

# Firm Heterogeneity in Production-Based Asset Pricing: The Role of Habit Sensitivity and Lumpy Investment

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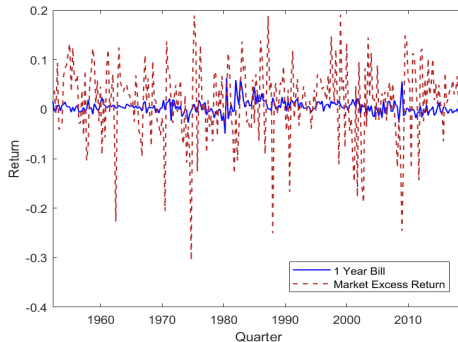
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December 14th, 2020

# Evidence 1: Risky and Risk-Free Assets

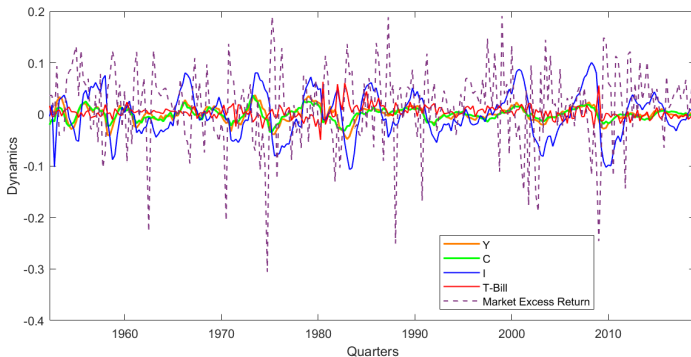
Figure 1: Risky and Risk-Free Asset



- ▶  $r^e - r^f$  is High and Volatile
- ▶  $r^f$  is Low and Smooth
- ▶ 1st Challenge: Market Excess Return is High & Volatile but Risk-Free Rate is Low & Smooth

# Evidence 2: Asset Prices & Real Economy

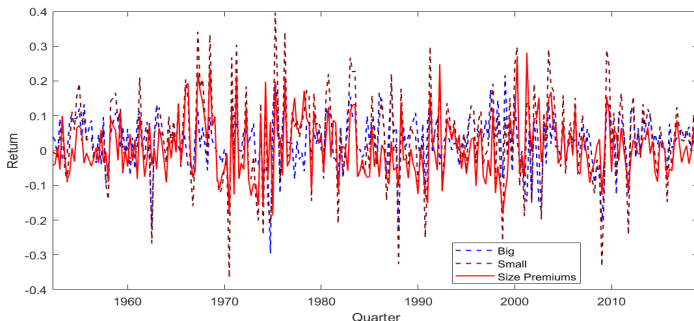
Figure 2: Macro-Asset Pricing Dynamics



- ▶ Market Excess Return is High but Consumption is Smooth  $\Rightarrow$  2nd Challenge
- ▶ Market Excess Return is High and Investment Dynamics are Volatile  $\Rightarrow$  3rd Challenge

# Evidence 3: Size Premiums

Figure 3: Size Premiums



- ▶ Small Firms earn Higher Expected Return than Big Firms  $\Rightarrow$  Size Premiums
- ▶ Size Premiums are no Small (0.66% Quarterly or 2.5%-7% Annually) and Volatile (8.69% Quarterly or 17-20% Annually)  $\Rightarrow$  4th Challenge

# Motivation: Two Aspects of Asset Pricing Puzzles

**Challenges:** The G.E. Model Has Difficulty in.....

- ▶ Matching Equity Risk Premiums and Risk-Free Rate jointly
  - ▶ Inter-temporal Effects on Risk-Free Rate Dominates...
- ▶ Matching Macro Quantities and Equity Risk Premiums jointly
  - ▶ Consumption Smoothing  $\Rightarrow$  Stochastic Discount Factor Less Volatile  
 $\Rightarrow$  Underestimate 1st/2nd-Order Moments of Risk Premiums
  - ▶ Adjustment Costs  $\Rightarrow$  Risk Premiums  $\uparrow$  and Investment Volatility  $\downarrow$
- ▶ Replicating the Time-Series and Cross-Sectional Stock Return jointly
  - ▶ Lumpy Investment Model + Decreasing Return to Scale  
 $\Rightarrow$  High Prod. Firms have High Market-to-Book Ratio but More Risky  
 $\Rightarrow$  Value Premiums might be Negative (Favilukis & Lin 2010WP)

**Inconsistency:**

- ▶ Most Production-Based Models Rely on Convex Costs  $\Rightarrow$  Empirically Rejected  
 $\Rightarrow$  Micro-Level Investment is Lumpy
- ▶ Calibration with Unreasonable Parameters.

# What I Did: Resolve Four Challenges in Both Aspects

## Build a G.E. Heterogeneous Firm Macro Model with Following Ingredients:

- ▶ Campbell & Cochrane (1999JPE)'s Habit + GHH Structure
- ▶ Khan & Thomas (2008Ecta)'s Fixed Capital Adjustment Costs
- ▶ New Habit Sensitivity Function:  $\lambda^S = \frac{1}{\bar{S}} - 1$  vs.  $\lambda^S = \frac{1}{\log \bar{S}} - 1$

**Results:** The Model Replicates

Table 1: Compare to Previous Literature

Facts	Equity +Premium + Volatility	Business Cycle	Micro Lumpy Investment	Cross-Sectional Stock Return
Cooper et al. (2006JF)	✓	✗	✓	✓
Herskovic (2020WP)	✓	✗	✗	✓
Favilukis&Lin (2015RFS)	✓	✓	✗	✗
Chen (2017RFS,2018RAPs)	✓	✓	✗	✗
Favilukis&Lin (2013JME)	✗	✓	✓	✗
Winberry(2020AER)	✗	✓	✓	✗
This Paper	✓	✓	✓	✓

## Micro Foundation:

- ▶ Fixed Capital Adjustment Costs + Reasonable Calibrated Values

# Literature Review

- ▶ Asset Pricing with Habits:

Boldrin et al. (1999AER), Jermann (1998JME); Campbell & Cochrane (1999JPE); Chen (2017RFS, RAPS), Luo (2019WP)

- ▶ Macro Implications of Non-Convex Costs:

Khan & Thomas (2008Ecta), House (2014JME), Clementi et al. (2017WP), Mongey Williams (2017WP), Winberry (2020AER), Koby & Wolf (2020WP)

- ▶ Asset Pricing with Non-Convex Costs:

Kogan (2004JFE), Carlson et al. (2004JF), Zhang (2005JF); Cooper(2006JF), Herskovic et al. (2020WP); Favilukis & Lin (2010WP)

- ▶ G.E. Model with Heterogeneous Firms:

Luo (2019WP); Ai et al. (2013RFS), Favilukis & Lin (2010WP, 2015RFS), Chen (2018RAPS); Gomes et al. (2003JPE), Garleanu et al. (2012JF)

- ▶ Model Set Up
  - ▶ Physical Environment, Firm, and Households Problem
  - ▶ Equilibrium Conditions and Numerical Algorithm
- ▶ Solution to Challenges
- ▶ Benchmark Calibration: Target Moments
- ▶ Model Evaluation
  - ▶ Time-Series Implications
  - ▶ Cross-Sectional Implications
- ▶ Conclusion
- ▶ Appendix



