Enhanced Lab08: Logistic Regression

Clarification Concepts for Logistic Regression

1. Logistic Regression Basics

What is Logistic Regression?

- Logistic regression is a classification algorithm used to predict categorical outcomes (e.g., spam or not spam)
- It estimates the probability that an instance belongs to a certain class using the sigmoid function

Why Not Use Linear Regression for Classification?

- Linear regression predicts continuous values, which can go beyond 0 and 1
- Logistic regression ensures outputs are bounded between 0 and 1, making it suitable for classification

2. The Sigmoid Function

Formula:

σ(z) = 1/(1 + e^(-z))

where z = wX + b

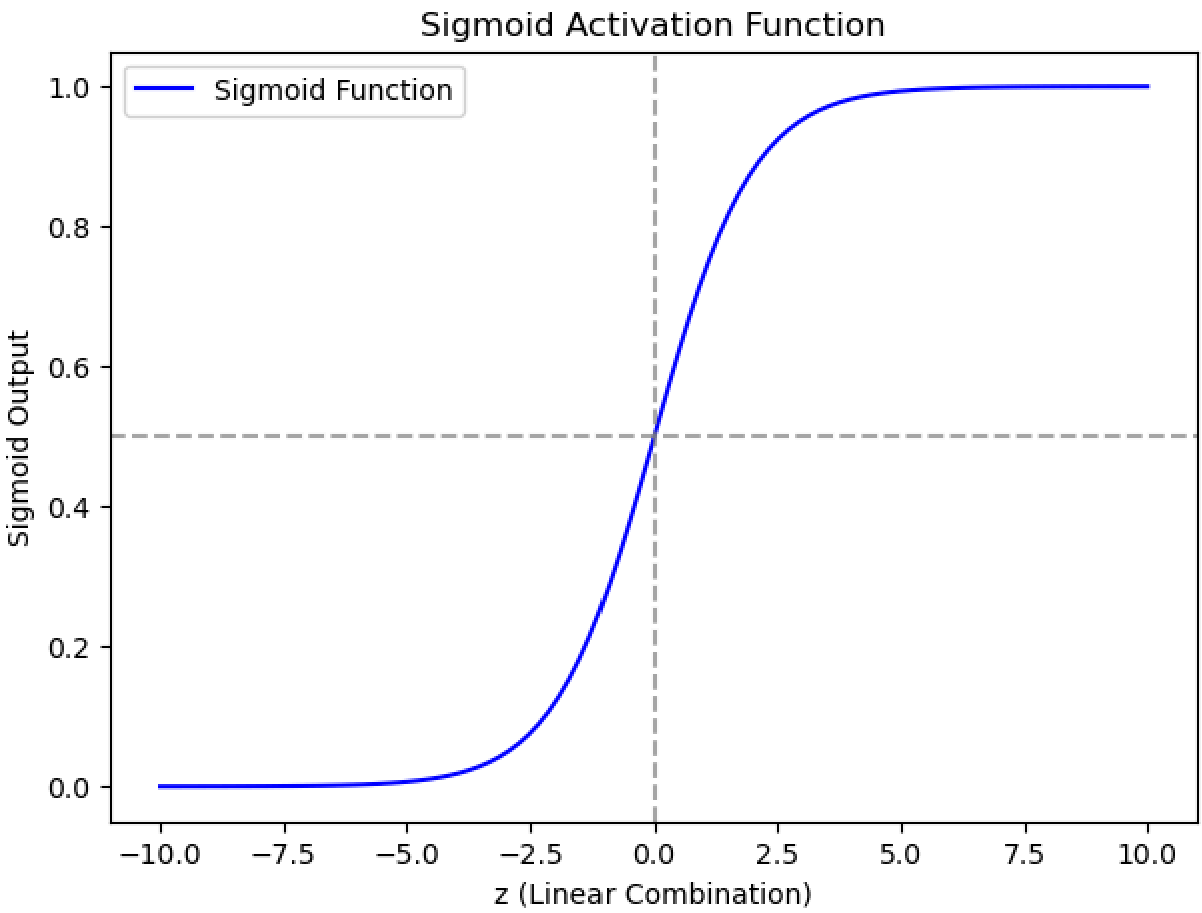
```
import numpy as np
import matplotlib.pyplot as plt

# Define sigmoid function
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

# Generate values for z
z = np.linspace(-10, 10, 100)
sigma = sigmoid(z)

# Plot sigmoid function
plt.figure(figsize=(7, 5))
plt.plot(z, sigma, label="Sigmoid Function", color='b')
```

```
plt.axvline(0, color='gray', linestyle='--', alpha=0.7)
plt.axhline(0.5, color='gray', linestyle='--', alpha=0.7)
plt.xlabel("z (Linear Combination)")
plt.ylabel("Sigmoid Output")
plt.title("Sigmoid Activation Function")
plt.legend()
plt.show()
```



2. Advanced Concepts For Logistics Regression

2.1 Binary Classification with Regularization (L1/L2) with Real-World Datsets

2.1.1 Import Real-World Dataset

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import accuracy_score, precision_score, recall_score, f1_score
from sklearn.metrics import confusion_matrix, roc_curve, auc
```

```
from sklearn.preprocessing import StandardScaler

np.random.seed(42)

# 1. Data Loading and Preprocessing
print("1. Data Loading and Initial Analysis")
print("-" * 50)

# Load the dataset
df = pd.read_csv('../Dataset/Lab_Enhancement_Lab8/emails.csv')
print(f"Dataset shape: {df.shape}")
print("\nClass distribution:")
print(df['spam'].value_counts(normalize=True))
```

```
1. Data Loading and Initial Analysis
--------------------------------------------------
Dataset shape: (5728, 2)

Class distribution:
spam
0    0.761173
1    0.238827
Name: proportion, dtype: float64
```

2.1.2 Feature Extraction

```
# 2. Feature Extraction
print("\n2. Feature Extraction")
print("-" * 50)

# Create TF-IDF Vectorizer
tfidf = TfidfVectorizer(max_features=1000, stop_words='english')
X = tfidf.fit_transform(df['text'])
y = df['spam'].astype(int)
```

```

2. Feature Extraction
--------------------------------------------------
```

2.1.3 Sampling Techniques for Imbalance Data

```
# Split the dataset
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)

# 2.1 Handling Imbalanced Data
print("\nHandling Imbalanced Data")
print("-" * 50)

from imblearn.over_sampling import SMOTE
from imblearn.under_sampling import RandomUnderSampler
from imblearn.combine import SMOTETomek
# Display original class distribution
```

```
print("Original class distribution in training set:")
print(pd.Series(y_train).value_counts(normalize=True))

# 1. SMOTE (Oversampling)
smote = SMOTE(random_state=42)
X_train_smote, y_train_smote = smote.fit_resample(X_train, y_train)
print("\nClass distribution after SMOTE:")
print(pd.Series(y_train_smote).value_counts(normalize=True))

# 2. Random Undersampling
undersampler = RandomUnderSampler(random_state=42)
X_train_under, y_train_under = undersampler.fit_resample(X_train, y_train)
print("\nClass distribution after Undersampling:")
print(pd.Series(y_train_under).value_counts(normalize=True))

