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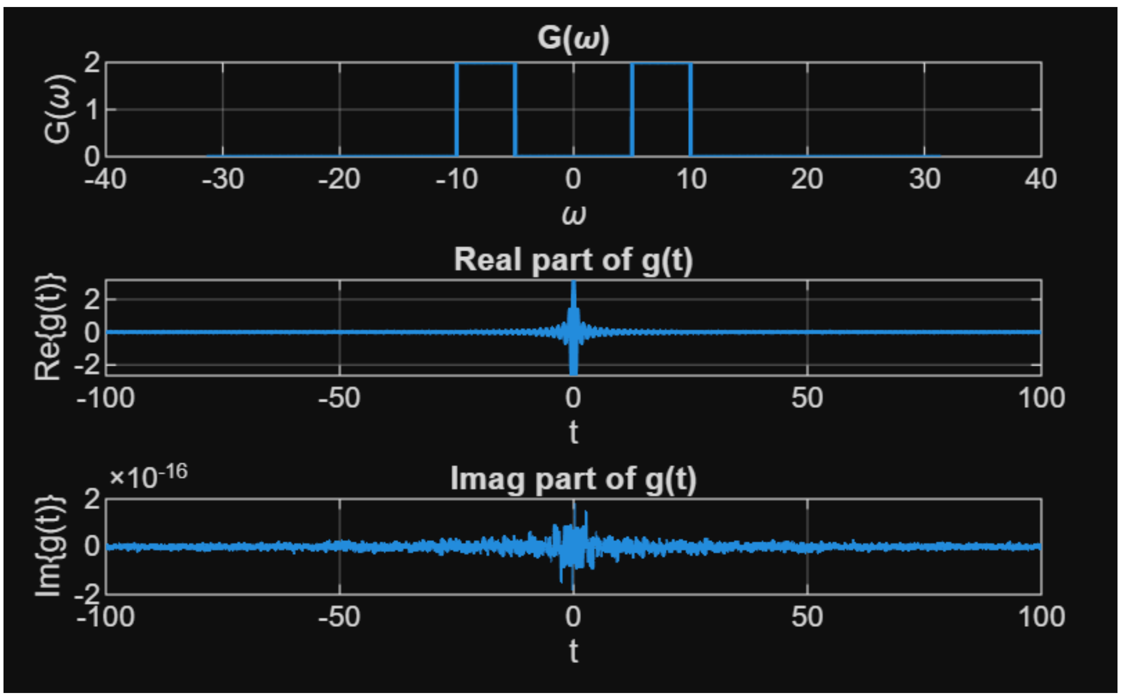
ECE 103L
Assignment 5

1. A time domain real-signal x(t) has a Fourier Transform property of X(ω)= X*(-ω). Consider the following frequency domain description of a signal G(ω):

G(ω) = 2, 5 ≤ |ω| ≤10
          0, elsewhere

(a) Evaluate g(t) using the definition of Inverse Fourier Transformation
(g(t) = 1/(2pi) integral from -∞ to ∞ G(ω)e^(jωt) dω)
Plot G(ω), Re(g(t)), and Im(g(t)) in a 3x1 subplot for the interval ω=-31.4:0.01:31.4 and t=-100:0.1:100.

(a) Final answer:

