Firm Heterogeneity in Production-Based Asset Pricing: the Role of Habit Sensitivity and Lumpy Investment

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Abstract

I study the interaction between lumpy investment and asset prices in both time-series and cross-section. To this end, I build a model with lumpy investment a variant of habit sensitivity function of Campbell & Cochrane (1999) habit preference. The model produces 100% equity volatility by generating volatile marginal utility and robustly matches investment dynamics under non-convex costs. Second, my model reproduces almost 100% equity premiums because the benchmark model assigns additional weights on precautionary savings and constrained firms, respectively. My model also generates considerable size premiums since small firms absorb more productivity risks. Finally, my model reasonably matches crucial moments of macro-dynamics and the cross-sectional investment rate.

Key Words: Habit Formation, Lumpy Investment, Firm Heterogeneity, Production-Based Asset Pricing, Size Premium

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1 Introduction

Equity risk premiums are high and volatile in contrast to smooth macroeconomic quantities such as consumption and output. Therefore, a crucial task for macro general equilibrium models is how to generate sizeable risk premiums with the low business cycle fluctuations in line with micro evidence, which refer to the equity premium puzzle (Mehra & Prescott 1985), equity volatility puzzle (Shiller 1981), and risk-free rate puzzle (Weil 1989). The main challenges come from two aspects. The first aspect is, asset pricing models may fail to match salient facts. A typical case is an asset pricing model has difficulty to reproduce the risk-free rate and equity premiums together, as the intertemporal substitution effects encourage the agent to borrow in the recession that rises the risk-free rate. Besides, replicating asset returns in general equilibrium models is challenging, as the household will smooth consumption that makes the stochastic discount factor and risk premiums less volatile. Matching high equity premiums also requires substantial adjustment costs, which may excessively smooth investment. Finally, recent literature rises the difficulties of macro theories to reproduce empirical evidence, as it asks the reproduction of the firm-level stock return.

The second aspect is, an asset pricing model is inconsistent with micro evidence even it can match empirical facts above. For example, many studies in production-based asset pricing assume convex capital adjustment costs (see Jermann 1998, Kaltenbrunner & Lochstoer 2010, Croce 2014, among others). However, the lumpiness nature of micro-level investment data (Doms & Dunne 1998, Cooper & Haltiwanger 2006) supports the assumption of fixed capital adjustment costs rather than convex capital adjustment costs, which has not been well understood in previous literature.

To this end, I provide a quantitative evaluation of fixed capital adjustment costs to asset prices and business cycle through the lens of a calibrated heterogeneous agent DSGE model in this paper. The model consists of Campbell & Cochrane (1999)'s habit preference and Khan & Thomas (2008)'s lumpy investment environment with one additional ingredient: a variant of the habit sensitivity function that captures the relationship between surplus-consumption ratio and consumption growth. With ingredients above, the model replicates critical moments of time-series asset pricing facts from Chen (2017), business cycle fluctuations from Boldrin et al. (1999), the cross-sectional distribution of investment rate from Winberry (2020), and size premiums from Fama & French (1993).

The quantitative success of my model hinges on two crucial assumptions: a variant of the sensitivity function of Campbell & Cochrane (1999) habit formation and micro frictions from Khan & Thomas (2008). To make the argument, I quantitatively show two failures of traditional sensitivity function of habit preference: (i) it has the potential to generate smooth and counter-cyclical aggregate investment; (ii) it produces a smooth marginal utility that mismatches the equity volatility; (iii) by picking appropriate parameters, it has the potential to match investment dynamics but still hard to generate volatile stock returns.

To solve the problem, I incorporate a logarithm surplus-consumption ratio into habit sensitivity. My sensitivity function implies an inverse relationship of habit to consumption. As suggested by Ljungqvist & Uhlig (2009, 2015), Chen (2017), the agent periodically destroys her habits with sharp consumption rebounds in the next period. Consequently, the surplus-consumption ratio fluctuates substantially such that it predicts large swings of the marginal utility and asset returns even the consumption is smooth. Besides, the novel sensitivity function robustly suggests impulse responsible functions of productivity shock to investment under the countercyclical real interest rate, which removes the irrelevance from Khan & Thomas (2008) and predicts pro-cyclical and large swings of investment dynamics.

Further, my variant of sensitivity function inherits the precautionary savings mechanim of Chen (2017) in a different way. Chen (2017) uses an external habit preference to endogenous time-varying consumption volatility risks. I alternatively find that a low stationary habit in the logarithm level can amplify equity risk premiums, as it assigns additional weights on precautionary saving motivations. However, these weights work oppositely to the weights allocated to inter-temporal substitution such that it predicts a low and smooth risk-free rate.

Moreover, I find that a lumpy investment model can account for large swings of aggregate investment and considerable equity premiums in both time-series and cross-sectional level. Raising the size of firm-level shock or the upper bound of fixed costs can assign a higher weight to unadjusted firms and increase overall adjustment hazards. Huge adjustment hazards would lead to lower fractions of firms to choose to adjust, and asset prices absorb more productivity risks such that equity premiums rise. Specifically, small firms absorb more productivity risks and earn more considerable risk premiums than large firms, as they have low productivity, insufficient capital, and tenser constraints. Thus, my model reproduces substantial equity premiums and size premiums at the same time. However, these two microeconomic frictions have opposite implications on aggregate investment swings. The size of idiosyncratic risks is positive to investment fluctuations, but the threshold of fixed capital adjustment costs is inversely proportional to investment volatility. Hence, the model predicts large swings of aggregate investment and considerable risk premiums at the same time.

My findings have several implications. First, the excess smooth of aggregate investment in the literature of asset pricing is avoidable, because two micro frictions have opposite effects on investment dynamics, but similar effects on asset returns. Second, a lumpy investment model with decrease return to scale production predicts that firms with high market-to-book ratio are riskier and generates negative value premiums. Alternatively, my model successfully captures crucial moments of size premiums such that it provides critical insights for a lumpy investment model to investigate cross-sectional stock returns. Besides, my paper supports Winberry (2020) that micro frictions do matter for aggregate investment, as my model predicts countercyclical responses of productivity shock to the real interest rate. Finally, the quantitative success of my model builds on a solid micro background in capturing the distribution of investment rate at firm-level. Overall, the benchmark model reasonably answers the main challenges in asset pricing puzzles from both aspects and supports Winberry (2020)'s viewpoint on the 'Irrelevance' debate.

Literature Review This study addresses four branches of macro-financial studies. First, this study contributes to the research of habit preference. The 'Keeping up with the Joneses' utility has the potential to overestimate risk-free rate volatility (see Boldrin et al. 1999, Jermann 1998, among others). To resolve this problem, Campbell & Cochrane (1999) introduces the surplus-consumption ratio in the habit preference. Chen (2017, 2018) applies Campbell & Cochrane (1999)'s utility in production economies and finds that equity premiums come from endogenous consumption growth volatility. My paper contributes to these studies by finding that the weight assigned to such volatility is also crucial. Besides, recent studies on the lumpy investment with habit preference suggest that Chen (2017)'s specification has the potential to produce counter-cyclical investment dynamics in a Khan & Thomas (2008) economy. For this reason, I develop a novel sensitivity function with the logarithm of stationary habit that can not only be robust in matching investment dynamics but also match asset prices variations with a wide range of parameters of steady-state habit level in the Khan & Thomas (2008) environment.

Additionally, this research contributes to the debate of firm-level fixed costs to its macro implications. Early macroeconomic studies, such as Caballero et al. (1995) and Caballero & Engel (1999), suggest that the extensive margin investment models pro-cyclically respond to productivity shocks under the fixed prices environment. On the contrary, Thomas (2002) and Khan & Thomas (2008) argue the opposite side because procyclical endogenous prices offset the investment demand. House (2014) supports Khan & Thomas (2008) in a case that the relative price of investment goods is remarkably sensitive. However, recent studies, such as Clementi et al. (2017), Mongey & Williams (2019), Winberry (2020), support early studies by arguing a counter-cyclical risk-free rate for it can reduce borrowing costs, thus amplifying the aggregate investment. Besides, Fang (2020) shows that frictions on micro-lumpy investments also determines the macro effects of monetary policy. My paper contributes to recent studies by showing the impact of habit sensitivity to aggregate investment dynamics, which provides new nights on the joint dynamics of micro-level investment and the real interest rate.

Moreover, this paper contributes to asset pricing studies of non-convex costs. Kogan (2004) proposes a model with irreversible and convex costs to study conditional value premiums. Carlson et al. (2004) presume fixed costs to explain the cross-sectional expected equity return. Zhang (2005) assumes the asymmetric and fixed costs to account for value premiums. However, the above literature pays less attention to aggregate risk premiums. Besides, Cooper (2006) develops a real options model with fixed and irreversible adjustment costs to match equity premiums and value premiums. Herskovic et al. (2020) match the equity premium, value premium, and size premium based on fixed and convex capital adjustment costs. However, most of the successful studies above assume partial equilibrium models which consumption is exogenous and mainly focus on asset returns. Favilukis & Lin (2010) introduce long-run risks and fixed costs into a general equilibrium model but they fail to match any of cross-sectional or aggregate risk premiums. By contrast, I look at aggregate and cross-sectional asset prices variations in a General Equilibrium model within a rich environment with realistic heterogeneity across firms.

Finally, this study belongs to the recent growth literature on asset prices of heterogeneous firms in the general equilibrium framework. Luo (2019) argues that the equipment investment has a stronger predictive power than the structural investment. Ai et al. (2013), Favilukis & Lin (2015), and Chen (2018) build general equilibrium models with heterogeneous capitals to explain equity premiums and value premiums. However, their models assume convex adjustment costs. Tong & Ying (2019) develop a model of dynamic agency theory to examine both cross-sectional and aggregate macro-asset pricing dynamics. Most of the models above under-predict value premiums except for the work of Ai et al. (2013). By contrast, I explore the size premiums instead of value premiums in the cross-sectional level. Gomes et al. (2003) and Garleanu et al. (2012) jointly examine equity premiums, value premiums, and size premiums in the general equilibrium framework. However, they merely explain a small fraction of the cross-sectional asset return. As a comparison, my paper is one of the earliest studies that successfully explain size premiums in a general equilibrium model.

2 Model

The baseline model starts with a representative household who has the Campbell & Cochrane (1999) habit formation preference and infinitely survives in the Khan & Thomas (2008) heterogeneous firm environment with discrete time.

2.1 Production Economy

The production sector is a standard Khan & Thomas (2008) environment that consists of a fixed unit mass of heterogeneous firms indexed by $i \in [0, 1]$ and two exogenous shocks (i) the aggregate productivity shock and (ii) idiosyncratic shock.

Production Suppose the i th firm adopts the capital $k_{i,t}$ to produce the final good with aggregate productivity risks x_t and idiosyncratic risks $z_{i,t}$. The production function is denoted by equation (1):

$$y_{i,t} = e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu} \tag{1}$$

where θ and ν define the elasticity of output concerning capital and labour, respectively. $\theta + \nu < 1$ implies a property of decreasing returns to scale. Besides, both of the aggregate TFP shock x_{t+1} and idiosyncratic shock $z_{i,t+1}$ are mean-reverting:

$$\begin{aligned} x_{t+1} = \log X_{t+1} = \rho^X \log X_t + \varepsilon^X_{i,t+1}, \varepsilon^X_{i,t+1} \backsim N(0, \sigma_x) \\ z_{i,t+1} = \log Z_{i,t+1} = \rho^Z \log Z_{i,t} + \varepsilon^z_{i,t+1}, \varepsilon^z_{i,t+1} \backsim N(0, \sigma_z) \end{aligned} \tag{2}$$

where ρ^X and ρ^Z represent coefficients of the persistence of TFP and idiosyncratic risks, respectively.

Capital Accumulation and Adjustment Cost The law of motion of the capital accumulation process is standard:

$$k_{i,t+1} = i_{i,t} + (1 - \delta) k_{i,t} \tag{3}$$

where δ denotes the depreciation rate. I presume a non-convex cost capital adjustment (see Khan & Thomas 2008, Favilukis & Lin 2013, Winberry 2018, 2020, among others) that in lies with the empirical evidence of micro-lumpy investment, making my asset pricing stand on a solid cornerstone. The fixed costs have an upper bound $\bar{\xi}$ and lower bound zero with the adjustment region $[-b, b]$. The firm merely adjusts in the case that the capital adjustment costs below the upper bound. Namely, if the investment-to-capital rate $\frac{i_{i,t}}{k_{i,t}}$ lies in a small interval $[-b, b]$, the firm does not need to pay costs. Otherwise, companies should pay $\xi_{i,t}$ in each unit of hours worked.

Firm Optimisation The firm maximisation problem heavily relies on Khan & Thomas (2008) and Winberry (2018). Under the profit maximisation problem, firm i optimises its value function over investment $i_{i,t}$ and capital adjustment $\xi_{i,t}$. I define $\mu_{i,t} = \mu(k_{i,t}, z_{i,t}, \xi_{i,t})$ as the idiosyncratic state, Ω_t as the aggregate state, $V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$ as the value of expected discount dividends of firm. The firm department makes two maximisation choices—static choice of labour and dynamic choice of extensive margin of investment, which satisfies:

$$\begin{aligned} V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t) = \lambda^c(\Omega_t) & \max_n \left\{ e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu} - w(\Omega_t) n_t \right\} \\ & + \max_i \{ V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t) \xi_{i,t} w(\Omega_t), V^n(k_{i,t}, z_{i,t}; \Omega_t) \} \end{aligned} \tag{4}$$

where $V^a(k_{i,t}, z_{i,t}; \Omega_t)$ and $V^n(k_{i,t}, z_{i,t}; \Omega_t)$ present the value function of unconstraint capital choice $k^a(k_{i,t}, z_{i,t}; \Omega_t)$ and constraint capital choice $k^n(k_{i,t}, z_{i,t}; \Omega_t)$, respectively. $\lambda^c(\Omega_t)$ captures the marginal utility of consumption. Specifically, value functions of adjust and unadjust firm are given by:

$$\begin{aligned} V^a(k_{i,t}, z_{i,t}; \Omega_t) &= \max_{k' \in R} - \lambda^c(\Omega_t) i(k_{i,t}, z_{i,t}; \Omega_t) + \beta E[\hat{V}(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) | k_{i,t}, z_{i,t}; \Omega_t] \\ V^n(k_{i,t}, z_{i,t}; \Omega_t) &= \max_{k' \in [-bk, bk]} - \lambda^c(\Omega_t) i(k_{i,t}, z_{i,t}; \Omega_t) + \beta E[\hat{V}(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) | k_{i,t}, z_{i,t}; \Omega_t] \end{aligned} \tag{5}$$

where the firm-level investment is denoted by:

$$i(k_{i,t}, z_{i,t}; \Omega_t) = k(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) - (1 - \delta) k(k_{i,t}, z_{i,t}; \Omega_t)$$

Each firm faces a binary choice: invest or not. If the condition

$$V^a(k_{i,t}, z_{i,t};\Omega_t) - \lambda^c(\Omega_t)\xi_{i,t}w(\Omega_t) \geq V^n(k_{i,t}, z_{i,t};\Omega_t) \tag{6}$$

is satisfied, the company would like to investment by paying the fixed costs ξ. The condition above implies a unique threshold value of fixed cost to make the options of binary choice be indifferent. The threshold is denoted by:

$$\tilde{\xi} = \frac{V^a(k_{i,t}, z_{i,t};\Omega_t) - V^n(k_{i,t}, z_{i,t};\Omega_t)}{\lambda^c w(\Omega_t)} \tag{7}$$

Further, consider $\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)$ as the range of value that one can choose in $\tilde{\xi}$:

$$\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t) = \arg\min\left\{\max\left\{0, \tilde{\xi}(k_{i,t}, z_{i,t};\Omega_t)\right\}, \bar{\xi}\right\} \tag{8}$$

Equation (8) suggests that firms face a binary choice that either pays fixed costs that range from zero to the upper bound if they would like to invest or pays nothing if they are unwilling to adjust. Assume $\hat{V}(k_{i,t}, z_{i,t};\Omega_t) = E_\xi V(k_{i,t}, z_{i,t};\Omega_t)$ as the expected value function and the fixed costs follow uniform distribution, I approximate $\hat{V}(k_{i,t}, z_{i,t};\Omega_t)$ analytically by the following equation:

$$\begin{aligned} \hat{V}(k_{i,t}, z_{i,t};\Omega_t) = {} & \lambda^c(\Omega_t)\max_n \left\{e^{z_{i,t}}e^{x_t}k_{i,t}^{\theta}n_{i,t}^{\nu} - w(\Omega_t)n_t\right\} \\ & + \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\left\{V^a(k_{i,t}, z_{i,t};\Omega_t) - \lambda^c(\Omega_t)w(\Omega_t)\frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{2}\right\} \\ & + \left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\right)V^n(k_{i,t}, z_{i,t};\Omega_t) \end{aligned} \tag{9}$$

where $\frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}$ denotes the adjustment probability (Winberry 2018).