# 3. SMOTE + Tomek (Combined approach)
smote_tomek = SMOTETomek(random_state=42)
X_train_smote_tomek, y_train_smote_tomek = smote_tomek.fit_resample(X_train, y_train)
print("\nClass distribution after SMOTE + Tomek:")
print(pd.Series(y_train_smote_tomek).value_counts(normalize=True))
```

```
2.1 Handling Imbalanced Data
--------------------------------------------------
Original class distribution in training set:
spam
0    0.764732
1    0.235268
Name: proportion, dtype: float64

Class distribution after SMOTE:
spam
0    0.5
1    0.5
Name: proportion, dtype: float64

Class distribution after Undersampling:
spam
0    0.5
1    0.5
Name: proportion, dtype: float64

Class distribution after SMOTE + Tomek:
spam
0    0.5
1    0.5
Name: proportion, dtype: float64
```

2.1.4 Model Evaluation

```
# Function to compare models with different sampling techniques
def train_and_evaluate_with_sampling(X_train, y_train, X_test, y_test, sampling_method):
    # Models to test
    models = {
```

```
        'Basic': LogisticRegression(random_state=42, max_iter=1000),
        'L1': LogisticRegression(penalty='l1', solver='liblinear', C=1.0, random_state=42
        'L2': LogisticRegression(penalty='l2', C=1.0, random_state=42, max_iter=1000)
    }

    results = {}
    for model_name, model in models.items():
        model.fit(X_train, y_train)
        y_pred = model.predict(X_test)

        results[model_name] = {
            'accuracy': accuracy_score(y_test, y_pred),
            'precision': precision_score(y_test, y_pred),
            'recall': recall_score(y_test, y_pred),
            'f1': f1_score(y_test, y_pred)
        }

    # Create comparison DataFrame
    results_df = pd.DataFrame(results).T
    print(f"\nResults with {sampling_method}:")
    print(results_df)
    return results_df
```

2.1.5 Comparison between sampling methods

```
# Compare performance with different sampling methods
print("\nComparing model performance with different sampling methods:")
results_original = train_and_evaluate_with_sampling(X_train, y_train, X_test, y_test, "Or
results_smote = train_and_evaluate_with_sampling(X_train_smote, y_train_smote, X_test, y_
results_under = train_and_evaluate_with_sampling(X_train_under, y_train_under, X_test, y_
results_smote_tomek = train_and_evaluate_with_sampling(X_train_smote_tomek, y_train_smote
```

```
Comparing model performance with different sampling methods:

Results with Original Data:
       accuracy  precision    recall        f1
Basic  0.979930   0.978495  0.941379  0.959578
L1     0.977312   0.974820  0.934483  0.954225
L2     0.979930   0.978495  0.941379  0.959578

Results with SMOTE:
       accuracy  precision   recall        f1
Basic  0.986911   0.969283  0.97931  0.974271
L1     0.985166   0.962712  0.97931  0.970940
L2     0.986911   0.969283  0.97931  0.974271

Results with Undersampling:
       accuracy  precision    recall        f1
Basic  0.981675   0.940984  0.989655  0.964706
L1     0.955497   0.852507  0.996552  0.918919
L2     0.981675   0.940984  0.989655  0.964706

Results with SMOTE+Tomek:
       accuracy  precision    recall        f1
```

```
      Basic  0.986911   0.969283  0.979310  0.974271
      L1     0.984293   0.962585  0.975862  0.969178
      L2     0.986911   0.969283  0.979310  0.974271
```

2.1.6 Visualization of Comparison of Different Sampling Methods

```
# Visualize the comparison
metrics = ['accuracy', 'precision', 'recall', 'f1']
sampling_methods = ['Original', 'SMOTE', 'Undersampling', 'SMOTE+Tomek']
model_types = ['Basic', 'L1', 'L2']

plt.figure(figsize=(15, 10))
for i, metric in enumerate(metrics):
    plt.subplot(2, 2, i+1)

    x = np.arange(len(sampling_methods))
    width = 0.25

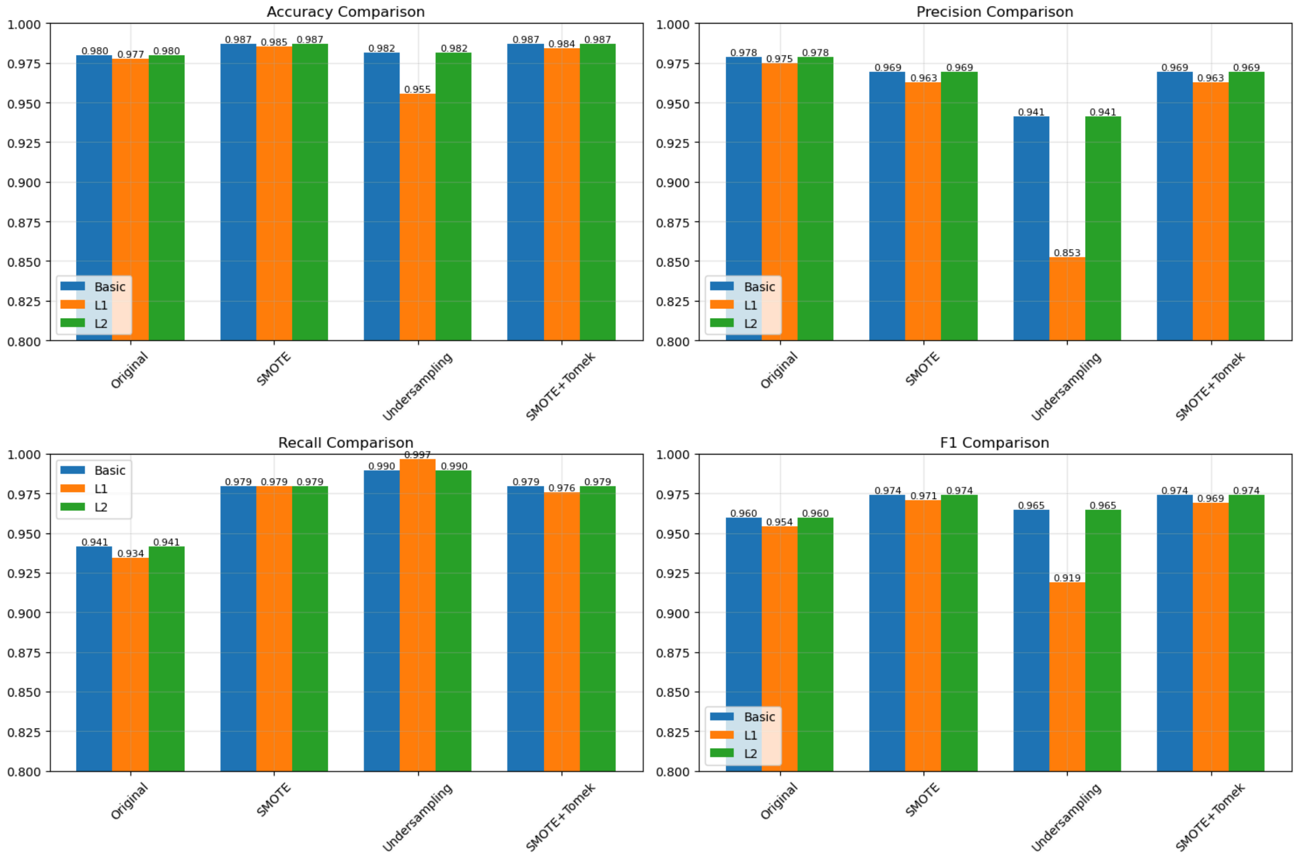
    for j, model_type in enumerate(model_types):
        metric_data = [
            results_original.loc[model_type, metric],
            results_smote.loc[model_type, metric],
            results_under.loc[model_type, metric],
            results_smote_tomek.loc[model_type, metric]
        ]

        bars = plt.bar(x + (j-1)*width, metric_data, width, label=model_type)

