A Novel Time and Frequency Synchronization Scheme for OFDM Systems

Guo Yi, Liu Gang and Ge Jianhua

Abstract — *A new training symbol is designed and a novel time and frequency synchronization scheme for orthogonal frequency division multiplexing systems is proposed. The time synchronization is accomplished by using the symmetric conjugate of the training symbol in time domain and the frequency synchronization is finished by utilizing the good autocorrelation of the training symbol in frequency domain. The simulation results show that compared with conventional schemes, the proposed scheme gives a more accurate estimate of symbol timing and carrier frequency offset and provides a wider acquisition range for the carrier frequency offset.*[1]

Index Terms — OFDM, time synchronization, frequency synchronization, training symbol.

I. INTRODUCTION

Synchronization has been a major research topic in orthogonal frequency division multiplexing (OFDM) systems due to the sensitivity to symbol timing and carrier frequency offset [1][2]. Several approaches have been proposed to estimate timing and frequency offset either jointly or individually [3] [4].

The most popular of the pilot-aided algorithms is the method proposed by Schmidl [5]. It uses a training symbol containing the same two halves to estimate the symbol timing offset. Schmidl's estimator provides a simple estimate for symbol timing offset. However, its timing metric has a plateau which causes a large variance in the timing offset estimation.

To reduce the uncertainty arising from the timing metric, Minn proposed a method as a modification to Schmidl's approach [6]. Minn's estimator performs well, but its estimation variance is quite large in ISI channels.

To improve the performance of timing synchronization, Park proposed a new synchronization scheme for OFDM systems. It produces an even sharper timing metric than Schmidl's and Minn's [7], but its timing metric has two large sidelobes which will affect the timing performance. It is noted that the frequency offset estimation ranges of the three methods are all narrow.

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In this paper, a new training symbol is designed and a simple timing and frequency offset estimation scheme for OFDM systems is proposed. The proposed scheme not only eliminates the sidelobes of Park's timing metric but also provides a wide range of frequency offset estimation, up to half the OFDM bandwidth.

The rest of this paper is organized as follows. In Section II the OFDM signal model is introduced and the existing timing and frequency offset estimation methods are described. Section III covers the proposed training symbol and presents the time and frequency synchronization scheme. In Section IV, the performance of the proposed scheme and the other schemes are compared in terms of mean-square error using computer simulation. Finally the conclusion is drawn in Section V.

II. SYSTEM DESCRIPTION

A. OFDM signal model

Consider a general case of OFDM systems using the standard complex-valued baseband equivalent signal model. The nth received sample has the standard form:

$$r_n = e^{j2\pi nf/N} \cdot \sum_{l=0}^{L-1} h_l \cdot x_{n-\varepsilon-\tau_l} + w_n \tag{1}$$

where N is the number of subcarriers, h_l is the channel impulse response, τ_l is the channel delay, L is the number of channel paths, f is the frequency offset as a fraction of the intercarrier spacing, ε is the integer-valued unknown arrival timing of a symbol, w_n is the AWGN and x_n is the time-domain OFDM signal expressed by

$$x_n = \frac{1}{\sqrt{N}} \cdot \sum_{k=0}^{N-1} C_k \cdot e^{j2\pi kn/N}, \qquad -N_g \le n \le N-1 \tag{2}$$

where C_k is the complex data modulating the kth active subcarrier.

B. OFDM Time and Frequency Synchronization

The goal of OFDM time and frequency synchronization is to estimate f and ε. Before we proceed, let us briefly describe the timing and frequency offset estimation methods presented in [5], [6] and [7].

1) Schmidl's Method

The form of the time-domain training symbol proposed by Schmidl is as follows:

← N/2 →	← N/2 →
A	A

where A represents samples of length N/2.

