) $\lim_{x\to 16}\frac{4-\sqrt{x}}{16x-x^2}$

(e) $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$

(f) $\lim_{h\to 0}\left(\frac{1}{h}-\frac{1}{h^2+h}\right)$