        # Add value labels on top of bars
        for bar in bars:
            height = bar.get_height()
            plt.text(bar.get_x() + bar.get_width()/2., height,
                    f'{height:.3f}',
                    ha='center', va='bottom', fontsize=8)

    plt.title(f'{metric.capitalize()} Comparison')
    plt.xticks(x, sampling_methods, rotation=45)
    plt.ylim(0.8, 1.0)  # Adjust y-axis range for better visualization
    plt.legend()
    plt.grid(True, alpha=0.3)

plt.tight_layout()
plt.show()
```




```
# Select the best sampling method based on F1 score
sampling_methods = {
    'Original': (X_train, y_train),
```

```
    'SMOTE': (X_train_smote, y_train_smote),
    'Undersampling': (X_train_under, y_train_under),
    'SMOTE+Tomek': (X_train_smote_tomek, y_train_smote_tomek)
}
```

2.1.7 Best Sampling Method

```
# Find best sampling method
best_f1_scores = {
    method: results_df['f1'].mean()
    for method, results_df in [
        ('Original', results_original),
        ('SMOTE', results_smote),
        ('Undersampling', results_under),
        ('SMOTE+Tomek', results_smote_tomek)
    ]
}

best_method = max(best_f1_scores.items(), key=lambda x: x[1])[0]
print(f"\nBest sampling method based on F1 score: {best_method}")
```

```
Best sampling method based on F1 score: SMOTE
```

```
# Use the best sampling method for further modeling
X_train_balanced, y_train_balanced = sampling_methods[best_method]

# Function to evaluate and print metrics
def evaluate_model(model, X_test, y_test, model_name):
    y_pred = model.predict(X_test)

    print(f"\nMetrics for {model_name}:")
    print(f"Accuracy: {accuracy_score(y_test, y_pred):.4f}")
    print(f"Precision: {precision_score(y_test, y_pred):.4f}")
    print(f"Recall: {recall_score(y_test, y_pred):.4f}")
    print(f"F1 Score: {f1_score(y_test, y_pred):.4f}")

    # Plot confusion matrix
    plt.figure(figsize=(6, 5))
    cm = confusion_matrix(y_test, y_pred)
    sns.heatmap(cm, annot=True, fmt='d', cmap='Blues')
    plt.title(f'Confusion Matrix - {model_name}')
    plt.ylabel('True Label')
    plt.xlabel('Predicted Label')
    plt.show()

    # Plot ROC curve
    y_pred_proba = model.predict_proba(X_test)[:, 1]
    fpr, tpr, _ = roc_curve(y_test, y_pred_proba)
    roc_auc = auc(fpr, tpr)

    plt.figure(figsize=(6, 5))
```

```
    plt.plot(fpr, tpr, color='darkorange', lw=2, label=f'ROC curve (AUC = {roc_auc:.2f})'
    plt.plot([0, 1], [0, 1], color='navy', lw=2, linestyle='--')
    plt.xlim([0.0, 1.0])
    plt.ylim([0.0, 1.05])
    plt.xlabel('False Positive Rate')
    plt.ylabel('True Positive Rate')
    plt.title(f'ROC Curve - {model_name}')
    plt.legend(loc="lower right")
    plt.show()

    return model.coef_
```

2.1.10 Comparison Between Basic, L1 & L2 Regularization

```
# 3. Basic Logistic Regression (Baseline)
print("\n3. Basic Logistic Regression (No Regularization)")
print("-" * 50)

baseline_model = LogisticRegression(random_state=42, max_iter=1000)
baseline_model.fit(X_train, y_train)
baseline_coef = evaluate_model(baseline_model, X_test, y_test, "Basic Logistic Regression

# 4. L1 Regularization (Lasso)
print("\n4. Logistic Regression with L1 Regularization")
print("-" * 50)

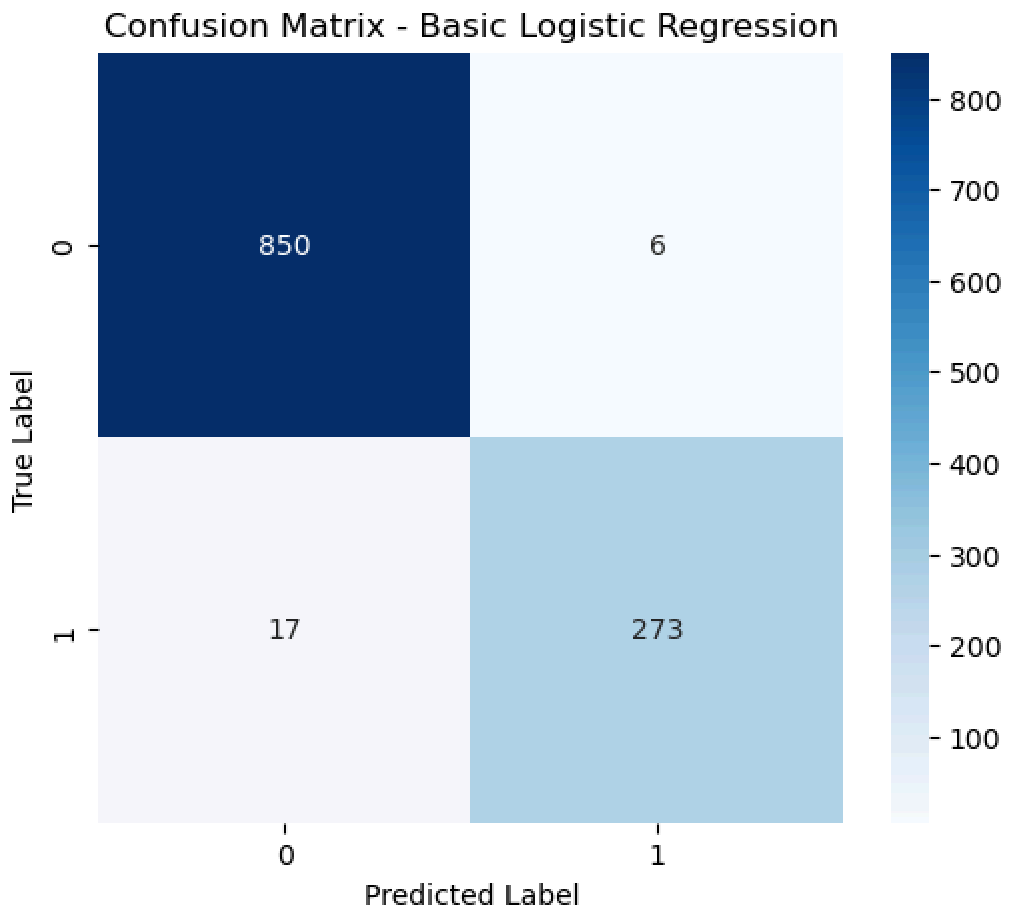
l1_model = LogisticRegression(penalty='l1', solver='liblinear', C=1.0, random_state=42)
l1_model.fit(X_train, y_train)
l1_coef = evaluate_model(l1_model, X_test, y_test, "L1 Regularization")