2.2 Households

Habit Formation The representative agent has a GHH (Greenwood et al. 1988) utility function U with aggregate consumption C_t, external habit H_t and labour N_t:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - H_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} \right\} \tag{10}$$

where β, σ, ψ, and α denote the subjective discount factor, curvature of utility, weight of disutility of labour supply, and inverse of Frisch elasticity of labour supply, respectively [2].

Surplus-Consumption Ratio Campbell & Cochrane (1999) introduce the surplus consumption ratio to capture the time-varying risk aversion, which is denoted by:

$$S_t \equiv \frac{C_t - H_t}{C_t}$$
$$s_{t+1} = \log S_{t+1} = \left(1 - \rho^S\right) \log \bar{S} + \rho^S \log S_t + \lambda^S \log(\frac{C_{t+1}}{C_t})$$

where λ^S, ρ^S, and $\bar{S}$ denote the sensitivity of habit to lag consumption, persistence of habits, and steady-state surplus-consumption ratio, respectively. Campbell & Cochrane (1999)'s specification argues that the habit stock of household depends on the surplus-consumption ratio that measures the geometric sum of past consumption.

Sensitivity Function Campbell & Cochrane (1999) choose the sensitivity function by the following rules: (i) the risk-free rate is constant, (ii) habit moves non-negatively with

[2] An alternative way to plug the habit formation would be KPR (King et al. 1988) preference:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right\}$$

I use GHH preference to guarantee a pro-cyclical dynamics on working hours with a simple assumption by removing the wealth effects of time-separable preference. To remove 'counter-cyclical labour' and keep the 'wealth effect' at the same time, one can assume the wage to be sticky — however, the labour market issues beyond the scope of this paper.

consumption, (iii) habit is predetermined by the steady-state [3]. Following Campbell & Cochrane (1999)' spirit of reverse engineering, this paper chooses a constant sensitivity of surplus consumption that also satisfies with three conditions: (i) the function should allow negative movements of habit to consumption; (ii) the function has the potential to generate a volatile marginal utility; (iii) the function generates consistent impulse response function of the risk-free rate and investment dynamics. Since the logarithm does not change the monotonicity, I specify the new sensitivity function by taking a logarithm of the denominator of the first item in Chen (2017, 2018) sensitivity function, which is given by:

$$\lambda^S = \frac{1}{\log \bar{S}} - 1. \tag{11}$$

Household Maximisation Let me restate Ω_t as the aggregate state. Assume $J(\Omega_t)$, $H(\Omega_t)$, $w(\Omega_t)$ to be the expected discounted utility, the level of habit stock, and the wage of the household sector given the current aggregate state Ω_t, the Bellman Equation of households satisfies:

$$J(\Omega_t) = \max \left\{ \frac{\left(C(\Omega_t) - H(\Omega_t) - \psi \frac{N(\Omega_t)^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} + \beta E_t[J(\Omega_{t+1})|\Omega_t] \right\} \tag{12}$$

[3] In Campbell & Cochrane (1999)'s endowment economy, λ^S usually is assumed to be

$$\lambda^S = \begin{cases} \frac{1}{\bar{S}} \sqrt{1 - 2(\log S_t - \log \bar{S})} - 1 & \log S_t \leq \log \bar{S} + (1 - \bar{S}^2)/2 \\ 0 & \log S_t > \log \bar{S} + (1 - \bar{S}^2)/2 \end{cases}$$

while in Chen (2017)'s production economy the sensitivity function is denoted by:

$$\lambda^S = \frac{1}{\bar{S}} - 1.$$

Note that the constant habit sensitivity has potential to generate inverse response of habit to consumption. See Appendix B.1

under the budget constraint:

$$\begin{aligned}&C(\Omega_t)+Q(\Omega_{t+1})A(\Omega_{t+1})+B(\Omega_{t+1})\\&=w\left(\Omega_t\right)N(\Omega_t)+(Q(\Omega_t)+\Pi(\Omega_t))A(\Omega_t)+(1+r(\Omega_t))\times B(\Omega_t)\end{aligned}\tag{13}$$

To solve the household optimisation problem, I rewrite the surplus-consumption ratio to be

$$C(\Omega_t)-H(\Omega_t)=C(\Omega_t)S(\Omega_t)$$

which delivers a simple stochastic discount factor

$$M(\Omega_{t+1}|\Omega_t)=\beta\left(\frac{C(\Omega_{t+1})S(\Omega_{t+1})-\psi\frac{N(\Omega_{t+1})^{1+\alpha}}{1+\alpha}}{C(\Omega_t)S(\Omega_t)-\psi\frac{N(\Omega_t)^{1+\alpha}}{1+\alpha}}\right)^{-\sigma}\tag{14}$$

and the consumption Euler Equation

$$E_t\left\{\beta\left(\frac{C(\Omega_{t+1})S(\Omega_{t+1})-\psi\frac{N(\Omega_{t+1})^{1+\alpha}}{1+\alpha}}{C(\Omega_t)S(\Omega_t)-\psi\frac{N(\Omega_t)^{1+\alpha}}{1+\alpha}}\right)^{-\sigma}\frac{Q(\Omega_{t+1})+\Pi(\Omega_{t+1})}{Q(\Omega_t)}\right\}=1\tag{15}$$

The derivation above suggests a unique stochastic discount factor which is identical to the inter-temporal marginal substitution rate of representative households.

2.3 Recursive Equilibrium

Equilibrium Characteristics Let me restate Ω_t as the aggregate state. In the rational expectation equilibrium, the agent maximises the value function $V(k_{i,t}, z_{i,t};\Omega_t)$ and collects dividends $\Pi(\Omega_t)$ given: (i) the associated policy functions $k(k_{i,t}, z_{i,t};\Omega_t)$, $n(k_{i,t}, z_{i,t};\Omega_t)$; (ii) cut-off rules $\tilde{\xi}$; (iii) the associated stochastic discount factor $M(\Omega_{t+1}|\Omega_t)$; (iv) labour demand $N(\Omega_t)$ and wages $w(\Omega_t)$. I specify the detailed equilibrium conditions below.

i. Household Given the aggregate prices vector $\{w(\Omega_t), \lambda^c(\Omega_t), \Pi(\Omega_t)\}$, the household consumption $C(\Omega_t)$ and labour demand $N(\Omega_t)$ resolve the aggregate utility maximisation problem in equation (12) and (13):

$$
\begin{aligned}
C(\Omega_t) = \int \Bigg\{ & e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu} + (1-\delta)k - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}} k^a(k_{i,t}, z_{i,t};\Omega_t) \\
& - \left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\right) k^n(k_{i,t}, z_{i,t};\Omega_t) \Bigg\} g(k_{i,t}, z_{i,t}) \mathrm{dzdk} \qquad (16) \\
\left(\frac{w(\Omega_t)}{\chi}\right)^{\frac{1}{\alpha}} = \int & \left(n(k_{i,t}, z_{i,t};\Omega_t) + \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)^2}{2\bar{\xi}} \right) g(k_{i,t}, z_{i,t}) \mathrm{dzdk}
\end{aligned}
$$

where the optimal choice of labour would be:

$$
n(k_{i,t}, z_{i,t};\Omega_t) = \left(\frac{e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} \nu}{w(\Omega_t)} \right)^{\frac{1}{1-\nu}} \qquad (17)
$$

Because I assume GHH utility in the household department, the labour market equilibrium only consists of real wage without the marginal utility.

ii. Firm Taking the aggregate prices vector $\{w(\Omega_t), \lambda^c(\Omega_t), \Pi(\Omega_t)\}$ as given, the value function $V(k_{i,t}, z_{i,t};\Omega_t)$, policy functions $k(k_{i,t}, z_{i,t};\Omega_t)$, $n(k_{i,t}, z_{i,t};\Omega_t)$, and the threshold bound value of fixed costs $\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)$ resolve the firm maximisation problem (4)-(9):

$$
\Pi(AS) = \int \left\{ \pi(\Omega_t) - \lambda^c(\Omega_t) \left(i(k_{i,t}, z_{i,t};\Omega_t) + \xi_{i,t} w(\Omega_t) \mathbf{1}\left\{ i(k_{i,t}, z_{i,t};\Omega_t) \notin [-bk, bk] \right\} \right) \right\} \mathrm{di} \qquad (18)
$$

where $\pi(k_{i,t}, z_{i,t}, \Omega_t) = \lambda^c(\Omega_t) \max_n \left(e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu} - w(\Omega_t) n_{i,t} \right)$ denotes the profit of each firm. Additionally, $i^a(k_{i,t}, z_{i,t};\Omega_t)$, $i^n(k_{i,t}, z_{i,t};\Omega_t)$ denote the unconstraint and constraint investment that satisfies:

$$
i(k_{i,t}, z_{i,t}, \xi_{i,t};\Omega_t) = \begin{cases} i^a(k_{i,t}, z_{i,t};\Omega_t) & \xi \leq \hat{\xi} \\ i^n(k_{i,t}, z_{i,t};\Omega_t) & \xi > \hat{\xi} \end{cases} \qquad (19)
$$

where the specific expressions are given by:

$$i^n(k_{i,t}, z_{i,t};\Omega_t) = \begin{cases} bk & i^a(k_{i,t}, z_{i,t};\Omega_t) > bk \\ i^a(k_{i,t}, z_{i,t};\Omega_t) & i^a(k_{i,t}, z_{i,t};\Omega_t) \in [-bk, bk] \\ -bk & i^a(k_{i,t}, z_{i,t};\Omega_t) < bk \end{cases} \tag{20}$$

iii. Asset Pricing The household substitute between the consumption and saving, lend the bond to the firm department, and collect the fixed-income return. Hence the real risk-free rate is defined as the reciprocal of the conditional mean of stochastic discount factor:

$$R^f(\Omega_{t+1}|\Omega_t) = 1 + r^f(\Omega_{t+1}|\Omega_t) = \frac{1}{E_t[M(\Omega_{t+1}|\Omega_t)]}. \tag{21}$$

Following Jermann (1998), the gross return of un-levered firm is given by [4]:

$$R(\Omega_{t+1}|\Omega_t) = \frac{Q(\Omega_{t+1}) + \Pi(\Omega_{t+1})}{Q(\Omega_t)} = \frac{\int V(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \tag{22}$$

that satisfies the asset pricing Euler equation:

$$E[M(\Omega_{t+1}|\Omega_t)R(\Omega_{t+1}|\Omega_t)] = 1 \tag{23}$$

Given the un-levered gross return on equity in Equation (23), one can compute the levered equity premia by:

$$(R^e(\Omega_{t+1}|\Omega_t) - R^f(\Omega_{t+1}|\Omega_t)) = (1 + \lambda^e) \times (R(\Omega_{t+1}|\Omega_t) - R^f(\Omega_{t+1}|\Omega_t)) \tag{24}$$

with the equity value ratio λ^e. The dividend process $D(\Omega_t)$ is determined by the profits $Y_t - \omega(\Omega_t)n_t$ removing investment I_t and capital adjustment costs $AC(\Omega_t)$:

$$D(\Omega_t) = \Pi(\Omega_t) = Y_t - \omega(\Omega_t)n_t - I_t - AC(\Omega_t) = C(\Omega_t) - \omega(\Omega_t)n_t \tag{25}$$

which refers to the endogenous dividend payout.

[4] An alternative way to compute gross return is define the value as a cum dividend value(Favilukis & Lin 2010): $R(\Omega_{t+1}|\Omega_t) = \int V(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} / \int (V(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di} - D(\Omega_t))$. Both methods produce similar values on equity premia and Sharpe ratio.

iv. Law of Motion for Cross-Sectional Distribution Following Winberry (2018), I Define $\mathbf{m}$ as the moment vector of for all $(k_{i,t}, z_{i,t})$,

$$g(k_{i,t}, z_{i,t};\Omega_t, \mathbf{m}) = \int\int\int \left\{ \begin{array}{c} 1\left\{\rho^Z z_t + \varepsilon^{z_{t+1}} = z_{t+1}\right\} \times \\ \left[\frac{\hat{\xi}(k_{i,t},z_{i,t};\Omega_t,\mathbf{m})}{\bar{\xi}} 1\left\{k^a(k_{i,t}, z_{i,t};\Omega_t, \mathbf{m}) = k_{t+1}\right\}\right. \\ \left. + \left(1 - \frac{\hat{\xi}(k_{i,t},z_{i,t};\Omega_t,\mathbf{m})}{\bar{\xi}}\right) 1\left\{k^n(k_{i,t}, z_{i,t};\Omega_t, \mathbf{m}) = k_{t+1}\right\}\right] \end{array} \right\} \times p(\varepsilon^{z_{t+1}}) g(k_{i,t}, z_{i,t}; \mathbf{m}) \mathrm{d}\varepsilon^{\mathrm{z_{t+1}}}\mathrm{dzdk} \tag{26}$$

where $p(\varepsilon^{z_{t+1}})$ denotes the probability distribution function of the idiosyncratic uncertainty.

v. Law of Motion for Aggregate Uncertainty The model consists of two aggregate uncertainties: the productivity shock and the surplus-consumption ratio, whose law of motions are presented by:

$$\begin{aligned} x_{t+1} &= \log X_{t+1} = \rho^X \log X_{t+1} + \varepsilon^{X_{t+1}}, \varepsilon^{X_{t+1}} \backsim N(0, \sigma_x) \\ s_{t+1} &= \log S_{t+1} = \log S_{t+1} = \left(1 - \rho^S\right) \log \bar{S} + \rho^S \log S_t + \lambda^S \Delta C_{t+1} \end{aligned} \tag{27}$$

Note that the surplus-consumption ratio is endogenous while productivity risks are exogenous. I use Winberry (2018)'s projection and second-order perturbation method to solve the model. For numerical details please see Appendix C.

3 Benchmark Calibration

Section 3 discusses benchmark calibration results of my asset pricing model. **Section 3.1** will present economic parameters selections, and **Section 3.2** presents the benchmark calibration for target moments. My calibration targets (i) three financial moments: equity premiums, equity volatility, and the Sharpe ratio; and (ii) three macroeconomic moments: the output fluctuation, the investment co-movement, and the relative fluctuations of investment to output.

3.1 Structure Parameters

Table 1: Parameters Selection: Microeconomic Structure

Parameter	Description	Value	Parameter	Description	Value
λ^e	Debt-to-Equity Ratio	0.54	θ	Capital Share	0.21
β	Discount Factor	0.99	ν	Labour Share	0.64
σ	Curvature	1	δ	Depreciation	0.025
α	Inverse Frisch Elasticity	1/2	$[-b, b]$	Bound of Region	0.011
N	Stationary Work Hour	1/3	ρ^Z	Persistence of Idio. Shock	0.859
ψ	Labour Disutility	1	σ^Z	s.d. of Idio. Shock	0.015

Structural Parameters Choices. **Table 1** reports structural parameters for quarterly calibration. I first compute the average debt-to-equity ratio to be 0.54 for the benchmark calibration [5]. For the GHH preference, I pick the subjective discount factor β to be 0.99 (Prescott 1986), the steady-state hours worked to be 1/3 (Juster & Stafford 1991), the Inverse Frisch Elasticity α to be 0.5 (Chetty et al. 2011), and the curvature σ to be one (Jaimovich & Rebelo 2009), which are standard in the RBC literature. Further, I pick the coefficient of dis-utility ψ to be one such that household can put the one third time to work.

The sum of the labour share and capital share should be smaller than one in the production side, as I assume the decreasing return on the scale in the production function. To this end, I pick capital share to be 0.64 (Prescott 1986), labour share to be 0.21 (Winberry 2016), and the region of fixed costs around -0.011 to 0.011 (Khan & Thomas 2008). For firm-level productivity risks, I follow Khan & Thomas (2008) again and choose the persistence and volatility of idiosyncratic shock to be 0.859 and 0.015, respectively. Empirical evidence

[5] Many macro-asset pricing studies assume various values on the financial leverage λ^e. Jermann (1998) assumes λ^e to be 0.4 or 0.6, Favilukis & Lin (2013) select 2/3 as λ^e, Croce (2014) presume $\lambda^e = 1$, Chen (2017) choose $\lambda^e = 0.3$. This study chooses λ^e based on empirical evidence instead of literature.

suggests that the quarterly depreciation rate is approximately equal to 0.025. I thus adopt this value in my benchmark calibration — all parameters above in line with microeconomic evidence or the real business cycle literature.

