Firm Heterogeneity in Production-Based Asset Pricing: The Role of Habit Sensitivity and Lumpy Investment

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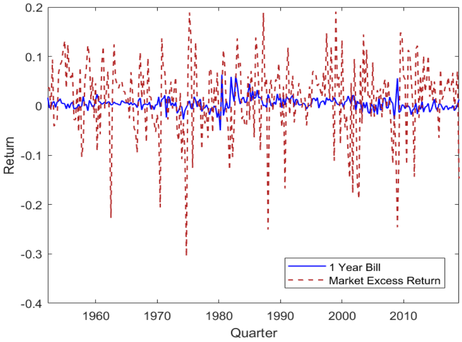
December 14th, 2020

Evidence 1: Risky and Risk-Free Assets

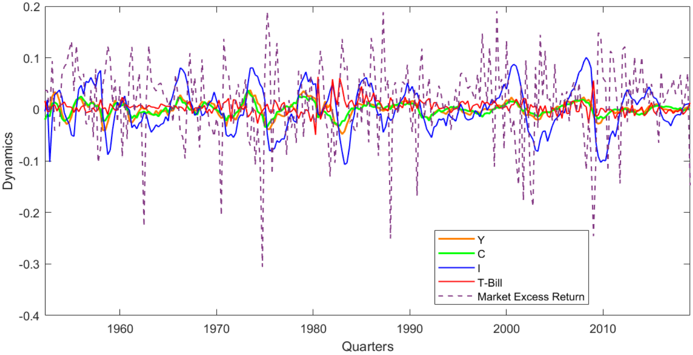
Figure 1: Risky and Risk-Free Asset



- $r^e - r^f$ is High and Volatile
- r^f is Low and Smooth
- 1st Challenge: Market Excess Return is High & Volatile but Risk-Free Rate is Low & Smooth

Evidence 2: Asset Prices & Real Economy

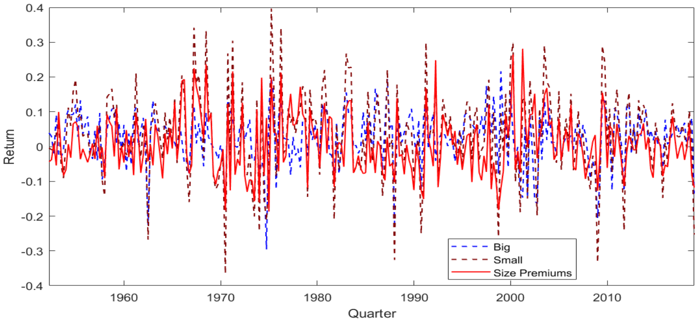
Figure 2: Macro-Asset Pricing Dynamics



- Market Excess Return is High but Consumption is Smooth ⇒ 2nd Challenge
- Market Excess Return is High and Investment Dynamics are Volatile ⇒ 3rd Challenge

Evidence 3: Size Premiums

Figure 3: Size Premiums



- Small Firms earn Higher Expected Return than Big Firms ⇒ Size Premiums
- Size Premiums are no Small (0.66% Quarterly or 2.5%-7% Annually) and Volatile (8.69% Quarterly or 17-20%Annually) ⇒ 4th Challenge

Motivation: Two Aspects of Asset Pricing Puzzles

Challenges: The G.E. Model Has Difficulty in......

- Matching Equity Risk Premiums and Risk-Free Rate jointly
  - Inter-temporal Effects on Risk-Free Rate Dominates...
- Matching Macro Quantities and Equity Risk Premiums jointly
  - Consumption Smoothing $\Rightarrow$ Stochastic Discount Factor Less Volatile $\Rightarrow$ Underestimate 1st/2nd-Order Moments of Risk Premiums
  - Adjustment Costs $\Rightarrow$ Risk Premiums $\uparrow$ and Investment Volatility $\downarrow$
- Replicating the Time-Series and Cross-Sectional Stock Return jointly
  - Lumpy Investment Model + Decreasing Return to Scale $\Rightarrow$ High Prod. Firms have High Market-to-Book Ratio but More Risky $\Rightarrow$ Value Premiums might be Negative (Favilukis & Lin 2010WP)

Inconsistency:

- Most Production-Based Models Rely on Convex Costs $\Rightarrow$ Empirically Rejected $\Rightarrow$ Micro-Level Investment is Lumpy
- Calibration with Unreasonable Parameters.

What I Did: Resolve Four Challenges in Both Aspects

Build a G.E. Heterogeneous Firm Macro Model with Following Ingredients:

- Campbell & Cochrane (1999JPE)'s Habit + GHH Structure
- Khan & Thomas (2008Ecta)'s Fixed Capital Adjustment Costs
- New Habit Sensitivity Function: $\lambda^S = \frac{1}{\bar{S}} - 1$ vs. $\lambda^S = \frac{1}{\log \bar{S}} - 1$

Results: The Model Replicates

Table 1: Compare to Previous Literature

Facts	Equity +Premium + Volatility	Business Cycle	Micro Lumpy Investment	Cross-Sectional Stock Return
Cooper et al. (2006JF)	√	×	√	√
Herskovic (2020WP)	√	×	×	√
Favilukis&Lin (2015RFS)	√	√	×	×
Chen (2017RFS,2018RAPS)	√	√	×	×
Favilukis&Lin (2013JME)	×	√	√	×
Winberry(2020AER)	×	√	√	×
This Paper	√	√	√	√

Micro Foundation:

- Fixed Capital Adjustment Costs + Reasonable Calibrated Values

Literature Review

- Asset Pricing with Habits:

  Boldrin et al. (1999AER), Jermann (1998JME); Campbell & Cochrane (1999JPE); Chen (2017RFS, RAPS), Luo (2019WP)

- Macro Implications of Non-Convex Costs:

  Khan & Thomas (2008Ecta), House (2014JME), Clementi et al. (2017WP), Mongey Williams (2017WP), Winberry (2020AER), Koby & Wolf (2020WP)

- Asset Pricing with Non-Convex Costs:

  Kogan (2004JFE), Carlson et al. (2004JF), Zhang (2005JF); Cooper(2006JF), Herskovic et al. (2020WP); Favilukis & Lin (2010WP)

- G.E. Model with Heterogeneous Firms:

  Luo (2019WP); Ai et al. (2013RFS), Favilukis & Lin (2010WP, 2015RFS), Chen (2018RAPS); Gomes et al. (2003JPE), Garleanu et al. (2012JF)