# Model Set Up

# Environment: Discrete Time and Infinite Horizons

## Household:

- ▶ Consumes Consumption Good  $C_t$
- ▶ Supplies Labor  $N_t$  at Competitive Wage  $W_t$
- ▶ Receives Aggregate Dividends  $\Pi_t$  from Firms

**Firms:** Continuum of Firms, Each with Idiosyncratic State  $\mu_{i,t} = \mu(k_{i,t}, z_{i,t}, \xi_{i,t})$

- ▶ Hire Labour  $n_{i,t}$  at Competitive Wage  $W_t$ ,
- ▶ Produce Goods with Labour  $n_{i,t}$  and Capital Stock  $k_{i,t}$ 
  - ▶ Production Function with Decreasing Returns to Scale
$$y_{i,t} = e^{z_{i,t}} e^{x_{i,t}} k_{i,t}^\theta n_{i,t}^\nu$$
- ▶ Choose whether to Adjust Capital or not
  - ▶ if Adjust  $\Rightarrow$  Pay  $\xi_{i,t}$  and Choose next Period's Capital Stock  $k_{i,t+1}$
- ▶ Pay Out Firm-Level Dividends  $\pi_t$

## Shock:

- ▶ Aggregate Shock:  $x_{t+1} = \log X_{t+1} = \rho^X \log X_t + \varepsilon_{i,t+1}^X, \varepsilon_{i,t+1}^X \sim N(0, \sigma_x)$
- ▶ Firm-Level Shock:  $z_{i,t+1} = \log Z_{i,t+1} = \rho^Z \log Z_{i,t} + \varepsilon_{i,t+1}^Z, \varepsilon_{i,t+1}^Z \sim N(0, \sigma_z)$

- ▶ Fixed Capital Adjustment Costs  $\Rightarrow$  Capture Microeconomic Lumpy Investment Behaviour.
- ▶ Each Firm Faces a Binary Choice: Invest or Not.
- ▶ Investment Project Brings Extra Value to Firm  $\Rightarrow$  Firm Chooses to Invest.
- ▶  $V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t) \xi_{i,t} w(\Omega_t) > V^n(k_{i,t}, z_{i,t}; \Omega_t)$
- ▶ Inaction Firms  $\Rightarrow$  Small Market Equities + Issue Dividends  $\Rightarrow$  Cash Flow Risks  $\uparrow \Rightarrow$  Size Premiums  $\uparrow$
- ▶ Adjustment Probability  $\downarrow \Rightarrow$  Expected Return  $\uparrow \Rightarrow$  Equity Premiums  $\uparrow$

# Household Problem

- ▶ Habit Formation:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left( C_t - H_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} \right\} \quad (1)$$

- ▶ Surplus Consumption Ratio:  $S_t \equiv \frac{C_t - H_t}{C_t}$

$$s_{t+1} = \log S_{t+1} = (1 - \rho^S) \log \bar{S} + \rho^S \log S_t + \lambda^S \log \left( \frac{C_{t+1}}{C_t} \right) \quad (2)$$

- ▶ Habit Sensitivity Function:  $\lambda^S = \frac{1}{\log \bar{S}} - 1$

- ▶ Stochastic Discount Factor:  $M_{t,t+1} = \beta \left( \frac{C_{t+1} S_{t+1} - \psi \frac{N_{t+1}^{1+\alpha}}{1+\alpha}}{C_t S_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha}} \right)^{-\sigma}$

# Equilibrium Conditions

An Equilibrium of This Model Consists of

- ▶ Firms' Value Function:  $V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$
- ▶ Firms' Policy Functions:  $k(k_{i,t}, z_{i,t}; \Omega_t)$ ,  $n(k_{i,t}, z_{i,t}; \Omega_t)$ , and  $\hat{\xi}$
- ▶ Households' Policies  $C(\Omega_t)$ ,  $N(\Omega_t)$ , and Stochastic Discount Factor  $M(\Omega_{t+1}|\Omega_t)$
- ▶ a Wage  $W(\Omega_t)$
- ▶ a Law of Motion for the Distribution of Firm  $g(k_{i,t}, z_{i,t}; \Omega_t)$

such that

- ▶ the Firms' Policies are Optimal
- ▶ the Households' Policies are Optimal
- ▶ the Labour Market Clears

$$\left(\frac{w(\Omega_t)}{\chi}\right)^{\frac{1}{\alpha}} = \int \left( n(k_{i,t}, z_{i,t}; \Omega_t) + \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)^2}{2\bar{\xi}} \right) g(k, z) dz dk \quad (3)$$

- ▶ the Law of Motion of the Distribution is Consistent with the Firm's Policies

# Algorithm: Reiter (2009JEDC) & Winberry (2018QE)

## Numerical Method: Projection+Perturbation Method

- ▶ A G.E. Production-Based Asset Pricing Model:
  - ▶ The Cross-Section Distribution of Firms is a State Variable
- ▶ Step One: Projection in Stationary Distribution
  - ▶ Approximate the Cross-Sectional Distribution with Finite Dimension Parametric Family  $\Rightarrow$  Reduce Dimension
  - ▶ Capture the Law of Motion of Distribution
  - ▶ Compute the Stationary Equilibrium with Projection Method
- ▶ Step Two: Perturbation in Aggregate Dynamics—Second-Order Perturbation

## Solution to Challenges

# 1. Equity Premium & Risk-Free Rate Puzzle

- ▶ Challenge: High & Volatile Equity Premium and Low & Smooth Risk-Free Rate
- ▶ Reason: the Intertemporal Substitution Effects on Risk-Free Rate
- ▶ Solution: Precautionary Saving Motivations Offset the Intertemporal Effects
- ▶ Intuition: Same Story in Chen (2017RFS, 2018RAPS)
  - ▶ SDF Decomposition  $\frac{C_{t+1}S_{t+1}}{C_t S_t}$  Plays a Key Role
  - ▶ Take Logarithm and Extend the  $\frac{C_{t+1}S_{t+1}}{C_t S_t}$

$$\left\{ (-\sigma) \left[ \left( \rho^S - 1 \right) (\log \bar{S} - \log S_t) + \frac{1}{\log \bar{S}} \mu(g_c) \right] + \frac{(-\sigma)^2}{2(\log \bar{S})^2} \sigma(g_c) \right\}$$

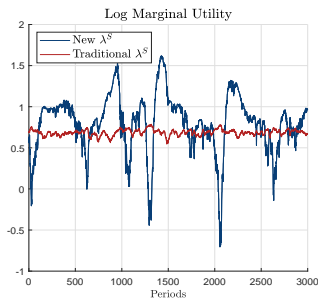
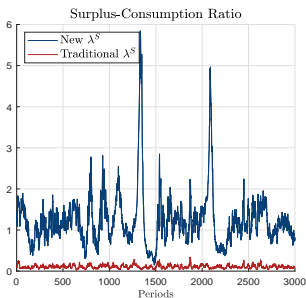
- ▶ The Sign of  $-\frac{\sigma}{\log \bar{S}}$  and  $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$  are Opposite  $\Rightarrow$  Low & Smooth Risk-Free Rate