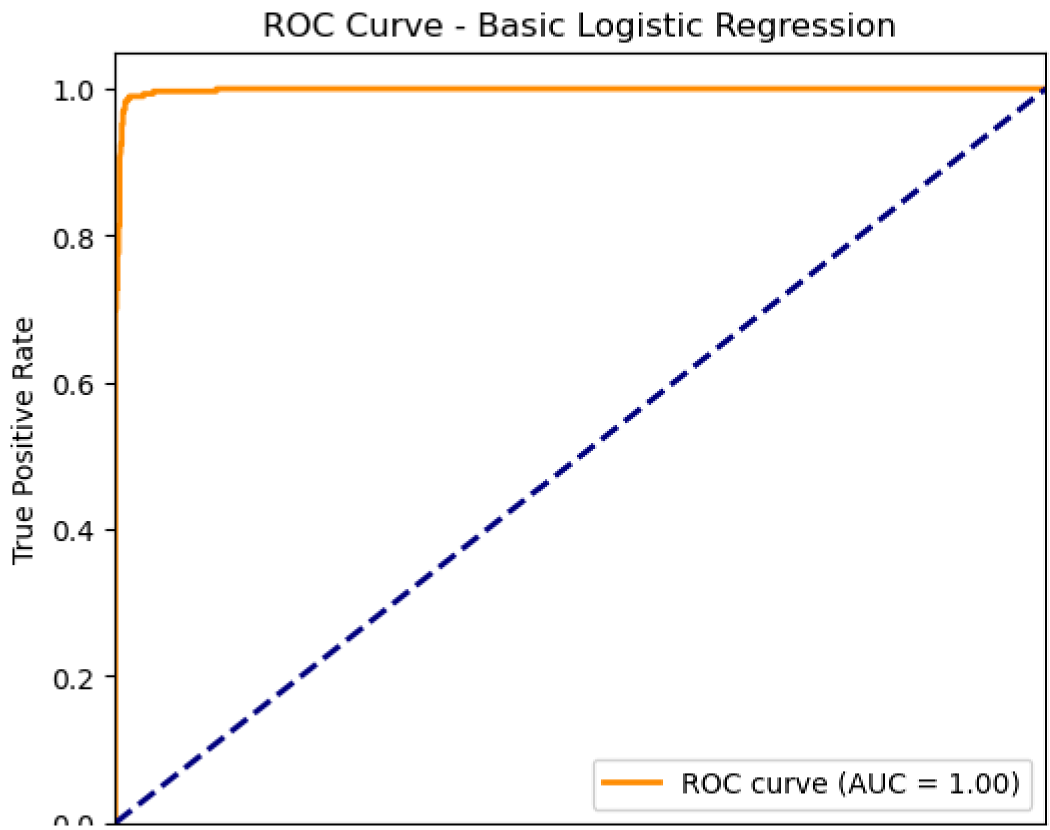
# 5. L2 Regularization (Ridge)
print("\n5. Logistic Regression with L2 Regularization")
print("-" * 50)

l2_model = LogisticRegression(penalty='l2', C=1.0, random_state=42, max_iter=1000)
l2_model.fit(X_train, y_train)
l2_coef = evaluate_model(l2_model, X_test, y_test, "L2 Regularization")
```

```
3. Basic Logistic Regression (No Regularization)
--------------------------------------------------

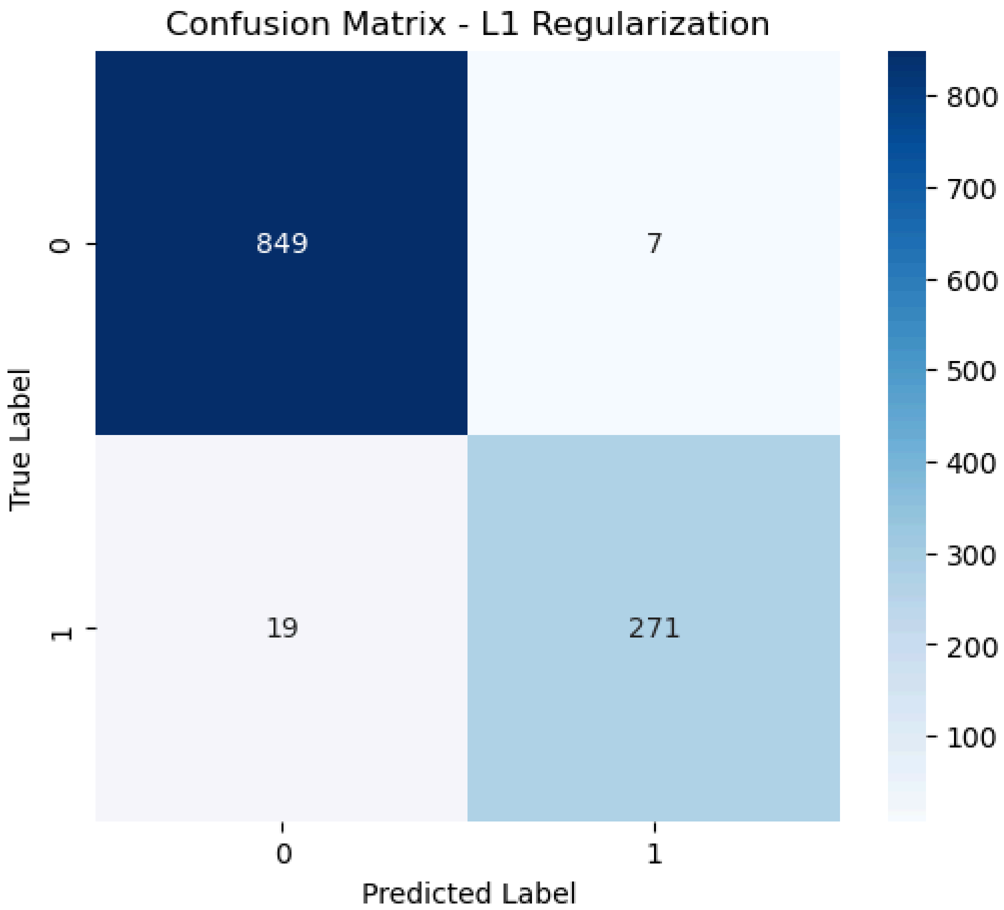
Metrics for Basic Logistic Regression:
Accuracy: 0.9799
Precision: 0.9785
Recall: 0.9414
F1 Score: 0.9596
```

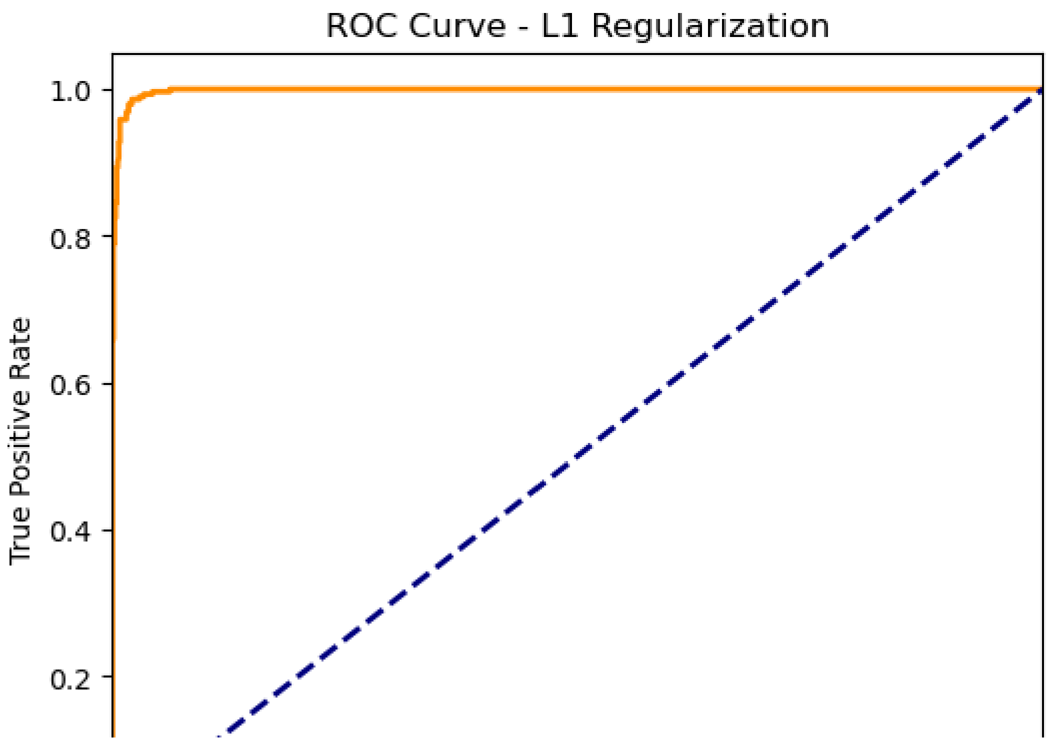


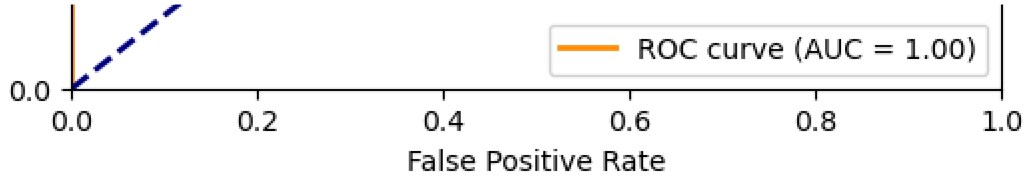