Outline

- Model Set Up
  - Physical Environment, Firm, and Households Problem
  - Equilibrium Conditions and Numerical Algorithm
- Solution to Challenges
- Benchmark Calibration: Target Moments
- Model Evaluation
  - Time-Series Implications
  - Cross-Sectional Implications
- Conclusion
- Appendix

Model Set Up

Environment: Discrete Time and Infinite Horizons

Household:

- Consumes Consumption Good C_t
- Supplies Labor N_t at Competitive Wage W_t
- Receives Aggregate Dividends Π_t from Firms

Firms: Continuum of Firms, Each with Idiosyncratic State $\mu_{i,t} = \mu(k_{i,t}, z_{i,t}, \xi_{i,t})$

- Hire Labour $n_{i,t}$ at Competitive Wage W_t,
- Produce Goods with Labour $n_{i,t}$ and Capital Stock $k_{i,t}$
  - Production Function with Decreasing Returns to Scale

    $y_{i,t} = e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu}$
- Choose whether to Adjust Capital or not
  - if Adjust $\Rightarrow$ Pay $\xi_{i,t}$ and Choose next Period's Capital Stock $k_{i,t+1}$
- Pay Out Firm-Level Dividends π_t

Shock:

- Aggregate Shock: $x_{t+1} = \log X_{t+1} = \rho^X \log X_t + \varepsilon^X_{i,t+1}, \varepsilon^X_{i,t+1} \backsim N(0, \sigma_x)$
- Firm-Level Shock: $z_{i,t+1} = \log Z_{i,t+1} = \rho^Z \log Z_{i,t} + \varepsilon^z_{i,t+1}, \varepsilon^z_{i,t+1} \backsim N(0, \sigma_z)$

Production Block: Khan & Thomas (2008Ecta) Economy

- Fixed Capital Adjustment Costs $\Rightarrow$ Capture Microeconomic Lumpy Investment Behaviour.
- The Firm's Aggregate State is $(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$
- Let $V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$ be the Discounted Present Value of the Firm.
- Each Firm Faces a Binary Choice: Invest or Not.
- Ivestment Project Brings Extra Value to Firm$\Rightarrow$ Firm Chooses to Invest.
- $V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t)\xi_{i,t} w(\Omega_t) > V^n(k_{i,t}, z_{i,t}; \Omega_t)$
- Inaction Firms $\Rightarrow$ Small Market Equities + Issue Dividends $\Rightarrow$ Cash Flow Risks $\uparrow$ $\Rightarrow$ Size Premiums $\uparrow$
- Adjustment Probability $\downarrow$ $\Rightarrow$ Expected Return $\uparrow$ $\Rightarrow$ Equity Premiums $\uparrow$

Household Block

- Habit Formation:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - H_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} \right\} \tag{1}$$

- Surplus Consumption Ratio: $S_t \equiv \frac{C_t - H_t}{C_t}$

$$s_{t+1} = \log S_{t+1} = \left(1 - \rho^S\right) \log \bar{S} + \rho^S \log S_t + \lambda^S \log(\frac{C_{t+1}}{C_t}) \tag{2}$$

- Habit Sensitivity Function: $\lambda^S = \frac{1}{\log \bar{S}} - 1$
- Household Problem

$$J(\Omega_t) = \max \left\{ \frac{\left(C(\Omega_t) - H(\Omega_t) - \psi \frac{N(\Omega_t)^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} + \beta E_t [J(\Omega_{t+1}) | \Omega_t] \right\} \tag{3}$$

st. $C(\Omega_t) + Q(\Omega_{t+1}) A(\Omega_{t+1}) + B(\Omega_{t+1}) =$
$w(\Omega_t) N(\Omega_t) + (Q(\Omega_t) + \Pi(\Omega_t)) A(\Omega_t) + (1 + r(\Omega_t)) \times B(\Omega_t)$

Equilibrium Conditions

An Equilibrium of This Model Consists of

- Firms' Value Function: $V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$
- Firms' Policy Functions: $k(k_{i,t}, z_{i,t}; \Omega_t)$, $n(k_{i,t}, z_{i,t}; \Omega_t)$, and $\hat{\xi}$
- Households' Policies $C(\Omega_t)$, $N(\Omega_t)$, and Stochastic Discount Factor
$$M(\Omega_{t+1}|\Omega_t) = \beta \left(\frac{C(\Omega_{t+1})S(\Omega_{t+1}) - \psi \frac{N(\Omega_{t+1})^{1+\alpha}}{1+\alpha}}{C(\Omega_t)S(\Omega_t) - \psi \frac{N(\Omega_t)^{1+\alpha}}{1+\alpha}} \right)^{-\sigma}$$
- a Wage $W(\Omega_t)$
- a Law of Motion for the Distribution of Firm $g(k_{i,t}, z_{i,t}; \Omega_t)$

such that

- the Firms' Policies are Optimal
- the Households' Policies are Optimal
- the Labour Market Clears

$$\left(\frac{w(\Omega_t)}{\chi} \right)^{\frac{1}{\alpha}} = \int \left(n(k_{i,t}, z_{i,t}; \Omega_t) + \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)^2}{2\bar{\xi}} \right) g(k, z) \mathrm{dzdk} \tag{4}$$

- the Law of Motion of the Distribution is Consistent with the Firm's Policies

Algorithm: Reiter (2009JEDC) & Winberry (2018QE)

Numerical Method: Projection+Perturbation Method

- A G.E. Production-Based Asset Pricing Model:
  - The Cross-Section Distribution of Firms is a State Variable
- Step One: Projection in Stationary Distribution
  - Approximate the Cross-Sectional Distribution with Finite Dimension Parametric Family $\Rightarrow$ Reduce Dimension
  - Capture the Law of Motion of Distribution
  - Compute the Stationary Equilibrium with Projection Method
- Step Two: Perturbation in Aggregate Dynamics—Second-Order Perturbation

Solution to Challenges

1. Equity Premium & Risk-Free Rate Puzzle

- Challenge: High & Volatile Equity Premium and Low & Smooth Risk-Free Rate
- Reason: the Intertemporal Substitution Effects on Risk-Free Rate
- Solution: Precautionary Saving Motivations Offset the Intertemporal Effects
- Intuition: Same Story in Chen (2017RFS, 2018RAPS)
  - SDF Decomposition $\frac{C_{t+1}S_{t+1}}{C_t S_t}$ Plays a Key Role
  - Take Logarithm and Extend the $\frac{C_{t+1}S_{t+1}}{C_t S_t}$

