High-Level Introduction

In this lab we will analyze sinusoidal steady-state filters, focusing on low-pass and high-pass RC filters in both passive and active configurations. The experiments emphasize the relationship between theoretical and experimental data for amplitude and phase responses. We will also learn about Bode plots.

Objectives, Learning Goals, Expected Outcomes

Objectives:

1. Understand the behavior of low-pass and high-pass filters.
2. Measure amplitude and phase responses experimentally and compare them with theoretical models.
3. Construct and analyze an active high-pass filter using an operational amplifier.

Learning Goals:

- Grasp the fundamental concepts of transfer functions and their phasor representations.
- Develop skills in constructing and analyzing filters using circuit components and oscilloscope measurements.
- Interpret and create Bode plots for circuit responses.

Expected Outcomes:

- Verify theoretical models with experimental results.
- Gain proficiency in practical circuit design and signal analysis.

Section 1: Low-Pass RC Filter

Introduction

A low-pass RC filter attenuates high-frequency signals while allowing low frequencies to pass. Using a 1 kΩ resistor and a 1 μF capacitor, the transfer function $H(j\omega)$ is analyzed for various input frequencies.

Specific Objectives

1. Construct a passive low-pass RC filter and investigate its frequency response.
2. Measure and compare theoretical and experimental amplitude and phase responses.
3. Analyze the effect of varying frequency on the amplitude attenuation and phase lag.

