21-120: Differential and Integral Calculus
Recitation #11 Outline: 10/01/24

1. (a) Recall that in Lecture 14 we defined e to be the real number which satisfies the following property:
$$\lim_{h\to 0}\frac{e^h-1}{h}=1.$$
Using this property, show that for any $a>0$,
$$\lim_{h\to 0}\frac{a^h-1}{h}=\ln a.$$
(b) Using the limit definition of the derivative and the fact above, prove that for any $a>0$, we have $\dfrac{d}{dx}\left(a^x\right)=a^x\ln a$.

2. Find the derivatives of the functions below. For at least one function of your choice among the given ones, find its derivative in two ways: without using logarithmic differentiation and by using logarithmic differentiation.[1]

(a) $f(x)=2^x\cdot\log_7(6x^4+3)^5$;
(b) $f(x)=3^{\sin(3x)}$;
(c) $f(x)=x^\pi\cdot\pi^x$;
(d) $f(x)=e^{x^3\ln x}$.

3. Find the derivatives of the following functions:[2]

(a) $f(x)=(3x)^{2x}$;
(b) $f(x)=x^{\log_2 x^3}$;
(c) $f(x)=(3x^2+4)^{\cos x}$;
(d) $f(x)=(\sin(2x))^{4x}$.

4. Consider the curve given by the following equation:
$$x^3-x\ln(y)+y^3=2x+5.$$
(a) Find the slope of the tangent line to this curve at the point $(x,y)=(2,1)$.
(b) Write an equation of the tangent line to this curve at the point $(x,y)=(2,1)$.

[1] In principle, you don't have to use logarithmic differentiation for any of the functions here, but note that in some cases, using it might be a better choice, as it can lead to a simpler solution.

[2] Note that since a variable is raised to a variable power in all these functions, the ordinary rules of differentiation do not apply! The rule $(a^t)'=a^t\ln(a)$ is only valid when a is a constant. If a is an expression containing variables, we cannot use this rule and must instead apply logarithmic differentiation. Similarly, the rule $(t^n)'=nt^{n-1}$ is only valid when n is a constant. If n is an expression containing variables, we cannot use this rule.