3.2 Target Moments

Target Moments: Selection. I target three financial moments: (i) average equity premiums, (ii) equity volatility, and (iii) the Sharpe ratio, as the central research questions of this study are the equity premium and equity volatility puzzle [6]. Second, what investors truly care is the risk premium for it measures how much compensation investors can gain by taking a position.

For business cycle moments, I follow Nezafat & Slavik (2015) and choose the relative volatility of investment to production as a macro target, as the asset prices will absorb more productive risks under the capital adjustment costs that makes the investment volatility less than observations (Guvenen 2009). Further, I pick the output fluctuation as macro target because a reasonable relative investment volatility should base on a plausible value on output fluctuations. Given that the traditional sensitivity function is not robust for investment co-movements, I also consider the investment cyclicity as a target moment.

Table 2: Parameters Selection: Identification

Parameter	Description	Value	Parameter	Description	Value
$\log \bar{S}$	Log Steady State Habit	0.1	ρ^X	Persistence of TFP Shock	0.95
ρ^S	Persistence of Habit Formation	0.975	σ^X	s.d of TFP Shock	0.007
λ^S	Sensitivity of Habit Formation	$\frac{1}{\log \bar{S}} - 1$	$\bar{\xi}$	Upper Bound of Fixed Costs	0.25

[6] Many macro studies target and match risk-free rate dynamics fairly well. For example, Chen (2017) and Nezafat & Slavik (2015) focus on the risk-free rate while Adam & Merkel (2019) target on the equity return. However, both Nezafat & Slavik (2015) and Adam & Merkel (2019) quantitatively generate no more than 30 % of equity premiums.

Target Moments: Strategy. The identification strategy follows the spirit of 'separate calibrations' [7]. Note that the productivity shock mainly contributes to macroeconomic dynamics and the habit preference drives variabilities of equity claims. I first choose TFP size to fit output and investment volatilities and then select the size of external habits for equity premiums, equity volatility, and investment co-movements. All macroeconomic moments are HP-filtered, while financial moments are not.

Following Cooley & Prescott (1995), King & Rebelo (1999) and Winberry (2020), I match the output volatility and the relative volatility of investment to output by choosing the persistence and standard deviation of productivity shock to be 0.95 and 0.007, respectively. To match high and volatile equity premia, I pick a low unconditional log surplus-consumption ratio $\log \bar{S}$ to be 0.1, and a high habit persistence ρ^S to be 0.975. I compute the sensitivity of habit to consumption λ^S by $\frac{1}{\log \bar{S}} - 1$ in a production economy. As the upper bound of fixed costs determines the adjustment probability $\frac{\hat{\xi}}{\bar{\xi}}$ and macro-financial statistics, I pick a moderate value between Khan & Thomas (2008) and Bachmann et al. (2013). Because I use six parameters to match six moments, the benchmark model is just-identified [8].

Table 3: Benchmark Model: Target Moments

Macro Quantities	Benchmark	Data	Asset Returns	Benchmark	Data
$\sigma(y_t)$	1.56%	1.51%	$E(r^e - r^f)$	1.79%	1.54%
$\sigma(i_t)/\sigma(y_t)$	2.92	2.89	$\sigma(r^e - r^f)$	7.72%	7.74%
$\rho(i_t, y_t)$	0.82	0.88	$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	0.20

[7] Winberry (2020) calibrate the real interest rate and aggregate investment separately while I calibrate macroeconomic fluctuations and financial moments separately.

[8] The habit sensitivity is determined by log steady-state surplus-consumption ratio, and equity premia and volatility co-determine the Sharpe Ratio. For this reason, this study calibrates five empirical targets with five parameters, and the baseline model is still just-identified.

Target Moments: Identification. Table 3 presents that the benchmark model closely matches target moments. Specifically, the baseline model exactly generates volatile risk premia (7.72% vs 7.74% in data), reasonably predicts average risk premiums (1.79% vs 1.54% in evidence) and the Sharpe ratio (0.23 vs 0.20 in data). The model also matches relative volatility (2.92 vs 2.89 in data) and co-movements (0.82 vs 0.88 in data) of aggregate investment, as well as the output volatility (1.56% vs 1.51% in data) reasonably well.

The benchmark calibration contributes to previous studies in several aspects. First, it can jointly match equity risk premiums and investment volatility, avoiding the excess investment smoothing from Guvenen (2009). Second, the benchmark model matches equity risk premiums merely using a log utility. Macro models usually calibrate the risk premiums by a high parameter of relative risk aversion. However, since GHH preference can amplify the TFP propagation, using the GHH preference implies the model can only choose a low curvature of utility, adding restrictions to fit empirical risk premiums [9]. Thus, calibrating macro-financial moments with σ=1 in GHH preference would be a sensible contribution.

4 Model Evaluation

Before illustrating the mechanism of my model, I demonstrate that the benchmark model performs well in non-target moments. Given that the model is designed to calibrate target moments, it could fit the target moments well. Hence, the fitness of non-target moments is an important criterion to evaluate the validation of the benchmark model—the more non-target moments that a model can match the better the model is.

[9]Without the GHH preference, Bansal & Yaron (2004) pick a relative risk aversion to be 10. With the GHH preference, Winberry (2020) picks a curvature value of 1 to match macroeconomic moments, and Nezafat & Slavik (2015) assume the risk aversion to match asset prices variations. However, Nezafat & Slavik (2015)'s model merely matches 24% of risk premiums and 67% of equity volatility. Even in habit preference, choosing $\sigma = 1$ as of utility curvature is not usual for asset pricing studies: Campbell & Cochrane (1999), Chen (2017), and Luo (2019) pick the curvature value to be 2.

4.1 Time-Series Implications

4.1.1 Asset Prices Dynamics

In the aggregate stock market, I value the model performance by financial moments, impulse response functions of TFP shock to asset returns, and the predictability of market risk premiums.

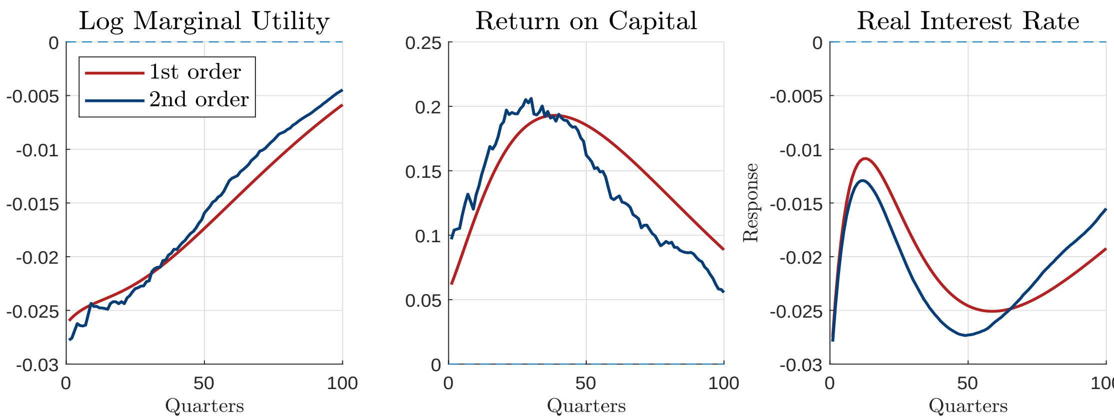
Table 4: Benchmark Model: Time-Series Asset Pricing Moments

Equity	Benchmark	Data	T-Bills	Benchmark	Data
$E(r^e - r^f)$	1.79%	1.54%	$E(r^f)$	0.88%	0.40%
$\sigma(r^e - r^f)$	7.72%	7.74%	$\sigma(r^f)$	0.57%	0.99%
$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	0.20			

Time-Series Asset Pricing Moments **Table 4** demonstrates the unconditional asset pricing statistics of both risky assets and risk-free assets. As predicted above, the benchmark model exactly matches equity volatility and slightly overestimates risk premiums with a log utility, which makes it exaggerates the Sharpe Ratio (0.23 vs 0.20 in data) a little. Additionally, the benchmark model also controls the mean and volatility of real interest rate at reasonably low levels. As illustrated in **Table 4**, the baseline model slightly over-predicts the risk-free rate (0.88% vs 0.40 % in data) and under-estimates the risk-free rate volatility (0.57% vs 0.99% in evidence). Although the calibrated model does not exactly fit risk-free rate moments, it still reasonably predicts low and smooth interest rates and avoids the risk-free rate puzzle [10].

[10] The 'risk-free rate puzzle' suggests that a model predicts large swings of the risk-free rate closing to the equity volatility (see Boldrin et al. 1999, Jermann 1998, many others), which is not in line with data.

Figure 1: Impulse Response Function of TFP Shock to Asset Return



Impulse Response Functions: Asset Prices. **Figure 1** presents the asset prices effects of the productivity level shock under the first and second-order perturbation with dark red and navy blue solid lines, respectively. I plot the steady-state level by light blue dot lines. The second-order approximation displays jagged impulse response functions to financial variables except for risk-free rate dynamics. By contrast, the first-order perturbation produces smooth impulse response functions with the same trend for it less captures the effects of fixed costs. **Figure 1** also explains the channel of risk premiums. To this end, I first present the equation to generate conditional risk premiums:

$$E_t[R^e_{t+1} - R^f_{t+1}] \approx -Cov_t(R^e_{t+1} - R^f_{t+1}, \lambda_{t+1}) \tag{28}$$

The ex-post excess return $R^e_{t+1} - R^f_{t+1}$ is procyclical, as the ex-post return on capital is procyclical but the risk-free rate is countercyclical. Second, the marginal utility inversely responds to the aggregate TFP shock. Hence, the covariance between the realised excess return and the marginal utility is negative, which delivers considerable risk premiums.

The Predictability of Market Excess Return Following Campbell & Cochrane (1999), the benchmark model adopts the dividend-price ratio as the predictor of risk premiums. The prediction equation is:

$$E_t\left[r_{t+k}\right] - r^f_{t+k} = a + \beta_{dp} \times \frac{d_t}{p_t} + \epsilon_{t+k} \tag{29}$$

where a, β_{dp}, $\frac{d_t}{p_t}$, and ϵ_{t+k} denote the regression intercept, slope, dependent variable, and residual respectively. k denotes the holing period of assets.

Table 5: Benchmark Model: Return Prediction

	Benchmark		Full Sample 1952-2018	
Horizons	Coefficients	R^2	Coefficients	R^2
1 Quarter	0.0462	0.0034	0.0178	0.0087
1 Year	0.2295	0.0255	0.0811	0.0377
2 Year	0.5739	0.0405	0.1401	0.0577
3 Year	1.0473	0.0610	0.1652	0.0640
4 Year	1.5592	0.0777	0.1745	0.0582
5 Year	1.9942	0.0832	0.2142	0.0683

Table 5 presents regression results of cumulative future realised real market excess returns to the dividend-price ratio from one quarter to five-year. The left panel illustrates the coefficient and regression fitness of simulated results with 268 quarters, and the right panel demonstrates the same target of empirical evidence from 1952Q1-2018Q4. Both observations and the model implied beta and regressive fitness rise with the increases in investment horizons, and the dividend-price ratio positively associates with risk premiums. Besides, the magnitudes of simulated regression results on coefficients and R-square are comparable and reasonably close to empirical evidence.

4.1.2 Macroeconomic Implications

I evaluate macro performances by business-cycle moments and impulse responsible functions of productivity shock to macro variables.

Table 6: Benchmark Model: Macroeconomic Implications

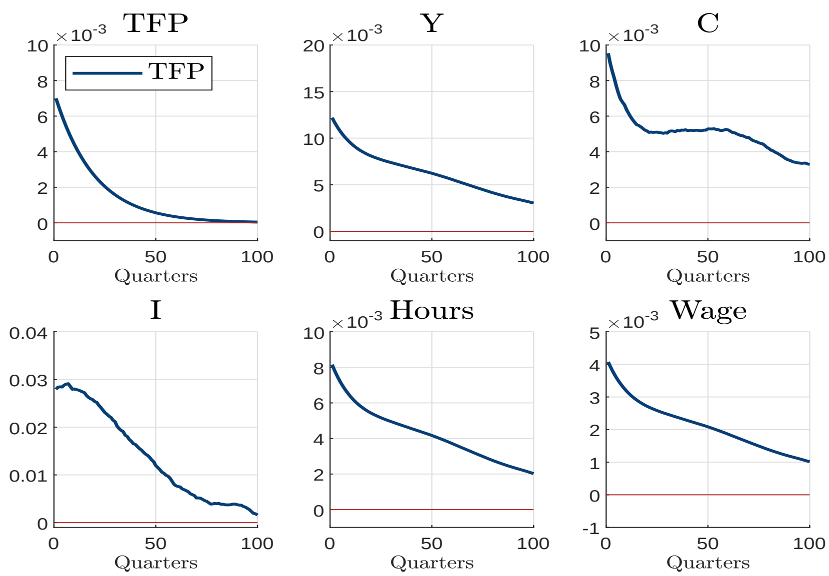
	Benchmark				Data			
Variables	$\sigma(x_t)$	$\sigma(x_t)/\sigma(y_t)$	$\rho(x_t, y_t)$	$\rho(x_t, x_{t-1})$	$\sigma(x_t)$	$\sigma(x_t)/\sigma(y_t)$	$\rho(x_t, y_t)$	$\rho(x_t, x_{t-1})$
y_t	1.56%	1	1	0.71	1.51%	1	1	0.85
c_t	1.28%	0.82	0.93	0.70	1.20%	0.80	0.85	0.81
i_t	4.54%	2.92	0.82	0.76	4.35%	2.89	0.88	0.80
n_t	1.04%	0.67	1.00	0.71	1.84%	1.21	0.87	0.90

Macroeconomic Moments **Table 6** presents the unconditional statistics of macro dynamics from the benchmark model and data. Except for the underprediction of labour fluctuations, the second and third columns of **Table 6** suggest that my calibrated model closely matches the volatilities of output, investment, and consumption as data. The fourth and fifth rows illustrate moments of business cycle comovements and persistence. Since there is mere a productivity level shock driving the aggregate dynamics, the baseline model slightly over-predicts most of the macro comovements but underestimates all moments of the business cycle persistence [11]. Although the adoption of GHH structure makes the working hours and output perfectly correlated, it avoids counter-cyclical labour co-movements (see Nezafat & Slavik 2015, Winberry 2020, among others). On average, the baseline model matches the macroeconomic aggregates to a large extent — the mismatch of working hours due to the standard RBC model mechanism that is beyond the scope of this study.

Impulse Responsible Functions: Business Cycle. **Figure 2** presents the macroeconomic effects of the productivity level shock in 100 quarters. I display the steady-state level

[11] Like the standard macro general equilibrium models (Boldrin et al. 1999, Khan & Thomas 2008, Gourio 2013, Basu & Bundick 2017), the benchmark model replicates around 60% volatility of hours worked. Since (Kydland & Prescott 1982), the single shock driven standard RBC model underestimates the macro persistence meanwhile overestimates comovements than macro data.

Figure 2: Impulse Responsible Function of TFP Shock to Business Cycle



with solid red lines and examine macro-dynamics based on such a reference level. Like the standard RBC model, the spike of TFP shock stimulates all macro variables in various extents. Because the aggregate uncertainty is a level shock, all business cycle variables revert to initial level as productivity risks die out eventually. The TFP shock excessively raises consumption, investment, and output, but it has marginal effects on the real wage and working hours under a GHH structure utility function.

4.2 Cross-Sectional Implications

4.2.1 The Distribution of Investment Rate

I evaluate the micro performance by the stationary distribution and patterns of investment rates for it captures the investment behaviour in the long-run balanced growth.