```
4. Logistic Regression with L1 Regularization
--------------------------------------------------

Metrics for L1 Regularization:
Accuracy: 0.9773
Precision: 0.9748
Recall: 0.9345
F1 Score: 0.9542
```

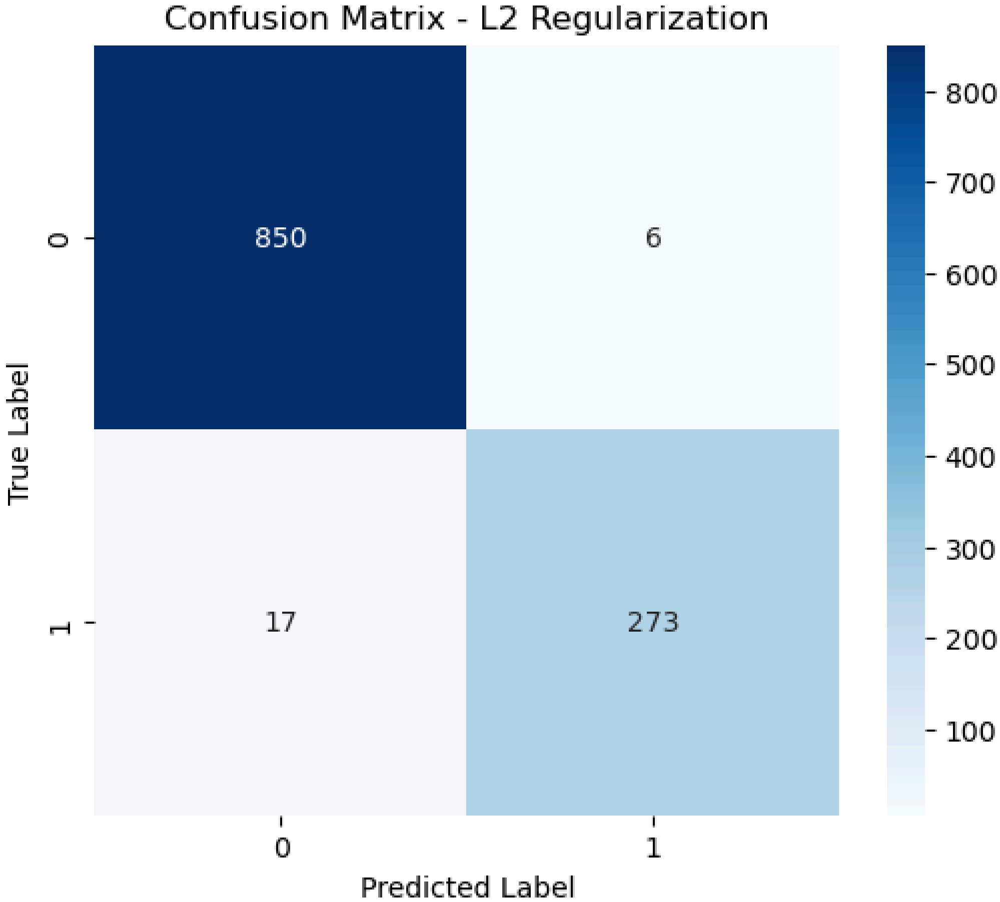


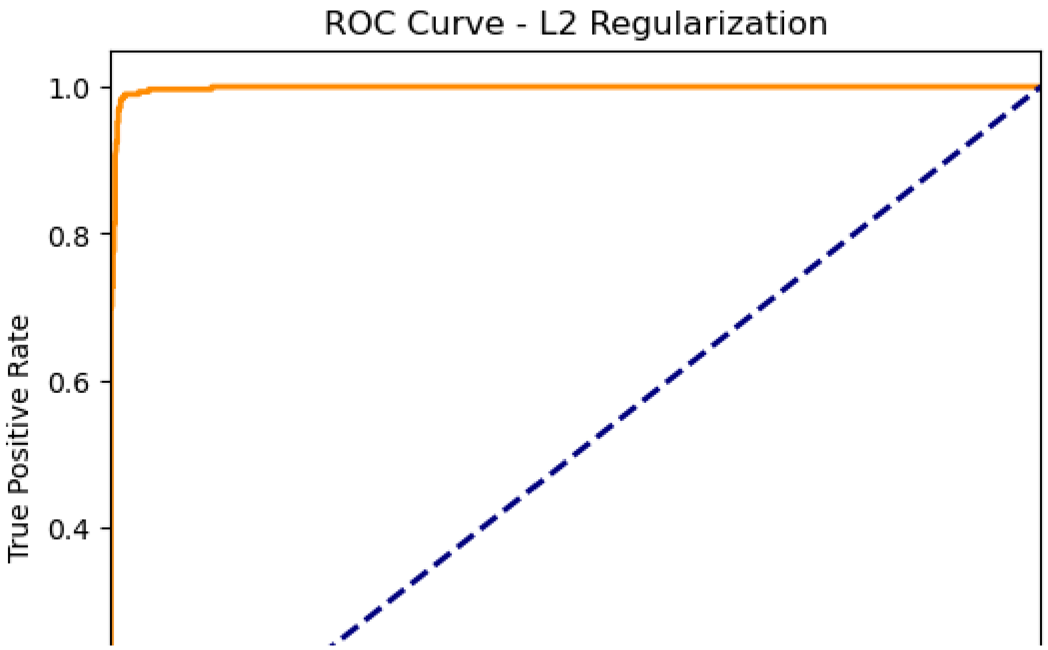