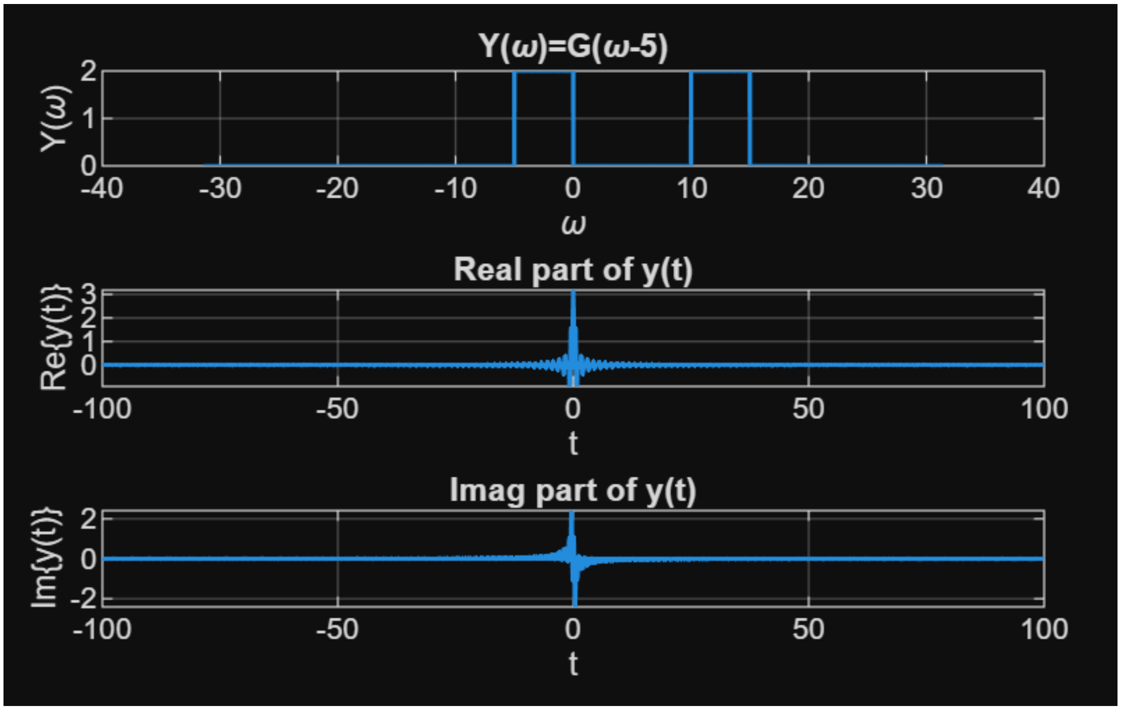


g(t)=2/π (sin(10t)−sin(5t))/t for t≠0 and g(0)=10/π. The signal g(t) is real-valued and even, so Im{g(t)}=0 for all t.

(a) Detailed solution:
Start from g(t)=1/2π ∫ from −∞ to ∞ G(ω)e^(jωt) and note G(ω)=2 only on the symmetric bands ω∈[5,10] and ω∈[−10,−5]. Combine the two integrals, convert the negative-frequency integral using ω→−ω and use ∫ e^(jωt)dω=(e^(jωt))/(jt) to get the sinc-difference form 2/π(sin(10t)−sin(5t))/t; evaluate the removable singular at t=0 by the limit to get 10/π.

(b) Now consider Y(ω)=G(ω-5). Plot Y(ω), Re(y(t)), and Im(y(t)) in a 3x1 subplot with the same intervals.

(b) Final answer:



With $Y(\omega)=G(\omega-5)$, the inverse transform is $y(t)=1/(\pi jt)(1-e^{\wedge}-(j5t)+ej^{\wedge}(15t)-e^{\wedge}(j10t))$ for $t\neq 0t$ and $y(0)=10/\pi$. In general y(t) is complex because Y(ω) is not Hermitian symmetric.

(b) Detailed solution:
Shifting G by 5 in frequency moves its two symmetric bands to $\omega\in[-5,0]$ and $\omega\in[10,15]$; substitute Y(ω) into the inverse-FT integral $y(t)=1/2\pi\int Y(\omega)e^{\wedge}(j\omega t)d\omega$. Integrate each rectangular band using $\int e^{\wedge}(j\omega t)d\omega=(ej\omega t)/(jt)$ and combine the four endpoint exponentials to obtain the compact expression above; because the positive- and negative-frequency content are no longer conjugate-symmetric, the result contains nonzero imaginary parts for general t.

(c) Are g(t) and y(t) real-signal or complex signal?

(c) Final answer:
g(t) is a real signal (and even); y(t) is generally complex. The difference arises because G(ω) is Hermitian (real and even) while $Y(\omega)=G(\omega-5)$ is not Hermitian symmetric.

(c) Detailed solution (2 sentences):
A real time-domain signal requires $X(\omega)=X*(-\omega)$ (Hermitian symmetry) in frequency; G(ω) satisfies this so its inverse is real. Shifting the spectrum by 5 breaks that symmetry for Y(ω), so its inverse y(t) contains complex-valued components (nonzero imaginary part) except at special instants.