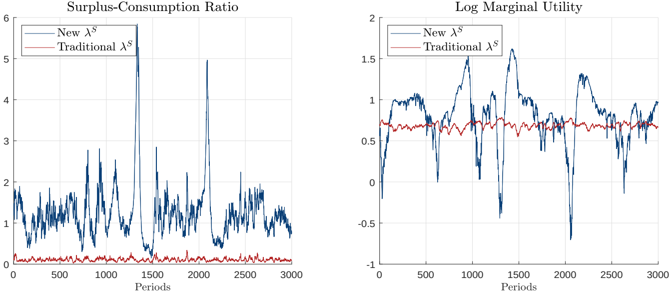
$$\left\{(-\sigma)\left[\left(\rho^S - 1\right)\left(\log \bar{S} - \log S_t\right) + \frac{1}{\log \bar{S}}\mu(g_c)\right] + \frac{(-\sigma)^2}{2(\log \bar{S})^2}\sigma(g_c)\right\}$$

  - The Sign of $-\frac{\sigma}{\log \bar{S}}$ and $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$ are Opposite $\Rightarrow$ Low & Smooth Risk-Free Rate

2. Equity Premium & Volatility Puzzle

- Challenge: High & Volatile Equity Premium under Consumption Smoothing
- Reason: Household Endogenously Smoothes Consumption in a G.E. Model
- Solution: Novel Habit Sensitivity Produces Volatile SDF
- Intuition: Habit Destruction and Negative Habit

Figure 4: λ^S and SDF: the Way to Replicate Equity Volatility



Mathematically

- $S_{t+1} = \bar{S}^{1-\rho_s} S_t^{\rho_s} \left(\frac{C_{t+1}}{C_t}\right)^{\left(\lambda^S\right)}$
- $\lambda^S = \frac{1}{\log \bar{S}} - 1$ vs $\lambda^S = \frac{1}{\bar{S}} - 1$.
- $\log \bar{S} < \bar{S}$

  $\Rightarrow \frac{1}{\log \bar{S}} - 1 > \frac{1}{\bar{S}} - 1$

  $$\Rightarrow Var\left\{\bar{S}^{1-\rho_s} S_t^{\rho_s} \left(\frac{C_{t+1}}{C_t}\right)^{\left(\frac{1}{log \bar{S}}-1\right)}\right\} > Var\left\{\bar{S}^{1-\rho_s} S_t^{\rho_s} \left(\frac{C_{t+1}}{C_t}\right)^{\left(\frac{1}{\bar{S}}-1\right)}\right\}$$

  given that $\frac{1}{log \bar{S}} - 1 > 0$ and $S_{t+1} > 1$.
- The Agent Destroys Her Habit Periodically (Dunlap 1928; Ljungqvist & Uhlig 2009, 2015)
- Consumption is Smooth but Marginal Utility is Volatile

  $\Rightarrow$ SDF is Volatile

  $\Rightarrow$ Risk Premiums are High: $E_t[R^e_{t+1} - R^f_{t+1}] = -Cov_t(R^e_{t+1} - R^f_{t+1}, \lambda_{t+1})$

  $\Rightarrow$ Risk Premiums are Volatile: $R^e_{t+1} = \frac{V_{t+1}+D_{t+1}}{V_t}$

  $\Rightarrow$ A Solution for the 2nd Challenge

3. High Risk Premiums with Volatile Aggregate Investment

Equity Premium Puzzle in the Production-Based Asset Pricing Literature

- Challenge: High Risk Premiums with Volatile Aggregate Investment
- Reason: Asset Prices rather than Investment Absorb the Productivity Shock
- Intuition: Idiosyncratic Risks Offset the Fixed Adjustment Costs
- Solution: A Story in Khan & Thomas (2008Ecta)

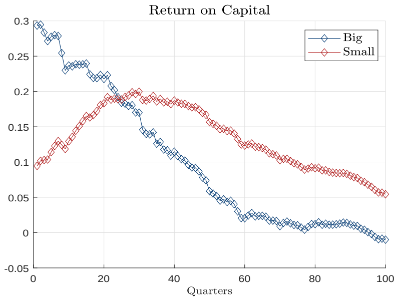
Micro Frictions: Adjustment Probabilities $\downarrow \Rightarrow$ Risk Premiums $\uparrow$

- Have Similar Implications on Risk Premiums
  - Upper Bound of Fixed Costs $\uparrow \Rightarrow$ Adjustment Probabilities $\downarrow$
  - Idiosyncratic Risks $\uparrow \Rightarrow$ Adjustment Probabilities $\downarrow$
- Have Opposite Implications on Aggregate Investment
  - Upper Bound of Fixed Costs $\uparrow \Rightarrow$ Investment Volatility $\downarrow$
  - Idiosyncratic Risks $\uparrow \Rightarrow$ Investment Volatility $\uparrow$

4. High Cross-Sectional Stock Return

- I Specify on the Lumpy Investment Model
- Challenge: Cross-Sectional Stock Return—Size and Value
- Reason: Operational Leverage $\Rightarrow$ Negative Value Premiums
- Solution: Change the Direction to Size Premium
- Intuition: Small Firms Absorb More Productivity Shock

Figure 5: The Impulse Response Function of TFP to Return on Big/Small Firm



Benchmark Calibration

Table 2: Benchmark Identification: Target Moments

Parameter	Value	Target Moments	Model	Data
ρ^X	0.95	$\sigma(y_t)$	1.56%	1.51%
σ^X	0.007	$\sigma(i_t)/\sigma(y_t)$	2.92	2.89
$\bar{\xi}$	0.25	$\rho(i_t, y_t)$	0.82	0.88
$\log \bar{S}$	0.1	$E(r^e - r^f)$	1.79%	1.54%
ρ^S	0.975	$\sigma(r^e - r^f)$	7.72%	7.74%
λ^S	$\frac{1}{\log \bar{S}} - 1$	$\frac{E(r^e - r^f)}{\sigma(r^e - r^f)}$	0.23	0.20

- Strategy & Targets
  - Asset Pricing: Moments of Risky Assets
  - Macroeconomics: Moments of Investment Cyclicity and Volatility
- Just-Identified: 6 Parameters vs. 6 Moments

Model Evaluation

Using Non-target Moments to Evaluate Performances

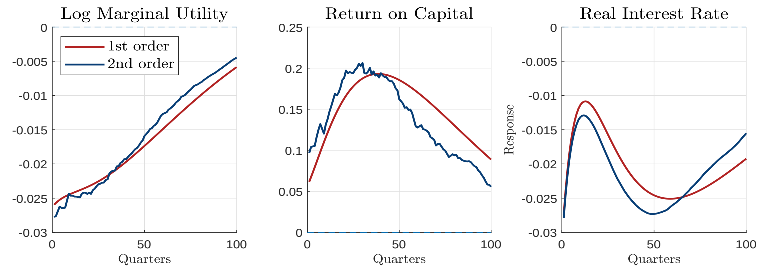
- Time-Series Implications
  - Asset Prices Dynamics
    - Asset Prices Moments
    - IRF of TFP to Asset Prices
    - The Predictability of Market Excess Return
  - Macroeconomic Implications
    - Business Cycle Moments
    - IRF of TFP to Macro Variables
- Cross-Sectional Implications
  - The Cross-Sectional Distribution of Investment Rate
  - Size Premiums