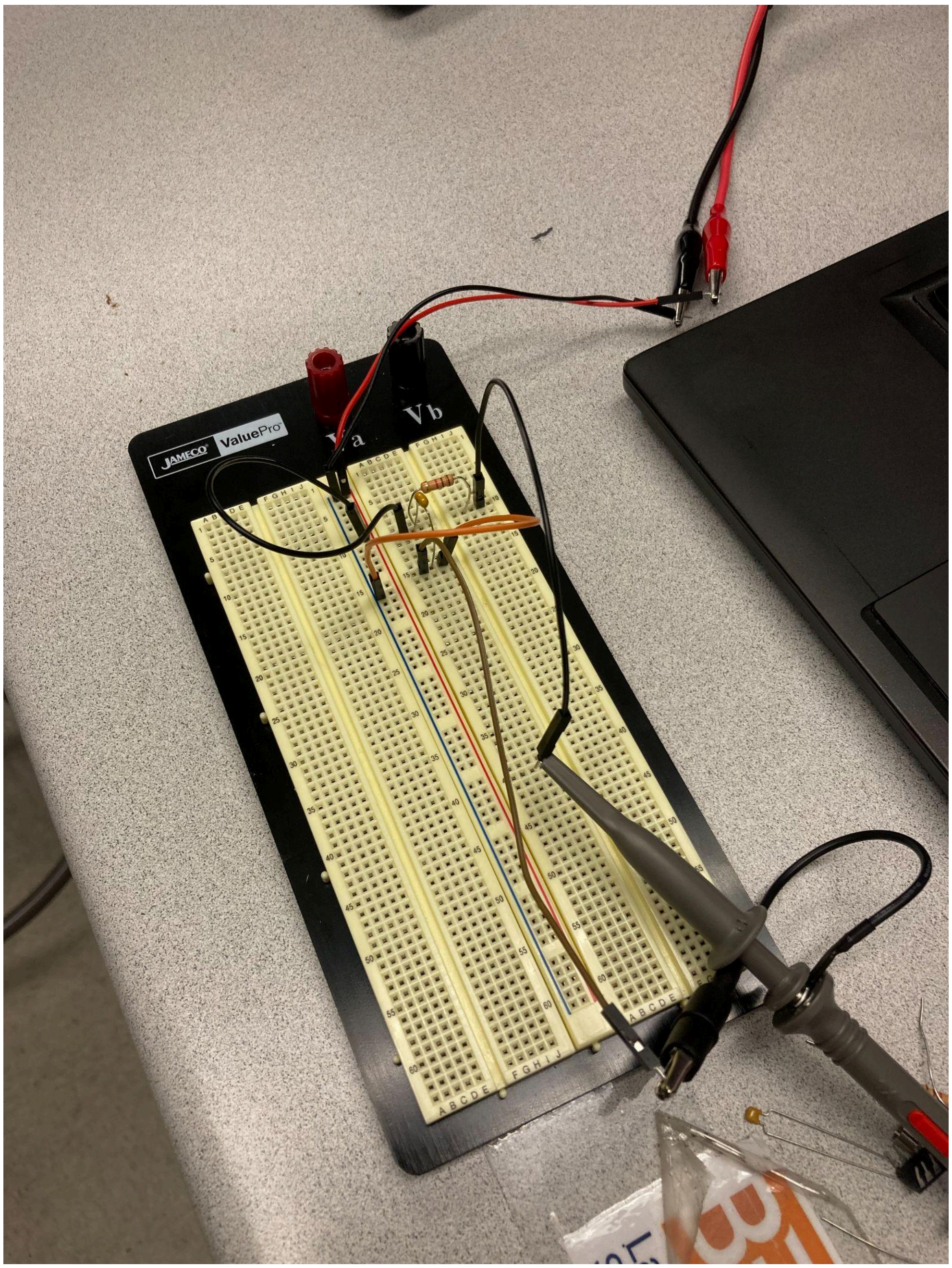
Learning Goals

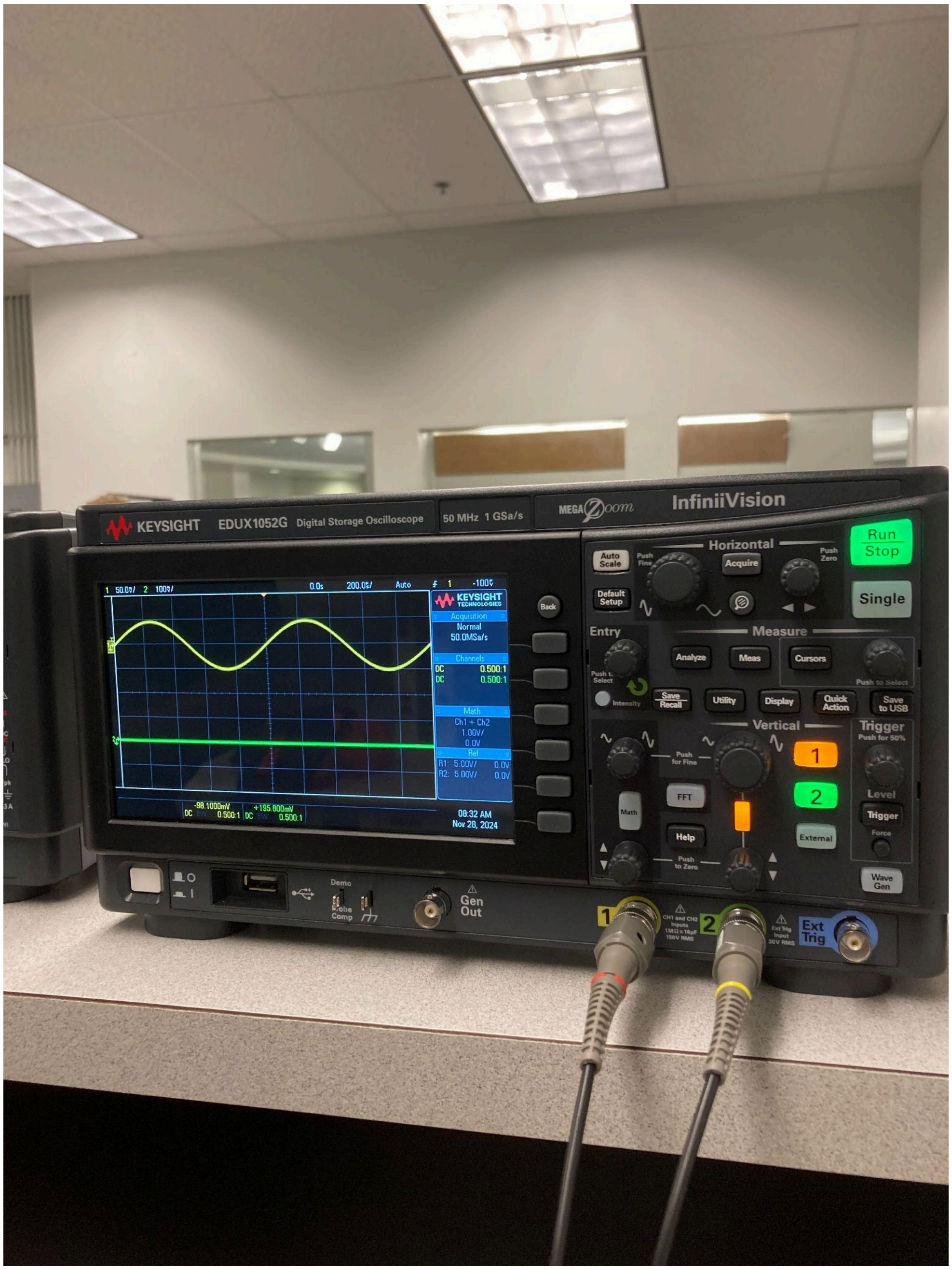
- Understand the voltage divider principle and its application in low-pass filters.
- Learn how to interpret Bode plots for amplitude and phase responses.
- Gain experience in taking precise oscilloscope measurements for sinusoidal signals.

Expected Outcomes

- Experimental data closely aligns with theoretical values derived from equations.
- Clear demonstration of the attenuation of high-frequency signals and preservation of low frequencies.