The Schmidl's timing estimator finds the starting point of the symbol at the maximum point of the timing metric given by

$$M(m) = \frac{|p(m)|^2}{(R(m))^2} \quad (3)$$

where

$$p(m) = \sum_{k=0}^{N/2-1} r^*(m+k) \cdot r(m+k+N/2) \quad (4)$$

$$R(m) = \sum_{k=0}^{N/2-1} |r(m+k+N/2|^2 \quad (5)$$

Thus the estimation of timing offset is

$$\hat{\varepsilon} = \arg\max_m (M(m)) \quad (6)$$

and the estimation of frequency offset only using one training symbol is

$$\hat{f} = angle(p(\hat{\varepsilon}))/\pi \quad (7)$$

The timing metric of Schmidl's method has a plateau which results in large MSE and the acquisition range for the carrier frequency offset is only $|\hat{f}| \le 1$ due to the periodicity of $angle(\bullet)$.

2) Minn's Method

In order to alleviate the uncertainty caused by the timing metric plateau and to improve the timing offset estimation, Minn proposed a modified training symbol. Minn's training symbol has the following form:

← N/4 →	← N/4 →	← N/4 →	← N/4 →
B	B	-B	-B

where B represents a PN sequence of length *N*/4.

Then the timing metric is expressed as

$$M(m) = \frac{|p(m)|^2}{(R(m))^2} \quad (8)$$

where

$$p(m) = \sum_{l=0}^{1} \sum_{k=0}^{N/4-1} r^*(k + lN/2 + m) \cdot r(k + lN/2 + N/4 + m) \quad (9)$$

$$R(m) = \sum_{l=0}^{1} \sum_{k=0}^{N/4-1} |r(k + lN/2 + N/4 + m)|^2 \quad (10)$$

Thus the estimation of timing offset is

$$\hat{\varepsilon} = \arg\max_m (M(m)) \quad (11)$$

and the estimation of frequency offset is

$$\hat{f} = angle(p(\hat{\varepsilon})) \cdot 2/\pi \quad (12)$$

Minn's method uses negative-valued samples at the second-half of training symbols to eliminate the peak plateau of the timing metric and has its peak at the correct starting point for the OFDM symbol. However, it is observed that the MSE of Minn's estimator is quite large in ISI channels from the results in [6] and the acquisition range for the carrier frequency offset is only $|\hat{f}| \le 2$.

3) Park's Method

To improve the performance of timing synchronization and enlarge the difference between the two adjacent values of the timing metric, Park proposed a modified training symbol which has the following form:

← N/4 →	← N/4 →	← N/4 →	← N/4 →
C	D	C	D

where C represents samples of length N/4, and D is designed to be symmetric with the conjugate of C.

Then the timing metric is expressed as

$$M(m) = \frac{|p(m)|^2}{(R(m))^2} \quad (13)$$

where

$$p(m) = \sum_{k=1}^{N/2-1} r(m+k) \cdot r(m+N-k) \quad (14)$$

$$R(m) = \sum_{k=1}^{N/2-1} |r(m+k)|^2 \quad (15)$$

Thus the estimation of timing offset is

$$\hat{\varepsilon} = \arg\max_m (M(m)) \quad (16)$$

and the estimation of frequency offset is

$$\hat{f} = angle(q(\hat{\varepsilon}))/\pi \quad (17)$$

where

$$q(\hat{\varepsilon}) = \sum_{k=0}^{N/4-1} r^*(\hat{\varepsilon}+k) \cdot r(\hat{\varepsilon}+k+N/2) \quad (18)$$

The $p(m)$ is designed such that there are N/2-1 different pairs of product between two adjacent values. Therefore, the timing metric of Park's method has its peak value at the correct symbol timing, while the values are almost small at all other positions. However, it is observed that the timing metric of Park's method has large sidelobes at the positions with ± N/4 samples spaced from the correct starting point of OFDM symbols, which will affect the accurateness of timing offset estimation. Moreover, the frequency offset estimation range of Park's method is only $|\hat{f}| \le 1$.

To improve the timing performance and enlarge the acquisition range for the carrier frequency offset, a new training symbol is designed and a simple timing and frequency offset estimation scheme is proposed.