Table 3: Benchmark Model: Time-Series Asset Pricing Moments

Equity	Benchmark	Data
$E(r^e - r^f)$	1.79%	1.54%
$\sigma(r^e - r^f)$	7.72%	7.74%
$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	0.20
T-Bills		
$E(r^f)$	0.88%	0.40%
$\sigma(r^f)$	0.57%	0.99%

What to Take Home: Successfully Avoids Risk-Free Rate Puzzle

Figure 6: Impulse Response Function of TFP Shock to Asset Return



- Marginal Utility is Counter-Cyclical
- Ex-Post Return is Pro-Cyclical⇒Expected Return is Positive
  - $E_t[R^e_{t+1} - R^f_{t+1}] = -Cov_t(R^e_{t+1} - R^f_{t+1}, \lambda_{t+1})$
- First-Order and Second-Order Perturbation
  - Second-Order Perturbation is Non-Smooth
    ⇔ Captures Infra-Marginal Properties of Value Function & Investment

Table 4: Benchmark Model: Return Prediction of Dividend-Price Ratio

	Benchmark		Full Sample 1952-2018	
Horizons	Coefficients	R^2	Coefficients	R^2
1 Quarter	0.0462	0.0034	0.0178	0.0087
1 Year	0.2295	0.0255	0.0811	0.0377
2 Year	0.5739	0.0405	0.1401	0.0577
3 Year	1.0473	0.0610	0.1652	0.0640
4 Year	1.5592	0.0777	0.1745	0.0582
5 Year	1.9942	0.0832	0.2142	0.0683

$$E_t\left[r_{t+k}\right] - r^f_{t+k} = a + \beta_{dp} \times \frac{d_t}{p_t} + \epsilon_{t+k} \tag{5}$$

What to Take Home: Predicts the Correct Sign and Trend

- Regressive Fitness Rises with the Increases in Investment Horizons
- Dividend Yields Positively Associate with Risk Premiums
- Magnitudes of Simulated Results are Comparable to Empirical Evidence

Table 5: Benchmark Model: Macroeconomic Implications

Variables	$\sigma(x_t)$	$\sigma(x_t)/\sigma(y_t)$	$\rho(x_t, y_t)$	$\rho(x_t, x_{t-1})$
		Benchmark		
y_t	1.56%	1	1	0.71
c_t	1.28%	0.82	0.93	0.70
i_t	4.54%	2.92	0.82	0.76
n_t	1.04%	0.67	1.00	0.71
		Data		
y_t	1.51%	1	1	0.85
c_t	1.20%	0.80	0.85	0.81
i_t	4.35%	2.89	0.88	0.80
n_t	1.84%	1.21	0.87	0.90

What to Take Home: Similar to a Single Shock RBC Model

- Matches Volatilities of Consumption, Investment, and Output
- Underestimates Co-movements and Overestimates Persistence

Table 6: Benchmark Model: Microeconomic Moments

Distribution	Benchmark	Data
Mean	9.93%	10.4%
Standard Deviation	12.03%	16.0%
Skewness	3.08	3.60
Excess Kurtosis	12.36	17.6
Investment Patterns		
Spike ($> 20\%$)	15%	14.4%
Positive($0\% - 20\%$)	85%	85.6%

What to Take Home: Assumes Non-negative Investment Rate $\Rightarrow$Spike

- Uses Depreciation Rate to Match Average Investment Rate
- Replicates Key Moments of Distribution and Patterns of Micro Investment

Table 7: Benchmark Model: Cross-Sectional Asset Pricing Moments

	Model	Data			
Moments	Half-Sort	Half-Sort	3 Portfolios	5 Portfolios	10 Portfolios
$E(r^s)$	3.02%	3.47%	3.39%	3.34%	3.33%
$E(r^b)$	1.91%	3.01%	2.76%	2.73%	2.68%
$E(r^s - r^b)$	1.11%	0.46%	0.63%	0.61%	0.66%
$std(r^s)$	6.74%	10.78%	11.51%	11.88%	12.27%
$std(r^b)$	7.13%	7.85%	7.76%	7.67%	7.61%
$std(r^s - r^b)$	4.48%	5.09%	6.76%	7.66%	8.69%
$\rho(r^s, r^b)$	0.79	0.89	0.82	0.78	0.71

What to Take Home:

- Matches Risk Premiums of Small Firms and Most Second-Order Moments
- Underestimates Risk Premiums of Large Firms$\Rightarrow$Overestimates Size Premiums

What to Take Home

Challenges: Lumpy Investment in G.E. Resolves Challenges Below

Table 8: Compare to Previous Literature

Facts	Equity +Premium + Volatility	Business Cycle	Micro Lumpy Investment	Cross-Sectional Stock Return
Cooper et al. (2006JF)	√	×	√	√
Herskovic (2020WP)	√	×	×	√
Favilukis&Lin (2015RFS)	√	√	×	×
Chen (2017RFS,2018RAPS)	√	√	×	×
Favilukis&Lin (2013JME)	×	√	√	×
Winberry(2020AER)	×	√	√	×
This Paper	√	√	√	√

- Novel Habit Sensitivity
  - $-\frac{\sigma}{\log \bar{S}}$ and $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$ are Opposite
  - Habit Destruction $\Rightarrow$ Unconditional Vol. of the SDF $\uparrow$ and Robust IRF
- Micro Frictions: Adjustment Probabilities $\downarrow$ $\Rightarrow$ Risk Premiums $\uparrow$
  - Have Similar (Opposite) Implications on Risk Premiums (Aggregate Investment)
  - Small Firms $\Rightarrow$ Un-adjust $\Rightarrow$ Higher Cash Flow Risks $\Rightarrow$ Size Premiums

Consistency: The Assumption of Fixed Costs is Consistent with Micro Evidence

Appendix

Firm Problem

Firm Problem: Khan & Thomas (2008Ecta) Economy

- The Firm's Aggregate State is $(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$
- Let $V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$ be the Discounted Present Value of the Firm.
- Firm Chooses to Adjust or Not

$$V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t) = \lambda^c(\Omega_t) \max_n \left\{ e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu} - w(\Omega_t) n_t \right\} \\ + \max_i \{ V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t) \xi_{i,t} w(\Omega_t), V^n(k_{i,t}, z_{i,t}; \Omega_t) \} \tag{6}$$

