Proceeds Divided Between Two Parties with Recouped Costs

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August 17, 2020

Abstract

This article investigates mathematical principles when two parties agree to share proceeds in the sale of an asset, with one party wishing to recoup invested costs.

1 Setting

There exist two parties, L and J, being equal shareholders in an asset A to be sold. The proceeds P from the sale are to be divided evenly after L first recoups costs C (which were invested into A, presumably to increase A's pre-sale value). The resultant proceeds belonging to L and J are denoted P_L and P_J, respectively.

Constraints

$$P \geq C \geq 0$$

$$P_L \geq P_J$$

2 Method to Calculate Proper Proceeds

1. Begin with total proceeds P.
2. Immediately subtract from P the costs C of L, allocating C to P_L.
3. The remaining proceeds, $P - C$, are to be divided equally with $\frac{P-C}{2}$ being allocated to both P_L and P_J.

3 Proper Proceeds

By following the aforementioned method, the respective *proper* proceeds are:

$$P_L = C + \frac{P - C}{2} \tag{1}$$

$$P_J = \frac{P - C}{2} \tag{2}$$

Note the following:

$$P_L = C + P_J \tag{3}$$

4 Relationships

4.1 Individual proceeds sum to P.

The sum of the individual proceeds for L and J equal total proceeds P:

$$P_L + P_J = P \tag{4}$$

proof:

$$\begin{aligned}
P_L + P_J &= \\
\left(C + \frac{P - C}{2}\right) + \left(\frac{P - C}{2}\right) &= \\
C + \frac{P - C}{2} + \frac{P - C}{2} &= \\
C + 2\left(\frac{P - C}{2}\right) &= \\
C + (P - C) &= \\
C + P - C &= \\
P \quad \blacksquare
\end{aligned}$$

4.2 Subtracting P_J from P_L leaves C.

To confirm that L recoups costs C entirely, it must be shown that:

$$P_L - P_J = C \tag{5}$$

Although (5) follows naturally from (3), it is nevertheless proven below.

$$\begin{aligned}
P_L - P_J &= \\
\left(C + \frac{P-C}{2}\right) - \left(\frac{P-C}{2}\right) &= \\
C + \frac{P-C}{2} - \frac{P-C}{2} &= \\
C + \left(\frac{P-C}{2}\right)(1-1) &= \\
C + \left(\frac{P-C}{2}\right)(0) &= \\
C \quad \blacksquare
\end{aligned}$$

4.3 When the costs to be recouped C are zero, P_L and P_J are both equal to split proceeds.

Given (5), it follows when $C = 0$ that:

$$P_L - P_J = 0$$

$$P_L = P_J$$

$$C + \frac{P - C}{2} = \frac{P - C}{2}$$

$$0 + \frac{P - 0}{2} = \frac{P - 0}{2}$$

$$\frac{P}{2} = \frac{P}{2} \quad \blacksquare$$

4.4 If L and J are forced to equally split P, then J shall owe to L exactly $\frac{C}{2}$.

Closing coordinators at title companies might not be aware of L and J's agreement to have L recoup costs C before evenly sharing the remaining proceeds $P - C$. In this case, L and J would be forced to split P in half, receiving *improper* proceeds P_i of:

$$P_i = \frac{P}{2} \tag{6}$$

with the notable property given $C > 0$:

$$P_i < P_L \tag{7}$$

In this situation, in order to make L's proceeds *proper,* J would need to pay a reconciliation R to L computed as:

$$R = P_L - P_i \tag{8}$$

namely,

$$\begin{aligned}
R &= P_L - P_i \\
&= \left(C + \frac{P - C}{2}\right) - \left(\frac{P}{2}\right) \\
&= C + \frac{P - C}{2} - \frac{P}{2} \\
&= C + \frac{P - C - P}{2} \\
&= C + \frac{-C}{2} \\
&= \frac{2C}{2} + \frac{-C}{2} \\
&= \frac{2C - C}{2} \\
&= \frac{C}{2} \quad \blacksquare
\end{aligned}$$