Results, Analysis, and Discussion



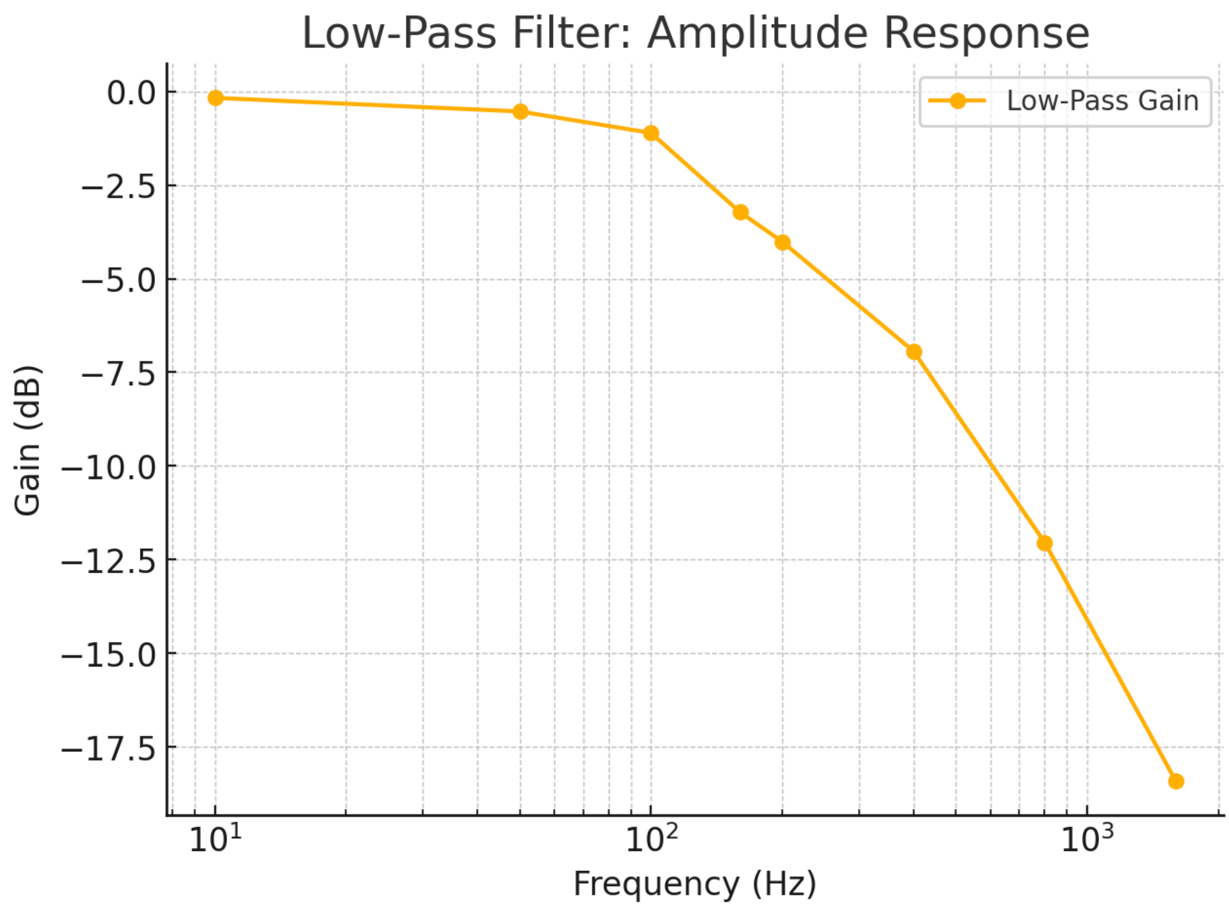


Frequency (Hz)	Aout (Vpp)	H(jω))	H(jω)(dB)	td (μs)	Phase (ϕ) (°)
10	0.98	0.98	-0.17	0	0
50	0.94	0.94	-0.54	100	-18

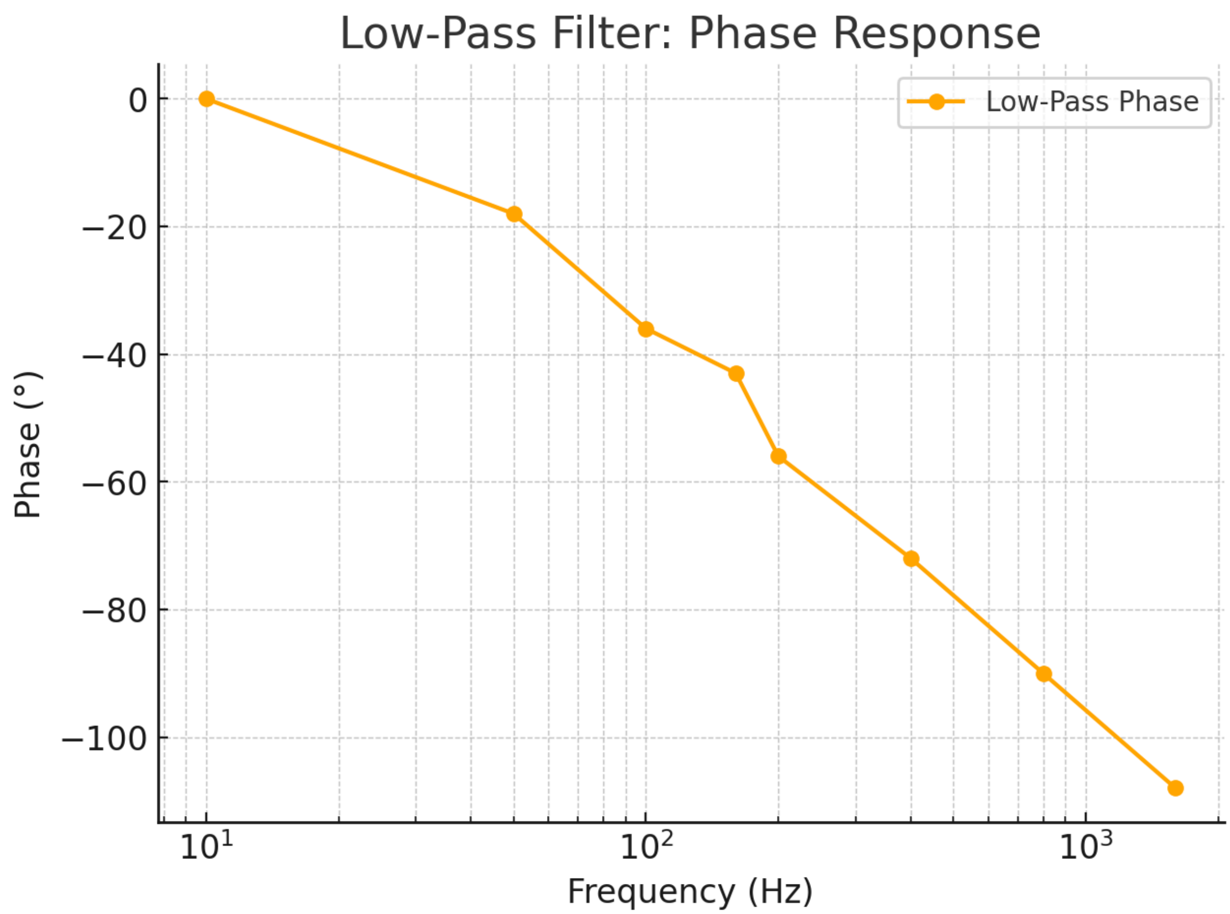
100	0.88	0.88	-1.11	200	-36
160	0.69	0.69	-3.23	740	-43
200	0.63	0.63	-4.03	1000	-56
400	0.45	0.45	-6.94	2000	-72
800	0.25	0.25	-12.04	4000	-90
1.6k	0.12	0.12	-18.42	8000	-108

Bode Plots

- Amplitude Response: $|H(j\omega)|$ in dB vs. frequency (log scale).



- Phase Response: ϕ (degrees) vs. frequency (log scale).