## 2. Equity Premium & Volatility Puzzle

- ▶ Challenge: High & Volatile Equity Premium under Consumption Smoothing
- ▶ Reason: Household Endogenously Smoothes Consumption in a G.E. Model
- ▶ Solution: Novel Habit Sensitivity Produces Volatile SDF
- ▶ Intuition: Habit Destruction and Negative Habit

Figure 4:  $\lambda^S$  and SDF: the Way to Replicate Equity Volatility



# Mathematically

$$\blacktriangleright S_{t+1} = \bar{S}^{1-\rho_s} S_t^{\rho_s} \left( \frac{C_{t+1}}{C_t} \right)^{(\lambda^S)}$$

$$\blacktriangleright \lambda^S = \frac{1}{\log \bar{S}} - 1 \text{ vs } \lambda^S = \frac{1}{\bar{S}} - 1.$$

$$\blacktriangleright \log \bar{S} < \bar{S}$$

$$\Rightarrow \frac{1}{\log \bar{S}} - 1 > \frac{1}{\bar{S}} - 1$$

$$\Rightarrow \text{Var} \left\{ \bar{S}^{1-\rho_s} S_t^{\rho_s} \left( \frac{C_{t+1}}{C_t} \right)^{\left( \frac{1}{\log \bar{S}} - 1 \right)} \right\} > \text{Var} \left\{ \bar{S}^{1-\rho_s} S_t^{\rho_s} \left( \frac{C_{t+1}}{C_t} \right)^{\left( \frac{1}{\bar{S}} - 1 \right)} \right\}$$

given that  $\frac{1}{\log \bar{S}} - 1 > 0$  and  $S_{t+1} > 1$ .

- ▶ The Agent Destroys Her Habit Periodically (Dunlap 1928; Ljungqvist & Uhlig 2009, 2015)

- ▶ Consumption is Smooth but Marginal Utility is Volatile

$\Rightarrow$  SDF is Volatile

$\Rightarrow$  Risk Premiums are High:  $E_t[R_{t+1}^e - R_{t+1}^f] = -\text{Cov}_t(R_{t+1}^e - R_{t+1}^f, \lambda_{t+1})$

$\Rightarrow$  Risk Premiums are Volatile:  $R_{t+1}^e = \frac{V_{t+1} + D_{t+1}}{V_t}$

$\Rightarrow$  A Solution for the 2nd Challenge

### 3. High Risk Premiums with Volatile Aggregate Investment

Equity Premium Puzzle in the Production-Based Asset Pricing Literature

- ▶ Challenge: High Risk Premiums with Volatile Aggregate Investment
- ▶ Reason: Asset Prices rather than Investment Absorb the Productivity Shock
- ▶ Intuition: Idiosyncratic Risks Offset the Fixed Adjustment Costs
- ▶ Solution: A Story in Khan & Thomas (2008)

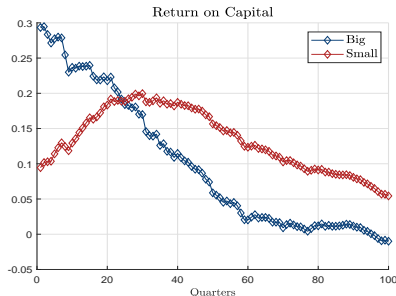
Micro Frictions: Adjustment Probabilities  $\downarrow \Rightarrow$  Risk Premiums  $\uparrow$

- ▶ Have Similar Implications on Risk Premiums
  - ▶ Upper Bound of Fixed Costs  $\uparrow \Rightarrow$  Adjustment Probabilities  $\downarrow$
  - ▶ Idiosyncratic Risks  $\uparrow \Rightarrow$  Adjustment Probabilities  $\downarrow$
- ▶ Have Opposite Implications on Aggregate Investment
  - ▶ Upper Bound of Fixed Costs  $\uparrow \Rightarrow$  Investment Volatility  $\downarrow$
  - ▶ Idiosyncratic Risks  $\uparrow \Rightarrow$  Investment Volatility  $\uparrow$

## 4. High Cross-Sectional Stock Return

- ▶ I Specify on the Lumpy Investment Model
- ▶ Challenge: Cross-Sectional Stock Return—Size and Value
- ▶ Reason: Operational Leverage  $\Rightarrow$  Negative Value Premiums
- ▶ Solution: Change the Direction to Size Premium
- ▶ Intuition: Small Firms Absorb More Productivity Shock

Figure 5: The Impulse Response Function of TFP to Return on Big/Small Firm



# Benchmark Calibration

Table 2: Benchmark Identification: Target Moments

Parameter	Value	Target Moments	Model	Data
$\rho^X$	0.95	$\sigma(y_t)$	1.56%	1.51%
$\sigma^X$	0.007	$\sigma(i_t)/\sigma(y_t)$	2.92	2.89
$\bar{\xi}$	0.25	$\rho(i_t, y_t)$	0.82	0.88
$\log \bar{S}$	0.1	$E(r^e - r^f)$	1.79%	1.54%
$\rho^S$	0.975	$\sigma(r^e - r^f)$	7.72%	7.74%
$\lambda^S$	$\frac{1}{\log \bar{S}} - 1$	$\frac{E(r^e - r^f)}{\sigma(r^e - r^f)}$	0.23	0.20

- Strategy & Targets
  - Asset Pricing: Moments of Risky Assets
  - Macroeconomics: Moments of Investment Cyclicity and Volatility
- Just-Identified: 6 Parameters vs. 6 Moments

# Model Evaluation

## Using Non-target Moments to Evaluate Performances

- ▶ Time-Series Implications
  - ▶ Asset Prices Dynamics
    - ▶ Asset Prices Moments
    - ▶ IRF of TFP to Asset Prices
    - ▶ The Predictability of Market Excess Return
  - ▶ Macroeconomic Implications
    - ▶ Business Cycle Moments
    - ▶ IRF of TFP to Macro Variables
- ▶ Cross-Sectional Implications
  - ▶ The Cross-Sectional Distribution of Investment Rate
  - ▶ Size Premiums

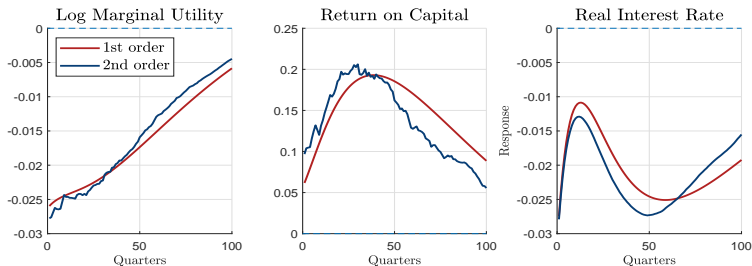
Table 3: Benchmark Model: Time-Series Asset Pricing Moments