```
5. Logistic Regression with L2 Regularization
--------------------------------------------------

Metrics for L2 Regularization:
Accuracy: 0.9799
Precision: 0.9785
Recall: 0.9414
F1 Score: 0.9596
```






0.2
0.0
ROC curve (AUC = 1.00)
0.0
0.2
0.4
0.6
0.8
1.0
False Positive Rate

```
#6. Comparing Feature Coefficients
print("\n6. Comparing Feature Coefficients")
print("-" * 50)

# Get feature names
feature_names = tfidf.get_feature_names_out()

# Create a DataFrame with coefficients
coef_df = pd.DataFrame({
    'Feature': feature_names,
    'Baseline': baseline_coef[0],
    'L1': l1_coef[0],
    'L2': l2_coef[0]
})

# Sort by absolute value of baseline coefficients
coef_df['Abs_Baseline'] = abs(coef_df['Baseline'])
coef_df = coef_df.sort_values('Abs_Baseline', ascending=False)
```

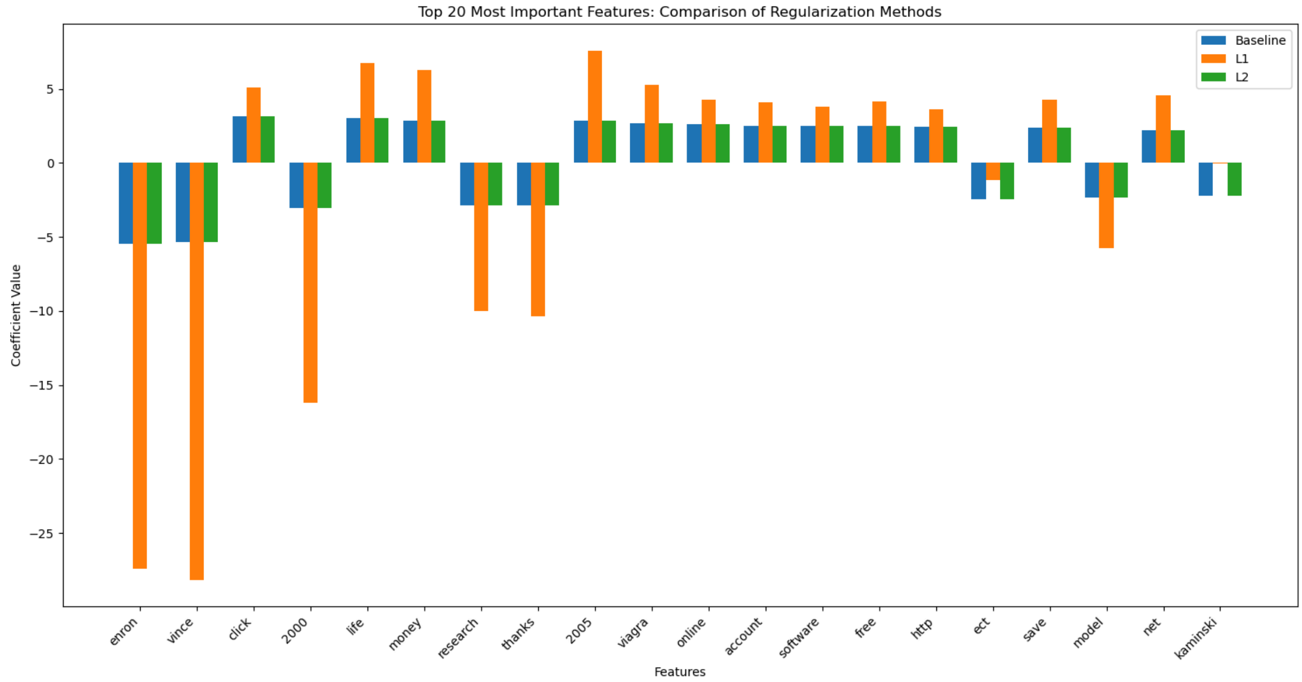
```
6. Comparing Feature Coefficients
--------------------------------------------------
```

2.1.9 Top 20 most important features

```
# Plot top 20 most important features
plt.figure(figsize=(15, 8))
top_features = coef_df.head(20)
x = np.arange(len(top_features))
width = 0.25

plt.bar(x - width, top_features['Baseline'], width, label='Baseline')
plt.bar(x, top_features['L1'], width, label='L1')
plt.bar(x + width, top_features['L2'], width, label='L2')

plt.xlabel('Features')
plt.ylabel('Coefficient Value')
plt.title('Top 20 Most Important Features: Comparison of Regularization Methods')
plt.xticks(x, top_features['Feature'], rotation=45, ha='right')
plt.legend()
plt.tight_layout()
plt.show()
```



2.1.10 Non-zero Coefficients

```
# Print number of non-zero coefficients
print("\nNumber of non-zero coefficients:")
print(f"Baseline: {np.sum(baseline_coef[0] != 0)}")
print(f"L1 Regularization: {np.sum(l1_coef[0] != 0)}")
print(f"L2 Regularization: {np.sum(l2_coef[0] != 0)}")
```

```
Number of non-zero coefficients:
Baseline: 1000
L1 Regularization: 133
```

```
L2 Regularization: 1000
```

2.1.11 Optimal Regularization Strength

```
# 7. Cross-validation to find optimal regularization strength
print("\n7. Finding Optimal Regularization Strength")
print("-" * 50)

from sklearn.model_selection import GridSearchCV

# Define parameter grid
param_grid = {'C': [0.001, 0.01, 0.1, 1, 10, 100]}

# L1 Regularization
l1_grid = GridSearchCV(
    LogisticRegression(penalty='l1', solver='liblinear', random_state=42),
    param_grid,
    cv=5,
    scoring='f1'
)
l1_grid.fit(X_train, y_train)

# L2 Regularization
l2_grid = GridSearchCV(
    LogisticRegression(penalty='l2', random_state=42, max_iter=1000),
    param_grid,
    cv=5,
    scoring='f1'
)
l2_grid.fit(X_train, y_train)

print("\nBest parameters:")
print(f"L1 Regularization: C={l1_grid.best_params_['C']}")
print(f"L2 Regularization: C={l2_grid.best_params_['C']}")

print("\nBest cross-validation scores:")
print(f"L1 Regularization: {l1_grid.best_score_:.4f}")
print(f"L2 Regularization: {l2_grid.best_score_:.4f}")
```

```
7. Finding Optimal Regularization Strength
--------------------------------------------------