Table 7: Benchmark Model: Microeconomic Moments

Distribution	Benchmark	Data	Investment Patterns	Benchmark	Data
Mean	9.93%	10.4%	Spike ($>20\%$)	15%	14.4%
Standard Deviation	12.03%	16.0%	Positive($0\% - 20\%$)	85%	85.6%
Skewness	3.08	3.60			
Excess Kurtosis	12.36	17.6			

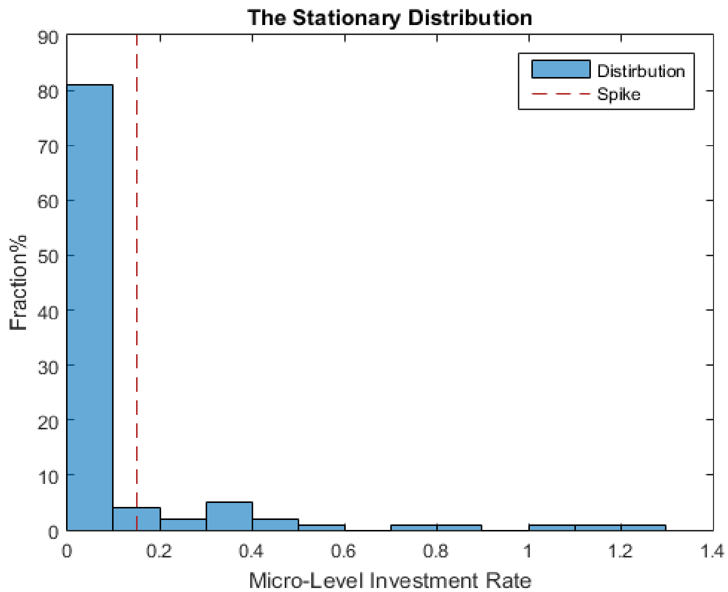
Table 7 reports model implied statistics of microeconomic level investment rates of the balanced growth path. I follow Winberry (2020), Zwick & Mahon (2017) and define the 'spike' if investment rates are greater than 20% and 'positive' if the investment rates are below 20%. The **Left Panel** presents the distribution of investment rates, and the **Right Panel** states investment patterns in the cross-sectional level. Like Clementi et al. (2017), Winberry (2016, 2020), I match the average cross-sectional investment rate to be 10% in Zwick & Mahon (2017) by choosing $\delta = 0.025$. The benchmark model suggests that investment rates are positive skewness, and it predicts reasonably closed statistics for distribution. However, the benchmark model underestimates the asymmetry of the right skewness and the standard deviations of investment rates. Second, the benchmark also matches the investment pattern that no more than 15% of firms choose to spike, and the non-spike investment rate is 85%.

Figure 3 displays the model implied distribution of cross-sectional investment rates more intuitively. Like the **Left Panel** of **Table 7**, only a few firms choose to adjust such that the distribution of investment rates is positive skewness.

4.2.2 Size Premiums

Given that the market value of adjustment firms V^a is higher than the non-adjustment firms V^n, I empirically sort firms into market capital groups and construct two, three, five, and ten value-weighted portfolios, respectively. I consider the quarterly return spread

Figure 3: Cross-Sectional Distributions of Investment Rate



between the first and last portfolio as the size premium [12]. Numerically, I compute the return of small and big firms by replacing the original value function as constrained firm's value V^n and unconstrained firm's value V^a, respectively [13].

Table 8 reports moments of size premiums from simulations and data. Both model and data predict that return on small firms $E(r^s)$ are more considerable than large firms $E(r^b)$. The magnitudes of most simulated moments are reasonably close to empirical evidence [14]. Quantitatively, the model with levered firms overestimates the return spread (1.11% vs

[12] One can alternatively sort firms into financial constraints for a lumpy investment model sets a threshold of fixed costs to bind the investment (Whited 2006). However, lumpy investment models ignore restrictions on issuing equity or bond (Whited & Wu 2006) such that it would not be a candidate in this case.

[13] Unlike the Krusell & Smith (2008)' algorithm that resolves a non-linear state space, I adopt Winberry (2018)'s algorithm and solve the model into a linear state space without requirements on aggregation. Hence, I follow Ai et al. (2019) to compute the cross-sectional return ex-ante rather than sorting the portfolio ex-post.

[14] Appendix 5.4 mathematically prove that small firms deliver high dividend-price ratio such that they are riskier and earn more risk premiums.

Table 8: Benchmark Model: Cross-Sectional Asset Pricing Moments

Moments		$E(r^s)$	$E(r^b)$	$E(r^s - r^b)$	$std(r^s)$	$std(r^b)$	$std(r^s - r^b)$	$\rho(r^s, r^b)$
Model	Levered Firm	3.02%	1.91%	1.11%	6.74%	7.13%	4.48%	0.79
Half-Sort	Un-levered Firm	1.96%	1.23%	0.73%	4.38%	4.63%	2.90%	0.79
Data	Half-Sort	3.47%	3.01%	0.46%	10.78%	7.85%	5.09%	0.89
	3 Portfolio Sort	3.39%	2.76%	0.63%	11.51%	7.76%	6.76%	0.82
	5 Portfolio Sort	3.34%	2.73%	0.61%	11.88%	7.67%	7.66%	0.78
	10 Portfolio Sort	3.33%	2.68%	0.66%	12.27%	7.61%	8.69%	0.71

maximum 0.66% in data) because it underestimates the return of big firms. However, it still fits most of the second-order moments except for the volatility of return in small firms.

Discussion: Investment Rate and Cross-Sectional Stock Return Recent literature draws into a new debate on the link between risk premiums and empirical investment rate in the cross-sectional level. Bai et al. (2019) account for value premiums by an accounting-based investment rate and calibrating highly asymmetric convex costs. However, Clementi & Palazzo (2019) suggest that an investment-based model only explains a small fraction of cross-sectional returns sorted by commonly used risk factors if it builds on an economic-based investment rate. Both sides of the debate assume partial equilibrium models.

Their arguments focus on two different aspects of asset pricing puzzles. Bai et al. (2019) show that a model can replicate stock return in cross-section regardless of how large the estimated parameters. In an extreme case, the value firms can face 800 times adjustment costs than growth firms, or an accounting-based investment rate can be a consideration. By contrast, Clementi & Palazzo (2019) focus on whether the micro-lumpy investment rate and calibrated parameters are reasonable. By contrast, I show that a general equilibrium model can at least account for the cross-sectional return spread between small and big firms, given an economic-based investment rate and reasonable calibrated parametric values.

5 Inspecting the Mechanism

In **Section 5**, I show that how the benchmark model resolves the asset pricing challenges and contribute to the debate of 'Irrelevance' between micro-lumpy investment and its aggregates.

5.1 External Habit and Precautionary Savings

The first asset pricing challenge implies a model should match the high and volatile equity premium, as well as a low and smooth real interest rate jointly. I resolve this challenge by calibrating high sensitivity and persistence of habit. To keep the high sensitivity and persistence, I investigate asset pricing implications of $\log \bar{S}$ from 0.05 to 1 and ρ^S from 0.6 to 1 with the spacing of 0.05 holding other parameters constant [15]. I find that not only the consumption growth volatility but also the weights assigned in precautionary savings and inter-temporal substitutions can account for asset prices moments. A relatively high weight on precautionary saving motivations helps to match equity premiums and equity volatility. Hence, the parameter choices of $\log \bar{S}$ play a central role in matching asset returns [16].

Decomposition of Stochastic Discount Factor I specify the behaviours of inter-temporal substitution and precautionary saving by decomposing its pricing kernel. In this study, I show that the weights assign to EIS and precautionary saving motivations are also essential

[15] Chen (2018) examines value premiums implications with various $\bar{S}$. My calibration deviates from his in two points: (i) I assume $\log \bar{S}$ in the sensitivity function, (ii) he jointly changes $\bar{S}$ and convex adjustment costs since low EIS requires high convex costs while I hold other variables constant.

[16] Golosov & Winberry (2019), Winberry (2020) suggest that the stochastic volatility or time-variant price of risks is critical for generating considerable risk premiums. However, Chen (2017), Jermann (1998), and I find that the stochastic volatility in the TFP shock is not a necessary ingredient. The typical Long-run risks literature (Bansal & Yaron 2004) that solves a constant price of risk can still account for sizeable risk premiums and the predictability of market excess return.

in fitting asset pricing statistics, which is the supplement of Chen (2017) [17]. I prove my argument by decomposing the stochastic discount factor as [18]:

$$\log M_{t+1} = \log\beta + (-\sigma)\left\{\log\left(\frac{C_{t+1}S_{t+1}}{C_tS_t}\right) + \log\left(\frac{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times N_{t+1}^{1+\alpha}}{C_tS_t - \frac{\psi}{1+\alpha}\times\frac{C_{t+1}S_{t+1}}{C_tS_t}\times N_t^{1+\alpha}}\right)\right\}$$

$$= \log\beta + \left\{(-\sigma)\left[\left(\rho^S - 1\right)\left(\log\bar{S} - \log S_t\right) + \frac{1}{\log\bar{S}}\mu(g_c)\right] + \frac{(-\sigma)^2}{2(\log\bar{S})^2}\sigma(g_c)\right\} \quad (30)$$

$$+ (-\sigma)\log\left(\frac{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha}\times\frac{C_{t+1}S_{t+1}}{C_tS_t}\times N_t^{1+\alpha}}\right)$$

From the decomposition above, I find that the habit persistence ρ^S, the log-level of $\bar{S}$, the mean and volatility of consumption growth (i.e., $\mu(g_c)$ and $\sigma(g_c)$), and their coefficients $-\frac{1}{\log\bar{S}}$ and $\frac{1}{2(\log\bar{S})^2}$ determine the stochastic discount factor hence drive asset pricing moments. Since my model enrolls the hours working, the unconditional mean and volatility of labour growth (i.e., $\mu(g_h)$ and $\sigma(g_h)$) should also be considerations in this case. Because the production economy hit by a level shock, the hours worked should be a stationary process and have marginal effects on financial moments. Mathematically, $\frac{C_{t+1}S_{t+1}}{C_tS_t}$ shares the same sign of $\log\left(\frac{C_{t+1}S_{t+1}-\frac{\psi}{1+\alpha}\times N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1}-\frac{\psi}{1+\alpha}\times\frac{C_{t+1}S_{t+1}}{C_tS_t}\times N_t^{1+\alpha}}\right)$ such that the second term and third term of Equation (30) move on the same direction. Given these two implications above, I give priority to analysing the second item of of Equation (30), particularly on $\log\bar{S}$ and ρ^S.

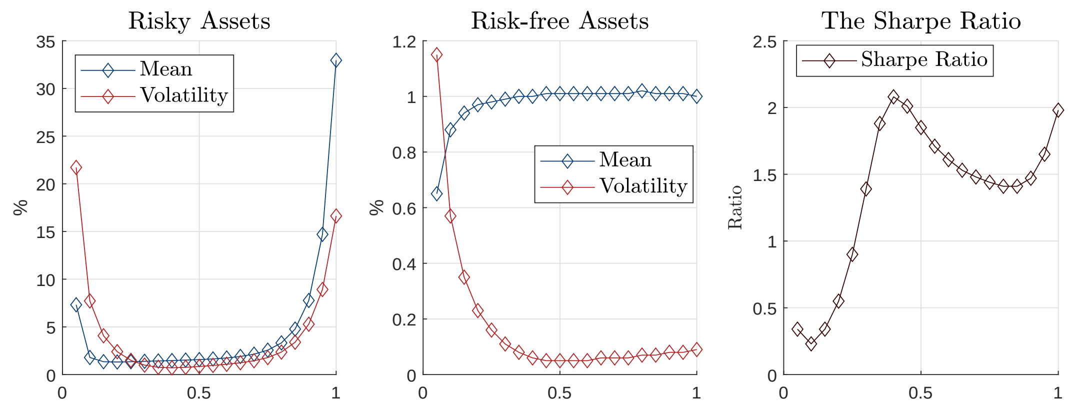
5.1.1 Role of Steady-State Surplus-Consumption.

Figure 4 displays the asset pricing effects of the Logarithm Steady-State Habits $\log\bar{S}$. Both of the aggregate risk premia and equity volatility are U-shaped with increases of

[17] Chen (2017) captures inter-temporal substitution by expected consumption growth and capture precautionary saving motivations by consumption growth volatility. I treat them as synonyms.

[18] I provide more details of the SDF decomposition in Appendix B.2. In my case, the hours worked does not join in external habits because we concern on whether consumption growth volatility can affect risk premia. By assuming labour joined habit formation, one can hardly decompose the effects of consumption and working hours.

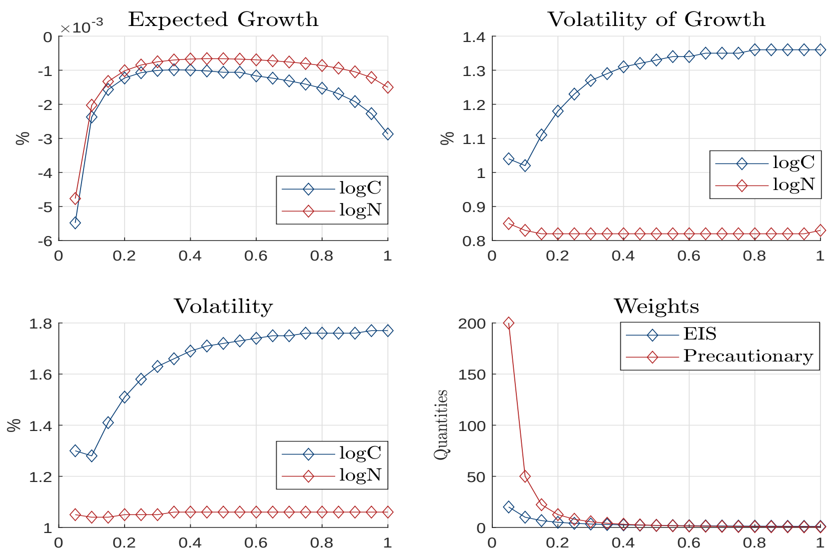
Figure 4: Asset Pricing Moments: Conditional on $log\bar{S}$



$\log \bar{S}$. In my case, each $\log \bar{S}$ corresponds to an equilibrium in the production economy and has different cyclicities. My calibrations suggest that (i) risk premiums are monotonically decreasing in $\log \bar{S} < 0.15$ and monotonically increasing afterwards, (ii) the equity volatility is monotonically decreasing in $\log \bar{S} < 0.4$ and monotonically increasing afterwards. As a consequent, the Sharpe ratio displays a creeping growth, even though it is monotonically decreasing in $\log \bar{S} < 0.1$. Second, unlike the constant real interest rate in Campbell & Cochrane (1999), the risk-free rate is endogenous in a production economy (Chen 2017). With the increase of $\log \bar{S}$, the average real interest rate is monotonically increasing, but the volatility of the risk-free rate is U-Shaped.

The conditional moments of $\log \bar{S}$ in **Figure 4** explain the failure of low habit sensitivity in matching risk premiums—the inflexion point of risk premia (the dark blue line in the first panel) requires a smaller value of $\log \bar{S}$ than equity volatility (the red line in the first panel). Second, the high $\log \bar{S}$ often overestimates the average risk-free rate for insufficient precautionary saving motivatons. The mismatch of risky assets moments is a crucial reason to renounce the low λ^S such that successful asset pricing models should pick low values of $\log \bar{S}$.

Figure 5: The Mechanism of $\log \bar{S}$



Why is large $\log \bar{S}$ difficult to produce volatile equity risk premiums? **Figure 5** provides a possible answer. The first three panels of **Figure 5** present the mean and volatility of consumption and labour growth, as well as unconditional volatilities of consumption and hours worked. As the expected growth of both variables and second-order moments of labour are tiny, they may have marginal asset pricing effects. Besides, I show that the low consumption volatility can also generate substantial risk premiums with $\log \bar{S} < 0.2$ while the similar values of consumption (growth) volatility can produce different equity premiums and volatilities under various $\log \bar{S} > 0.5$. Thus, arguing consumption fluctuations as the mere source of asset pricing statistics is insufficient. The fourth panel of **Figure 5** displays that the absolute value of the weight of precautionary saving motivations decreases more sharply (from 200 to 0.5) than its peer of inter-temporal substitutions (from 20 to 1) in the raise of $\log \bar{S}$. Thus, calibrating a low $\log \bar{S}$ suggests that we put more weights on precautionary saving motives rather than the EIS. In this case, even though the consumption volatility is small, it could produce sizeable and volatile risk premia, as well as flat risk-free rate dynamics. My calibrations quantitatively argue that putting more weights on precautionary saving motivations is more critical than generalising higher consumption volatility, and thus supplementarily contributing to Campbell & Cochrane (1999), Chen (2017). An additional implication is an opposite side between $-\frac{\sigma}{\log \bar{S}}$ and $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$ make the precautionary saving behaviours offset the inter-temporal substitutions, which smoothes the risk-free rate dynamics.

