Review of twelve-tone theory

We've seen how pitch-class set analysis can be used to trace the use of motives in music by Schoenberg, Berg, Webern and others. In the music of other composers, including Bartók, Messiaen, and Stravinsky, we've observed the use of symmetry as an organizing principle: the use of symmetrical sets or scales (octatonic, whole-tone, etc.) and exact or pitch-class inversion of melodic and harmonic materials.

Schoenberg's twelve-tone method combines aspects of both motivic and symmetrical ways of organizing music. Simply put, a twelve-tone composition takes one ordering of all twelve chromatic pitch classes as a basic shape (German: *Grundgestalt),* and repeats it in varied forms throughout the piece. The result is that such works cycle continuously through all twelve pitch classes—the overall pitch-class *content* remains largely consistent throughout. Variety is achieved by changing the order of the twelve-tone row according to some strict rules: the row may be played backwards (retrograde), upside-down (inversion), or both backwards and upside-down (retrograde inversion), as well as in its original or prime form. The 48 possible forms of any given row can be arranged in compact form as a 12x12 matrix.

Making a twelve-tone matrix

1) Choose one complete form of the row and write it out across the top row of a 12x12 grid. This will be considered the **prime** or **P** form of the row. The label that applies to the row shown below (from Schoenberg's *Phantasy)* is $P_{B♭}$: that is, the prime form that starts on B♭. If we read the row backwards, from right to left, the result will a **retrograde** (**R**) row. All retrograde rows carry the same pitch-class label as the P row that they reverse: so the label for this row read right-to-left is $R_{B♭}$, not (as you might think) R_D.

B♭ A C# B F G E C G# E♭ F# D

2) Down the leftmost column, write the **inversion** or **I** form of the row, starting with the same pitch class as the prime form in the top row. In this matrix, this row will be labeled $I_{B♭}$: the inverted form that starts on B♭. The starting pitch (B♭) will be the axis of inversion between the two rows. In this example, A (the second pitch of $P_{B♭}$) inverts to B (the second pitch of $I_{B♭}$), C# inverts to G, B inverts to A, etc…

B♭ A C# B F G E C G# E♭ F# D

B

G

A

E♭

etc…

The **retrograde inversion** (**RI**) row can be found by reading any inverted row from bottom-to-top instead of top-to-bottom. In this case, the vertical row in the leftmost column read from top to bottom would be labeled $RI_{B♭}$—the label indicates clearly that it's the retrograde of $I_{B♭}$.

3) Fill in the rows one by one, transposing the prime form to begin on each pitch in the leftmost column. It's often convenient to work by reference to the adjacent row or column instead of referring all the way back to the prime form. For example, while writing out the third row (starting on G), I think of "a major third below the second row" rather than "a minor third below the prime form." If you prefer, you can fill in columns instead, transposing versions of the inverted row as you work from left to right. One way to check your work is to make sure that the upper-left to lower-right diagonal repeats the same pitch class (in this case, B♭).

B♭	A	C#	B	F	G	E	C	G#	E♭	F#	D
B	B♭	D	C	F#	G#	F	C#	A	E	G	E♭
G	F#	B♭	G#	D	E	C#	A	F	C	E♭	B
A	G#	C	B♭	E	F#	E♭	B	G	D	F	C#
E♭	D	F#	E	B♭	C	A	F	C#	G#	B	G
C#	C	E	D	G#	B♭	G	E♭	B	F#	A	F
E	E♭	G	F	B	C#	B♭	F#	D	A	C	G#
G#	G	B	A	E♭	F	D	B♭	F#	C#	E	C
C	B	D#	C#	G	A	F#	D	B♭	F	G#	E
F	E	G#	F#	C	D	B	G	E♭	B♭	C#	A
D	C#	F	E♭	A	B	G#	E	C	G	B♭	F#
F#	F	A	G	C#	E♭	C	G#	E	B	D	B♭

Summary of labels

Prime (P): read left to right, label with the first pitch class

Retrograde (R): read right to left, label with the *last* pitch class

Inversion (I): read top to bottom, label with the first pitch class

Retrograde inversion (RI): read bottom to top, label with the *last* pitch class

Analyzing the row

Composers working with the twelve-tone method find many ways of using the row—sometimes melodic, sometimes harmonic, and often in combination with different row forms. A certain amount of analysis on the row before composition begins can suggest properties of the row that may be useful in a piece.

1) Successive pitch class intervals

Identifying the intervals between each pitch class in the row can be very useful: to the composer, a tally of intervals will be useful in understanding the music potential of each part of the row, and for analysts the intervals aid in row identification.

