21-120: Differential and Integral Calculus
Recitation #14 Outline: 10/10/24

1. Use Rolle's Theorem to prove the Mean Value Theorem (MVT).
2. Verify that the function satisfies the hypotheses of the MVT on the given interval. Then find all numbers c that satisfy the conclusion of the MVT.
   (a) $f(x) = \cos(x)$, $[\pi/2, 5\pi/2]$
   (b) $f(x) = \dfrac{x}{x+1}$, $[0,3]$
3. Let $f(x) = 2 - |2x-1|$. Show that there is no value of c such that $f(3) - f(0) = f'(c)(3-0)$. Why does this not contradict the MVT?
4. Show that the equation $2x + \cos(x) = 0$ has at most one real root.
5. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?
6. A function $f(x)$ is called *Lipschitz* if there exists a number $L \geq 0$ so that for all x and y
$$|f(x) - f(y)| \leq L|x - y|.$$
Show that if $f(x)$ is a differentiable function so that $|f'(x)| \leq M$ for all x, then f is Lipschitz.