5.1.2 Role of Habit Persistence

Figure 6: Asset Pricing Moments: Conditional on ρ^S

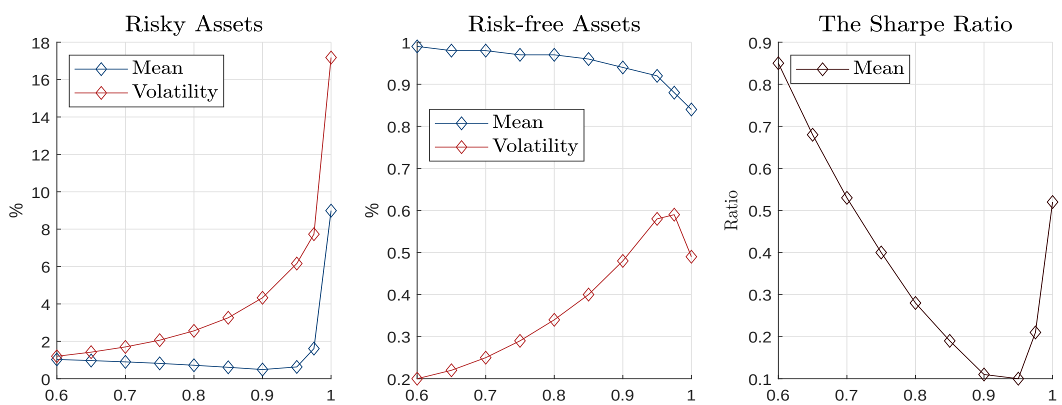
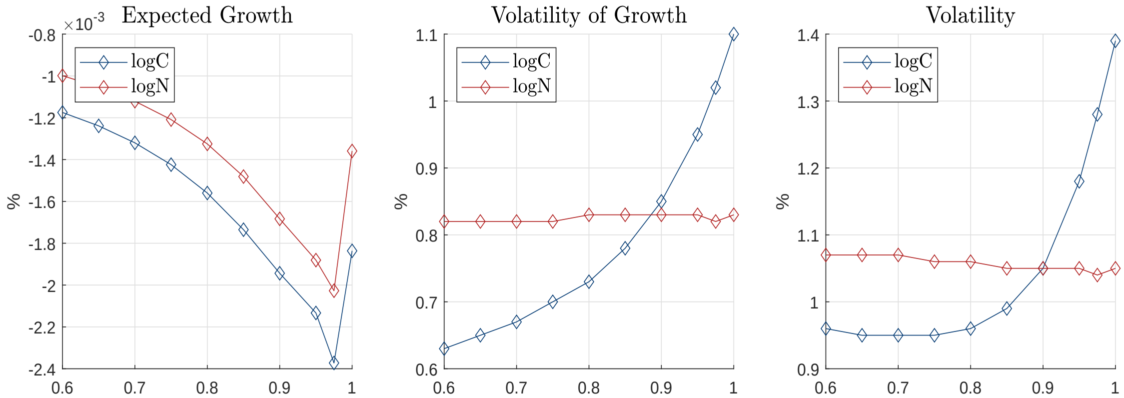


Figure 6 quantitatively shows the financial effects of habit persistence ρ^S. I find that with the increase of habit persistence, volatilities in both risk-free and risky assets grow while the risk-free rate declines. The risk premiums and Sharpe Ratio behave likely as U-Shaped. Second, because the standard deviation of risk premiums increase at $\rho^S = 0.9$, thus calibrating a low ρ^S does not help to interpret the equity volatility puzzle.

Figure 7 displays the mechanism of habit persistence. The consumption volatility reverts since $\rho^S = 0.75$ but the risk premia still decline in this case, and thus they do not perform a monotonically relationship. **Panel 1 of Figure 6** suggests that equity premia revert till $\rho^S > 0.9$. As discussed above, only when $\rho^S > 0.9$ can the model predicts positive

Figure 7: The Mechanism of ρ^S



relationship among consumption growth volatility, equity premia and volatility. Thus, Chen (2017)'s story about aggregate risk premia only happens in case of $\rho^S > 0.9$. On average, I resolve the first asset pricing challenge mainly by the precautionary saving channel, as the novel sensitivity function inherits the advantages of traditional habit sensitivity.

5.2 Habit Sensitivity: Traditional vs. Novel

The second asset pricing challenge suggests that the consumption smoothing makes the stochastic discount factor less risky. As a solution, I show that the novel sensitivity function can produce large swings to the marginal utility by contrast to the traditional one. An additional implication is, my habit sensitivity also contributes to the debate of 'irrelevance'.

5.2.1 Traditional vs. Novel: Calibration

In **Section 5.2.1**, I highlight the advantage of the new habit sensitivity function in explaining target moments before studying its mechanism.

Figure 9 demonstrates the statistics of target moments under different sensitivity functions and parameters. In this comparison, I pick three group parameters: (i) the

Table 9: Investment Dynamics and Risk Premiums: Different Sensitivity Functions

Sensitivity	$\frac{1}{\bar{S}}-1$			$\frac{1}{\log \bar{S}}-1$			Data
Parameters	Benchmark	$\bar{S}=0.65$	$\bar{\xi}=0.44$	Benchmark	$\log \bar{S}=0.65$	$\bar{\xi}=0.44$	
		$\rho^S=0.95$	$\rho^z=0.90$		$\rho^S=0.95$	$\rho^z=0.90$	
$\sigma(y_t)$	1.61%	1.61%	1.60%	1.56%	1.59%	1.56%	1.51%
$\rho(i_t, y_t)$	-0.65	-0.40	0.09	0.82	0.98	0.89	0.88
$\sigma(i_t)/\sigma(y_t)$	0.51	0.37	0.35	2.92	0.52	3.09	2.89
$E(r^e-r^f)$	1.56%	1.59%	1.73%	1.79%	1.93%	173.07%	1.54%
$std(r^e-r^f)$	1.43%	1.72%	1.43%	7.73%	1.30%	245.33%	7.74%
$\frac{E(r^e-r^f)}{\sigma(r^e-r^f)}$	1.09	0.92	1.21	0.23	1.49	0.71	0.20

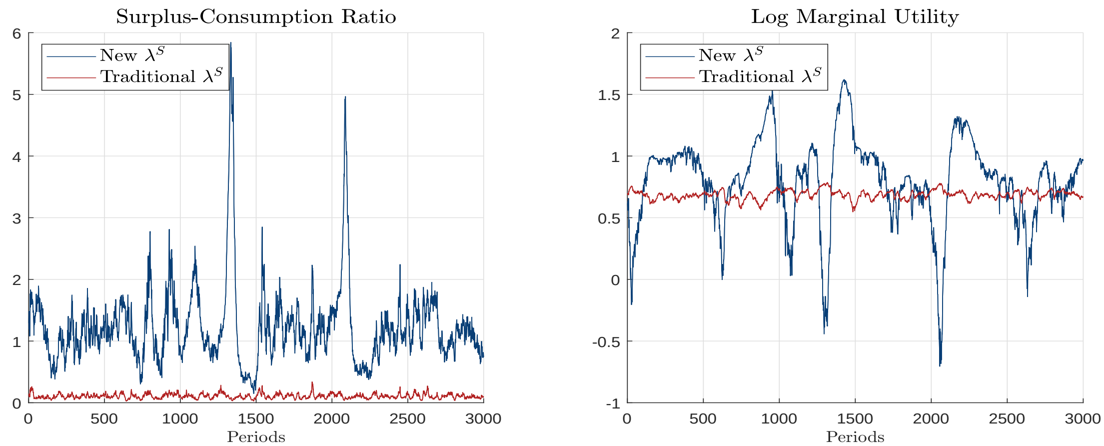
Benchmark Model; (ii) the preference parameters from Winberry (2016); and (iii) the high threshold of fixed costs and persistence of firm-level shock from Winberry (2016), respectively. Given the traditional habit sensitivity function, the left panel suggests that changing the firm-level parameters has the potential to match the pro-cyclical aggregate investment. However, such parameterisation still fails to generate sizable equity volatility (1.43% vs 7.74% in data). The right panel suggests that the new sensitivity function has the potential to predict the correct sign and comparative magnitudes of all target moments, given a wide range of parameter choices.

5.2.2 Volatility of Pricing Kernel: Traditional vs. Novel

In **Section 5.2.2**, I simulate 3000 periods of the log marginal utility and surplus-consumption ratio. The simulations show that the novel sensitivity function can address the second asset pricing puzzles by producing large swings of the marginal utility.

In my model economy, the habit is inverse to consumption once the surplus-consumption ratio is higher than one. Hence, the surplus-consumption fluctuates more substantially than

Figure 8: λ^S and SDF: the Way to Replicate Equity Volatility



the previous sensitivity function, driving the marginal utility more volatile. As the increase in $-Cov_t(R^e_{t+1} - R^f_{t+1}, \lambda_{t+1})$ and the large swings of value function, risk premiums become considerable and volatile, which solves the second challenge of asset pricing.

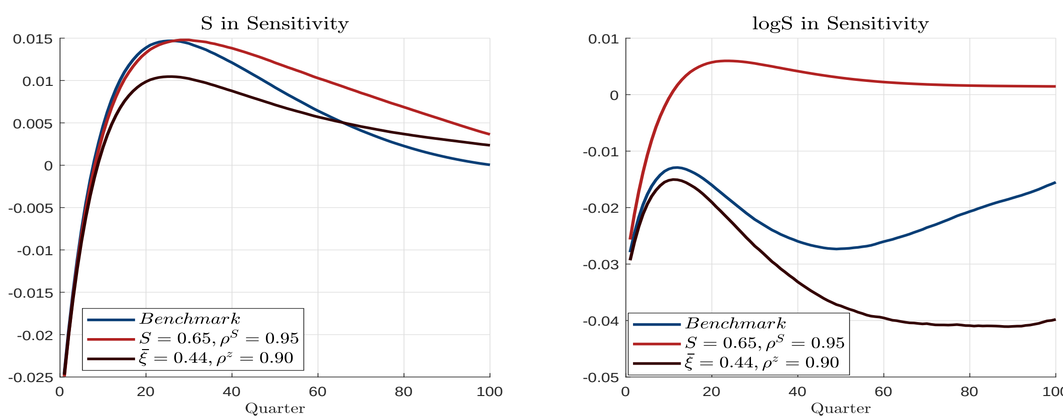
Discussion The quantitative success of my model base on two channels from the perspective of the utility function. Both of Chen (2017) and I calibrate a high sensitivity function to offer additional precautionary savings motivations. However, my model implies a new mechanism based on the destruction of habit. In like Chen (2017)'s work, these two channels can hardly be coincident, as a high stationary surplus-consumption ratio implies low habit sensitivity. Thus, Chen (2017)'s specification only has the potential to use habit destruction channel, but it is not the case in his calibration. In my case, a substantial $\bar{S}$ delivers low $\log \bar{S}$, such that habit sensitivity still considerable.

Why does traditional habit sensitivity provide a smooth stochastic discount factor in the lumpy investment model? A possible explanation is I solve the model by the second-order perturbation such that it has difficulty to capture the time-varying consumption volatility risks like Chen (2017). However, the second-order perturbation is the cutting edge in the heterogeneous macro model in Winberry (2018)'s algorithm, and it still captures the unconditional volatility of consumption. As a comparison, Oh (2011) adopts the Krusell-Smith global solution to compute an asset pricing model of traditional habit sensitivity and lumpy capital adjustment. However, the model seems misspecified in lumpy economics, as he still struggles against a smooth stochastic discount factor.

5.2.3 Debate on 'Irrelevance'

Khan & Thomas (2008) argue the 'Irrelevance' between micro friction and aggregate investment, as a procyclical risk-free rate cut the firm-level investment demand in expansion. By contrast, Winberry (2020) argue that the aggregate implication of micro-lumpy investment is significant, as he finds that the U.S real interest rate is countercyclical in the past 70 years. I address this debate by showing the impulse response function of productivity shock to the risk-free rate and aggregate investment.

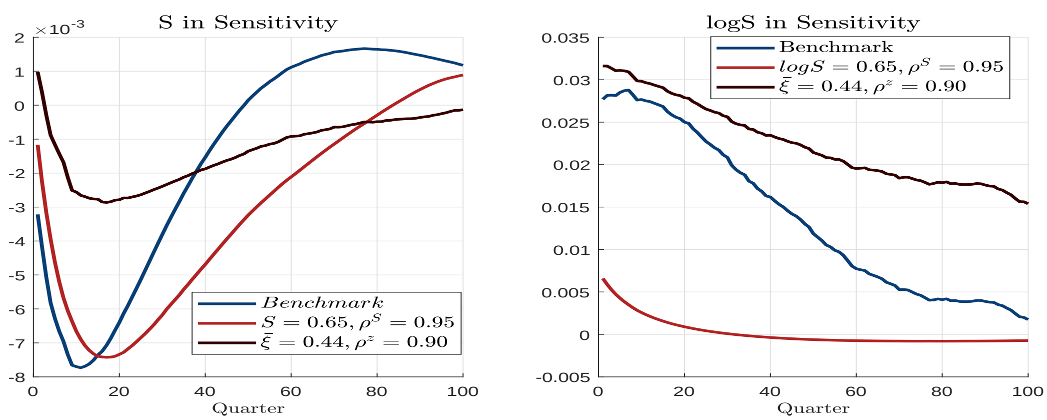
Figure 9: Interest Rate Responses: Different Sensitivities



The IRFs of TFP to Real Interest Rate **Figure 9** demonstrates the impulse responses functions of productivity risks to the real interest rate under both sensitivity specifications.

The blue, red, and brown line suggest three groups of parameters in the calibration mentioned above. The **Left Panel** indicates that the real risk-free rate does not inversely respond to productivity at all time. The positive response of the real interest rate has the potential to cut the investment demand. By contrast, the novel sensitivity function negatively responds to the TFP shock, which encourages the investment demand in good time and produces strongly procyclical investment dynamics.

Figure 10: Investment Responses: Different Sensitivities



The IRFs of Aggregate Investment **Figure** 10 illustrates the impulse responses functions of TFP shock to aggregate investment. The **Left Panel** indicates that the traditional sensitivity function cannot guarantee procyclical investment dynamics, as the investment rebounds after an initial drop, corresponding to the response of risk-free rate above. The offset effect also cut investment demand. Second, a small response to TFP smoothes investment dynamics. By contrast, the **Right Panel** demonstrates that investment dynamics positively and largely response to TFP shock such that the novel sensitivity function matches the cyclicality and fluctuations of investment, given a wide range of parameters.

5.3 Microeconomic Frictions and Cash Flow Risks

The third challenge of asset pricing literature states the incompatibility of high equity premiums and large swings of investment. I answer this question by showing that the upper bound of fixed costs $\bar{\xi}$ and the volatility of firm-level shock σ_z have similar effects on stock return but opposite impacts on investment fluctuations.

5.3.1 Role of Fixed Adjustments Costs

Mathematical Implications of Fixed Costs Let me restate the transform value function in a standard Khan & Thomas (2008) environment as:

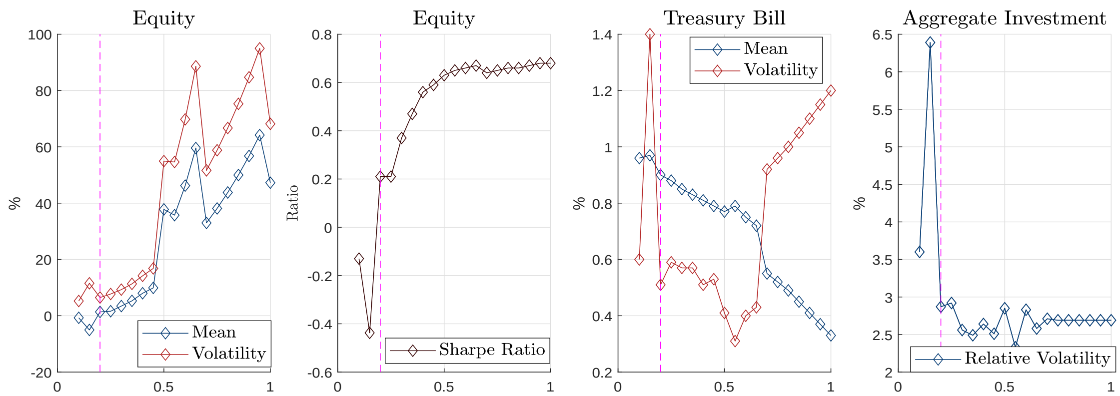
$$
\begin{aligned}
\hat{V}(k_{i,t}, z_{i,t}; \Omega_t) = \; & \lambda^c(\Omega_t) \max_n \left\{ e^{z_{i,t}} e^{x_t} k_{i,t}^{\theta} n_{i,t}^{\nu} - w(\Omega_t) n_t \right\} \\
& + \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}} \left\{ V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t) w(\Omega_t) \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{2} \right\} \\
& + \left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}} \right) V^n(k_{i,t}, z_{i,t}; \Omega_t)
\end{aligned}
$$

where the decision threshold and its arrange are given by:

$$
\begin{aligned}
\tilde{\xi} &= \frac{V^a(k_{i,t}, z_{i,t}; \Omega_t) - V^n(k_{i,t}, z_{i,t}; \Omega_t)}{\lambda^c w_t} \\
\hat{\xi} &= \arg\min \left\{ \max \left\{ 0, \tilde{\xi} \right\}, \bar{\xi} \right\} = \arg\min \left\{ \max \left\{ 0, \frac{V^a(k_{i,t}, z_{i,t}; \Omega_t) - V^n(k_{i,t}, z_{i,t}; \Omega_t)}{\lambda^c w} \right\}, \bar{\xi} \right\}
\end{aligned}
$$

Let me restate the $\frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}}$ as the adjustment probability in Winberry (2018). Note that if $\bar{\xi} < \tilde{\xi}$ then $\hat{\xi} = \bar{\xi}$, only adjusted firms are considerations and the adjustment probability is identical to one such that the upper bound of fixed costs has marginal effects on value function and risk premiums. Otherwise, $\hat{\xi} = \tilde{\xi} = \frac{V^a(k_{i,t}, z_{i,t}; \Omega_t) - V^n(k_{i,t}, z_{i,t}; \Omega_t)}{\lambda^c w_t} < \bar{\xi}$. Thus, the relative weights assigned to adjusted and constraint firms are $\frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}}$ and $1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}}$, respectively. Intuitively, the raise of upper bound of fixed costs will assign higher probabilities on constraint firms that are less likely to reinvest but have a higher willingness to issue dividends. Thus, constraint firms face higher cash-flow risks that amplify aggregate risk premiums. I quantitatively show this process below.