- Decision Threshold: $\tilde{\xi} = \frac{V^a(k_{i,t}, z_{i,t}; \Omega_t) - V^n(k_{i,t}, z_{i,t}; \Omega_t)}{\lambda^c w(\Omega_t)}$

Firm Problem: Khan & Thomas (2008Ecta) Economy

- Range of The Threshold:
  $\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t) = \arg\min\left\{\max\left\{0, \tilde{\xi}(k_{i,t}, z_{i,t};\Omega_t)\right\}, \bar{\xi}\right\}$
- Adjustment Probability: $\frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}} \in [0,1]$
- Firm Value

$$
\begin{aligned}
\hat{V}(k_{i,t}, z_{i,t};\Omega_t) = {} & \lambda^c(\Omega_t)\max_n\left\{e^{z_{i,t}}e^{x_t}k_{i,t}^{\theta}n_{i,t}^{\nu} - w(\Omega_t)n_t\right\} \\
& + \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\left\{V^a(k_{i,t}, z_{i,t};\Omega_t) - \lambda^c(\Omega_t)w(\Omega_t)\frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{2}\right\} \\
& + \left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\right)V^n(k_{i,t}, z_{i,t};\Omega_t)
\end{aligned}
\tag{7}
$$

Benchmark Calibration

Table 9: Parameters Selection: Microeconomic Structure

Parameter	Description	Value
λ^e	Debt-to-Equity Ratio	0.54
β	Discount Factor	0.99
σ	Curvature	1
α	Inverse Frisch Elasticity	1/2
N	Stationary Work Hour	1/3
ψ	Labour Disutility	1
θ	Capital Share	0.21
ν	Labour Share	0.64
δ	Depreciation	0.025
$[-b, b]$	Bound of Region	0.011
ρ^Z	Persistence of Idio. Shock	0.859
σ^Z	s.d. of Idio. Shock	0.015

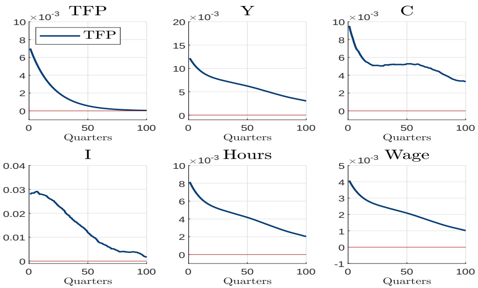
Table 10: Investment Dynamics and Risk Premiums: Different Sensitivity Functions

Parameters	Moments	Benchmark	$\log \bar{S} = 0.65$ $\rho^S = 0.95$	$\bar{\xi} = 0.44$ $\rho^z = 0.90$	Data
$\frac{1}{S} - 1$	$\sigma(y_t)$	1.61%	1.61%	1.61%	1.51%
	$\rho(i_t, y_t)$	-0.65	-0.40	0.09	0.88
	$\sigma(i_t)/\sigma(y_t)$	0.51	0.37	0.35	2.89
	$E(r^e - r^f)$	1.56%	1.59%	1.73%	1.54%
	$std(r^e - r^f)$	1.43%	1.72%	1.43%	7.74%
	$E(r^e - r^f)/\sigma(r^e - r^f)$	1.09	0.92	1.21	0.20
$\frac{1}{\log \bar{S}} - 1$	$\sigma(y_t)$	1.56%	1.59%	1.56%	1.51%
	$\rho(i_t, y_t)$	0.82	0.98	0.89	0.88
	$\sigma(i_t)/\sigma(y_t)$	2.92	0.52	3.09	2.89
	$E(r^e - r^f)$	1.79%	1.93%	173.07%	1.54%
	$std(r^e - r^f)$	7.73%	1.30%	245.33%	7.74%
	$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	1.49	0.71	0.20

Traditional Habit Sensitivity

- Has Potential to Generate (Smooth) and Counter-Cyclical Investment Dynamics
- Fails to Generate Volatile Risk Premiums

Figure 7: Impulse Response Function of TFP Shock to Business Cycle



GHH Utility

- Amplifies Aggregate Consumption, Investment, and Output
- Has Marginal Effects on Wages and Hours Worked

Inspecting the Mechanism

Challenge 1: Mathematical Proof

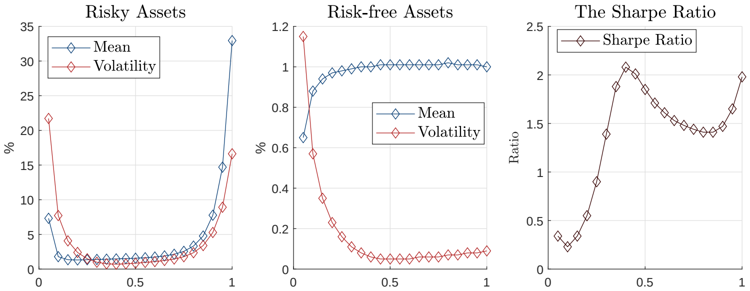
- SDF Decomposition

$$
\begin{aligned}
&\log M_{t+1} \\
&= \log\beta + (-\sigma)\left\{\log\left(\frac{C_{t+1}S_{t+1}}{C_tS_t}\right) + \log\left(\frac{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times N_{t+1}^{1+\alpha}}{C_tS_t - \frac{\psi}{1+\alpha}\times\frac{C_{t+1}S_{t+1}}{C_tS_t}\times N_t^{1+\alpha}}\right)\right\} \\
&= \log\beta + \left\{(-\sigma)\left[\left(\rho^S - 1\right)\left(\log\bar{S} - \log S_t\right) + \frac{1}{\log\bar{S}}\mu(g_c)\right] + \frac{(-\sigma)^2}{2(\log\bar{S})^2}\sigma(g_c)\right\} \\
&\qquad + (-\sigma)\log\left(\frac{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times\frac{C_{t+1}S_{t+1}}{C_tS_t}\times N_t^{1+\alpha}}\right)
\end{aligned} \tag{8}
$$

- $\frac{C_{t+1}S_{t+1}}{C_tS_t}$ shares the same sign of *log* $\left(\frac{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times\frac{C_{t+1}S_{t+1}}{C_tS_t}\times N_t^{1+\alpha}}\right)$
- $-\frac{\sigma}{\log\bar{S}}$ and $\frac{(-\sigma)^2}{2(\log\bar{S})^2}$ are Opposite

