Moore determinant

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Definition 0.1. *Let $\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ be a set of m elements of $\mathbb{F}_{q^m}$. Then the determinant*

$$\det M(\alpha_1, \ldots, \alpha_m) = \begin{vmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_m \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_m^2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^{q^{m-1}} & \alpha_2^{q^{m-1}} & \cdots & \alpha_m^{q^{m-1}} \end{vmatrix}$$

is called the Moore determinant of the set $\{\alpha_1, \ldots, \alpha_m\}$.

Theorem 0.2. *Let $\{\alpha_1, \ldots, \alpha_m\}$ be a set of m elements of $\mathbb{F}_{q^m}$. Then*

$$\det M(\alpha_1, \ldots, \alpha_m) = \alpha_1 \prod_{i=1}^{m-1} \prod_{c_1, \ldots, c_i \in \mathbb{F}_q} (\alpha_{i+1} - \sum_{j=1}^{i} c_j a_j).$$

Example 0.2.1. *Let $\{a_1, a_2, a_3\}$ be a set of elements of $\mathbb{F}_{2^3}$. Then*

$$\det M(a_1, a_2, a_3) = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_1^2 & a_2^2 & a_3^2 \\ a_1^4 & a_2^4 & a_3^4 \end{vmatrix} = a_1 a_2 (a_2 - a_1) a_3 (a_3 - a_1)(a_3 - a_2)(a_3 - a_1 - a_2)$$

Proposition 0.3. *Let $\{\alpha_1, \ldots, \alpha_m\}$ be a normal basis of $\mathbb{F}_{q^m}$ over $\mathbb{F}_q$. Then we have that*

$$\det M(\alpha_1, \ldots, \alpha_m) = 1.$$

Proof. We note that the elements which appear in the product of the Moore determinant are in fact all the elements of the field $\mathbb{F}_{q^m}$, since we have all the possible linear combinations of the normal basis $\{\alpha_1, \ldots, \alpha_m\}$ with coefficients over $\mathbb{F}_q$. Since the product of the elements of a field is 1, using the Theorem 0.2 we obtained that the Moore determinant is equal to 1. $\square$

References

[1] Gary L. Mullen, Daniel Panario *Handbook of finite fields*, CRC Press, 2013.