Successive pitch class intervals for the Schoenberg row, $P_{B\flat}$

B♭ A C# B F G E C G# E♭ F# D

-1 +4 -2 6 +2 -3 -4 -4 -5 +3 -4

Successive pitch class intervals for the Schoenberg row, $R_{B\flat}$

D F# E♭ G# C E G F B C# A B♭

+4 -3 +5 +4 +4 +3 -2 6 +2 -4 +1

Successive pitch class intervals for the Schoenberg row, $I_{B\flat}$

B♭ B G A E♭ C# E G# C F D F#

+1 -4 +2 6 -2 +3 +4 +4 +5 -3 +4

Successive pitch class intervals for the Schoenberg row, $RI_{B\flat}$

F# D F C G# E C# E♭ A G B B♭

-4 +3 -5 -4 -4 -3 +2 6 -2 -4 -1

- Each row has just one instance of the interval classes 1, 5, and 6, two each of interval classes 2 and 3, and four of interval class 4. This distribution of intervals lends the row a characteristic sound (lots of major thirds/minor sixths) and also makes some intervals more useful than others for signaling the identity of a row. If we hear a row that begins with an ascending minor second, we can assume that it's an I form—and if a row begins with a descending minor second, we can assume that it's a P form.

- The successive pitch class intervals can be used to identify rows. If we hear a row beginning with the pitches A-F-G#, we can note the intervals between the pitch classes (-4 then +3) and match that pattern to the start of the RI form of the row.

2) Trichords

Breaking the row into its discrete (non-overlapping) trichords can be a useful way of exploring its musical potential. The chart below shows the set class of each trichord of the $P_{B♭}$ row. (A similar analysis could be done on the discrete tetrachords (four-note sets) of the row.)

B♭	A	C#	B	F	G	E	C	G#	E♭	F#	D

pitches 1-3	B♭	A	C#	(014)
4-6	B	F	G	(026)
7-9	E	C	G#	(048)
10-12	E♭	F#	D	(014)

- Note that both the first and last trichords are of the set class (014). This means that the row tends to begin and end with a similar sonority—it also gives the composer the option of overlapping rows by using the end of this row (E♭ F# D) as the beginning of another prime or retrograde row form.

- The augmented triad in the second hexachord is a particularly recognizable sonority. Because the augmented triad is a symmetrical sonority that keeps its identity when transposed by +4 or -4, there are twelve different row forms that share the same augmented triad. Theorists describe this property be saying that the augmented triad is held *invariant* in these different row forms.

We can also note a few non-discrete trichords that are of musical interest:

3-5	B	C#	F	(026)	same set class as pitches 4-6 (see above)
6-8	G	E	C	(047)	major triad
8-10	C	G#	E♭	(047)	major triad

Note that since the prime form of the row includes two major triads, the I and RI forms will each include two *minor* triads.

3) Hexachords and combinatoriality

$P_{B\flat}$

pitches 1-6	B♭	A	C#	B	F	G

normal form: F G A B♭ B C# (024568)

pitches 7-12	E	C	G#	E♭	F#	D

normal form: C D E♭ E F# G# (023468)

An analysis of the two hexachords of the $P_{B\flat}$ row shows that they're inversions of each other. Because the two hexachords are inversions of one another, it follows that there's some I form of the row which begins with the same six pitch classes as the second hexachord of this row. That row is $I_{E\flat}$:

$I_{E\flat}$

pitches 1-6	E♭	E	C	D	G#	F#

normal form: C D E♭ E F# G# (023468)

pitches 7-12	A	C#	F	B♭	G	B

normal form: F G A B♭ B C# (024568)

This relationship, where two different forms of the row divide into identical hexachords, is called ***combinatoriality.*** This feature is especially useful for composers who wish to maintain a complete chromatic scale at all times: by combining $P_{B\flat}$ and $I_{E\flat}$ at the same time, *aggregates* (sets of all twelve notes) will be formed between the first hexachords of both rows *and* the second hexachords of both rows:

$P_{B\flat}$:	B♭	A	C#	B	F	G	E	C	G#	E♭	F#	D
$I_{E\flat}$:	E♭	E	C	D	G#	F#	A	C#	F	B♭	G	B

|--------------------aggregate-------------------| |--------------------aggregate-------------------|

If this combinatorial property occurs between $P_{B\flat}$ and $I_{E\flat}$, it will also apply to any other pair of sets in the same relationship: for example, P_B and I_E or R_C and RI_F.

More complex combinatorial relationships can also occur—they're particularly likely when a twelve-tone row is made up of hexachords with a high degree of symmetry: for example, a row using the set class (014589) as a hexachord is combinatorial with eleven other row forms.

Derived rows

A technique frequently used by Webern is the use of a *derived row*—a twelve-tone row that is made up of many transposed and/or inverted versions of the same trichord or (less frequently) tetrachord. In the row for Webern's Concerto, op. 24, every discrete trichord belongs to either the set class (014) or its inversion, (034).

Row from Webern, Concerto, op. 24

G	B	B♭	E♭	D	F#	E	F	C#	C	G#	A

pitches 1-3	G	B	B♭	(034)
4-6	E♭	D	F#	(014)
7-9	E	F	C#	(034)
10-12	C	G#	A	(014)

Multiple rows

Berg's *Lyric Suite* uses two different rows in combination. This is relatively rare among the Second Viennese School composers, but does occur occasionally in other works by Berg. The two rows are shown below:

I)	F	F#	B♭	E	C#	A	D	G#	G	E♭	C	B
II)	F	E	C#	D	C	B	F#	B♭	A	G#	G	E♭