Equity	Benchmark	Data
$E(r^e - r^f)$	1.79%	1.54%
$\sigma(r^e - r^f)$	7.72%	7.74%
$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	0.20
T-Bills		
$E(r^f)$	0.88%	0.40%
$\sigma(r^f)$	0.57%	0.99%

**What to Take Home:** Successfully Avoids Risk-Free Rate Puzzle



Figure 6: Impulse Response Function of TFP Shock to Asset Return



- ▶ Marginal Utility is Counter-Cyclical
- ▶ Ex-Post Return is Pro-Cyclical  $\Rightarrow$  Expected Return is Positive
  - ▶  $E_t[R_{t+1}^e - R_{t+1}^f] = -Cov_t(R_{t+1}^e - R_{t+1}^f, \lambda_{t+1})$
- ▶ First-Order and Second-Order Perturbation
  - ▶ Second-Order Perturbation is Non-Smooth  
 $\Leftrightarrow$  Captures Infra-Marginal Properties of Value Function & Investment

Table 4: Benchmark Model: Return Prediction of Dividend-Price Ratio

Horizons	Benchmark		Full Sample 1952-2018	
	Coefficients	$R^2$	Coefficients	$R^2$
1 Quarter	0.0462	0.0034	0.0178	0.0087
1 Year	0.2295	0.0255	0.0811	0.0377
2 Year	0.5739	0.0405	0.1401	0.0577
3 Year	1.0473	0.0610	0.1652	0.0640
4 Year	1.5592	0.0777	0.1745	0.0582
5 Year	1.9942	0.0832	0.2142	0.0683

$$E_t[r_{t+k}] - r_{t+k}^f = a + \beta_{dp} \times \frac{d_t}{p_t} + \epsilon_{t+k} \quad (4)$$

**What to Take Home:** Predicts the Correct Sign and Trend

- ▶ Regressive Fitness Rises with the Increases in Investment Horizons
- ▶ Dividend Yields **Positively** Associate with Risk Premiums
- ▶ Magnitudes of Simulated Results are Comparable to Empirical Evidence

**Table 5:** Benchmark Model: Macroeconomic Implications

Variables	Benchmark			
	$\sigma(x_t)$	$\sigma(x_t)/\sigma(y_t)$	$\rho(x_t, y_t)$	$\rho(x_t, x_{t-1})$
$y_t$	1.56%	1	1	0.71
$c_t$	1.28%	0.82	0.93	0.70
$i_t$	4.54%	2.92	0.82	0.76
$n_t$	1.04%	0.67	1.00	0.71
	Data			
	$\sigma(x_t)$	$\sigma(x_t)/\sigma(y_t)$	$\rho(x_t, y_t)$	$\rho(x_t, x_{t-1})$
$y_t$	1.51%	1	1	0.85
$c_t$	1.20%	0.80	0.85	0.81
$i_t$	4.35%	2.89	0.88	0.80
$n_t$	1.84%	1.21	0.87	0.90

**What to Take Home:** Similar to a Single Shock RBC Model

- ▶ Matches Volatilities of Consumption, Investment, and Output
- ▶ Underestimates Co-movements and Overestimates Persistence

**Table 6:** Benchmark Model: Microeconomic Moments

Distribution	Benchmark	Data
Mean	9.93%	10.4%
Standard Deviation	12.03%	16.0%
Skewness	3.08	3.60
Excess Kurtosis	12.36	17.6
Investment Patterns		
Spike ( $> 20\%$ )	15%	14.4%
Positive( $0\% - 20\%$ )	85%	85.6%

**What to Take Home:** Assumes Non-negative Investment Rate  $\Rightarrow$  Spike

- ▶ Uses Depreciation Rate to Match Average Investment Rate
- ▶ Replicates Key Moments of Distribution and Patterns of Micro Investment

Table 7: Benchmark Model: Cross-Sectional Asset Pricing Moments

Moments	Model		Data		
	Half-Sort	Half-Sort	3 Portfolios	5 Portfolios	10 Portfolios
$E(r^s)$	3.02%	3.47%	3.39%	3.34%	3.33%
$E(r^b)$	1.91%	3.01%	2.76%	2.73%	2.68%
$E(r^s - r^b)$	1.11%	0.46%	0.63%	0.61%	0.66%
$std(r^s)$	6.74%	10.78%	11.51%	11.88%	12.27%
$std(r^b)$	7.13%	7.85%	7.76%	7.67%	7.61%
$std(r^s - r^b)$	4.48%	5.09%	6.76%	7.66%	8.69%
$\rho(r^s, r^b)$	0.79	0.89	0.82	0.78	0.71

### What to Take Home:

- ▶ Matches Risk Premiums of Small Firms and Most Second-Order Moments
- ▶ Underestimates Risk Premiums of Large Firms  $\Rightarrow$  Overestimates Size Premiums

# What to Take Home

**Challenges:** Lumpy Investment in G.E. Resolves Challenges Below

**Table 8:** Compare to Previous Literature

Facts	Equity +Premium + Volatility	Business Cycle	Micro Lumpy Investment	Cross-Sectional Stock Return
Cooper et al. (2006JF)	✓	×	✓	✓
Herskovic (2020WP)	✓	×	×	✓
Favilukis&Lin (2015RFS)	✓	✓	×	×
Chen (2017RFS,2018RAPS)	✓	✓	×	×
Favilukis&Lin (2013JME)	×	✓	✓	×
Winberry(2020AER)	×	✓	✓	×
This Paper	✓	✓	✓	✓

► Novel Habit Sensitivity

►  $-\frac{\sigma}{\log \bar{S}}$  and  $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$  are Opposite

► Habit Destruction  $\Rightarrow$  Unconditional Vol. of the SDF  $\uparrow$  and Robust IRF

► Micro Frictions: Adjustment Probabilities  $\downarrow \Rightarrow$  Risk Premiums  $\uparrow$

► Have Similar (Opposite) Implications on Risk Premiums (Aggregate Investment)

► Small Firms  $\Rightarrow$  Un-adjust  $\Rightarrow$  Higher Cash Flow Risks  $\Rightarrow$  Size Premiums

**Consistency:** The Assumption of Fixed Costs is Consistent with Micro Evidence

# Appendix

# Firm Problem



# Firm Problem: Khan & Thomas (2008) Economy

- ▶ The Firm's Aggregate State is  $(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$
- ▶ Let  $V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t)$  be the Discounted Present Value of the Firm.
- ▶ Firm Chooses to Adjust or Not

$$\begin{aligned} V(k_{i,t}, z_{i,t}, \xi_{i,t}; \Omega_t) = & \lambda^c(\Omega_t) \max_n \left\{ e^{z_{i,t}} e^{x_t} k_{i,t}^\theta n_{i,t}^\nu - w(\Omega_t) n_t \right\} \\ & + \max_i \{ V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t) \xi_{i,t} w(\Omega_t), V^n(k_{i,t}, z_{i,t}; \Omega_t) \} \end{aligned} \quad (5)$$

- ▶ Decision Threshold:  $\tilde{\xi} = \frac{V^a(k_{i,t}, z_{i,t}; \Omega_t) - V^n(k_{i,t}, z_{i,t}; \Omega_t)}{\lambda^c w(\Omega_t)}$