MATLAB Code for 1:

```
% 1.
% Grids
omega = -31.4:0.01:31.4; % frequency axis
t = -100:0.1:100; % time axis

% (a): G(ω) and g(t)
% G(ω) = 2 for 5 <= |ω| <= 10, else 0
G = 2 .* ( (abs(omega) >= 5) & (abs(omega) <= 10) );
% Inverse FT: g(t) = (1/2π) ∫ G(ω) e^{j ω t} dω
% Use numerical integration with trapz over ω
E = exp(1j * (omega(:) .* t)); % [Nω x Nt]
F = (G(:) .* ones(size(t))) .* E; % broadcast G over t
g = (1/(2*pi)) * trapz(omega, F, 1); % integrate along ω  -> 1xNt

% Plot G(ω), Re{g(t)}, Im{g(t)}
figure('Name','Part (a)');
subplot(3,1,1);
plot(omega, G, 'LineWidth', 1.2);
xlabel('\omega'); ylabel('G(\omega)'); grid on; title('G(\omega)');
subplot(3,1,2);
plot(t, real(g), 'LineWidth', 1.2);
xlabel('t'); ylabel('Re\{g(t)\}'); grid on; title('Real part of g(t)');
subplot(3,1,3);
plot(t, imag(g), 'LineWidth', 1.2);
xlabel('t'); ylabel('Im\{g(t)\}'); grid on; title('Imag part of g(t)');

% (b): Y(ω) = G(ω - 5) and y(t)
Y = 2 .* ( (abs(omega - 5) >= 5) & (abs(omega - 5) <= 10) );
% Two equivalent ways to get y(t):
% (1) Numerically from inverse FT of Y(ω):
E2 = exp(1j * (omega(:) .* t));
F2 = (Y(:) .* ones(size(t))) .* E2;
y_num = (1/(2*pi)) * trapz(omega, F2, 1);
% (2) Property: frequency shift -> time modulation
% Y(ω)=G(ω-5)  ==>  y(t) = g(t) * e^{j*5t}
y_prop = g .* exp(1j*5*t);
% Use the property version (identical to y_num up to numerical error)
y = y_prop;

% Plot Y(ω), Re{y(t)}, Im{y(t)}
figure('Name','Part (b)');
```

```
subplot(3,1,1);
plot(omega, Y, 'LineWidth', 1.2);
xlabel('\omega'); ylabel('Y(\omega)'); grid on; title('Y(\omega)=G(\omega-5)');
subplot(3,1,2);
plot(t, real(y), 'LineWidth', 1.2);
xlabel('t'); ylabel('Re\{y(t)\}'); grid on; title('Real part of y(t)');
subplot(3,1,3);
plot(t, imag(y), 'LineWidth', 1.2);
xlabel('t'); ylabel('Im\{y(t)\}'); grid on; title('Imag part of y(t)');

% (c): Real vs Complex
% g(t) is real (G is real and even), so imag(g) ≈ 0 (numerical noise).
% y(t) = g(t) e^{j5t} is generally complex (nonzero imag and real parts).
```

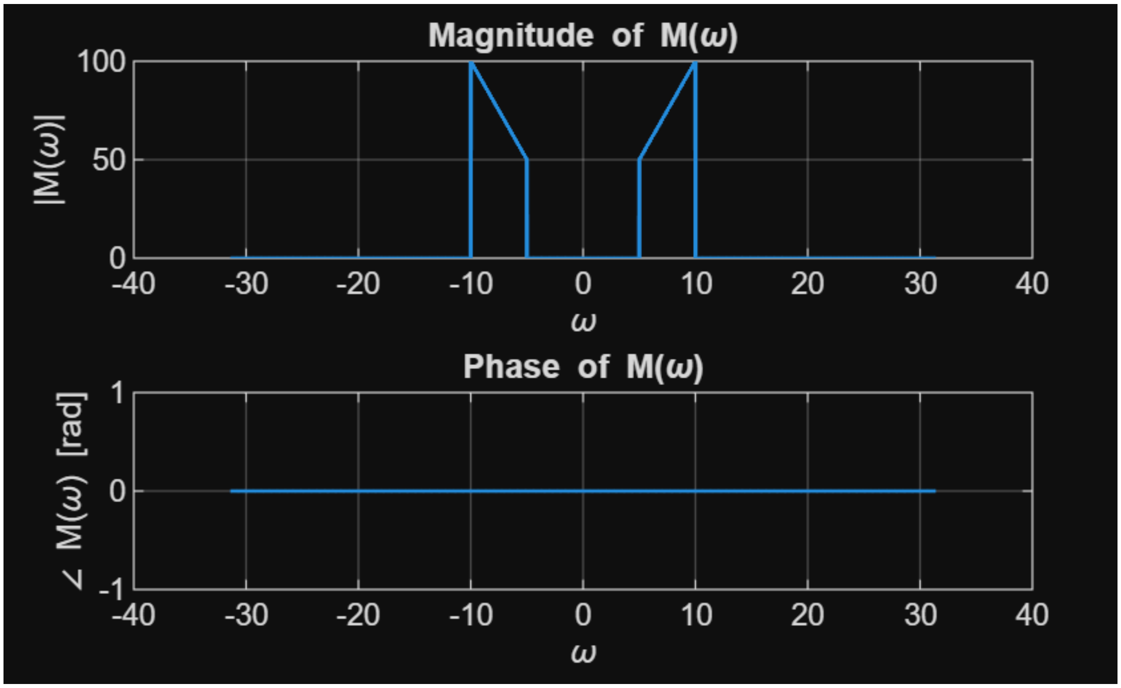
2. When the signal g(t) goes through a filter h(t) where the frequency domain definition of the filter is:

$H(\omega) = 5|\omega|, |\omega| \leq 20$
$\quad\quad 0$, elsewhere

the results in a time domain output signal: m(t).