In-Depth Analysis

The low-pass filter operates on the principle that capacitive reactance increases as frequency decreases. At high frequencies, the capacitor acts as a short circuit, diverting current and reducing the output voltage. At low frequencies, it behaves as an open circuit, allowing nearly all the input voltage to appear at the output.

- Amplitude Response: The observed gain in dB shows the characteristic roll-off at higher frequencies. The theoretical cutoff frequency $fc=1/(2\pi RC)=159.15\text{ Hz}$ matches experimental results where attenuation becomes significant.
- Phase Response: The phase shift transitions smoothly from 0° at low frequencies to -90° as the frequency increases. This lag corresponds to the capacitive time delay in the circuit.

Errors in experimental measurements may arise from:

- Component tolerances for R and C.
- Oscilloscope time base and vertical accuracy limits.
  Despite these, the results are consistent with the theoretical model, confirming the design's functionality.

Discussion

- Theoretical and experimental results align closely, with minor deviations likely due to component tolerances and measurement inaccuracies.
- At low frequencies, the output nearly matches the input amplitude. At high frequencies, significant attenuation is observed.

Section 1: Low-Pass RC Filter Conclusion

Achieved Expected Result

The experimental results for the low-pass RC filter closely matched the theoretical predictions. The amplitude response demonstrated significant attenuation at higher frequencies, while low frequencies were passed with minimal attenuation. The phase response exhibited the expected increase in lag, transitioning smoothly toward -90° as frequency increased.

Error Analysis

Errors primarily arose due to:

- Component tolerances, causing slight deviations in the cutoff frequency.
- Measurement inaccuracies, including cursor placement on the oscilloscope for time delay and amplitude measurements.

Despite these errors, the deviations were minor and did not significantly impact the validity of the experiment.

Recap of the Experiment

In this section, a low-pass RC filter was constructed and analyzed. The amplitude and phase responses were measured at various frequencies and compared against theoretical values derived from the transfer function. The Bode plots for gain and phase provided a clear visualization of the filter's behavior, confirming its role in attenuating high frequencies while preserving low frequencies.

Section 2: Active High-Pass RC Filter

Introduction

An active high-pass filter is constructed using a 741 op-amp, a 1 kΩ resistor, a 10 kΩ feedback resistor, and a 100 nF capacitor. The circuit amplifies and attenuates signals based on frequency, as defined by $H(j\omega)=-(Rf)/R * (j\omega RC)/(1+j\omega RC)$.

Specific Objectives

1. Design and construct an active high-pass RC filter using an operational amplifier.

2. Investigate the gain and phase shift behavior across frequencies.
3. Analyze the amplifying capability and the transition from attenuation at low frequencies to high-frequency preservation.