Figure 11: Asset Pricing Moments: Conditional on $\bar{\xi}$

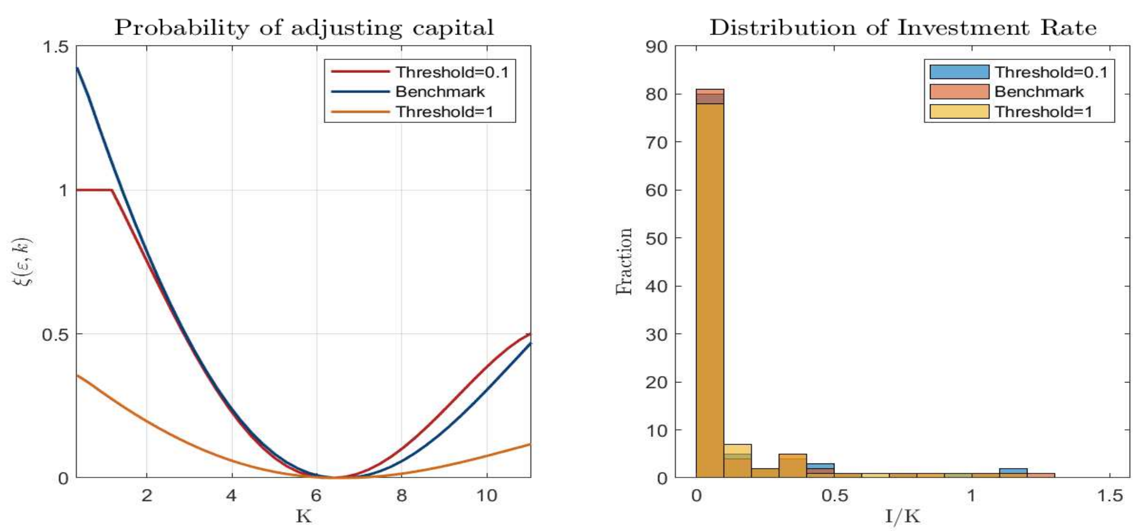


Numerical Effects of Investment Lumpiness **Figure 11** displays the asset pricing implications of the upper bound of fixed adjustment costs varieties from 0.1 to 1. The responses of asset prices to fixed costs are non-smooth and jagged because of investment lumpiness. To simplify, I discuss the numerical results in two intervals $[0.1, 0.2]$ and $[0.2, 1]$ by a pink dotted line. Since the low $\bar{\xi}$ (i.e., $\bar{\xi}$ lies in $[0.1, 0.2]$) implies that treasury bills are riskier and aggregate equity premiums are negative. With the increase of $\bar{\xi}$, firms are more likely to be inactive. Thus, the productivity has higher potential to transfer to future dividend such that it yields more substantial and volatile equity risk premiums. Note that equity premiums, volatility, and the Sharpe ratio respond to threshold $\bar{\xi}$ positively.

The situation of treasury bills is more complicated. On average, with larger values of $\bar{\xi}$, the real interest rate inversely associates with $\bar{\xi}$ while its volatility is positively proportional to $\bar{\xi}$. It is because firms are less likely to adjust and more willing to hold bonds. Namely, additional precautionary saving motivations reduce real interest rate but raise its volatility. The particular case comes from $\bar{\xi} = 0.15$ in which the bond is riskier than stock because of a large fraction of adjustment. In this case, investors are reluctant to hold bond but willing to hold stock such that the inverse 'precautionary saving' motivations reduce stock return but raise stock volatility, as well as the real interest rate and its volatility. Like most models of convex capital costs, an increase of fixed costs will smooth the aggregate investment.

The Adjustment Probability and Threshold Figure 12 displays how the upper bound of fixed costs $\bar{\xi}$ impacts on adjustment probabilities $\frac{\hat{\xi}(k_{i,t},z_{i,t};\Omega_t)}{\bar{\xi}}$ and the distribution of investment rates. I compare three examples with $\bar{\xi} = 0.1$, 0.25, and 1, respectively. The left panel presents the adjustment probabilities conditional on the capital stock with various upper bounds of fixed costs. With the increase of the threshold, the adjustment probability will shift down on average. Hence, firms are less likely to adjust and a substantial fraction of productivity transfers to future cash flows which raise equity risk premia. Such effects are not significant when $\bar{\xi}$ raises from 0.1 to 0.25 while they are distinct if $\bar{\xi}$ increases to 1. The left panel also illustrates that adjustment probabilities have the potential to be one.

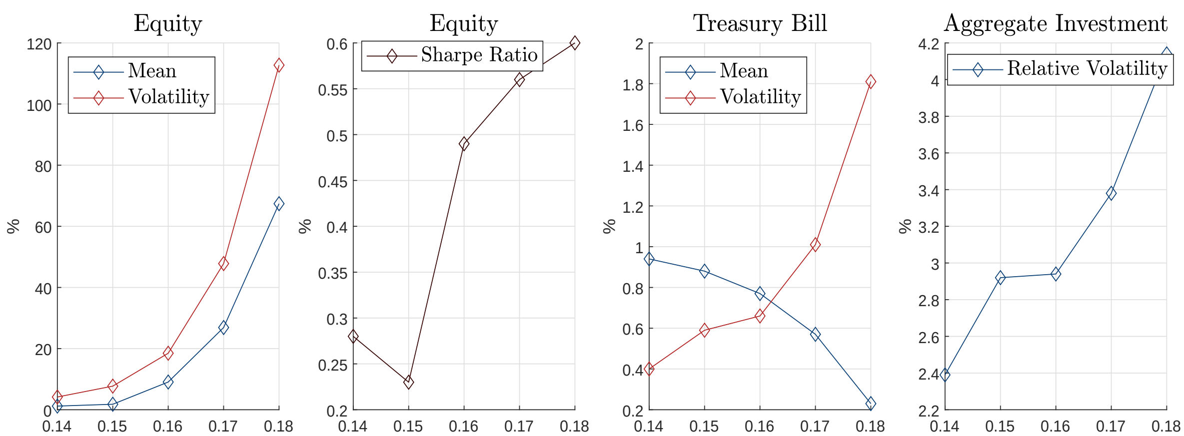
Figure 12: The Mechanism of $\bar{\xi}$



However, the right panel argues that changes in the upper bound of fixed costs have marginal effects on the investment pattern. All distributions in the right panel suggest similar investment patterns in micro-level. Hence, the adjustment probabilities of investment play an essential role in producing substantial equity risk premiums under Khan & Thomas (2008)' environment.

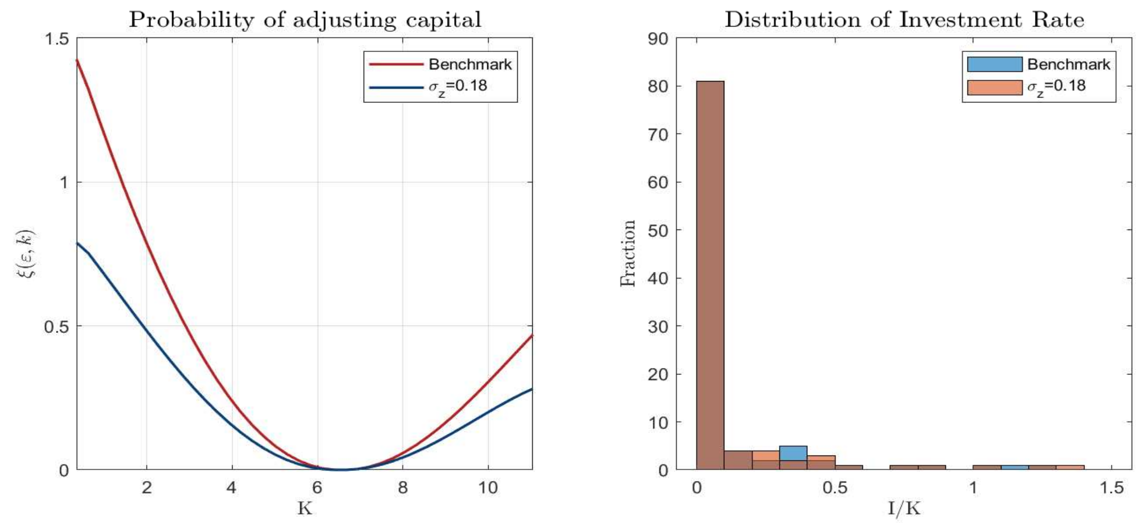
5.3.2 Role of Idiosyncratic Risks

Figure 13: Macro-Asset Pricing Moments: Conditional on σ_z



Macro-Asset Pricing Effects of Firm-Level Risks **Figure 13** presents asset pricing and investment effects of σ_z from 0.14 to 0.18 by five grids with spacing of 0.01. With the increase of idiosyncratic uncertainties, both equity premia and equity volatility raise, and they are sensitive to σ_z. The Sharpe Ratio also raises since the equity premiums increase faster than its volatility. Rasing the dispersion of firm-level productivity will decrease the interest rate but increase the risk-free rate fluctuations for it makes firms riskier such that investors are willing to hold safer assets that reduce the interest rate. Panel 4 suggests that, on the macro-level, raising the fluctuation of firm-level productivity can deviate investment, offsetting the effects of adjustment costs that avoids the investment excess smooth in Guvenen (2009).

Figure 14: The Mechanism of σ_z



The Adjustment Probability and Idiosyncratic Risks **Figure 14** explains the mechanism beyond the firm-level shock with small and large size in the idiosyncratic volatility. Panel 1 and 2 share the same spirit as threshold changes of fixed costs in micro-level. Namely, the adjustment probabilities play a more critical role than the stationary distribution of investment rates. Unlike the upper bound of fixed costs, the idiosyncratic risks can directly affect value function in the approximation process such that they are more sensitive to risk premiums. On average, the fixed costs and firm-level productivity have the similar effects on the stock return through the adjustment probability channel, but have opposing implications on aggregate investment. Thus, the excess smooth in investment volatility is avoidable.

5.4 Return on Constrained and Unconstrained Firm

The final challenge of asset pricing is an asset pricing model can hardly match the cross-sectional stock return. Like Favilukis & Lin (2010), some models may even produce an inverse premium in contrast to data. In **Section 5.4**, I first prove that the constrained (small) firm ask more risk premiums as compensations than unconstrained (large) firm in math, which guarantee the correct sign of size premiums. Let me restate the value function to be:

$$\begin{aligned}
\hat{V}(k_{i,t}, z_{i,t};\Omega_t) = &\lambda^c(\Omega_t)\max_n \left\{e^{z_{i,t}}e^{x_t}k_{i,t}^{\theta}n_{i,t}^{\nu} - w(\Omega_t)n_t\right\} \\
&+ \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\left\{V^a(k_{i,t}, z_{i,t};\Omega_t) - \lambda^c(\Omega_t)w(\Omega_t)\frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{2}\right\} \\
&+ \left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\right)V^n(k_{i,t}, z_{i,t};\Omega_t)
\end{aligned}$$

Thus, the return on unconstrained (big) firm and constrained (small) firm is given by:

$$\begin{aligned}
R^a(\Omega_{t+1}|\Omega_t) &= \frac{\frac{\hat{\xi}}{\bar{\xi}}\int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + \frac{\hat{\xi}}{\bar{\xi}}D(\Omega_{t+1})}{\frac{\hat{\xi}}{\bar{\xi}}\int V^a(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
&= \frac{\int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}} \\
R^n(\Omega_{t+1}|\Omega_t) &= \frac{\left(1-\frac{\hat{\xi}}{\bar{\xi}}\right)\int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + \left(1-\frac{\hat{\xi}}{\bar{\xi}}\right)D(\Omega_{t+1})}{\left(1-\frac{\hat{\xi}}{\bar{\xi}}\right)\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
&= \frac{\int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}}
\end{aligned} \tag{31}$$

and the return spread between small and big firm is:

$$\begin{aligned}
&R^n(\Omega_{t+1}|\Omega_t) - R^a(\Omega_{t+1}|\Omega_t) \\
=&\frac{\int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} - \frac{\int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}} \\
=&\frac{\left[\int V^n(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})\right]\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
&- \frac{\left[\int V^a(k_{i,t+1}, z_{i,t+1};\Omega_{t+1})\mathrm{di} + D(\Omega_{t+1})\right]\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
=&\frac{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}D(\Omega_{t+1}) - \int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} \\
=&\frac{\int\left[V^a(k_{i,t}, z_{i,t};\Omega_{t+1}) - V^n(k_{i,t}, z_{i,t};\Omega_t)\right]\mathrm{di}D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t};\Omega_{t+1})\mathrm{di}\int V^n(k_{i,t}, z_{i,t};\Omega_t)\mathrm{di}} > 0
\end{aligned} \tag{32}$$

The constrained firms are riskier and earn more premiums, as the value of adjustment firm $V^a(k_{i,t}, z_{i,t}; \Omega_{t+1})$ is larger than the value of constrained firm $V^n(k_{i,t}, z_{i,t}; \Omega_{t+1})$ (Whited 2006).

Figure 15: The Impulse Response Function of TFP to Return on Big/Small Firm

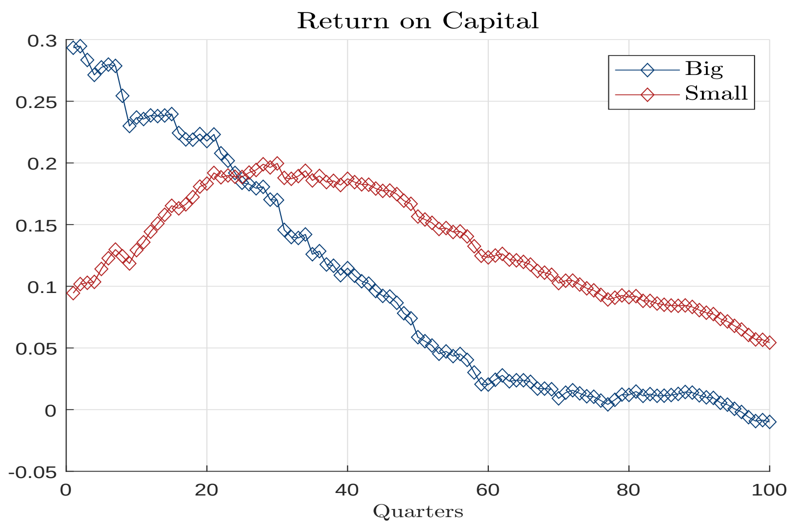


Figure 15 presents the impulse response functions of productivity shock to the return on small and big firms. The big firms respond to productivity strongly at first but die out faster, and it even has negative responses at the final periods. Small firms positively respond to productivity shocks from beginning to end. Thus, small firms earn higher returns, as its accumulation responses to productivity shock are larger. The intuition is, the representative household receives the same amount of aggregate dividends. Firms with small market capital have identical dividend payout as big firms, such that they absorb more productivity risks and become riskier. However, the inverted-U shape response implies a smooth return on small firms, which leads to the underestimation of fluctuations in return on small firms.