III. Proposed Time and Frequency Synchronization Scheme

A. Proposed Training Symbol

The samples of the new training symbol in frequency domain are designed to be of the form

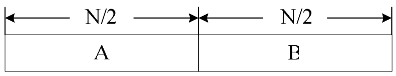
$$\mathbf{C}_{preamble} = [\underbrace{0,\cdots,0}_{n_f}, C_1, C_2, \cdots, C_K, \underbrace{0,\cdots,0}_{n_h}]$$

$$N = n_f + K + n_h \quad (19)$$

and the corresponding time-domain complex baseband training symbol samples are

$$\mathbf{s}_{preamble} = [s_0, s_1, \cdots, s_{N-1}] \quad (20)$$

where N is the number of subcarriers, K is the number of active subcarriers, $C_i, i = 1,2,\cdots,K$ is a real-valued PN sequence, the $n_f + n_h$ zeros are interposed as the guard band. Since the values of $C_i, i = 1,2,\cdots,K$ are real, $s_{N-n} = s_n^*$, $n = 1,\cdots,N/2-1$. The form of the designed training symbol in time domain is as follows:



where A represents samples of length N/2, and B is symmetric with the conjugate of A.

B. Time Synchronization

To make use of the property that B is symmetric with the conjugate of A, the timing metric is defined as follows:

$$M(m) = \frac{|p(m)|^2}{(R(m))^2} \tag{21}$$

where

$$p(m) = \sum_{k=1}^{N/2-1} r(m+k) \cdot r(m+N-k) \tag{22}$$

$$R(m) = \sum_{k=1}^{N/2-1} |r(m+k)|^2 \tag{23}$$

Thus the estimation of timing offset is

$$\hat{\varepsilon} = \arg\max_m (M(m)) \tag{24}$$

The $p(m)$ is designed such that the timing metric of the proposed method has its peak value at the correct symbol timing, while the values are almost zero at all other positions. Compared with Park's method, the proposed method eliminates the sidelobes.

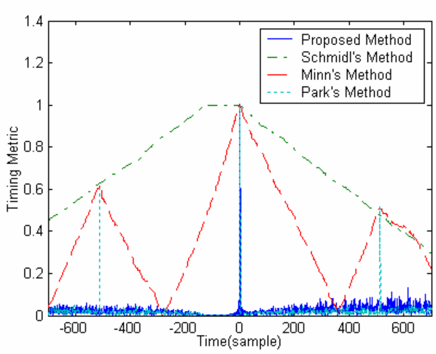


Fig. 1. Comparison of the timing metric of estimators.

Fig. 1 shows an example of the timing metric under no noise and no channel distortion with 2048 subcarriers and 128 cyclic prefix. The correct timing point is indexed 0 in the figure. The proposed timing metric is compared with those of Schmidl's, Minn's and Park's. As seen in the Fig. 1, Schmidl's method creates a plateau for the whole interval of cyclic prefix. The timing metric from Minn's method reduces the plateau, and yields a sharp timing metric. Park's method has an impulse-shaped timing metric, but large sidelobes occur at points indexed ± 512. As expected, the proposed method eliminates the sidelobes, allowing it to achieve a more accurate timing offset estimation.

C. Frequency Synchronization

In general, the carrier frequency offset f is divided into fractional frequency offset f_{fine} and integral frequency offset f_{int}. The fractional frequency offset can be estimated by ML estimator [3] using the redundant information contained within the cyclic prefix:

$$\hat{f}_{fine} = angle\left\{\sum_{k=1}^{N_g} r^*(\hat{\varepsilon}-k) \cdot r(\hat{\varepsilon}-k+N)\right\} \Big/ 2\pi \tag{25}$$

To improve the performance of estimation, averaging over many estimated values of the fractional frequency offset can be used.

The estimation of the integral frequency offset can be divided into two steps: the coarse estimate and the fine estimate.

1) Coarse estimate by minimum energy detection

Since the proposed training symbol in frequency domain contains null parts whose energy is very small in comparison with that of data, the end point of null part, N_e, can be obtained by

$$N_e = \arg\min_k \left\{\sum_{t=0}^{T-1} \left|Z_{(k-t)_N}\right|\right\} \tag{26}$$

where $Z_i, i = 0,1,2,\cdots,N-1$ is the received training symbol through FFT, $T = n_f + n_h$ is the size of window, k is amount of cyclic shift and $(\bullet)_N$ is the modulo-N operator. Due to channel environments such as noise and fading, the location of end point for null part may have error.