Challenge 1: Quantitative Implications

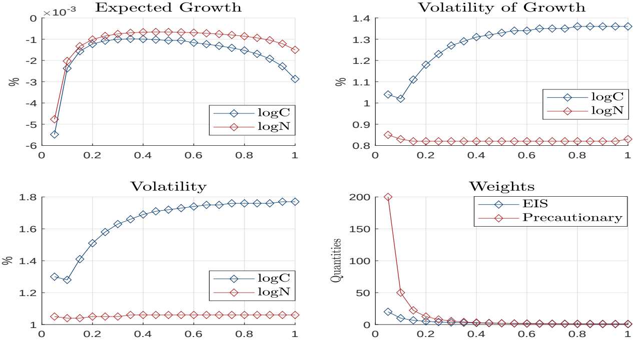
Figure 8: Asset Pricing Moments: Conditional on $log\bar{S}$



- $\log \bar{S} \uparrow \Rightarrow$ U-Shaped Risk Premium and Volatility $\Leftrightarrow$ When $\log \bar{S} > 0.25$, $E(r^e - r^f) > std(r^e - r^f)$
- $\log \bar{S} \uparrow \Rightarrow E(r^f) \uparrow$ and $std(r^f) \downarrow$

Challenge 1: Quantitative Implications

Figure 9: The Mechanism of $\log \bar{S}$



- The Weight between Precautionary Saving Motivations and EIS Plays a Key Role
- Consumption Volatility Plays another Key Role

Challenge 2: Compare to Chen (2017RFS)'s Work

- Traditional Habit Sensitivity $\lambda^S = \frac{1}{\bar{S}} - 1$
  - Large $\bar{S}$ to Make the Agent Destroys Her Habit Periodically
    $\Rightarrow \lambda^S \downarrow$ Assign a Small Weight to $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$
    $\Rightarrow$ Equity Volatility $\downarrow$
  - A Small $\bar{S}$ Cannot Produce a Volatile SDF under Lumpy Economics
    $\Rightarrow$ Equity Volatility $\downarrow$

  This Paper: Take the Advantages of Both Mechanisms with $\lambda^S = \frac{1}{\log \bar{S}} - 1$
  - Picking Small $\log \bar{S} \Longleftrightarrow$ Picking Large $\bar{S}$ to Make the Habit Destruction
    $\Rightarrow$ but λ^S still $\uparrow$
- Why Traditional Model Fails to....?
  - Numerical Method: Reiter (2009JEDC) and Winberry (2018QE)
  - Misspecified: Oh (2011WP) with Krusell & Smith (1998JPE)

Challenge 3: Mathematical Proof

Value Function

$$\hat{V} = \lambda^c \max_n \left\{ e^z e^x k^\theta n^\nu - wn \right\} + \frac{\hat{\xi}}{\bar{\xi}} \left\{ V^a - \lambda^c w \frac{\hat{\xi}}{2} \right\} + \left(1 - \frac{\hat{\xi}}{\bar{\xi}} \right) V^n$$

where

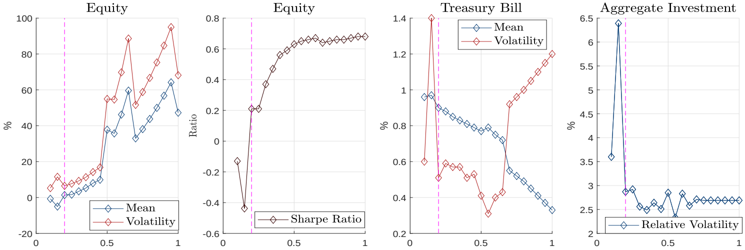
$$\tilde{\xi} = \frac{V^a - V^n}{\lambda^c w}$$

$$\hat{\xi} = \arg\min \left\{ \max \left\{ 0, \tilde{\xi} \right\}, \bar{\xi} \right\} = \arg\min \left\{ \max \left\{ 0, \frac{V^a - V^n}{\lambda^c w} \right\}, \bar{\xi} \right\}$$

- If $\bar{\xi} < \tilde{\xi}$ then $\hat{\xi} = \bar{\xi}$, only Adjusted Firms are Considered and the Adjustment Probability is One
- If $\hat{\xi} = \tilde{\xi} = \frac{v^a - v^n}{\lambda^c w} < \bar{\xi} \Rightarrow$ the Relative Weights Assigned to Adjusted and Constraint Firms are $\frac{\hat{\xi}}{\bar{\xi}}$ and $1 - \frac{\hat{\xi}}{\bar{\xi}} \Rightarrow \frac{\hat{\xi}}{\bar{\xi}}$ is Adjustment Probability
- $\bar{\xi} \uparrow \Rightarrow$ Adjustment Probability $\frac{\hat{\xi}}{\bar{\xi}} \uparrow \Rightarrow$ Assign Higher Weights on Constraint Firms $\Rightarrow$ Future Dividends $\uparrow$ Risk Premiums$\uparrow$

Challenge 3: Quantitative Implications

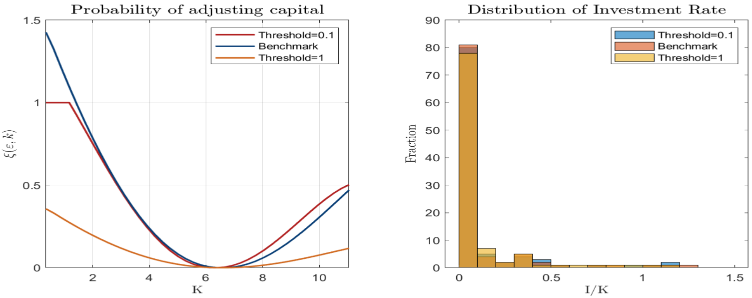
Figure 10: Asset Pricing Moments: Conditional on $\bar{\xi}$



- $\bar{\xi} \uparrow \Rightarrow E(r^e - r^f)$, $std(r^e - r^f)$, and Sharp Ratio $\uparrow$
- $\bar{\xi} \uparrow \Rightarrow E(r^f) \downarrow$ and Investment Smooth
- Small $\bar{\xi} \Rightarrow$ All Firms Adjust $\Rightarrow$ Equity is Less Risky than T-Bill $\Rightarrow$ Negative Premiums

