MECH 309: Numerical Methods in Mechanical Engineering
Department of Mechanical Engineering, McGill University

Final Project: Murman-Cole Scheme for the Transonic Small Disturbance Equation
Due 6pm, 4th. December, 2019

Instructions: Write a report (LateX, LibreOffice, Word...) with comments, equations, figures, and algorithms. Save it as a .pdf file which should neither exceed 2 Mb nor 12 pages. Fontsize should be 11 pt. You are free to organize the plots in an effective manner. Submit all your Matlab scripts in a .zip archive into which you will also include your report. The name of the archive should be a concatenation of the three family names of the members of your team. Submit only one archive per team. Unless stated otherwise, the use of advanced Matlab commands is prohibited. Equally important is the prohibited use of the symbolic toolbox in Matlab. All plots must have both x- and y-axis labels, a legend clearly describing the various lines, and Figure captions. Plots generated with MS Excel are not acceptable. Provide every step of your derivations. Please submit a single PDF file.

Grading scheme: Below is the grading scheme used only for the final report to be submitted on April 7th:

1. Comprehension of information and concepts of fundamental engineering sciences (10%)
2. Critical evaluation of the validity and accuracy of solution methods (45%)
3. Appropriate selection of solution techniques and resources (30%)
4. High quality written engineering report (15%)

Solve the transonic small disturbance (TSD) theory over a circular arc airfoil at various Mach numbers using the Murman-Cole method. The TSD equation is simplified from the two-dimensional full potential equations and can be written as

$$\left[\left(1-M_\infty^2\right)-(\gamma+1)M_\infty^2\frac{\phi_x}{U_\infty}\right]\phi_{xx}+\phi_{yy}=0$$

where $\gamma = 1.4$ is the specific heat ratio for air, the gas constant, R $= 287.058$ J kg^{-1} K^{-1}, the freestream static temperature and pressures are $T_\infty = 293$ K and $p_\infty = 100$ kN/m^2, and lastly, $(x, y) \in [0, 50.0]^2$ spans the two-dimensional domain, and the following boundary conditions hold:

$$\phi(x,y)=0,\ \ \forall\ \ (x,50),(j,0),\ \text{and}\ (j,50) \tag{1}$$

$$\frac{\partial\phi}{\partial n}=0\ \ \forall\ \ (x,0)\notin 20\le x\le 21 \tag{2}$$

$$\frac{\partial\phi}{\partial n}=U_\infty\frac{dy}{dx}\ \ \forall\ \ (x,0)\in 20\le x\le 21. \tag{3}$$

The airfoil is defined by the following equation for a circular-arc:

$$y(x)=(t/c)(-2x^2+82x-840),\ \ \forall(x,0)\in 20\le x\le 21$$

where, $t/c = 0.08$ is the thickness ratio, and x_c is the position of the mid chord. Employ a constant grid spacing in the x and y directions ~~over the airfoil but an exponential or polynomial stretching of the grid along the x and y directions~~. You may initialize the flow with, $\phi = 0$. Either specify the freestream Mach number or velocity as an input parameter. Use the isentropic equations to evaluate Mach, velocity, or pressure in the domain, where $U_\infty = M_\infty\sqrt{\gamma R T_\infty}$.

Provide the following in a written report:

1. Solve the TSD using the Murman-Cole method on the computational grid and use $\Delta x = \Delta y = 0.1$. ~~with at least 60 grid points in the y-direction. The first grid spacing adjacent to the airfoil in the y-direction should be $dy = \frac{t/c}{2}$.~~ Use the Gauss-Seidel method to solve the equation along each column (y-direction).

2. Provide a plot of the pressure coefficient along the airfoil surface with the negative pointing upwards. Vary the freestream Mach number between $[0.75, 0.85]$ with 0.02 increments. For each case, provide a convergence plot of the L_∞-norm, surface pressure coefficient as a function of x, and pressure contour for $x \in [20, 21]$ and $y \in [0, 1]$. Discuss your findings. A four order reduction in the residual is sufficient for the Gauss-Seidel method. Ensure that the residual reaches at least 1×10^{-4}.

3. Vary the grid size, by doubling it in each direction ~~as well as the number of points on the airfoil surface~~. Produce a coarse, medium, and fine grid. Plot the surface coefficient of pressure as a function of x on the airfoil surface for the three grids on the same plot at Mach 0.80. Discuss your findings. Does the shock location change.

4. For a select grid size, plot the coefficient of pressure along the airfoil surface for Mach number between $[0.75, 0.85]$ with 0.02 increments on the same plot. Discuss your observations. What is the effect of increasing the Mach number.