21-120: Differential and Integral Calculus
Lecture #24 Outline

Read: Section 4.7 of the textbook

Objectives and Concepts:

- Optimization problems involve situations where there is a function that you wish to maximize or minimize.
- A procedure for solving optimization problems involves first understanding the problem, drawing a diagram, finding the quantity to be optimized in terms of an independent variable, and then finding the absolute max/min of that function on its domain.
- When the quantity to be optimized is initially expressed in terms of more than one independent variable, look for information (perhaps a constraint) that will allow you to write the quantity in terms of a single variable.

Suggested Textbook Exercises:

- 4.7: 311-314 all, 315-355 odd.

More Applied Optimization Problems

Example 1: A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for the base costs \$10 per square meter. Materials for the sides costs \$6 per square meter. Find the cost of the materials for the cheapest such container.

Example 2: A man is in a canoe 4 miles from a straight shoreline, and wants to know the quickest way to get to a house, which is 10 miles down the shoreline from the point on the shore closest to the boat. If the man can row at a rate of 2 miles per hour and can walk at a rate of 3 miles per hour, to what point on the shoreline should he row to minimize the time needed for the trip?

If $C(x)$ represents the cost of producing x units of a certain product, the **marginal cost** function is the derivative $C'(x)$.

In marketing, let $p(x)$ be the price per unit that a company can charge if it sells x units. This is commonly called the **demand function**. As the price p increases, (usually) fewer people are interested in purchasing the product, so we would expect $p(x)$ to be a decreasing function of x. If we sell x units at a price of p dollars, then the total revenue is

$$R(x) = xp(x)$$

and R is called the **revenue function**. Its derivative $R'(x)$ is called the **marginal revenue** function. The total profit from selling x units is

$$P(x) = R(x) - C(x)$$

and P is called the **profit function**. Its derivative $P'(x)$ is called the **marginal profit** function.

Example 3: A store has been selling 200 iPhones a week at $350 each. A market survey indicates that for each $10 rebate offered to buyers, the number of units sold will increase by 20 per week. Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenues?

Example 4: A baseball team plays in a stadium that holds 55000 spectators. With ticket prices at \$10, hte average attendance had been 27000. When ticket prices were lowered to \$8, the average attendance rose to 33000.

(a) Find the demand function, assuming that it is linear.

(b) How should ticket prices be set to maximize revenue?