2) Fine estimate by partial correlation

To reduce the error of coarse estimate, correlation of several samples around null part is used. It is obtained by

$$N_c = \arg\max_{d \in D} \left\{\left|\sum_{w=1}^{W} (Z_{N_e+w+d} \bullet C_w)\right|\right\} \tag{27}$$

where W is the number of samples taking correlation.

Therefore the estimation of integral frequency offset f_{int} would be

$$\hat{f}_{\text{int}} = N_e + N_c - n_f \tag{28}$$

and the estimation of the total frequency offset f would be

$$\hat{f} = \hat{f}_{\text{int}} + \hat{f}_{fine} \tag{29}$$

Table 1 gives the comparison of the frequency offset estimation ranges of Schmidl's, Minn's, Park's and the proposed methods. It can be seen that the frequency offset estimation range of the proposed method is much larger than that of the others.

TABLE 1
COMPARISON OF THE FREQUENCY OFFSET ESTIMATION RANGE

Method	**Schmidl's Method**	**Minn's Method**	**Park's Method**	**Proposed Method**
frequency offset estimation range	$\lvert\hat{f}\rvert \le 1$	$\lvert\hat{f}\rvert \le 2$	$\lvert\hat{f}\rvert \le 1$	$\lvert\hat{f}\rvert \le N/2$

IV. SIMULATION RESULTS

In this section, the performance of the proposed time and frequency synchronization scheme is evaluated by mean-square error (MSE), and is compared with that of Minn's and Park's methods. The performance of Schmidl's method is not evaluated here because it is worse than that of Minn's and Park's methods [7].

The algorithm is investigated using the Monte-Carlo simulation method in a multipath fading channel whose channel impulse response is shown in Fig. 2. We assume the OFDM system has 2048 subcarriers among which there are 1536 active subcarriers with a 1 KHz inter-carriers pacing in total. The length of guard interval is 128 samples.

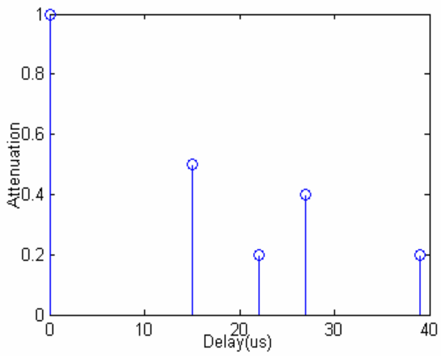
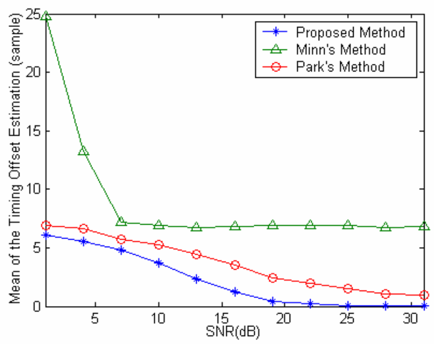
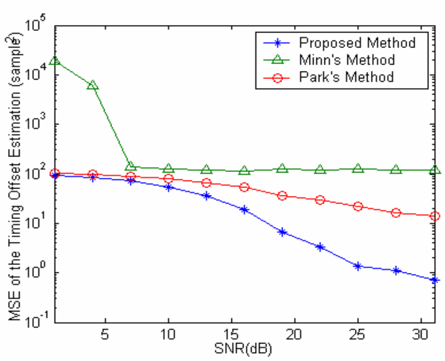


Fig. 2. Channel impulse response of the multipath fading channel



(a) Mean of the timing offset estimation



(b) MSE of the timing offset estimation

Fig. 3. Mean and MSE of the timing offset estimation in multipath fading channels

Fig. 3 shows the means and variances for the timing offset estimation in multipath fading channels. It can be seen that the proposed timing offset estimation method has much smaller mean timing offset estimation error and MSE than the others. This improvement can be inferred from the elimination of the large sidelobes and the impulse-like shape of the timing metric of the proposed estimator.

