21-120: Differential and Integral Calculus
Recitation #26 Outline: 12/5/24

1. Evaluate the following limits (if they exist):

   (a) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 - 3x + 2}$

   (b) $\lim\limits_{x \to 0^+} \dfrac{e^{x^2 + 1}}{\sin(x)}$

   (c) $\lim\limits_{x \to \infty} \dfrac{-2x}{\sqrt{2x^2 - 3}}$

2. For which values of a and b is the following function continuous and differentiable? (Hint: use left and right sided limits to evaluate the derivative at 0)

$$g(x) := \begin{cases} a^x & x < 0 \\ x + b & x \geq 0 \end{cases}$$

3. A spherical balloon is being inflated. Given that the volume of air in the ballon is increasing at a rate of $5\,cm^3$ per second when the volume is $100\,cm^3$, at what rate is the radius increasing?

4. Let a, b, c real numbers. Show that there exists $x \in (0, 1)$ such that:

$$4ax^3 + 3bx^2 + 2cx = a + b + c.$$

5. Determine the following antiderivatives :

   (a) $\displaystyle\int \dfrac{dt}{\sqrt{t} + \sqrt{t^3}}$

   (b) $\displaystyle\int \dfrac{\ln t}{t + t(\ln t)^2}\,dt$

   (c) $\displaystyle\int \dfrac{e^{2t}}{e^t + 1}\,dt$

6. Calculate the following integrals:

   (a) $\displaystyle\int_0^1 \arctan t\,dt$

   (b) $\displaystyle\int_0^{1/2} \arcsin t\,dt$

   (c) $\displaystyle\int_0^1 t \arctan t\,dt$

7. Let f be a continuous function. The function $F(x)$ is defined as follows

$$F(x) = \int_1^x \left(t \int_1^t f(s)\,ds \right) dt.$$

Compute $F'(1)$.