Best parameters:
L1 Regularization: C=10
L2 Regularization: C=10

Best cross-validation scores:
L1 Regularization: 0.9552
L2 Regularization: 0.9670
```

2.1.12 Final Model Comparison

```
# 8. Final model comparison with optimal parameters
print("\n8. Final Model Comparison with Optimal Parameters")
print("-" * 50)

# Train final models with best parameters
final_l1_model = LogisticRegression(penalty='l1', solver='liblinear', C=l1_grid.best_para
final_l2_model = LogisticRegression(penalty='l2', C=l2_grid.best_params_['C'], random_sta

final_l1_model.fit(X_train, y_train)
final_l2_model.fit(X_train, y_train)

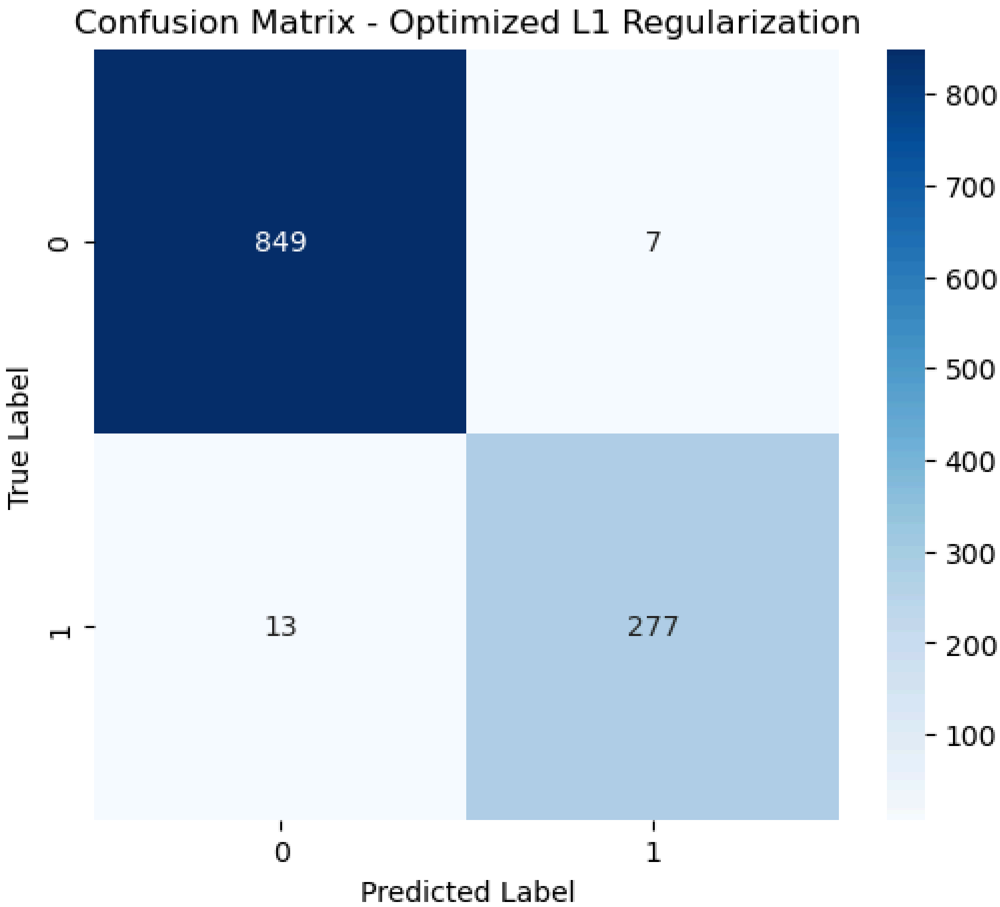
print("\nFinal L1 Model Performance:")
evaluate_model(final_l1_model, X_test, y_test, "Optimized L1 Regularization")

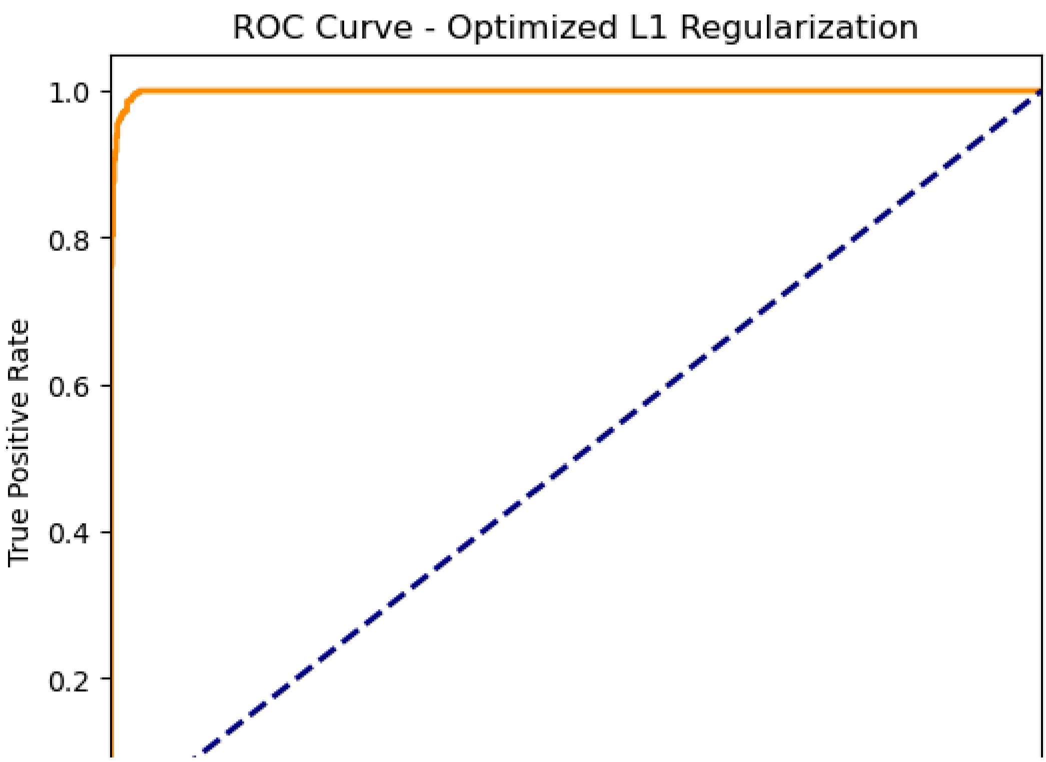
print("\nFinal L2 Model Performance:")
evaluate_model(final_l2_model, X_test, y_test, "Optimized L2 Regularization")
```

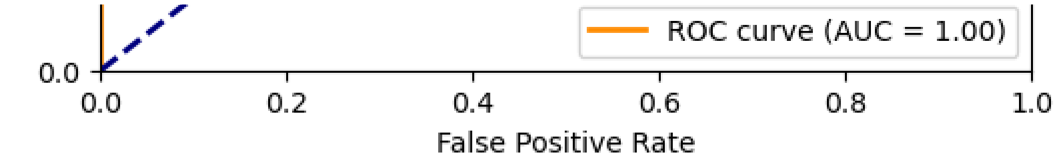
```
8. Final Model Comparison with Optimal Parameters
--------------------------------------------------

Final L1 Model Performance:

Metrics for Optimized L1 Regularization:
Accuracy: 0.9825
Precision: 0.9754
Recall: 0.9552
F1 Score: 0.9652
```

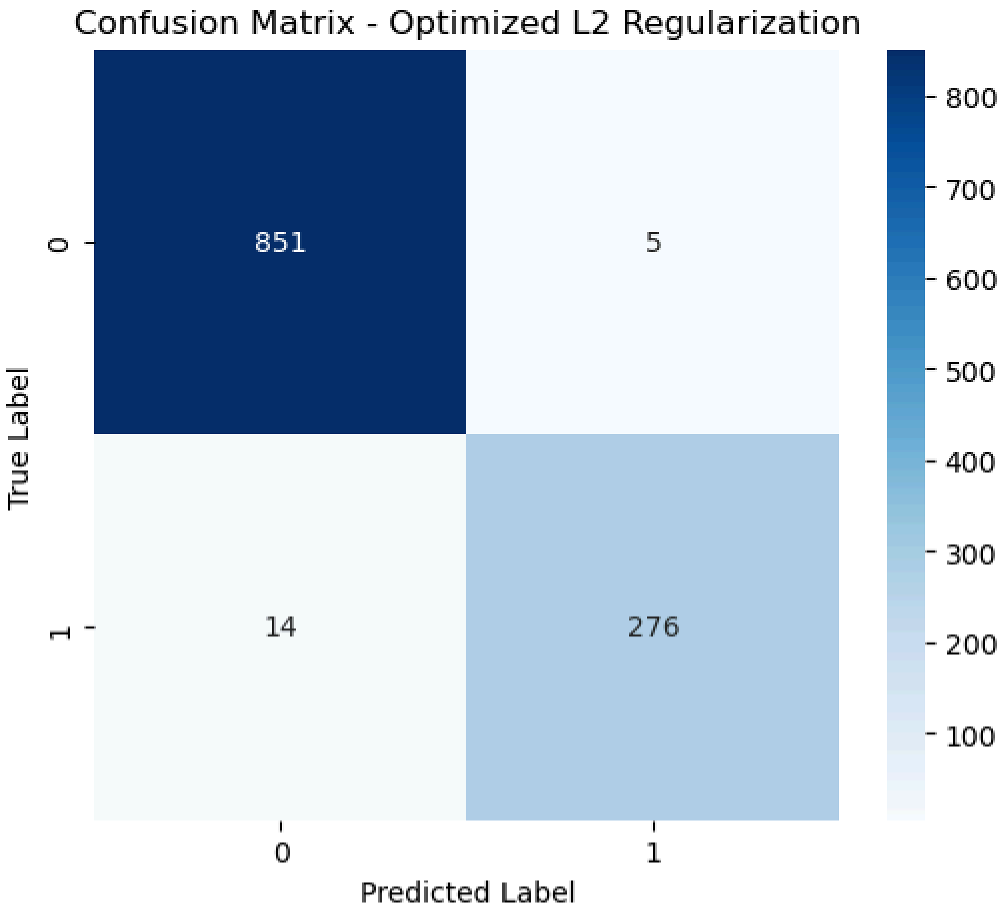


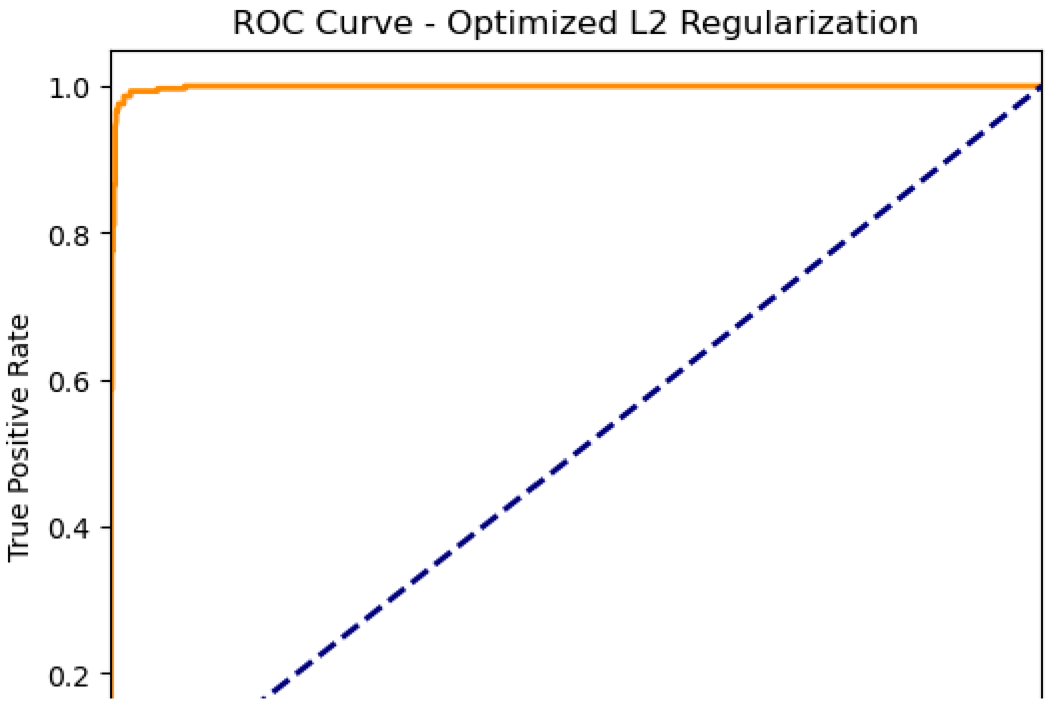


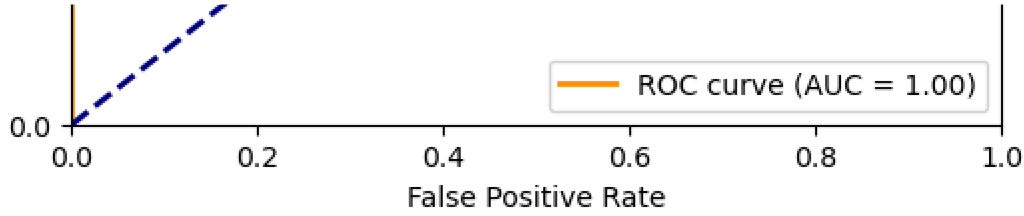



```
Final L2 Model Performance:

Metrics for Optimized L2 Regularization:
Accuracy: 0.9834
Precision: 0.9822
Recall: 0.9517
F1 Score: 0.9667
```








```
array([[-4.33711257e-01,  1.10911303e+00, -1.02553210e+00,
         7.02076560e-01, -1.08319863e+00, -9.52637833e-01,
        -6.13046200e-01, -9.37132962e-01,  3.24197016e-01,
         4.41916051e-01, -2.84572322e-01,  7.03882065e-01,
         2.69977911e+00, -1.46479915e+00, -4.89109655e-01,
        -8.00116689e-01, -1.79352004e-01,  8.27523312e-01,
        -7.77938980e-01, -6.84555125e-02, -8.81614227e-01,
        -2.50728372e-01, -1.62192831e+00,  4.82543429e-01,
        -6.18158673e+00, -3.94821455e+00,  1.84531575e+00,
         5.86263876e+00,  1.17054495e+00, -1.16230117e+00,
         5.66424333e-01, -1.19741270e+00,  2.76631521e+00,
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