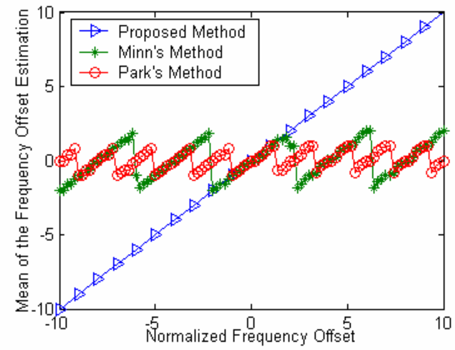
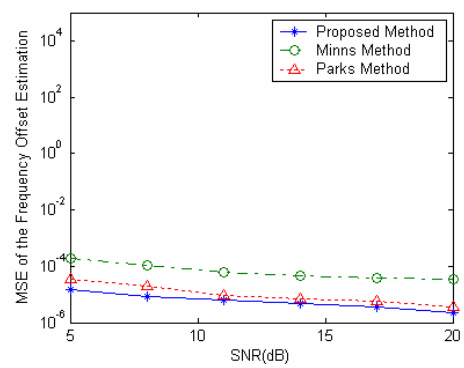
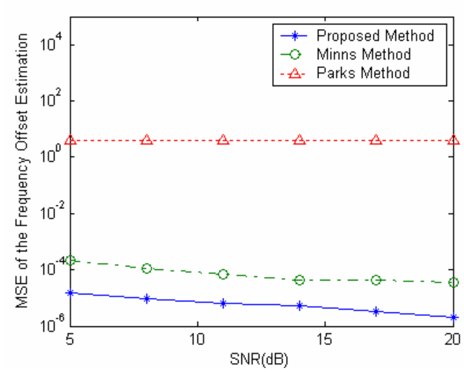


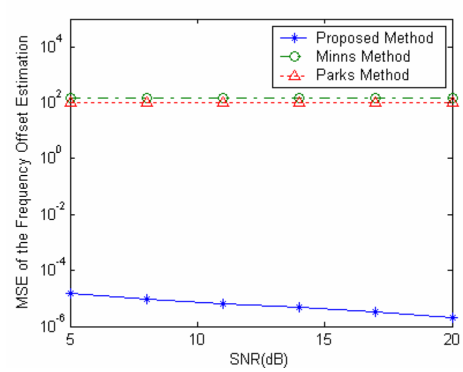
Fig. 4. Mean of the frequency offset estimation in multipath fading channels



(a) f=0.3 carrier spacing



(b) f=1.3 carrier spacing



(c) f=10.3 carrier spacing

Fig. 5. MSE of the frequency offset estimation in multipath fading channels

Fig. 4 shows the means of the frequency offset estimation using Minn's, Park's and the proposed methods (W=256) in multipath fading channels when SNR is 15dB. We observe that the frequency offset estimation ranges of Minn's and Park's methods are only $\left|\hat{f}\right| \leq 2$ and $\left|\hat{f}\right| \leq 1$ respectively, while the frequency offset estimation range of the proposed method is much larger than that of the others.

Fig. 5 shows the MSE of the frequency offset estimation in multipath fading channels where three frequency offsets are used. It is observed that three methods perform well when the frequency offset is 0.3 subcarrier spacing. With the increase of the frequency offset, the performance of Park's and Minn's degrade rapidly. When the frequency offset approaches 10.3 subcarrier spacing, only the proposed method has good performance. This is due to the frequency offset estimation range of the proposed method is much larger than that of the others.

The simulation results make it clear that the proposed timing and frequency offset estimation scheme performs much better than the others, therefore it is a more favorable option for the time and frequency synchronization of OFDM systems.

V. Conclusion

A new training symbol is designed and a simple time and frequency synchronization scheme is proposed in this paper. The scheme gives very accurate estimates of symbol timing and carrier frequency offset and provides a very wide acquisition range for the carrier frequency offset. Therefore, it is suitable for the time and frequency synchronization of OFDM systems.

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