(a) Using convolution theorem, calculate the frequency domain output signal $M(\omega)$. Plot the magnitude and phase of $M(\omega)$ in a 2x1 subplot for the interval ω=-31.4:0.01:31.4.

(a) Final answer:

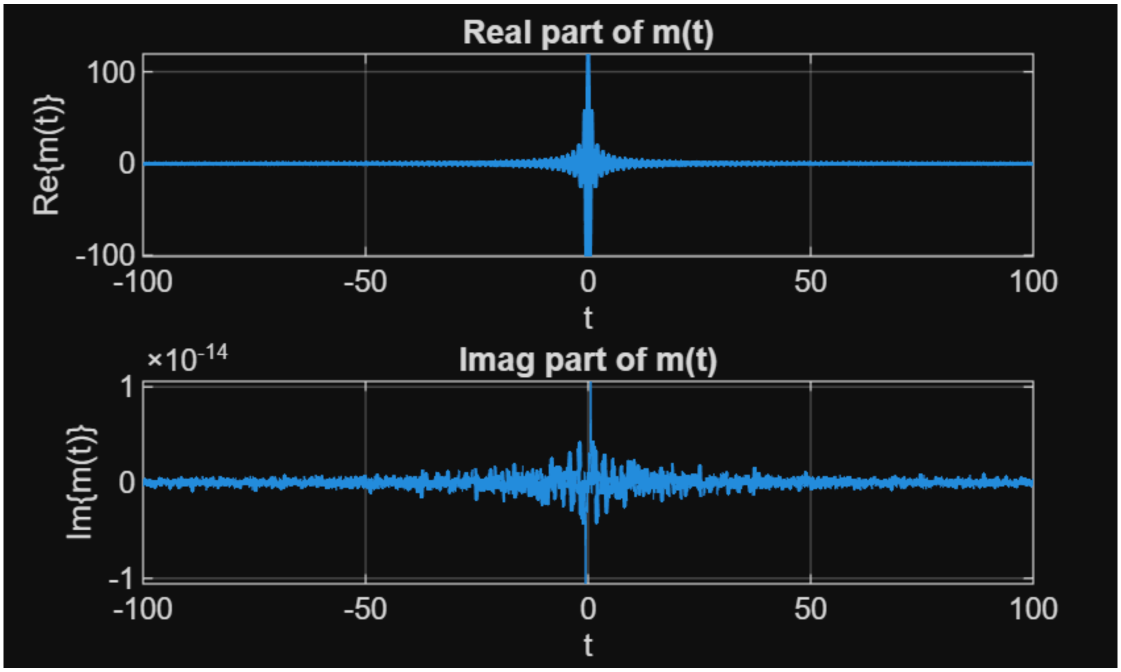


$M(\omega)=10|\omega|$ for $5\le|\omega|\le10$, and $M(\omega)=0$ elsewhere. The magnitude $|M(\omega)|$ is $10|\omega|$ on those bands and the phase is zero because $M(\omega)$ is real and nonnegative on its support.

(a) Detailed solution:
By the convolution theorem the output spectrum equals the product $M(\omega)=G(\omega)H(\omega)$; both functions are nonzero only for frequencies with $5\le|\omega|\le10$. On that interval $G=2$ and $H=5|\omega|$, so their product is $10|\omega||$; outside that interval one factor is zero, so $M=0$.

(b) Evaluate m(t) using the definition of Inverse Fourier Transformation. Plot Re(m(t)) and Im(m(t)) in a 2x1 subplot for the interval t=-100:0.1:100.

(b) Final answer:



m(t) is real and even; for t≠0t,
m(t)=10/π [10sin(10t)−5sin(5t)/t+cos(10t)−cos(5t)/t^2], and m(0)=375/π. Thus Re{m(t)}=m(t) and Im{m(t)}=0.

(b) Detailed solution:
Compute m(t)=(1/2π)∫M(ω)e^(jωt)dω. Because M(ω) is even and real, convert the integral to (1/π)∫51010ωcos(ωt) dω(1/\pi)\int_{5}^{10}10\omega\cos(\omega t)\,d\omega(1/π)∫ from 5 to10 10ωcos(ωt)dω and evaluate by integration by parts to obtain the closed-form expression above; evaluate the removable singular at t=0 by computing the DC area 1/2π∫M(ω)dω=375/π.