6 Conclusions

In this paper, I document two aspects of asset pricing puzzles in the previous literature: (i) the difficulties in matching asset pricing facts; (ii) the inconsistency in the micro foundation. To this end, I formulate a different habit sensitivity to match critical macro-asset pricing facts broadly. The periodical destruction of habit and precautionary saving motivations play essential roles, as they produce volatile marginal utility but smooth consumption jointly, which corresponds to the first aspect of asset pricing puzzles.

Second, a model of fixed costs matches sizable risk premiums and large swings of investment dynamics together, as the firm-level risks and adjustment costs have similar results on the stock return but opposite effects on investment fluctuations. An additional impact is the benchmark model reproduce size premiums, as small firms face higher cash flow risks. Besides, the fixed costs model also capture the critical moments of micro-lumpy investment, making the baseline model build on a solid micro background. Thus, a lumpy investment model contributes to both aspects of asset pricing puzzles.

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A Appendix: Data

For macroeconomic evidence, I collect quarterly observations of the aggregate consumption, investment, and output from BEA historical table 1.1.5. All samples are collected from 1952Q1-2018Q4. Further, I deflate all data from nominal variables to real variables by related deflators in BEA statistics table 1.1.9 and de-trend them by the HP-filter with a parameter of 1600. Besides, I collect the aggregate working hours of non-farm business sector from the Fed series HOANBS. Working hours are also recorded in the quarterly frequency, deflated by relative deflators and de-trended by HP-filter with a parameter of 1600.

For empirical cross-sectional investment rates, the debate still exists. Clementi & Palazzo (2019) and Bai et al. (2019) share completely different evidence and statistics in firm-level investment rates. To simplify, I follow Winberry (2020) and pick IRS firm-level investment rates from Zwick & Mahon (2017) from 1998 to 2010, and their Table B.1 in the online appendix displays statistics of microeconomic moments.

For the aggregate capital market, I take the monthly interest rate from CRSP one-year treasury-bill observations, and monthly equity return from CRSP value-weighted market portfolio data include and exclude dividends. Data lengths are from January 1952 to December 2018. I reshape the data from a monthly to a quarterly frequency. For the dividend-price ratio forecasting, I take return data excluded and included dividend of CRSP value-weighted portfolio to construct the net dividend and dividend-price ratio. For financial leverage, I take the empirical debt to equity ratio since 1952Q1 to 2018Q4 of non-financial business from the Fed and compute the average.

Second, I take the data of cross-sectional return from Professor Kenneth French' web. To construct the two portfolios sorted by the market capital, I directly compute the size premium with the six Book-to-Market and Size portfolios by:

$$\begin{aligned} \text{SMB} = &\frac{1}{3}\left(\text{Small Value} + \text{Small Neutral} + \text{Small Growth}\right) \\ &- \frac{1}{3}\left(\text{Big Value} + \text{Big Neutral} + \text{Big Growth}\right) \end{aligned} \tag{33}$$

In other cases, I compute the size premium by finding the differences between the first portfolio and last portfolio, given the data of one-way portfolios sort return.

B Appendix: Derivation

B.1 The Properties of New Sensitive Function

The Monotonicity of Sensitivity Given the first-order condition of traditional sensitivity function $\lambda^S = \frac{1}{\bar{S}} - 1$ is $\lambda^{S'} = -\frac{1}{\bar{S}^2}$, the tangent of new sensitivity function $\lambda^S = \frac{1}{\log \bar{S}} - 1$ is

$$\lambda^{S'} = -\frac{1}{\bar{S} \log \bar{S}^2} \tag{34}$$

Since the logarithm does not change the monotonicity, the new sensitivity function negatively moves to the surplus-consumption ratio, which is identical to the original sensitivity function.

The Monotonicity of Habits Following the initial sensitivity function of Campbell & Cochrane (1999), find the derivation of habit to consumption as follow:

$$\frac{dh_{t+1}}{dc_{t+1}} = 1 - \frac{\lambda(s_t)}{e^{-s_{t+1}} - 1} \approx 1 - \frac{\lambda(s_t)}{e^{-s_t} - 1} \tag{35}$$

Given $s_t = \bar{s}$, I simplify the derivation as:

$$\frac{dh_{t+1}}{dc_{t+1}} = 1 - \frac{\lambda^S}{e^{-s_{t+1}} - 1} \tag{36}$$

Thus, the habit might move negatively to consumption in my new specification which is in line with Chen (2017), Ljungqvist & Uhlig (2009, 2015).

Habits Stock: The Geometric Sum of Consumption If the habit is positive, we can log-linearise the surplus consumption ratio as:

$$s_t = \log S_t = \log\left(\frac{C_t - H_t}{C_t}\right) = \log\left(1 - \frac{H_t}{C_t}\right)$$
$$= \log[1 - exp(h_t - c_t)] \approx \kappa - (\lambda^s)^{-1}(h_t - c_t) \tag{37}$$

Plug it into the mean-reverting process of surplus consumption ratio we have:

$$h_{t+1} = (Constants) + \rho^S h_t + \left(1 - \rho^S\right) c_t - (\lambda^s)^2 \Delta c_{t+1}$$
$$= (Constants) + \left(1 - \rho^S\right) \sum_{j=0}^{\infty} \rho_j^S c_{t-j} + (\lambda^s)^2 \Delta c_{t+1} \tag{38}$$
$$\approx (Constants) + \left(1 - \rho^S\right) \sum_{j=0}^{\infty} \rho_j^S c_{t-j}$$

Thus, the habit is still the geometric sum of the consumption, as only the persistence affects the habit process—the sensitivity function merely plays a marginal role. The second case is, if the habit is negative, we log-linearise the surplus-consumption ratio as:

$$(-h_{t+1}) \approx (Constants) + \left(1 - \rho^S\right) \sum_{j=0}^{\infty} \rho_j^S c_{t-j} \tag{39}$$

The absolute value of habit is still the geometric sum of consumption.

B.2 Stochastic Discount Factor Decomposition

The stochastic discount factor is given by:

$$\beta \left(\frac{C_{t+1} S_{t+1} - \psi \frac{N_{t+1}^{1+\alpha}}{1+\alpha}}{C_t S_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha}}\right)^{-\sigma} \tag{40}$$

Take the logarithm of SDF we have:

$$\log \beta + (-\sigma) \log \left(\frac{C_{t+1} S_{t+1} - \psi \frac{N_{t+1}^{1+\alpha}}{1+\alpha}}{C_t S_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha}}\right)$$
$$= \log \beta + (-\sigma) \left\{\log \left(C_{t+1} S_{t+1} - \psi \frac{N_{t+1}^{1+\alpha}}{1+\alpha}\right) - \log \left(C_t S_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha}\right)\right\} \tag{41}$$

I rearrange the second component of equation (41) to be:

$$
\begin{aligned}
&\log\left(C_{t+1}S_{t+1} - \psi\frac{N_{t+1}^{1+\alpha}}{1+\alpha}\right) - \log\left(C_tS_t - \psi\frac{N_t^{1+\alpha}}{1+\alpha}\right) \\
&= \log\left(C_{t+1}S_{t+1}\left(1 - \frac{\psi}{1+\alpha} \times \frac{N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1}}\right)\right) - \log\left(C_tS_t\left(1 - \frac{\psi}{1+\alpha} \times \frac{N_t^{1+\alpha}}{C_tS_t}\right)\right) \\
&= \{\log(C_{t+1}S_{t+1}) - \log(C_tS_t)\} \\
&+ \left\{\log\left(1 - \frac{\psi}{1+\alpha} \times \frac{N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1}}\right) - \log\left(1 - \frac{\psi}{1+\alpha} \times \frac{N_t^{1+\alpha}}{C_tS_t}\right)\right\}
\end{aligned}
\tag{42}
$$

I also rearrange the second component of equation (42) as:

$$
\begin{aligned}
&\left\{\log\left(1 - \frac{\psi}{1+\alpha} \times \frac{N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1}}\right) - \log\left(1 - \frac{\psi}{1+\alpha} \times \frac{N_t^{1+\alpha}}{C_tS_t}\right)\right\} \\
&= \log\left(\frac{1 - \frac{\psi}{1+\alpha} \times \frac{N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1}}}{1 - \frac{\psi}{1+\alpha} \times \frac{N_t^{1+\alpha}}{C_tS_t}}\right) = \log\left(\frac{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha} \times N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha} \times \frac{C_{t+1}S_{t+1}}{C_tS_t} \times N_t^{1+\alpha}}\right)
\end{aligned}
\tag{43}
$$

We know $\frac{C_{t+1}S_{t+1}}{C_tS_t}$ moves on the same direction as $\log\left(\frac{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha} \times N_{t+1}^{1+\alpha}}{C_{t+1}S_{t+1} - \frac{\psi}{1+\alpha} \times \frac{C_{t+1}S_{t+1}}{C_tS_t} \times N_t^{1+\alpha}}\right)$. Therefore, one can focus on the analysis of $\log\left(\frac{C_{t+1}S_{t+1}}{C_tS_t}\right)$. Following the standard literature of habit formation, I decompose $\log\left(\frac{C_{t+1}S_{t+1}}{C_tS_t}\right)$ as:

$$
\begin{aligned}
\{\log(C_{t+1}S_{t+1}) - \log(C_tS_t)\} &= (\log C_{t+1} - \log C_t) + (\log S_{t+1} - \log S_t) \\
&= (\log C_{t+1} - \log C_t) + \left(\rho^S - 1\right)\log S_t + \left(1 - \rho^S\right)\log \bar{S} \\
&+ \left(\frac{1}{\log \bar{S}} - 1\right)\log(\frac{C_{t+1}}{C_t}) \\
&= \left(\rho^S - 1\right)\left(\log \bar{S} - \log S_t\right) + \frac{1}{\log \bar{S}}\log(\frac{C_{t+1}}{C_t}) \\
&= \left(\rho^S - 1\right)\left(\log \bar{S} - \log S_t\right) + \frac{1}{\log \bar{S}}\mu(g_c) + \frac{1}{2(\log \bar{S})^2}\sigma(g_c)
\end{aligned}
\tag{44}
$$

Plug the utility curvature to equation (44) we have:

$$
(-\sigma)\left[\left(\rho^S - 1\right)\left(\log \bar{S} - \log S_t\right) + \frac{1}{\log \bar{S}}\mu(g_c)\right] + \frac{(-\sigma)^2}{2(\log \bar{S})^2}\sigma(g_c) \tag{45}
$$

Hence, the coefficient of consumption growth $-\frac{\sigma}{\log \bar{S}}$ and consumption growth volatility $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$ work oppositely and capture the inter-temporal substitution and precautionary saving behaviours, respectively.

B.3 Producing Risk Premiums

As Cochrane (2005) suggests, the price of equity risk premiums is identical to zero, which delivers

$$E_t[\frac{\lambda_{t+1}}{\lambda_t}\left(R^e_{t+1} - R^f_{t+1}\right)] = 0 \Rightarrow E_t[\lambda_{t+1}\left(R^e_{t+1} - R^f_{t+1}\right)] = 0 \times \lambda_t = 0 \tag{46}$$

Thus, the conditional expected risk premiums is

$$\begin{aligned} E_t[\lambda_{t+1}\left(R^e_{t+1} - R^f_{t+1}\right)] &= E_t[\lambda_{t+1}]E_t[\left(R^e_{t+1} - R^f_{t+1}\right)] + Cov_t(\lambda_{t+1}, R^e_{t+1} - R^f_{t+1}) = 0 \\ \Rightarrow E_t[R^e_{t+1} - R^f_{t+1}] &= -\frac{Cov_t(R^e_{t+1} - R^f_{t+1}, \lambda_{t+1})}{E_t[\lambda_{t+1}]} \approx Cov_t(R^e_{t+1} - R^f_{t+1}, \lambda_{t+1}) \end{aligned} \tag{47}$$

C Appendix: Numerical Algorithm

This paper employs Winberry (2018)'s solution method. First, I approximate the cross-sectional distribution with finite dimension parametric family. Further, I compute the stationary equilibrium with projection method by combining the Chebyshev Polynomial approximation and Gaussian quadrature. At the end of the calculation, I solve the aggregate dynamics with second-order perturbation method. The advantage of such approach is it generates linear spaces in solution thus allows the VAR and Bayesian estimation. Additionally, Winberry (2018)'s approach does not require aggregate approximations.

C.1 Approximate Steady State with Projection

Using the finite-dimensional approach, I approximate the cross-sectional distribution of firm density, firm's value function and market-clearing conditions in this stage. More specifically, I approximate the law of motion for distribution by Winberry (2018)'s parametric family methodology and the value function by standard Chebyshev Polynomials. One step further, I approximate the equilibrium conditions with Gaussian quadrature. According to Khan & Thomas (2008), one can multiply the marginal utility with the value function before solving the whole system.

i. Logarithm Density Define $g(k,z)$ as the density definition of firm and $\mathbf{x}$ as the draws of aggregate shock, I follow Algan et al. (2008) and Winberry (2018) to approximate the firm density function concerning the log capital and idiosyncratic uncertainties under a functional form:

$$g(k,z) \cong g_0 exp\left\{ g_1^1(z-m_1^1) + g_1^2(\log - m_1^2) + \sum_{i=2}^{n_g}\sum_{j=0}^{i} g_i^j \left[(z-m_1^1)^{i-j}(\log k - m_1^2)^j - m_i^j\right]\right\} \tag{48}$$

where $\mathbf{g} = \left\{g_i^j\right\}_{i,j=(1,0)}^{(n_g,i)}$ and $\mathbf{m} = \left\{m_i^j\right\}_{i,j=(1,0)}^{(n_g,i)}$ represent the parameters vector and the vector of critical moments, respectively. Such a two-dimensional functional form contains a family of parameters. As Winberry (2018) predicts, a higher-order polynomials ($e.g., n_g > 2$) with this method can capture both of the skewness and excess kurtosis of the logarithm density functions.

