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CSE 107
Lab 5

In this project you will construct the CDF of a random variable $Z = Y1 + Y2$, where both $Y1$ and $Y2$ are exponential random variables with parameter $\lambda = 1$. You will present your CDF as a table similar in form to the Standard Normal CDF table posted on the class webpage. We will show in lecture that Z has a well-known distribution called the Erlang distribution. Here you will construct your CDF by performing 20,000 trials of the following simple experiment. Obtain samples $Y1$ and $Y2$ from the exponential distribution, then compute the sum $Z = Y1 + Y2$. Using these trials, compute the relative frequencies of the events $\{Z \leq z\}$ for all $z \in \{0.0, 0.1, 0.2, 0.3, \ldots \ldots, 9.7, 9.8, 9.9\}$, which are 100 equally spaced points in the range 0.0 to 9.9, at distance 0.1 apart.

Code in Python:

```
import numpy as np

# Parameters
num_trials = 20000  # Number of trials
lambda_param = 1    # Parameter for exponential distribution
z_values = np.arange(0.0, 10.0, 0.1)  # z values from 0.0 to 9.9 in steps of 0.1

# Function to generate exponential random variables from uniform random variables
def generate_exponential_samples(lambda_param, size):
    uniform_samples = np.random.uniform(0, 1, size)
    exponential_samples = -np.log(1 - uniform_samples) / lambda_param
    return exponential_samples

# Generate Y1 and Y2 samples
Y1 = generate_exponential_samples(lambda_param, num_trials)
Y2 = generate_exponential_samples(lambda_param, num_trials)

# Compute Z = Y1 + Y2
Z = Y1 + Y2

# Initialize the CDF table
cdf_table = np.zeros((10, 10))  # 10 rows (0.0 to 9.0) and 10 columns (.0 to .9)

# Compute the relative frequencies for the CDF table
for z in z_values:
    row = int(z)  # Row index (0 to 9)
    col = int(round((z - row) * 10))  # Column index (0 to 9)
    if col == 10:  # Handle edge case where z = 9.9
        col = 9
```

```
    cdf_table[row, col] = np.mean(Z <= z)

# Print the CDF table
print("Sum of Exponentials CDF:")
print("          .0       .1       .2       .3       .4       .5       .6       .7       .8       .9")
for row in range(10):
    print(f"{row}.0 |", end=" ")
    for col in range(10):
        print(f"{cdf_table[row, col]:.4f}", end=" ")
    print()
```

Output:

```
Sum of Exponentials CDF:
          .0       .1       .2       .3       .4       .5       .6       .7       .8       .9
0.0 | 0.0000 0.0043 0.0167 0.0369 0.0600 0.0889 0.1211 0.1565 0.1898 0.2255
1.0 | 0.2614 0.3019 0.3389 0.3731 0.4072 0.4417 0.4731 0.5061 0.5353 0.5661
2.0 | 0.5959 0.6226 0.6469 0.6694 0.6922 0.7127 0.7317 0.7504 0.7680 0.7839
3.0 | 0.7985 0.8129 0.8278 0.8410 0.8522 0.8633 0.8733 0.8821 0.8905 0.8988
4.0 | 0.9057 0.9132 0.9197 0.9262 0.9313 0.9365 0.9419 0.9470 0.9510 0.9548
5.0 | 0.9584 0.9613 0.9645 0.9680 0.9706 0.9732 0.9754 0.9770 0.9787 0.9802
6.0 | 0.9817 0.9832 0.9842 0.9852 0.9865 0.9875 0.9887 0.9896 0.9903 0.9912
7.0 | 0.9922 0.9928 0.9937 0.9943 0.9947 0.9952 0.9957 0.9959 0.9961 0.9963
8.0 | 0.9966 0.9967 0.9970 0.9974 0.9976 0.9977 0.9979 0.9980 0.9980 0.9981
9.0 | 0.9983 0.9984 0.9986 0.9987 0.9987 0.9989 0.9990 0.9990 0.9990 0.9991
```

Conjecture:

By simulating the sum of two exponential random variables (Y1 and Y2) many times, this experiment aims to estimate the Cumulative Distribution Function (CDF) of the resulting random variable Z. The simulation involves generating random numbers from a uniform distribution, transforming them into exponential random variables, and summing them. By calculating how often Z falls below certain values, we can approximate the probability $P(Z \leq z)$ for various z values, and use these values to construct a CDF table. Key takeaways include an understanding of how to construct a CDF through simulation, how to transform uniform random variables into exponential random variables, and an approximation of the Erlang distribution (which Z theoretically follows) by computing the relative frequency of events given the number of trials that are performed.