The Human Resources Problem

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0.1 Motivation and Overview

Recently, multiple news sources reported that Goldman Sachs would start a material amount of layoffs right after a period of record deal-making, revenues, and (anecdotally) analyst overwork for investment banking as an industry. At first glance, this doesn't make much sense - how could it be beneficial to overwork employees and then lay them off? Why not hire more (or less) to begin with? In this post I work out a simple profit-max problem showing that 1) imperfect forecasting of future dealflow and 2) sticky hiring make it optimal to overwork analysts during good times and underwork (+ fire) them during bad.

In subsequent blog posts I will add 2 additional complications

1. Directors with variable compensation can increase deal-flow but are still limited by industry-wide trends
2. Mid level bankers increase the effectivity of analysts but do not increase deal flow

Sources of a huge amount of deals

https://www.investmentbankingcouncil.org/blog/the-future-of-investment-banking-2022-and-beyond

News publicizing goldman sachs firing people

https://www.economist.com/finance-and-economics/2022/09/28/investment-banks-are-sharpening-the-axe

https://nypost.com/2022/09/26/goldman-sachs-begins-layoffs-targeting-mid-level-bankers/

https://www.washingtontimes.com/news/2022/sep/27/investment-bank-goldman-sachs-begins-mass-layoffs-/

0.2 Problem v1 - Optimal Analyst Overwork

Problem Setup

Intuitively, the investment bank wants to maximize expected revenues. But, they are uncertain about next year's deaflow and can only pick this year's hiring one year in advance. If they hire too many analysts, they will do all the deals availble to them (under utilizing some analysts). If they don't hire enough, they will do as many deals as the analyst productivity lets them do.

Mathematically, the firm's profit function in year t is

$$p_t = r \min \{ f(\ell_{t-1}), D_t \} - w\ell_{t-1}$$

Where ℓ_{t-1} is the amount of analyst labor hired last year (to do this year's deals), $f(\ell)$ is the analyst production function (how many deals ℓ analysts can complete), D_t is the amount of deals, w is the analyst wage, and r is the revenue per deal.

The firm's dynamic objective is pick a hiring plan that maximizes discounted profits over its whole lifetime

$$\max_{\{\ell\}} \mathbb{E}\left(\sum_{t=0}^{\infty} \beta^t p_t\right)$$

For simplicity, assume that D_t is binomial and depends only on the value of D_{t-1}. There are 2 possible values that D_t can take on, $D_t = D_h$ (high number of deals) and $D_t = D_l$ (low number of deals).

$$P\left(D_t = D_h \mid D_{t-1} = D_h\right) = p_{h,h}$$

$$P\left(D_t = D_h \mid D_{t-1} = D_l\right) = p_{l,h}$$

The solution to the objective function is to pick ℓ_{t-1} that maximizes p_t (in both the $D_{t-1} = D_h$ regime and $D_{t-1} = D_l$ regime).

Solution

We want to maximize

$$p_t = \mathbb{E}\left(\underbrace{r \min\left\{f\left(\ell_{t-1}\right), D_t\right\}}_{\text{Revenues}} - \underbrace{w\ell_{t-1}}_{\text{Wages Paid}}\right)$$

Since the objective function has a kink where $f\left(\ell_{t-1}\right) = D_t$ we have to consider a few cases depending on what D_t actually is before we go about solving the optimization.

Claim: It's reasonable to assume $D_l \leq f\left(\ell_{t-1}\right) \leq D_h$

Case 1: $f\left(\ell_{t-1}\right) > D_h$

Note that this is strictly sub optimal. This means that the bank is wasting wages in both the high number of deals and low number of deals state. They would be better off in both states reducing ℓ_{t-1} such that $f\left(\ell_{t-1}\right) = D_h$.

Case 2: $f\left(\ell_{t-1}\right) < D_l$

This is actually possible (depending on the functional form of f, value of D_l, and value of r), though not interesting. Intuitively, it means that the number of deals in the "low deals" state is still so high that the firm still wouldn't be able to capture all of the deals. So, I won't consider this case.

Without loss of generality, let's assume we're in the low state l. For an interior solution $D_l < f\left(\ell_{t-1}\right) < D_h$, the the profit function is

$$p_t = p_{l,h} \cdot rf\left(\ell_{t-1}\right) + p_{l,l}D_l - w\ell_{t-1}$$

If f is differentiable, strictly increasing, with decreasing returns to scale then notice that p_t has a unique solution described by the first order conditions:

$$\left[\ell_{t-1}\right] : p_{l,h} r f'\left(\ell_{t-1}\right) = w\ell_{t-1}$$

Which means the marginal profit of hiring another analyst equals the wage. Note that the marginal profit is also scaled by $p_{l,h}$ which is the odds that we turn out in the high deal state (where we will get maximum analyst utilization). Hiring more analysts yields no profits in the low deal state because the number of deals is lower than analyst productivity.

Just for the sake of it, assume $f(\ell) = \log \ell$. Then explicitly solved

$$\ell_{t-1} = \frac{r \cdot p_{l,h}}{w}$$

Since r, w are constants, we then see that the time series for ℓ_{t-1} will directly follow the time series for $p_{l,h}$ (the probabilities we end up in a high dealflow state).

Economically,

1. Analyst hiring follows the dealflow (economic) cycle
2. The firm will hemorrage money in the low state, over hiring, in preparation for capturing more deal flow in the high state
3. If $p_{l,h}$ is sufficiently small, the firm will also fire analysts (but not so many that it over-fits to the low regime) because expected future dealflow is lower.

For the exterior solutions, I'm just going to assert the firm will never pick them because the transition probabilities are always nonzero.