21-120: Differential and Integral Calculus
Recitation #10 Outline: 09/26/24

1. For each of the following, find $(f^{-1})'(a)$:

   (a) $f(x) = x^2 + 3x + 2,\ x \geq -\dfrac{3}{2},\ a = 2$

   (b) $f(x) = x - \dfrac{2}{x},\ x < 0,\ a = 1$

   (c) $f(x) = x + \sin x,\ a = 0$

   (d) $f(x) = x + \sqrt{x},\ a = 2$

2. In the problem above, why do some parts have restrictions on x while others don't? What would go wrong if we removed the requirement $x \geq -\dfrac{3}{2}$ in part (a)?

3. For each function f below, find the equation of the tangent line to the graph of f^{-1} at the specified point P, *without* directly using the Inverse Function Theorem. That is, first write an equation for the tangent line for f at the appropriate point, and then convert the equation into an equation of the tangent line for f^{-1} at the point P.

   (a) $f(x) = (x^3 + 1)^4,\ P(16, 1)$

   (b) $\sqrt{x-4},\ P(2, 8)$

4. For each function f below, find the equation of the tangent line to the graph of f^{-1} at the specified point P, using the Inverse Function Theorem. Check that your answers agree with the answers to the previous problem.

   (a) $f(x) = (x^3 + 1)^4,\ P(16, 1)$

   (b) $\sqrt{x-4},\ P(2, 8)$

5. Find the derivatives of the following functions:

   (a) $y = \arccos(\sqrt{x})$

   (b) $y = \sec^{-1}(-x)$

   (c) $y = \sqrt{\csc^{-1}(x)}$

   (d) $y = x \csc^{-1}(x)$

6. For each of the following, use the given values to find $(f^{-1})'(a)$: functions:

   (a) $f(\pi) = 0,\ f'(\pi) = -1,\ a = 0$

   (b) $f(6) = 2,\ f'(6) = 1/3,\ a = 2$

   (c) $f(1) = 0,\ f'(1) = -2,\ a = 0$

   (d) $f(\sqrt{3}) = 1/2,\ f'(\sqrt{3}) = 2/3,\ a = 1/2$

7. Suppose $f(t) = t^3 + 4t + 2$. Find the slope of the tangent line to the graph of $g(x) = xf^{-1}(x)$ at the point $x = 7$.