# Firm Problem: Khan & Thomas (2008) Economy

- Range of The Threshold:

$$\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t) = \arg \min \left\{ \max \left\{ 0, \tilde{\xi}(k_{i,t}, z_{i,t}; \Omega_t) \right\}, \bar{\xi} \right\}$$

- Adjustment Probability:  $\frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}} \in [0, 1]$

- Firm Value

$$\begin{aligned} \hat{V}(k_{i,t}, z_{i,t}; \Omega_t) = & \lambda^c(\Omega_t) \max_n \left\{ e^{z_{i,t}} e^{x_t} k_{i,t}^\theta n_{i,t}^\nu - w(\Omega_t) n_t \right\} \\ & + \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}} \left\{ V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t) w(\Omega_t) \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{2} \right\} \\ & + \left( 1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}} \right) V^n(k_{i,t}, z_{i,t}; \Omega_t) \end{aligned} \quad (6)$$

# Benchmark Calibration

Table 9: Parameters Selection: Microeconomic Structure

Parameter	Description	Value
$\lambda^e$	Debt-to-Equity Ratio	0.54
$\beta$	Discount Factor	0.99
$\sigma$	Curvature	1
$\alpha$	Inverse Frisch Elasticity	1/2
$N$	Stationary Work Hour	1/3
$\psi$	Labour Disutility	1
$\theta$	Capital Share	0.21
$\nu$	Labour Share	0.64
$\delta$	Depreciation	0.025
$[-b, b]$	Bound of Region	0.011
$\rho^Z$	Persistence of Idio. Shock	0.859
$\sigma^Z$	s.d. of Idio. Shock	0.015

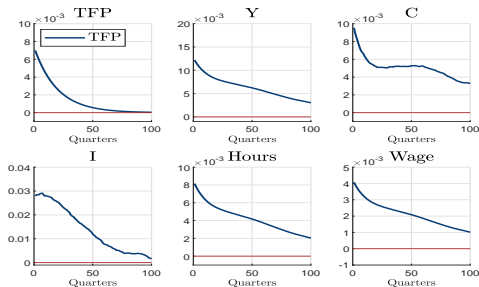
**Table 10:** Investment Dynamics and Risk Premiums: Different Sensitivity Functions

Parameters	Moments	Benchmark	$\log \bar{S} = 0.65$ $\rho^S = 0.95$	$\bar{\xi} = 0.44$ $\rho^Z = 0.90$	Data
$\frac{1}{\bar{S}} - 1$	$\sigma(y_t)$	1.61%	1.61%	1.61%	1.51%
	$\rho(i_t, y_t)$	-0.65	-0.40	0.09	0.88
	$\sigma(i_t)/\sigma(y_t)$	0.51	0.37	0.35	2.89
	$E(r^e - r^f)$	1.56%	1.59%	1.73%	1.54%
	$std(r^e - r^f)$	1.43%	1.72%	1.43%	7.74%
	$E(r^e - r^f)/\sigma(r^e - r^f)$	1.09	0.92	1.21	0.20
$\frac{1}{\log \bar{S}} - 1$	$\sigma(y_t)$	1.56%	1.59%	1.56%	1.51%
	$\rho(i_t, y_t)$	0.82	0.98	0.89	0.88
	$\sigma(i_t)/\sigma(y_t)$	2.92	0.52	3.09	2.89
	$E(r^e - r^f)$	1.79%	1.93%	173.07%	1.54%
	$std(r^e - r^f)$	7.73%	1.30%	245.33%	7.74%
	$E(r^e - r^f)/\sigma(r^e - r^f)$	0.23	1.49	0.71	0.20

### Traditional Habit Sensitivity

- Has Potential to Generate (Smooth) and Counter-Cyclical Investment Dynamics
- Fails to Generate Volatile Risk Premiums

Figure 7: Impulse Response Function of TFP Shock to Business Cycle



### GHH Utility

- ▶ Amplifies Aggregate Consumption, Investment, and Output
- ▶ Has Marginal Effects on Wages and Hours Worked

## Inspecting the Mechanism

# Challenge 1: Mathematical Proof

## ► SDF Decomposition

$\log M_{t+1}$

$$\begin{aligned} &= \log \beta + (-\sigma) \left\{ \log \left( \frac{C_{t+1} S_{t+1}}{C_t S_t} \right) + \log \left( \frac{C_{t+1} S_{t+1} - \frac{\psi}{1+\alpha} \times N_{t+1}^{1+\alpha}}{C_t S_t - \frac{\psi}{1+\alpha} \times \frac{C_{t+1} S_{t+1}}{C_t S_t} \times N_t^{1+\alpha}} \right) \right\} \\ &= \log \beta + \left\{ (-\sigma) \left[ \left( \rho^S - 1 \right) (\log \bar{S} - \log S_t) + \frac{1}{\log \bar{S}} \mu(g_c) \right] + \frac{(-\sigma)^2}{2(\log \bar{S})^2} \sigma(g_c) \right\} \\ &\quad + (-\sigma) \log \left( \frac{C_{t+1} S_{t+1} - \frac{\psi}{1+\alpha} \times N_{t+1}^{1+\alpha}}{C_{t+1} S_{t+1} - \frac{\psi}{1+\alpha} \times \frac{C_{t+1} S_{t+1}}{C_t S_t} \times N_t^{1+\alpha}} \right) \end{aligned} \tag{7}$$

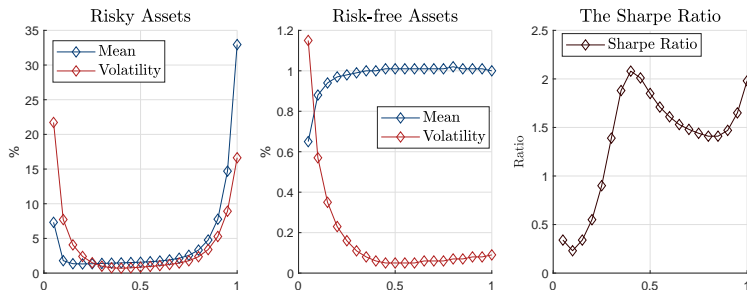
►  $\frac{C_{t+1} S_{t+1}}{C_t S_t}$  shares the same sign of  $\log \left( \frac{C_{t+1} S_{t+1} - \frac{\psi}{1+\alpha} \times N_{t+1}^{1+\alpha}}{C_{t+1} S_{t+1} - \frac{\psi}{1+\alpha} \times \frac{C_{t+1} S_{t+1}}{C_t S_t} \times N_t^{1+\alpha}} \right)$

►  $-\frac{\sigma}{\log \bar{S}}$  and  $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$  are Opposite



# Challenge 1: Quantitative Implications

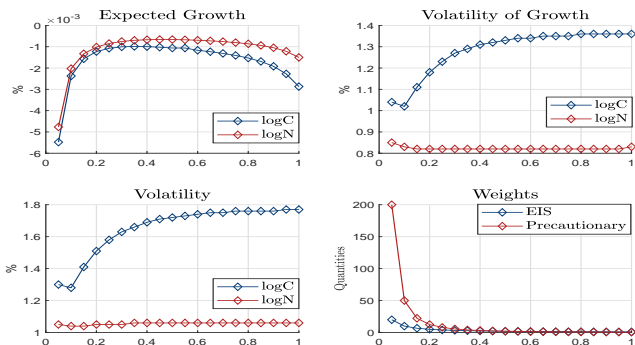
Figure 8: Asset Pricing Moments: Conditional on  $\log \bar{S}$