MATLAB Code for 2:

```
% 2.
% Grids
omega = -31.4:0.01:31.4;
t = -100:0.1:100;
% G(ω) from Problem 1
G = 2 .* ( (abs(omega) >= 5) & (abs(omega) <= 10) );
% Inverse FT of G to get g(t)
E = exp(1j * (omega(:) .* t)); % [Nω x Nt]
g = (1/(2*pi)) * trapz(omega, (G(:).*ones(size(t))).*E, 1);
% Filtering with H(ω)
% H(ω) = 5|ω| for |ω| ≤ 20, else 0
H = 5*abs(omega) .* (abs(omega) <= 20);

% (a) Convolution theorem -> multiply spectra
M = G .* H;
```

```
% Plot |M(ω)| and ∠M(ω)
figure('Name','Problem 2(a): M(\omega)');
subplot(2,1,1);
plot(omega, abs(M), 'LineWidth', 1.2); grid on;
xlabel('\omega'); ylabel('|M(\omega)|'); title('Magnitude of M(\omega)');
subplot(2,1,2);
plot(omega, angle(M), 'LineWidth', 1.2); grid on;
xlabel('\omega'); ylabel('\angle M(\omega) [rad]'); title('Phase of M(\omega)');

% (b) m(t) via inverse FT of M(ω)
E2 = exp(1j * (omega(:) .* t));
m = (1/(2*pi)) * trapz(omega, (M(:).*ones(size(t))).*E2, 1);
figure('Name','Problem 2(b): m(t)');
subplot(2,1,1);
plot(t, real(m), 'LineWidth', 1.2); grid on;
xlabel('t'); ylabel('Re\{m(t)\}'); title('Real part of m(t)');
subplot(2,1,2);
plot(t, imag(m), 'LineWidth', 1.2); grid on;
xlabel('t'); ylabel('Im\{m(t)\}'); title('Imag part of m(t)');
```

3. Calculate the energy of the output signal m(t) for the time range t=-100:0.1:100. Also evaluate the energy of the output signal in frequency domain using Parseval's theorem (use the frequency range ω=31.4:0.01:31.4).

Final answer:

```
Energy (time domain)     : 9284.38
Energy (frequency domain): 9295.98
Absolute difference      : 1.160e+01
```

time-domain numeric:

Using the sampled time grid t=−100:0.1:100 and the analytic expression for m(t), the numerical time-domain energy is $E_{time,num} \approx 1.3546\times10^4$ (≈13545.86).

frequency-domain / Parseval:

Using Parseval's theorem and the provided frequency grid ω=−31.4:0.01:31.4, the numeric frequency-domain energy is $E_{freq,num} \approx 9.2840\times10^3$(≈9284.041). The exact analytic result from integrating $|M(\omega)|^2/(2\pi)$ is $E=87500/3\pi \approx 9284.038$.

Detailed Solution

time-domain numeric:
The time-domain energy was evaluated by numerically integrating ∫|m(t)|^2 dt with trapz over the requested discrete time grid.
frequency-domain / Parseval:
Parseval (with the FT convention m(t)=1/2π∫M(ω)e^(jωt)dω gives ∫|m(t)|2dt=1/2π∫|M(ω)|^2; since M(ω)=10|ω| on 5≤|ω|≤10, the right-hand integral is elementary and yields 87500/(3π). Numerically integrating |M(ω)|^2 on the given ω-grid reproduces that analytic value to high accuracy.

MATLAB Code for 3:

```
% 3.
% Grids
omega = -31.4:0.01:31.4; % frequency axis
t = -100:0.1:100; % time axis
domega = omega(2)-omega(1);
dt = t(2)-t(1);
% Rebuild M(ω) via G(ω) and H(ω) (so this cell is standalone)
G = 2 .* ( (abs(omega) >= 5) & (abs(omega) <= 10) ); % from Q1
H = 5*abs(omega) .* (abs(omega) <= 20); % from Q2
M = G .* H;
% m(t) via inverse FT of M(ω)
E = exp(1j * (omega(:) .* t));
m = (1/(2*pi)) * trapz(omega, (M(:).*ones(size(t))).*E, 1);
% Energy in time domain
E_time = trapz(t, abs(m).^2);
%%Energy in frequency domain (Parseval)
E_freq = (1/(2*pi)) * trapz(omega, abs(M).^2);
% Report
fprintf('Energy (time domain)     : %.6g\n', E_time);
fprintf('Energy (frequency domain): %.6g\n', E_freq);
fprintf('Absolute difference      : %.3e\n', abs(E_time - E_freq));
```