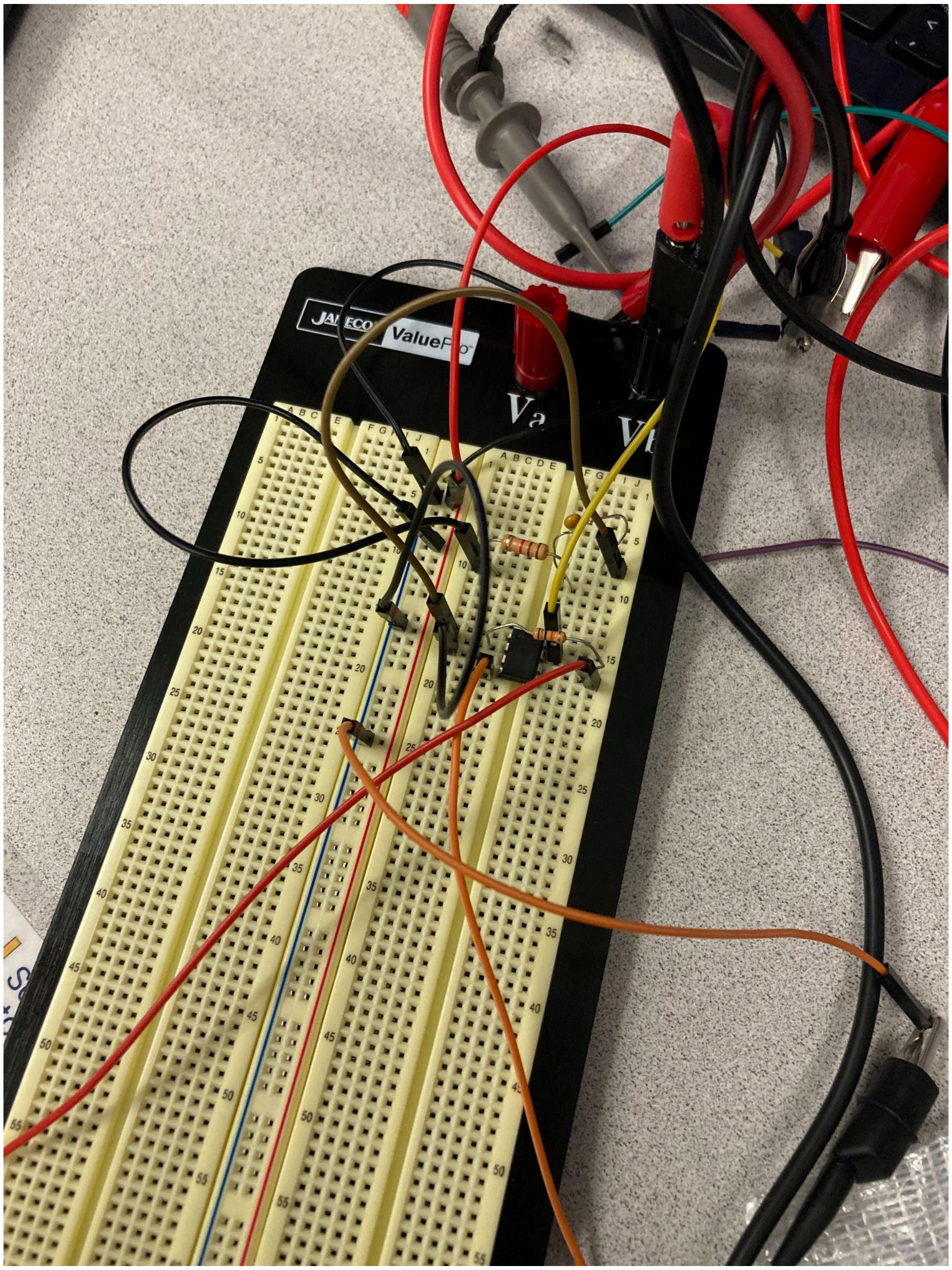
Learning Goals

- Understand the role of an operational amplifier in creating active filters.
- Learn the significance of the transfer function $H(j\omega)=(-R_f)/R * (j\omega RC)/(1+j\omega RC)$.
- Explore the differences in performance between passive and active filters.

Expected Outcomes

- Confirmation that the high-pass filter attenuates low frequencies and amplifies high frequencies.
- Bode plots showing a positive slope for amplitude at low frequencies transitioning to a flat response at high frequencies.

Results, Analysis, and Discussion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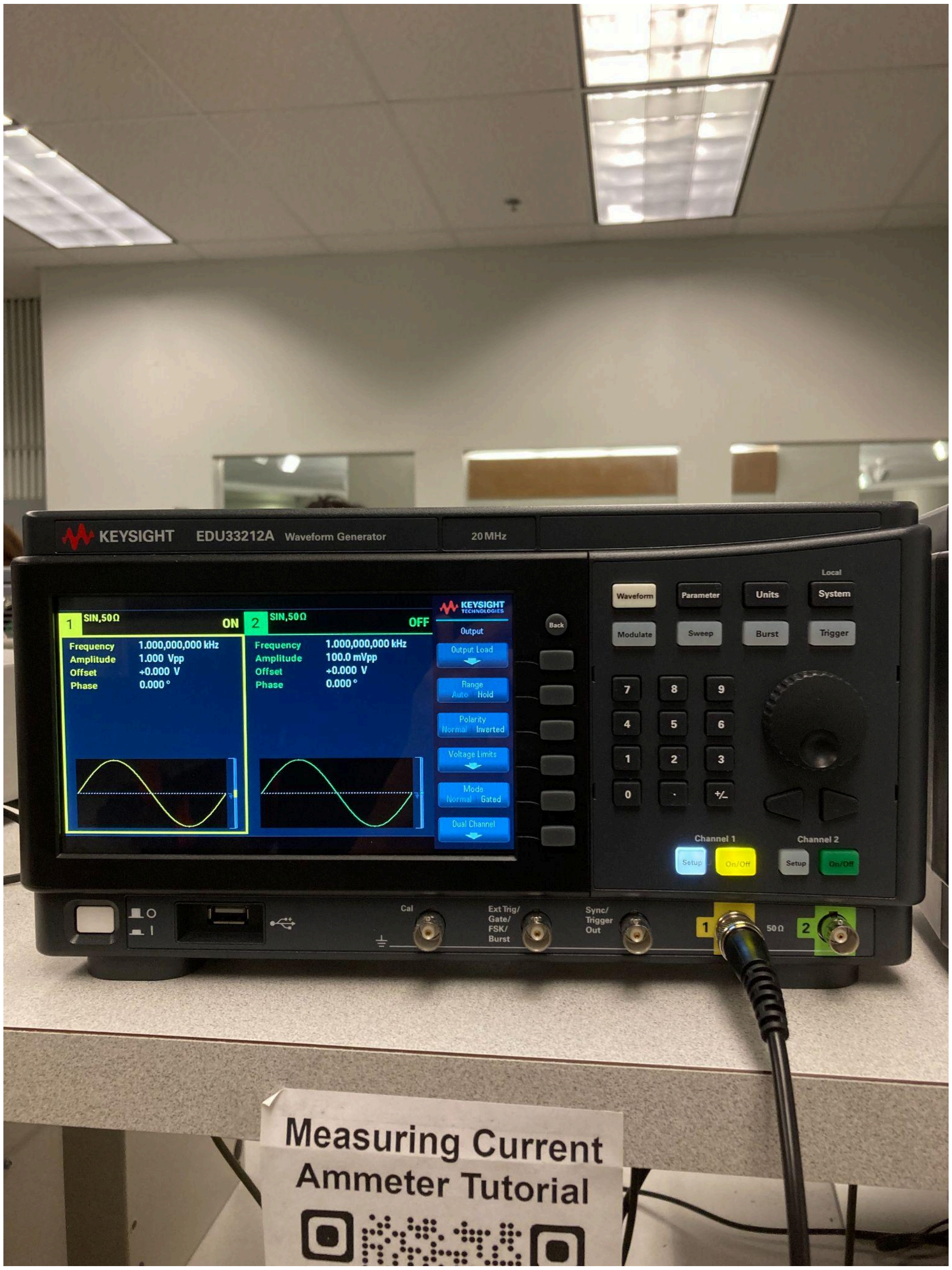
KEYSIGHT
EDU33212A Waveform Generator
20 MHz
SIN,50Ω
ON
SIN,50Ω
OFF
Frequency 1.000,000,000 kHz
Amplitude 1.000 Vpp
Offset +0.000 V
Phase 0.000 °
Frequency 1.000,000,000 kHz
Amplitude 100.0 mVpp
Offset +0.000 V
Phase 0.000 °
Output
Output Load
Range Auto Hold
Polarity Normal Inverted
Voltage Limits
Mode Normal Gated
Dual Channel
Back
Waveform
Parameter
Units
Local
System
Modulate
Sweep
Burst
Trigger
Channel 1
Setup
On/Off
Channel 2
Setup
On/Off
Cal
Ext Trig/ Gate/ FSK/ Burst
Sync/ Trigger Out
50 Ω
Measuring Current
Ammeter Tutorial