- ▶  $\log \bar{S} \uparrow \Rightarrow$  U-Shaped Risk Premium and Volatility  $\Leftrightarrow$  When  $\log \bar{S} > 0.25$ ,  $E(r^e - r^f) > std(r^e - r^f)$
- ▶  $\log \bar{S} \uparrow \Rightarrow E(r^f) \uparrow$  and  $std(r^f) \downarrow$

# Challenge 1: Quantitative Implications

Figure 9: The Mechanism of  $\log \bar{S}$



- ▶ The Weight between Precautionary Saving Motivations and EIS Plays a Key Role
- ▶ Consumption Volatility Plays another Key Role

## Challenge 2: Compare to Chen (2017RFS)'s Work

- ▶ Traditional Habit Sensitivity  $\lambda^S = \frac{1}{\bar{S}} - 1$ 
  - ▶ Large  $\bar{S}$  to Make the Agent Destroys Her Habit Periodically  
 $\Rightarrow \lambda^S \downarrow$  Assign a Small Weight to  $\frac{(-\sigma)^2}{2(\log \bar{S})^2}$   
 $\Rightarrow$  Equity Volatility  $\downarrow$
  - ▶ A Small  $\bar{S}$  Cannot Produce a Volatile SDF under Lumpy Economics  
 $\Rightarrow$  Equity Volatility  $\downarrow$

This Paper: Take the Advantages of Both Mechanisms with  $\lambda^S = \frac{1}{\log \bar{S}} - 1$

- ▶ Picking Small  $\log \bar{S} \iff$  Picking Large  $\bar{S}$  to Make the Habit Destruction  
 $\Rightarrow$  but  $\lambda^S$  still  $\uparrow$
- ▶ Why Traditional Model Fails to....?
  - ▶ Numerical Method: Reiter (2009JEDC) and Winberry (2018QE)
  - ▶ Misspecified: Oh (2011WP) with Krusell & Smith (1998JPE)

# Challenge 3: Mathematical Proof

## Value Function

$$\hat{V} = \lambda^c \max_n \left\{ e^z e^x k^\theta n^\nu - wn \right\} + \frac{\hat{\xi}}{\xi} \left\{ V^a - \lambda^c w \frac{\hat{\xi}}{2} \right\} + \left( 1 - \frac{\hat{\xi}}{\xi} \right) V^n$$

where

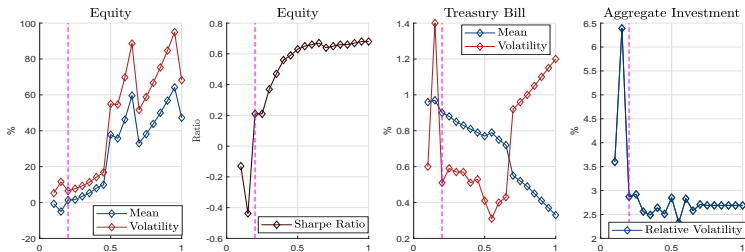
$$\tilde{\xi} = \frac{V^a - V^n}{\lambda^c w}$$

$$\hat{\xi} = \arg \min \left\{ \max \left\{ 0, \tilde{\xi} \right\}, \bar{\xi} \right\} = \arg \min \left\{ \max \left\{ 0, \frac{V^a - V^n}{\lambda^c w} \right\}, \bar{\xi} \right\}$$

- ▶ If  $\bar{\xi} < \tilde{\xi}$  then  $\hat{\xi} = \bar{\xi}$ , only Adjusted Firms are Considered and the Adjustment Probability is One
- ▶ If  $\hat{\xi} = \tilde{\xi} = \frac{V^a - V^n}{\lambda^c w} < \bar{\xi} \Rightarrow$  the Relative Weights Assigned to Adjusted and Constraint Firms are  $\frac{\hat{\xi}}{\xi}$  and  $1 - \frac{\hat{\xi}}{\xi} \Rightarrow \frac{\hat{\xi}}{\xi}$  is Adjustment Probability
- ▶  $\bar{\xi} \uparrow \Rightarrow$  Adjustment Probability  $\frac{\hat{\xi}}{\xi} \uparrow \Rightarrow$  Assign Higher Weights on Constraint Firms  $\Rightarrow$  Future Dividends  $\uparrow$  Risk Premiums  $\uparrow$

# Challenge 3: Quantitative Implications

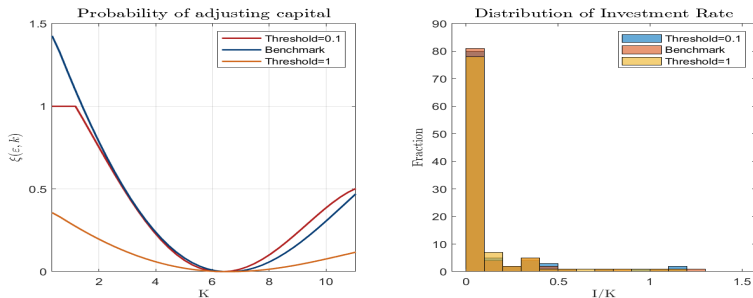
Figure 10: Asset Pricing Moments: Conditional on  $\bar{\xi}$



- ▶  $\bar{\xi} \uparrow \Rightarrow E(r^e - r^f)$ ,  $std(r^e - r^f)$ , and Sharp Ratio  $\uparrow$
- ▶  $\bar{\xi} \uparrow \Rightarrow E(r^f) \downarrow$  and Investment Smooth
- ▶ Small  $\bar{\xi} \Rightarrow$  All Firms Adjust  $\Rightarrow$  Equity is Less Risky than T-Bill  $\Rightarrow$  Negative Premiums

# Challenge 3: Quantitative Implications

Figure 11: The Mechanism of  $\bar{\xi}$

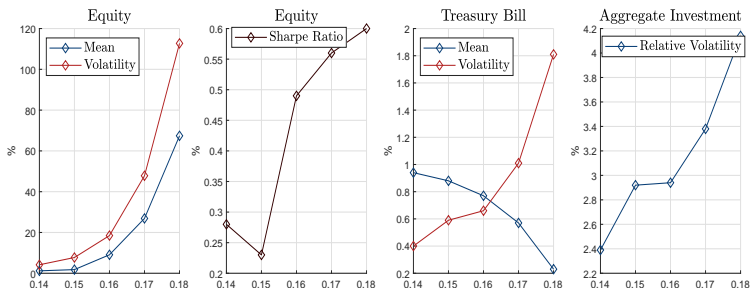


$\bar{\xi} \uparrow \Rightarrow \text{Adjustment Probability } \xi(\epsilon, k) = \bar{\xi} \downarrow$