Additionally, either the parameters vector $\mathbf{g}$ or moment vector $\mathbf{m}$ should be consistent with each other, which suggests that one could adopt parameters to denote the moments:

$$\begin{aligned} m_1^1 &= \int\int z g(k,z)\mathrm{dzdk} \\ m_1^2 &= \int\int \log k g(k,z)\mathrm{dzdk} \\ m_i^j &= \int\int (z-m_1^1)^{i-j}(\log k - m_1^2)^j g(k,z)\mathrm{dzdk}, i=2,...,n_g; j=0,...,i. \end{aligned} \tag{49}$$

Algan et al. (2008) propose a brief and robust approach to compute the system above. Given such consistency holds, one can resolve the law of motion for moment vectors by the next period's parameter vector:

$$\begin{aligned}
m_1^{1'} &= \int\int\int \left(\rho^Z z + \varepsilon^{z'}\right) g(k,z;m)p(\varepsilon^{z'})d\varepsilon^{z'}\text{dzdk} \\
m_1^{2'} &= \int\int\int \left[\frac{\hat{\xi}(k,z;\mathbf{x},\mathbf{m})}{\bar{\xi}} \log k^a(k,z;\mathbf{x},\mathbf{m}) \right. \\
&\quad \left. + \left(1 - \frac{\hat{\xi}(k,z;\mathbf{x},\mathbf{m})}{\bar{\xi}}\right) \log k^n(k,z;\mathbf{x},\mathbf{m})\right] g(k,z;\mathbf{m})p(\varepsilon^{z'})d\varepsilon^{z'}\text{dzdk} \\
m_i^{j'} &= \int\int\int (\rho^Z z + \varepsilon^{z'} - m_1^1)^{i-j} \left\{\frac{\hat{\xi}(k,z;\mathbf{x},\mathbf{m})}{\bar{\xi}} \left(\log k^a(k,z;\mathbf{x},\mathbf{m}) - m_1^{2'}\right)^j \right. \\
&\quad \left. + \left(1 - \frac{\hat{\xi}(k,z;\mathbf{x},\mathbf{m})}{\bar{\xi}}\right) \left(\log k^n(k,z;\mathbf{x},\mathbf{m}) - m_1^{2'}\right)^j\right\} g(k,z;\mathbf{m})p(\varepsilon^{z'})d\varepsilon^{z'}\text{dzdk}
\end{aligned} \tag{50}$$

The system above maps the future moments into current states, Winberry (2018) argues that the system of law of motion for moments can converge in the steady state moment vector $\mathbf{m}^*$ by using iterations even this mapping is non-linear.

ii. Value Function Using the typical orthogonal polynomial (usually Chebyshev Polynomials), the general form ex-ante value function is approximated by:

$$\hat{V}(k,z;\mathbf{x},m) \approx \sum_{i=1}^{n_z}\sum_{j=1}^{n_k} \theta_{ij}(\mathbf{x},m)T_i(z)T_j(k) \tag{51}$$

where n_z and n_k, $T_i(z)$ and $T_j(k)$, and θ_{ij} denote the order of approximation, the Chebyshev Polynomials, and the Chebyshev coefficients, respectively. In this case, I approximate the Bellman Equation by sets of grid points below:

$$\begin{aligned}
\hat{V}(k,z;\mathbf{x},\mathbf{m}) = {}& \lambda^c \max_n \left\{e^z e^x k^\theta n^\nu - w(\mathbf{x},\mathbf{m})n\right\} + (1-\delta)k \\
& + \frac{\hat{\xi}(k,z;\mathbf{x},\mathbf{m})}{\bar{\xi}} \left\{\lambda^c(-k^a(k,z\mathbf{x},\mathbf{m}) - w(\mathbf{x},\mathbf{m})\frac{\hat{\xi}(k,z;\mathbf{x},\mathbf{m})}{2}) \right. \\
& \left. + \beta E_{\mathbf{x}'|\mathbf{x}}[\int \hat{V}^a(\rho^Z z_i + \varepsilon^{z'}, k^{a'}; \mathbf{x}', \mathbf{m}')p(\varepsilon^{z'})d\varepsilon^{z'}]\right\} \\
& + \left(1 - \frac{\hat{\xi}(k,z;\mathbf{x},\mathbf{m})}{\bar{\xi}}\right) \left\{-\lambda^c k^n(k,z;\mathbf{x},\mathbf{m}) + \beta E_{\mathbf{x}'|\mathbf{x}}[\int \hat{V}^n(\rho^Z z_i + \varepsilon^{z'}, k^{n'}; \mathbf{x}', \mathbf{m}')p(\varepsilon^{z'})d\varepsilon^{z'}]\right\}
\end{aligned}$$

(52)

in which we compute the decision rules by derive the first-order conditions of the value function.

iii. Market-clearing Conditions Given the approximations above, this study computes the Markovian equilibrium with minimum residual methods. This study replaces the true aggregate state $(\mathbf{x}, g)$, Bellman Equation (52), and law of motion for firm distribution (49) with approximate aggregate state $AS = (\mathbf{x}, \mathbf{m})$, Chebyshev Tensor products (51), and parametric family approximations (50). Let f to be the whole system function with both state and control variables, the expectation of residual function f is denoted by:

$$E_{\varepsilon^{z'}}[f(y', y, AS', AS; \psi) = 0] \tag{53}$$

where $\mathbf{x}$ denotes the aggregate shock vector and AS denote the aggregate state vector. In the control variables vector $y = (\theta_t, g_t, \mathbf{k}^a, w, \lambda^c)$, $\mathbf{k}^a$ presents the target capital stock. To solve the expectation operator, one can employ the Gaussian quadrature. In this case, we adopt two-dimensional Gauss-Legendre quadrature to approximate the cross-sectional distribution and one-dimensional Gauss-Hermite quadrature to compute the idiosyncratic shocks.

C.2 Resolve the Equilibrium with Idiosyncratic Shocks

To compute the stationary equilibrium, one should first assume the aggregate shock vector $\mathbf{x}$ to be zero such that the aggregate state $AS^* = (0, \mathbf{m}^*)$. Hence, the corresponding control variable vector tends to be $y = (\theta^*, g^*, \mathbf{k}^a*, w^*, \lambda^{c*})$, which results in the stationary equilibrium system:

$$f(y^*, y^*, AS^*, AS^*; 0) = \mathbf{0} \tag{54}$$

Note that in Winberry (2018)'s method, we merely solve the steady state wage by root-finding method to avoid wasting the computation time.

C.3 Compute the Macro-Financial Dynamics with Perturbation

At the final step, we resolve the dynamic equilibrium conditions in equation (54) with perturbation approach. Only the productivity shock hit the economy, which is summarized by aggregate shock vector $\mathbf{x}$ above. A possible solution to macro dynamics is given by:

$$
\begin{aligned}
&y = g(x, \psi) \\
&\mathbf{x}' = h(\mathbf{x}; \psi) + \psi \times \eta \varepsilon_{\mathbf{x}}^{'}, \varepsilon_{\mathbf{x}}^{'} = (\mathbf{1}_3, \mathbf{0}_{n_g \times 1})'
\end{aligned}
$$

yielding the first-order Taylor Expansions around the point $\psi = 0$

$$
\begin{aligned}
&g(\mathbf{x}; 1) = g_{\mathbf{x}}(\mathbf{x}^*; 0)(\mathbf{x} - \mathbf{x}^*) + g_{\psi}(\mathbf{x}^*; 0) \\
&h(\mathbf{x}; 1) = h_{\mathbf{x}}(\mathbf{x}^*; 0)(\mathbf{x} - \mathbf{x}^*) + h_{\psi}(\mathbf{x}^*; 0) + \eta \varepsilon_{\mathbf{x}}^{'}
\end{aligned}
$$

Second-order Perturbation The logic of using higher-order perturbations is analogous. I follow Winberry (2018) to solve the partial derivatives $g_{\mathbf{x}}, g_{\psi}, h_{\mathbf{x}}, h_{\psi}$ given the partial derivative of equilibrium conditions $f_{\mathbf{x}}, f_{\mathbf{x}'}, f_y f_{y'} f_{\psi}$ and then present the algorithm of second-order perturbation with $\psi \neq 0$ below:

$$
g(\mathbf{x}; \psi) = g_{\psi}(\mathbf{x}^*; 0) + g_{\mathbf{x}}(\mathbf{x}^*; 0)(\mathbf{x} - \mathbf{x}^*) + g_{\psi}(\mathbf{x}^*; 0)\psi + \frac{1}{2}(\mathbf{x} - \mathbf{x}^*; 0) H_g \begin{pmatrix} \mathbf{x} - \mathbf{x}^* \\ \psi \end{pmatrix}
$$

$$
h(\mathbf{x}; 1) = h_{\psi}(\mathbf{x}^*; 0) + h_{\mathbf{x}}(\mathbf{x}^*; 0)(\mathbf{x} - \mathbf{x}^*) + h_{\psi}(\mathbf{x}^*; 0)\psi + \frac{1}{2}(\mathbf{x} - \mathbf{x}^*; 0) H_h \begin{pmatrix} \mathbf{x} - \mathbf{x}^* \\ \psi \end{pmatrix}
$$

where

$$
H_g \begin{pmatrix} \mathbf{x} - \mathbf{x}^* \\ \psi \end{pmatrix} = \begin{pmatrix} g_{xx}(\mathbf{x}^*; 0) & \cdots & g_{x\psi}(\mathbf{x}^*; 0) \\ \cdots & \cdots & \cdots \\ g_{\psi x}(\mathbf{x}^*; 0) & \cdots & g_{\psi\psi}(\mathbf{x}^*; 0) \end{pmatrix}
$$

$$
H_h \begin{pmatrix} \mathbf{x} - \mathbf{x}^* \\ \psi \end{pmatrix} = \begin{pmatrix} h_{xx}(\mathbf{x}^*; 0) & \cdots & h_{x\psi}(\mathbf{x}^*; 0) \\ \cdots & \cdots & \cdots \\ h_{\psi x}(\mathbf{x}^*; 0) & \cdots & h_{\psi\psi}(\mathbf{x}^*; 0) \end{pmatrix}
$$

D Appendix: Comparative Statistics

In section D, I study the mechanisms that impact on macro-financial fluctuations by comparing the calibration results of several critical specifications. I first evaluate model performances of various utility functions in the GHH framework. Besides, I examine the aggregate implications of various additional micro frictions in recent studies of lumpy investment.

D.1 Alternative Household Preferences

Habits with Actual Consumption In section **D.1**, I examine the model performance with various habit preferences. Let me restate the utility function as:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - H_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} \right\} \tag{55}$$

with habit stock of actual consumption and related marginal utility:

$$H_t = \tau C_{t-1}$$
$$\lambda_t^c = \left(C_t - \tau C_{t-1} - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{-\sigma}$$

Habits with Net Consumption Given the same utility function, I follow Winberry (2020) to introduce the net consumption bundle $\hat{C}_t = C_t - \psi \frac{N_{t-1}^{1+\alpha}}{1+\alpha}$ into external habits and redefine the habit stock and marginal utility as:

$$H_t = \tau \left(C_{t-1} - \psi \frac{N_{t-1}^{1+\alpha}}{1+\alpha} \right)$$
$$\lambda_t^c = \left(C_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} - \tau \left(C_{t-1} - \psi \frac{N_{t-1}^{1+\alpha}}{1+\alpha} \right) \right)^{-\sigma}$$

No Habits The model without habit formation is a special case of the 'Keeping up with the Joneses' habit formation with $\tau = 0$. It delivers the following marginal utility:

$$\lambda_t^c = \left(C_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{-\sigma}$$

Discussions **Table 10** displays comparative statistics with different utility functions. For the 'Catching up with the Joneses' preference, I follow Winberry (2020) and pick τ to be 0.73. Numerical simulations suggest that the benchmark model outperforms other alternatives for (i) it can produce sizable equity volatility and fit the Sharpe ratio better, (ii) it is robust in matching investment dynamics, (iii) it smoothes risk-free rate dynamics.

Table 10: Habit Formation: Moment Comparisons

Models	CC		KJ		No Habits	Data
	$\lambda^S = \frac{1}{\log S} - 1$	$\lambda^S = \frac{1}{S} - 1$	actual C_t	$\hat{C}_t$		
$\sigma(y_t)$	1.56%	1.61%	1.78%	1.59%	1.58%	1.51 %
$\sigma(c_t)/\sigma(y_t)$	0.82	1.23	1.16	0.60	0.77	0.80
$\sigma(i_t)/\sigma(y_t)$	2.92	0.51	4.40	3.39	2.38	2.89
$\sigma(n_t)/\sigma(y_t)$	0.67	0.67	0.67	0.67	0.67	1.21
$\rho(c_t, y_t)$	0.93	0.99	0.80	0.99	0.99	0.85
$\rho(i_t, y_t)$	0.82	-0.65	0.33	0.99	0.99	0.88
$\rho(n_t, y_t)$	1.00	1.00	1.00	1.00	1.00	0.87
$\rho(c_{t-1}, c_t)$	0.70	0.73	0.94	0.77	0.72	0.81
$\rho(i_{t-1}, i_t)$	0.76	0.88	0.76	0.68	0.71	0.80
$\rho(n_{t-1}, n_t)$	0.71	0.72	0.67	0.72	0.72	0.90
$\rho(y_{t-1}, y_t)$	0.71	0.72	0.76	0.72	0.72	0.85
$E(r^e - r^f)$	1.79%	1.56%	1.78%	6.05%	1.77%	1.54%
$std(r^e - r^f)$	7.72%	1.43%	1.58%	11.31%	1.18%	7.74%
$E(r^f)$	0.88%	1.01%	1.02%	0.90%	0.98%	0.40%
$std(r^f)$	0.57%	0.09%	0.15%	0.26%	0.12%	0.99%
$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	1.09	1.13	0.53	1.51	0.20

D.2 Microeconomic Frictions

The Investment-Specific Shock In Section D.2, I investigate the role of microeconomic frictions by introducing the IST shock, micro-uncertainty, and convex costs additional to the benchmark model. Following Winberry (2018), I start with the investment-specific shock and jointly define the capital accumulation to be:

$$\begin{aligned} k_{i,t+1} &= (1-\delta)k_{i,t} + e^{q_t} i_{i,t} \\ q_{t+1} &= \rho^q q_t + \sigma^q \varepsilon^q_{t,t+1} + \sigma^{qx} \varepsilon^{qz}_{t,t+1} \end{aligned} \tag{56}$$

where ρ^q, σ^q, and σ^{qx} define the persistence, volatility, and the correlation to TFP shock of the investment-specific shock.

The Uncertainty Shock To introduce the time-varying uncertainty of firm dispersion, I follow Mongey & Williams (2019) to modify the idiosyncratic shock as $z_{i,t+1} = \rho^Z z_i + \varepsilon^z_{i,t+1}, \varepsilon^z_{i,t+1} \backsim N(0, X^U_t \bar{\sigma})$ and denote the uncertainty shock to be an AR(1) process:

$$X^U_{t+1} = \rho^U \log X^U_t + \varepsilon^U_{t+1}, \varepsilon^U_{t+1} \backsim N(0, \sigma^U) \tag{57}$$

where ρ^U and σ^U present the persistence and volatility of micro uncertainty.

The Convex Capital Adjustment Costs The convex costs is denoted by:

$$\Phi(I_{i,t}/k_{i,t}) = \frac{\lambda_x}{2}\left(\frac{I_{i,t}}{k_{i,t}}\right)^2 k_{i,t}$$

which also delivers a new value function as:

$$
\begin{aligned}
\hat{V}(k_{i,t}, z_{i,t};\Omega_t) = {} & \lambda^c(\Omega_t)\max_n \left\{e^{z_{i,t}}e^{x_t}k_{i,t}^{\theta}n_{i,t}^{\nu} - w(\Omega_t)n_t\right\} \\
& + \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\left\{V^a(k_{i,t}, z_{i,t};\Omega_t) - \lambda^c(\Omega_t)w(\Omega_t)\frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{2}\right\} \\
& + \left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\right)V^n(k_{i,t}, z_{i,t};\Omega_t) \\
& - \lambda^c(\Omega_t)\frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}} \times \frac{\lambda_x}{2}(\frac{k^a(k_{i,t}, z_{i,t};\Omega_t)}{k_{i.t}} - (1-\delta))^2 \\
& - \lambda^c(\Omega_t)\left(1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t};\Omega_t)}{\bar{\xi}}\right) \times \frac{\lambda_x}{2}(\frac{k^n(k_{i,t}, z_{i,t};\Omega_t)}{k_{i,t}} - (1-\delta))^2
\end{aligned}
\tag{58}
$$

I solve the new value function with the same numerical algorithm and consider fixed capital adjustment costs as the special case with $\lambda_x = 0$.

Discussions **Table 11** presents calibration results with various micro frictions. I pick parametric values of the IST shock and micro uncertainty estimated by Winberry (2018) and Mongey & Williams (2019), respectively. All additional micro frictions have marginal effects on macro-asset pricing empirics. The benchmark model has similar performance in matching business cycle dynamics and dominates alternative models in fitting financial moments, particularly the Sharpe ratio.

Table 11: The Microeconomic Frictions: Comparisons

Models	Benchmark	IST Shock	Uncertainty Shock	Convex Costs	Data
		$\rho_z = 0.9695$	$\rho_z = 0.775$	$\lambda^x = 0.005$	
		$\sigma_z = 0.006 \sigma_{qz} = -0.0043$	$\sigma_z = 0.002$		
$\sigma(y_t)$	1.56%	1.58%	1.58 %	1.58 %	1.51 %
$\sigma(c_t)/\sigma(y_t)$	0.82	0.80	0.80	0.73	0.80
$\sigma(i_t)/\sigma(y_t)$	2.92	2.79	2.67	2.97	2.89
$\sigma(n_t)/\sigma(y_t)$	0.67	0.67	0.67	0.67	1.21
$\rho(c_t, y_t)$	0.93	0.95	0.96	0.94	0.85
$\rho(i_t, y_t)$	0.82	0.88	0.88	0.85	0.88
$\rho(n_t, y_t)$	1.00	1.00	1.00	1.00	0.87
$\rho(c_{t-1}, c_t)$	0.70	0.70	0.70	0.71	0.81
$\rho(i_{t-1}, i_t)$	0.76	0.76	0.76	0.74	0.80
$\rho(n_{t-1}, n_t)$	0.71	0.71	0.71	0.71	0.90
$\rho(y_{t-1}, y_t)$	0.71	0.71	0.71	0.71	0.85
$E(r^e - r^f)$	1.79%	1.57%	1.39%	2.66%	1.54%
$std(r^e - r^f)$	7.72%	5.70%	4.95%	7.76%	7.74%
$E(r^f)$	0.88%	0.96%	0.97%	0.88%	0.40%
$std(r^f)$	0.59%	0.48%	0.45%	0.49%	0.99%
$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	0.27	0.28	0.34	0.20