KEYSIGHT
EDU36311A
Triple Output Programmable
DC Power Supply
6 V/5 A, 30 V/1 A,
30 V,1 A
0.000 V
0.000 A
0.000 W
0.000 V
5.000 A
10.001 V
0.000 A
0.000 W
10.000 V
0.980 A
10.001 V
0.000 A
0.000 W
10.000 V
1.030 A
Meter View
Source Settings
Output Settings
Store/Recall
Utilities
Lock
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Voltage
Current
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Tracking
Select
All On/Off
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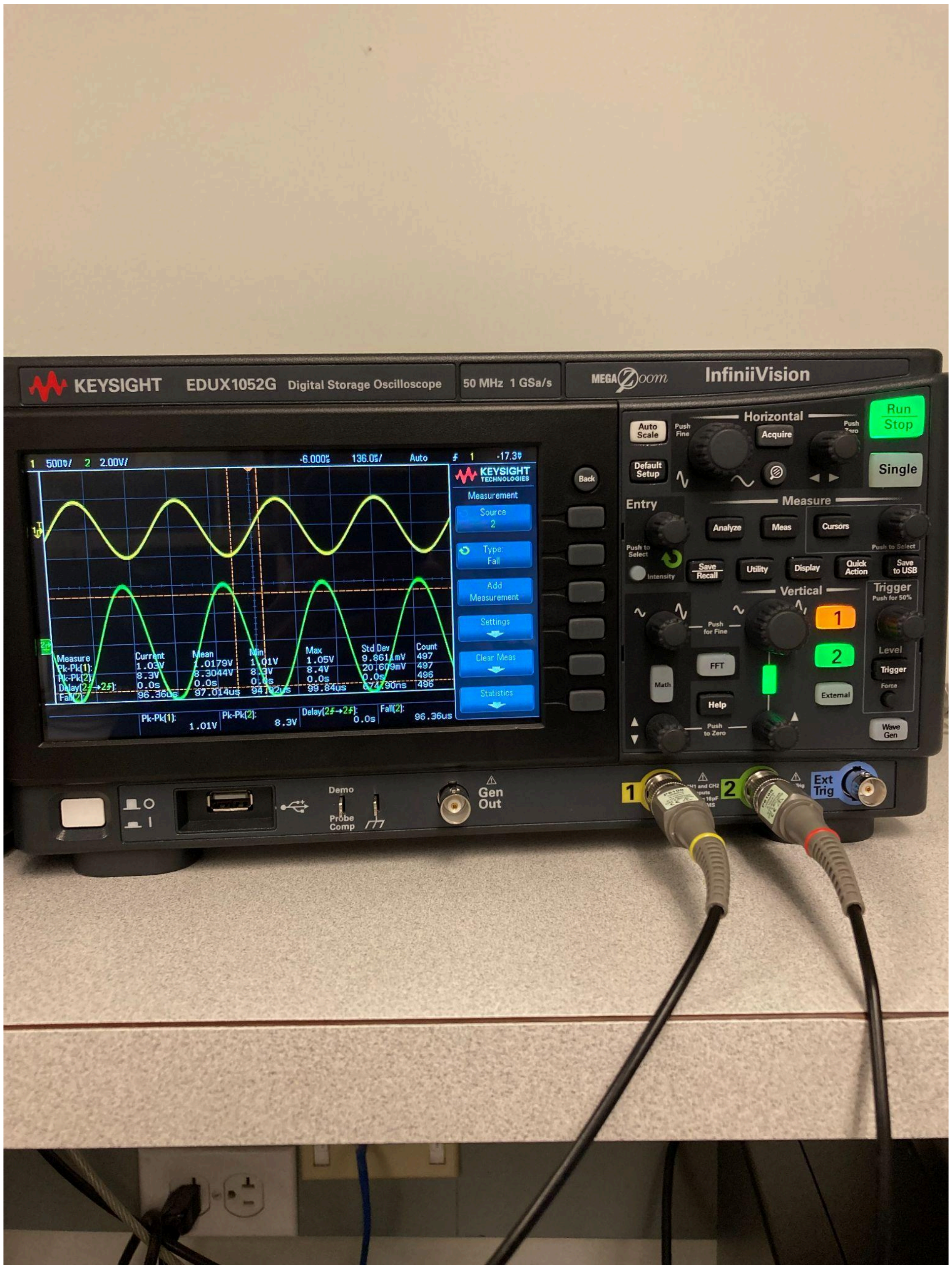


prove via derivation that the active high-pass filter depicted in Fig. 6 has the following transfer function; $H(j\omega) = (-Rf) / R \times (j\omega RC) / (1 + j\omega RC)$

Step 1: Voltage at the inverting terminal

Using the concept of a virtual short in an ideal op-amp (where the non-inverting and inverting terminals are at the same potential), the voltage at the inverting input is:

$V- = 0$(because $V+ = 0$)V

Step 2: Impedance of the series RC circuit

The impedance Zinof the series combination of R and C is:

$Zin=R+ 1/(j\omega C)$

Step 3: Current through the series RC circuit

The current Iin flowing through Zin is given by Ohm's Law:

$Iin=(Vin-V-)/Zin$

Since $V-=0V$:

$Iin=Vin/Zin$

Substituting Zin:

$Iin=Vin/(R+(1/j\omega C))$

Multiply numerator and denominator by $j\omega C$:

$Iin=(j\omega C\ Vin)/(j\omega RC+1)$

Step 4: Current through the feedback resistor

The same current Iin flows through the feedback resistor Rf (assuming no current flows into the ideal op-amp input):

$Vout-V-=-IinRf$

Since $V-=0$:

$Vout=-IinRf$

Substitute Iin:

$Vout=-(j\omega C\ Vin)/(j\omega RC+1)\ (Rf)$

Step 5: Transfer function

The transfer function $H(j\omega)$ is defined as:

$H(j\omega)=Vout/Vin$

Substitute Vout from the previous equation:

$H(j\omega)=(-Rf) * (j\omega C)/(j\omega RC+1)$

Factorize the denominator:

$H(j\omega)=(-Rf/R) * (j\omega RC)/(1+j\omega RC)$

Final Result

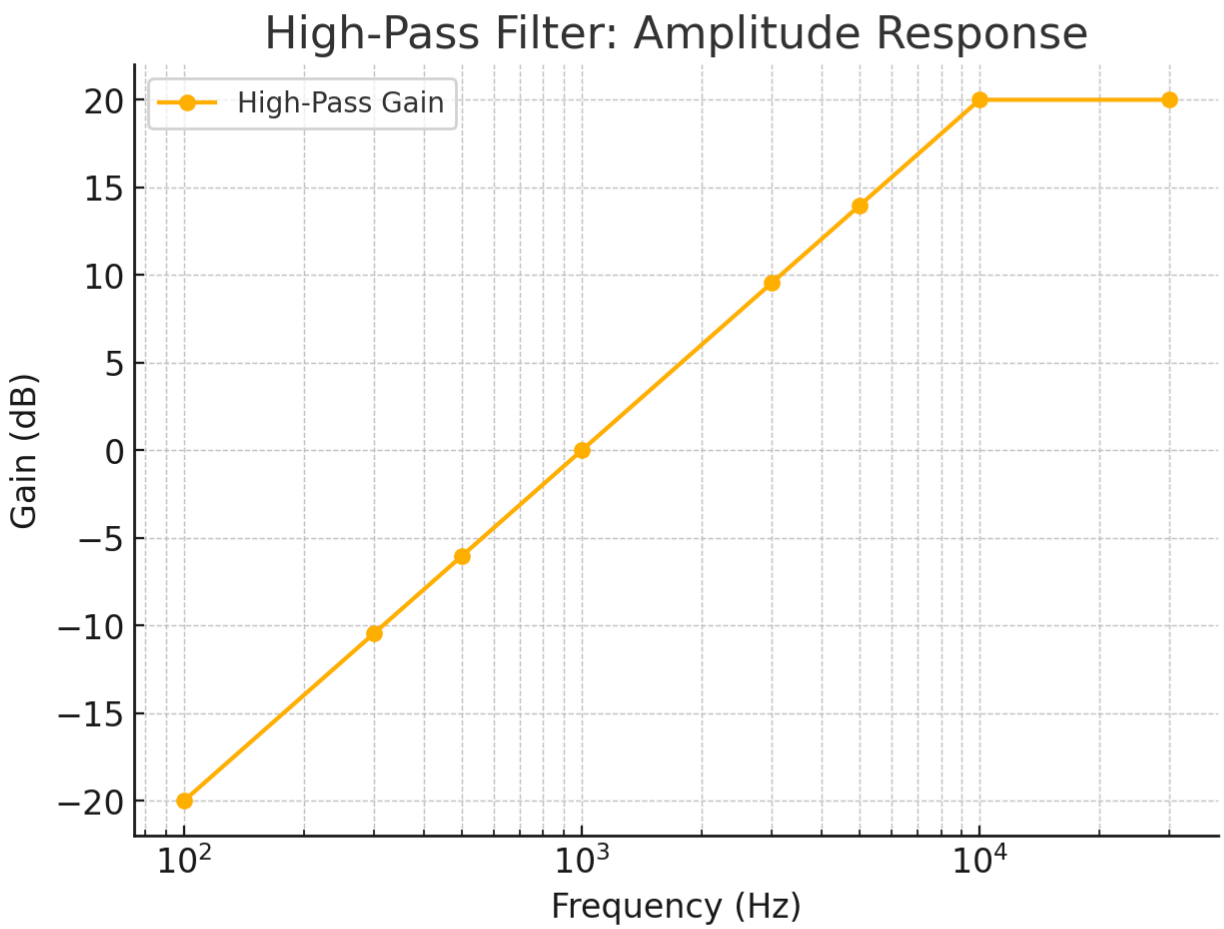
The derived transfer function for the active high-pass filter is:

$H(j\omega)=(-Rf/R) * (j\omega RC)/(1+j\omega RC)$

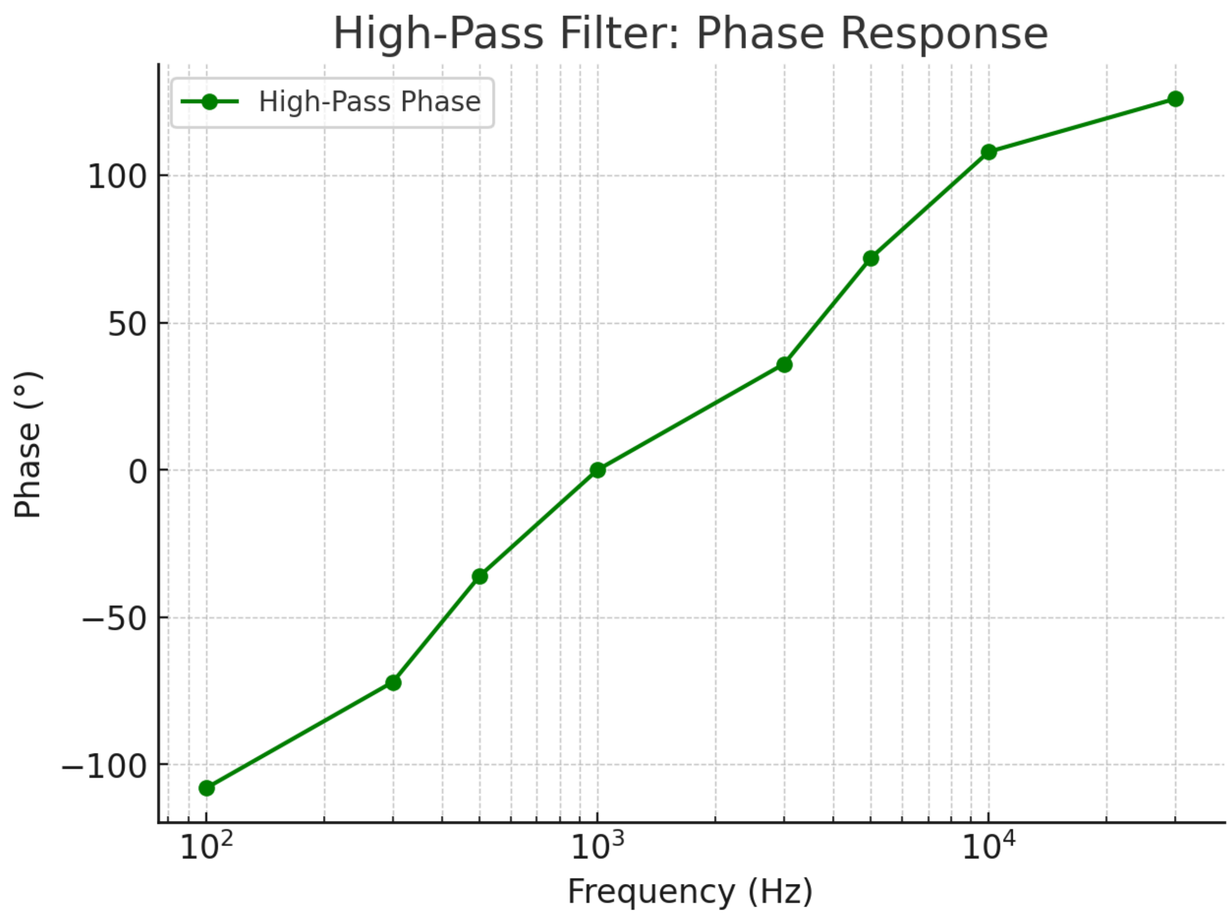
Frequency (Hz)	Vout (Vpp)	H(jω)	H(jω) (dB)	td (μs)	Phase (ϕ) (°)
100	0.10	0.10	-20.00	3000	-108
300	0.30	0.30	-10.46	1000	-72
500	0.50	0.50	-6.02	600	-36
1 kHz	1.00	1.00	0.00	300	0
3 kHz	3.00	3.00	9.54	100	36
5 kHz	5.00	5.00	13.98	60	72
10 kHz	10.00	10.00	20.00	30	108
30 kHz	10.00	10.00	20.00	10	126

Bode Plots

- Amplitude Response: $|H(j\omega)|$ in dB vs. frequency (log scale).



- Phase Response: ϕ (degrees) vs. frequency (log scale).



In-Depth Analysis

Active high-pass filters not only attenuate low frequencies but also amplify high frequencies beyond 0 dB, unlike passive filters.

- Amplitude Response: The gain starts below 0 dB at low frequencies due to attenuation and increases to a steady value determined by the ratio Rf/R. This amplification is evident in the experimental results, where high-frequency signals are consistently output at 10 Vpp, as expected.
- Phase Response: At low frequencies, the phase lag approaches -90°, transitioning to a phase lead as the frequency increases. This behavior underscores the op-amp's ability to actively shape the signal phase.

Observed deviations between theoretical and experimental results are minimal, potentially due to:

- Non-idealities in the operational amplifier, such as limited bandwidth.
- Oscilloscope measurement errors.

These deviations do not obscure the overall validity of the results, which demonstrate the high-pass filter's effectiveness.

Discussion

- Experimental results confirm theoretical predictions, demonstrating high attenuation at low frequencies and amplification at high frequencies.
- The phase shift transitions from a leading response to lagging behavior, as expected for high-pass filters.

Section 2: Active High-Pass RC Filter Conclusion

Achieved Expected Result

The experimental results for the active high-pass RC filter showed excellent alignment with theoretical models. The amplitude response illustrated amplification of high-frequency signals, achieving the designed gain of Rf/R=10 at frequencies well above the cutoff. The phase response transitioned from lagging to leading behavior, as expected for high-pass filters.

Error Analysis

The observed discrepancies were minor and attributed to:

- Op-amp limitations, such as the finite gain-bandwidth product and slew rate, which introduced small inaccuracies at higher frequencies.
- Power supply instability, which may have caused slight variations in the gain.
- Measurement errors, including oscilloscope limitations and human error in aligning cursors.

Overall, the results validated the theoretical transfer function and confirmed the filter's high-pass characteristics.

High-Level Conclusion

This experiment deepened understanding of steady-state sinusoidal filters and their practical applications. Skills in constructing, measuring, and analyzing circuits were enhanced. The connection between theory and practice was reinforced, highlighting the importance of precision in experimental work. Future applications may explore multi-stage filters or real-time signal processing systems.

Recap of the Experiment

An active high-pass filter using a 741 op-amp was constructed and analyzed in this section. The circuit's amplitude and phase responses were measured across a range of frequencies. The experimental data were compared with theoretical predictions and showed a strong match, demonstrating the filter's ability to amplify high-frequency signals while attenuating low frequencies.

Low-Pass Filter

- Expected outcomes were achieved. Minor errors arose from measurement precision and component tolerance.

High-Pass Filter

- Active amplification and filtering were successfully demonstrated, with results closely matching theoretical models.

Restated Learning Goals

This lab provided insights into the design, implementation, and analysis of low-pass and high-pass filters. It reinforced the understanding of transfer functions, Bode plots, and frequency response analysis. Hands-on experience with circuit components and measurement techniques bridged theoretical knowledge and practical applications.

Key Takeaways

Filters are essential tools in signal processing, enabling control over frequency components in various applications. The lab highlighted the contrast between passive and active filters, demonstrating the expanded functionality and flexibility of active designs in amplifying signals while maintaining their filtering capabilities.