$\Rightarrow \text{Firms Choose Inaction} \Rightarrow \text{Future Dividend} \uparrow \Rightarrow \text{Risk Premiums} \uparrow$

# Challenge 3: Quantitative Implications

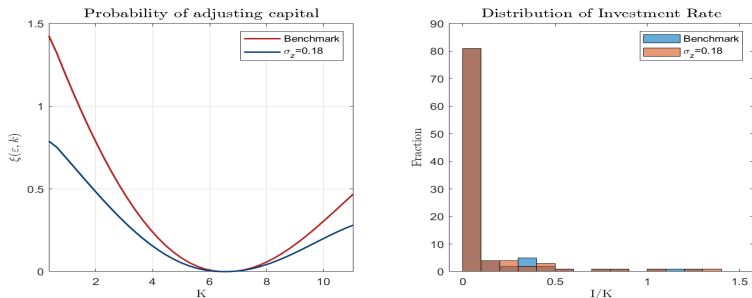
Figure 12: Macro-Asset Pricing Moments: Conditional on  $\sigma_z$



- ▶  $\sigma_z \uparrow \Rightarrow E(r^e - r^f)$ ,  $std(r^e - r^f)$ , Sharp Ratio, and  $\sigma(i_t) \uparrow$
- ▶  $\bar{\xi} \uparrow \Rightarrow E(r^f) \downarrow$

# Challenge 3: Quantitative Implications

Figure 13: The Mechanism of  $\sigma_z$



Idiosyncratic Risks  $\sigma_z \uparrow \Rightarrow$  Adjustment Probability  $\xi(\epsilon, k) = \xi^* \downarrow$

$\Rightarrow$  Cash Flow Risks  $\Rightarrow$  Risk Premiums  $\uparrow$



## Challenge 4: Mathematical Proof

Recall the value function:

$$\begin{aligned}\hat{V}(k_{i,t}, z_{i,t}; \Omega_t) &= \lambda^c(\Omega_t) \max_n \left\{ e^{z_{i,t}} e^{x_t} k_{i,t}^\theta n_{i,t}^\nu - w(\Omega_t) n_t \right\} \\ &\quad + \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}} \left\{ V^a(k_{i,t}, z_{i,t}; \Omega_t) - \lambda^c(\Omega_t) w(\Omega_t) \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{2} \right\} \\ &\quad + \left( 1 - \frac{\hat{\xi}(k_{i,t}, z_{i,t}; \Omega_t)}{\bar{\xi}} \right) V^n(k_{i,t}, z_{i,t}; \Omega_t)\end{aligned}$$

The return on unconstrained (big) firm and constrained (small) firm is given by:

$$\begin{aligned}R^a(\Omega_{t+1}|\Omega_t) &= \frac{\frac{\hat{\xi}}{\bar{\xi}} \int V^a(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + \frac{\hat{\xi}}{\bar{\xi}} D(\Omega_{t+1})}{\frac{\hat{\xi}}{\bar{\xi}} \int V^a(k_{i,t}, z_{i,t}; \Omega_t) di} \\ &= \frac{\int V^a(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di} \\ R^n(\Omega_{t+1}|\Omega_t) &= \frac{\left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) \int V^n(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + \left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) D(\Omega_{t+1})}{\left(1 - \frac{\hat{\xi}}{\bar{\xi}}\right) \int V^n(k_{i,t}, z_{i,t}; \Omega_t) di} \\ &= \frac{\int V^n(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + D(\Omega_{t+1})}{\int V^n(k_{i,t}, z_{i,t}; \Omega_t) di}\end{aligned} \tag{8}$$

## Challenge 4: Mathematical Proof

The return spread between small and big firm is:

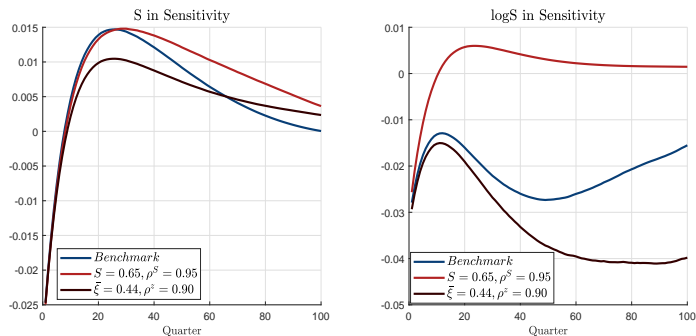
$$\begin{aligned} & R^n(\Omega_{t+1}|\Omega_t) - R^a(\Omega_{t+1}|\Omega_t) \\ &= \frac{\int V^n(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + D(\Omega_{t+1})}{\int V^n(k_{i,t}, z_{i,t}; \Omega_t) di} - \frac{\int V^a(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di} \\ &= \frac{[\int V^n(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + D(\Omega_{t+1})] \int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di}{\int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di \int V^n(k_{i,t}, z_{i,t}; \Omega_t) di} \\ &\quad - \frac{[\int V^a(k_{i,t+1}, z_{i,t+1}; \Omega_{t+1}) di + D(\Omega_{t+1})] \int V^n(k_{i,t}, z_{i,t}; \Omega_t) di}{\int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di \int V^n(k_{i,t}, z_{i,t}; \Omega_t) di} \\ &= \frac{\int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di D(\Omega_{t+1}) - \int V^n(k_{i,t}, z_{i,t}; \Omega_t) di D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di \int V^n(k_{i,t}, z_{i,t}; \Omega_t) di} \\ &= \frac{\int [V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) - V^n(k_{i,t}, z_{i,t}; \Omega_t)] di D(\Omega_{t+1})}{\int V^a(k_{i,t}, z_{i,t}; \Omega_{t+1}) di \int V^n(k_{i,t}, z_{i,t}; \Omega_t) di} > 0 \end{aligned} \tag{9}$$

## Challenge 5: Debate on Irrelevance

- ▶ Debate: Micro Frictions Has (Marginal) Effects on Aggregate Investment
- ▶ Reason: Procyclical Risk-Free Rate Offset the Aggregate Investment Demand  
⇒ Irrelevance
- ▶ Solution: Novel Habit Sensitivity Produces Robust IRF of TFP
- ▶ Intuition: The Similar Story in Winberry (2020AER)
  - ▶ IRF of TFP to Risk-Free Rate is Negative  
⇒ Counter-Cyclical Risk-Free Rate  
⇒ Investment Demand  $\uparrow$
  - ▶ IRF of TFP to Aggregate Investment is Robustly Positive  
⇒ Procyclical Investment  
⇒ Investment Volatility  $\uparrow$