Challenge 3: Quantitative Implications

Figure 11: The Mechanism of $\bar{\xi}$

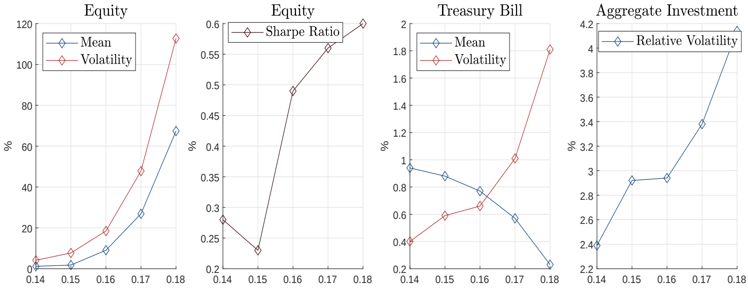


$\bar{\xi} \uparrow \Rightarrow$ Adjustment Probability$\xi(\epsilon, k) = \frac{\hat{\xi}}{\bar{\xi}} \downarrow$

$\Rightarrow$ Firms Choose Inaction $\Rightarrow$ Future Dividend $\uparrow \Rightarrow$ Risk Premiums $\uparrow$

Challenge 3: Quantitative Implications

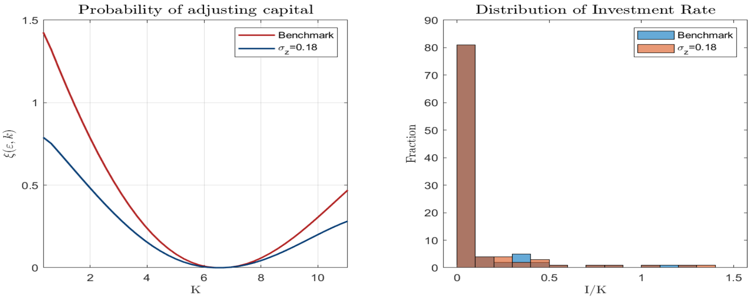
Figure 12: Macro-Asset Pricing Moments: Conditional on σ_z



- $\sigma_z \uparrow \Rightarrow E(r^e - r^f)$, $std(r^e - r^f)$, Sharp Ratio, and $\sigma(i_t) \uparrow$
- $\bar{\xi} \uparrow \Rightarrow E(r^f) \downarrow$

Challenge 3: Quantitative Implications

Figure 13: The Mechanism of σ_z



Idiosyncratic Risks $\sigma_z \uparrow \Rightarrow$ Adjustment Probability$\xi(\epsilon, k) = \frac{\hat{\xi}}{\bar{\xi}} \downarrow$

$\Rightarrow$ Cash Flow Risks $\Rightarrow$ Risk Premiums $\uparrow$

Challenge 4: Mathematical Proof

Recall the value function:

$$\hat{V}(k_{i,t}, z_{i,t};\Omega_t) = \lambda^c(\Omega_t)\max_n \left\{ e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu} - w(\Omega_t) n_t \right\}$$
$$+ \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}} \left\{ V^a(k_{i,t}, z_{i,t};\Omega_t) - \lambda^c(\Omega_t) w(\Omega_t) \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{2} \right\}$$
$$+ \left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\right) V^n(k_{i,t}, z_{i,t};\Omega_t)$$

The return on unconstrained (big) firm and constrained (small) firm is given by:

$$\begin{aligned}
R^a(\Omega_{t+1}|\Omega_t) &= \frac{\frac{\hat{\xi}}{\bar{\xi}} \int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + \frac{\hat{\xi}}{\bar{\xi}} D(\Omega_{t+1})}{\frac{\hat{\xi}}{\bar{\xi}} \int V^a(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
&= \frac{\int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}} \\
R^n(\Omega_{t+1}|\Omega_t) &= \frac{\left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) \int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + \left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) D(\Omega_{t+1})}{\left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
&= \frac{\int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}}
\end{aligned} \tag{9}$$

Challenge 4: Mathematical Proof

The return spread between small and big firm is:

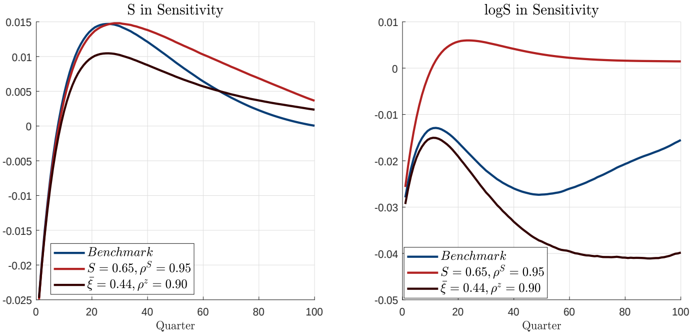
$$
\begin{aligned}
& R^n(\Omega_{t+1}|\Omega_t) - R^a(\Omega_{t+1}|\Omega_t) \\
= & \frac{\int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} - \frac{\int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}} \\
= & \frac{\left[\int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})\right] \int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di} \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
& - \frac{\left[\int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})\right] \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di} \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
= & \frac{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}D(\Omega_{t+1}) - \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di} \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
= & \frac{\int \left[V^a(k_{i,t}, z_{i,t};\Omega_{t+1}) - V^n(k_{i,t}, z_{i,t};\Omega_t)\right] \mathrm{di}D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di} \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} > 0
\end{aligned}
\tag{10}
$$

Challenge 5: Debate on Irrelevance

- Debate: Micro Frictions Has (Marginal) Effects on Aggregate Investment
- Reason: Procyclical Risk-Free Rate Offset the Aggregate Investment Demand $\Rightarrow$ Irrelevance
- Solution: Novel Habit Sensitivity Produces Robust IRF of TFP
- Intuition: The Similar Story in Winberry (2020AER)
  - IRF of TFP to Risk-Free Rate is Negative
    $\Rightarrow$ Counter-Cyclical Risk-Free Rate
    $\Rightarrow$ Investment Demand $\uparrow$
  - IRF of TFP to Aggregate Investment is Robustly Positive
    $\Rightarrow$ Procyclical Investment
    $\Rightarrow$ Investment Volatility $\uparrow$

Challenge 5: Quantitative Implications

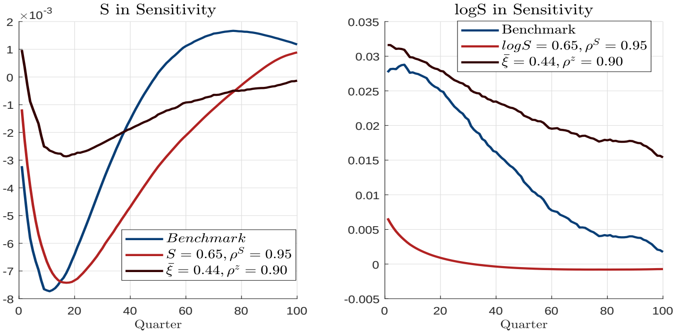
Figure 14: IRF of TFP to Risk-Free Rate



- Counter-Cyclical Risk-Free Rate $\Rightarrow$ Investment Demand $\uparrow$

Challenge 5: Quantitative Implications

Figure 15: Investment IRFs: (log) Steady-State Surplus-Consumption Ratio



- U-Shaped IRF ⇒ Counter-Cyclical & Smooth Investment Dynamics

Comparative Statistics

Alternative Preferences: Keeping Up with the Joneses

Recall the Utility Function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - H_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} \right\}$$