# Challenge 5: Quantitative Implications

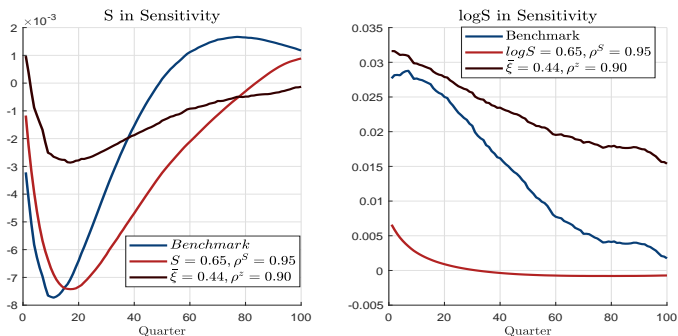
Figure 14: IRF of TFP to Risk-Free Rate



► Counter-Cyclical Risk-Free Rate  $\Rightarrow$  Investment Demand  $\uparrow$

# Challenge 5: Quantitative Implications

Figure 15: Investment IRFs: (log) Steady-State Surplus-Consumption Ratio



► U-Shaped IRF  $\Rightarrow$  Counter-Cyclical & Smooth Investment Dynamics

# Comparative Statistics

# Alternative Preferences: Keeping Up with the Joneses

Recall the Utility Function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left( C_t - H_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{1-\sigma}}{1-\sigma} \right\}$$

► Habit with Actual Consumption

$$H_t = \tau C_{t-1}$$

$$\lambda_t^c = \left( C_t - \tau C_{t-1} - \psi \frac{N_t^{1+\alpha}}{1+\alpha} \right)^{-\sigma}$$

► Habit with Net Consumption

$$H_t = \tau \left( C_{t-1} - \psi \frac{N_{t-1}^{1+\alpha}}{1+\alpha} \right)$$

$$\lambda_t^c = \left( C_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha} - \tau \left( C_{t-1} - \psi \frac{N_{t-1}^{1+\alpha}}{1+\alpha} \right) \right)^{-\sigma}$$

Table 11: Habit Formation: Moment Comparisons

Models	CC		KJ		No Habits	Data
	$\frac{1}{\log S} - 1$	$\frac{1}{S} - 1$	$C_t$	$C_t - \psi \frac{N_t^{1+\alpha}}{1+\alpha}$		
$\sigma(y_t)$	1.56%	1.61%	1.78%	1.59%	1.58%	1.51 %
$\sigma(c_t)/\sigma(y_t)$	0.82	1.23	1.16	0.60	0.77	0.80
$\sigma(i_t)/\sigma(y_t)$	2.92	0.73	4.40	3.39	2.38	2.88
$\sigma(n_t)/\sigma(y_t)$	0.67	0.67	0.67	0.67	0.67	1.21
$\rho(c_t, y_t)$	0.93	0.99	0.80	0.99	0.99	0.85
$\rho(i_t, y_t)$	0.82	-0.47	0.33	0.99	0.99	0.88
$\rho(n_t, y_t)$	1.00	1.00	1.00	1.00	1.00	0.87
$\rho(c_{t-1}, c_t)$	0.70	0.73	0.94	0.77	0.72	0.81
$\rho(i_{t-1}, i_t)$	0.76	0.84	0.76	0.68	0.71	0.80
$\rho(n_{t-1}, n_t)$	0.71	0.72	0.67	0.72	0.72	0.90
$\rho(y_{t-1}, y_t)$	0.71	0.72	0.76	0.72	0.72	0.85
$E(r^e - r^f)$	1.79%	1.59%	1.78%	6.05%	1.77%	1.54%
$std(r^e - r^f)$	7.72%	1.43%	1.58%	11.31%	1.18%	7.74%
$E(r^f)$	0.88%	1.01%	1.02%	0.90%	0.98%	0.40%
$std(r^f)$	0.57%	0.09%	0.15%	0.26%	0.12%	0.99%
$\frac{E(r^e - r^f)}{\sigma(r^e - r^f)}$	0.23	1.12	1.13	0.53	1.51	0.20



# Additional Micro Frictions

## ► Investment-Specific Shock

$$\begin{aligned}k_{i,t+1} &= (1 - \delta)k_{i,t} + e^{q_t} i_{i,t} \\ q_{t+1} &= \rho^q q_t + \sigma^q \varepsilon_{t,t+1}^q + \sigma^{qx} \varepsilon_{t,t+1}^{qx}\end{aligned}\tag{10}$$

## ► Uncertainty Shock

$$\begin{aligned}z_{i,t+1} &= \rho^Z z_i + \varepsilon_{i,t+1}^Z, \varepsilon_{i,t+1}^Z \sim N(0, X_t^U \bar{\sigma}) \\ X_{t+1}^U &= \rho^U \log X_t^U + \varepsilon_{t+1}^U, \varepsilon_{t+1}^U \sim N(0, \sigma^U)\end{aligned}\tag{11}$$

## ► Convex Costs

$$\Phi(l_{i,t}/k_{i,t}) = \frac{\lambda_x}{2} \left( \frac{l_{i,t}}{k_{i,t}} \right)^2 k_{i,t}\tag{12}$$

and its Value Function

$$\begin{aligned}\hat{V} = & \lambda^c \max_n \left\{ e^z e^x k^\theta n^\nu - wn \right\} + \frac{\hat{\xi}}{\xi} \left\{ V^a - \lambda^c w \frac{\hat{\xi}}{2} \right\} + \left( 1 - \frac{\hat{\xi}}{\xi} \right) V^n \\ & - \lambda^c \frac{\hat{\xi}}{\xi} \times \frac{\lambda_x}{2} \left( \frac{k^a}{k} - (1 - \delta) \right)^2 - \lambda^c \left( 1 - \frac{\hat{\xi}}{\xi} \right) \times \frac{\lambda_x}{2} \left( \frac{k^n}{k} - (1 - \delta) \right)^2\end{aligned}\tag{13}$$

Table 12: The Microeconomic Frictions: Comparisons

Models	Benchmark	IST	Uncertainty Shock	Convex Costs	Data
$\sigma(y_t)$	1.56%	1.58%	1.58 %	1.58 %	1.51 %
$\sigma(c_t)/\sigma(y_t)$	0.82	0.80	0.80	0.73	0.80
$\sigma(i_t)/\sigma(y_t)$	2.92	2.79	2.67	2.97	2.88
$\sigma(n_t)/\sigma(y_t)$	0.67	0.67	0.67	0.67	1.21
$\rho(c_t, y_t)$	0.93	0.95	0.96	0.94	0.85
$\rho(i_t, y_t)$	0.82	0.88	0.88	0.85	0.88
$\rho(n_t, y_t)$	1.00	1.00	1.00	1.00	0.87
$\rho(c_{t-1}, c_t)$	0.70	0.70	0.70	0.71	0.81
$\rho(i_{t-1}, i_t)$	0.76	0.76	0.76	0.74	0.80
$\rho(n_{t-1}, n_t)$	0.71	0.71	0.71	0.71	0.90
$\rho(y_{t-1}, y_t)$	0.71	0.71	0.71	0.71	0.85
$E(r^e - r^f)$	1.79%	1.57%	1.39%	2.66%	1.54%
$std(r^e - r^f)$	7.72%	5.70%	4.95%	7.76%	7.74%
$E(r^f)$	0.88%	0.96%	0.97%	0.88%	0.40%
$std(r^f)$	0.59%	0.48%	0.45%	0.49%	0.99%
$\frac{E(r^e - r^f)}{\sigma(r^e - r^f)}$	0.23	0.27	0.28	0.34	0.20

Additional Micro Frictions have merely Marginal Contribution