- Habit with Actual Consumption

$$H_t = \tau C_{t-1}$$

$$\lambda_t^c = \left(C_t - \tau C_{t-1} - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{-\sigma}$$

- Habit with Net Consumption

$$H_t = \tau \left(C_{t-1} - \psi \frac{N_{t-1}^{1+\alpha}}{1+\alpha} \right)$$

$$\lambda_t^c = \left(C_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} - \tau \left(C_{t-1} - \psi \frac{N_{t-1}^{1+\alpha}}{1+\alpha} \right) \right)^{-\sigma}$$

Table 11: Habit Formation: Moment Comparisons

Models	CC	KJ			No Habits	Data
	$\frac{1}{\log \bar{S}} - 1$	$\frac{1}{\bar{S}} - 1$	C_t	$C_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha}$		
$\sigma(y_t)$	1.56%	1.61%	1.78%	1.59%	1.58%	1.51 %
$\sigma(c_t)/\sigma(y_t)$	0.82	1.23	1.16	0.60	0.77	0.80
$\sigma(i_t)/\sigma(y_t)$	2.92	0.73	4.40	3.39	2.38	2.88
$\sigma(n_t)/\sigma(y_t)$	0.67	0.67	0.67	0.67	0.67	1.21
$\rho(c_t, y_t)$	0.93	0.99	0.80	0.99	0.99	0.85
$\rho(i_t, y_t)$	0.82	-0.47	0.33	0.99	0.99	0.88
$\rho(n_t, y_t)$	1.00	1.00	1.00	1.00	1.00	0.87
$\rho(c_{t-1}, c_t)$	0.70	0.73	0.94	0.77	0.72	0.81
$\rho(i_{t-1}, i_t)$	0.76	0.84	0.76	0.68	0.71	0.80
$\rho(n_{t-1}, n_t)$	0.71	0.72	0.67	0.72	0.72	0.90
$\rho(y_{t-1}, y_t)$	0.71	0.72	0.76	0.72	0.72	0.85
$E(r^e - r^f)$	1.79%	1.59%	1.78%	6.05%	1.77%	1.54%
$std(r^e - r^f)$	7.72%	1.43%	1.58%	11.31%	1.18%	7.74%
$E(r^f)$	0.88%	1.01%	1.02%	0.90%	0.98%	0.40%
$std(r^f)$	0.57%	0.09%	0.15%	0.26%	0.12%	0.99%
$\frac{E(r^e - r^f)}{\sigma(r^e - r^f)}$	0.23	1.12	1.13	0.53	1.51	0.20

Additional Micro Frictions

- Investment-Specific Shock

$$\begin{aligned} k_{i,t+1} &= (1-\delta)k_{i,t} + e^{q_t} i_{i,t} \\ q_{t+1} &= \rho^q q_t + \sigma^q \varepsilon^q_{t,t+1} + \sigma^{qx} \varepsilon^{qz}_{t,t+1} \end{aligned} \tag{11}$$

- Uncertainty Shock

$$\begin{aligned} z_{i,t+1} &= \rho^Z z_i + \varepsilon^z_{i,t+1}, \varepsilon^z_{i,t+1} \backsim N(0, X^U_t \bar{\sigma}) \\ X^U_{t+1} &= \rho^U \log X^U_t + \varepsilon^U_{t+1}, \varepsilon^U_{t+1} \backsim N(0, \sigma^U) \end{aligned} \tag{12}$$

- Convex Costs

$$\Phi(I_{i,t}/k_{i,t}) = \frac{\lambda_x}{2}\left(\frac{I_{i,t}}{k_{i,t}}\right)^2 k_{i,t} \tag{13}$$

and its Value Function

$$\begin{aligned} \hat{V} =& \lambda^c \max_n \left\{ e^z e^x k^\theta n^\nu - wn \right\} + \frac{\hat{\xi}}{\bar{\xi}} \left\{ V^a - \lambda^c w \frac{\hat{\xi}}{2} \right\} + \left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) V^n \\ & - \lambda^c \frac{\hat{\xi}}{\bar{\xi}} \times \frac{\lambda_x}{2} \left(\frac{k^a}{k} - (1-\delta)\right)^2 - \lambda^c \left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) \times \frac{\lambda_x}{2} \left(\frac{k^n}{k} - (1-\delta)\right)^2 \end{aligned} \tag{14}$$

Table 12: The Microeconomic Frictions: Comparisons

Models	Benchmark	IST	Uncertainty Shock	Convex Costs	Data
$\sigma(y_t)$	1.56%	1.58%	1.58 %	1.58 %	1.51 %
$\sigma(c_t)/\sigma(y_t)$	0.82	0.80	0.80	0.73	0.80
$\sigma(i_t)/\sigma(y_t)$	2.92	2.79	2.67	2.97	2.88
$\sigma(n_t)/\sigma(y_t)$	0.67	0.67	0.67	0.67	1.21
$\rho(c_t, y_t)$	0.93	0.95	0.96	0.94	0.85
$\rho(i_t, y_t)$	0.82	0.88	0.88	0.85	0.88
$\rho(n_t, y_t)$	1.00	1.00	1.00	1.00	0.87
$\rho(c_{t-1}, c_t)$	0.70	0.70	0.70	0.71	0.81
$\rho(i_{t-1}, i_t)$	0.76	0.76	0.76	0.74	0.80
$\rho(n_{t-1}, n_t)$	0.71	0.71	0.71	0.71	0.90
$\rho(y_{t-1}, y_t)$	0.71	0.71	0.71	0.71	0.85
$E(r^e - r^f)$	1.79%	1.57%	1.39%	2.66%	1.54%
$std(r^e - r^f)$	7.72%	5.70%	4.95%	7.76%	7.74%
$E(r^f)$	0.88%	0.96%	0.97%	0.88%	0.40%
$std(r^f)$	0.59%	0.48%	0.45%	0.49%	0.99%
$\frac{E(r^e - r^f)}{\sigma(r^e - r^f)}$	0.23	0.27	0.28	0.34	0.20

Additional Micro Frictions have merely Marginal Contribution