Line Rasterization

CS418 Computer Graphics

John C. Hart

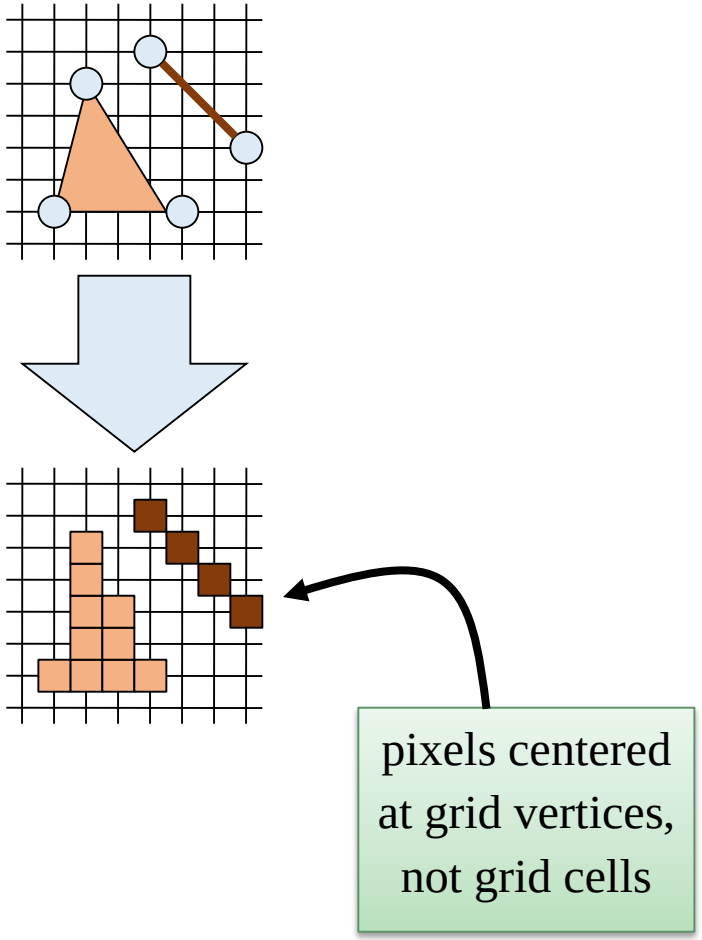
Rasterization

Converts

- lines and triangles
- with floating point vertices
- in viewport (screen) coordinates

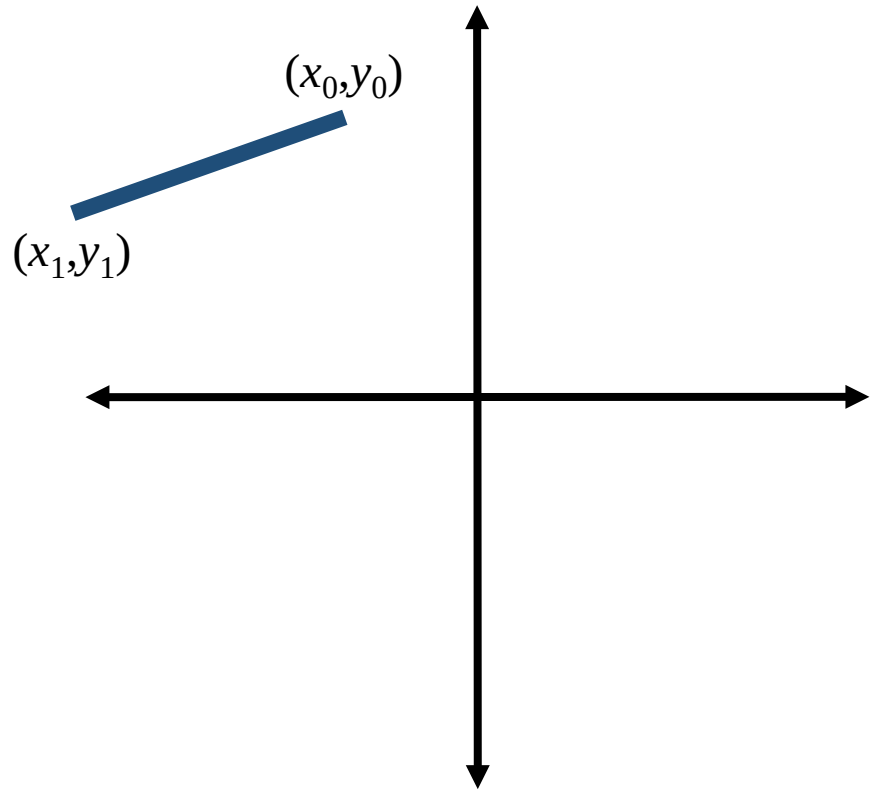
into

- pixels
- with integer coordinates
- in viewport (screen) coordinates



Line Rasterization

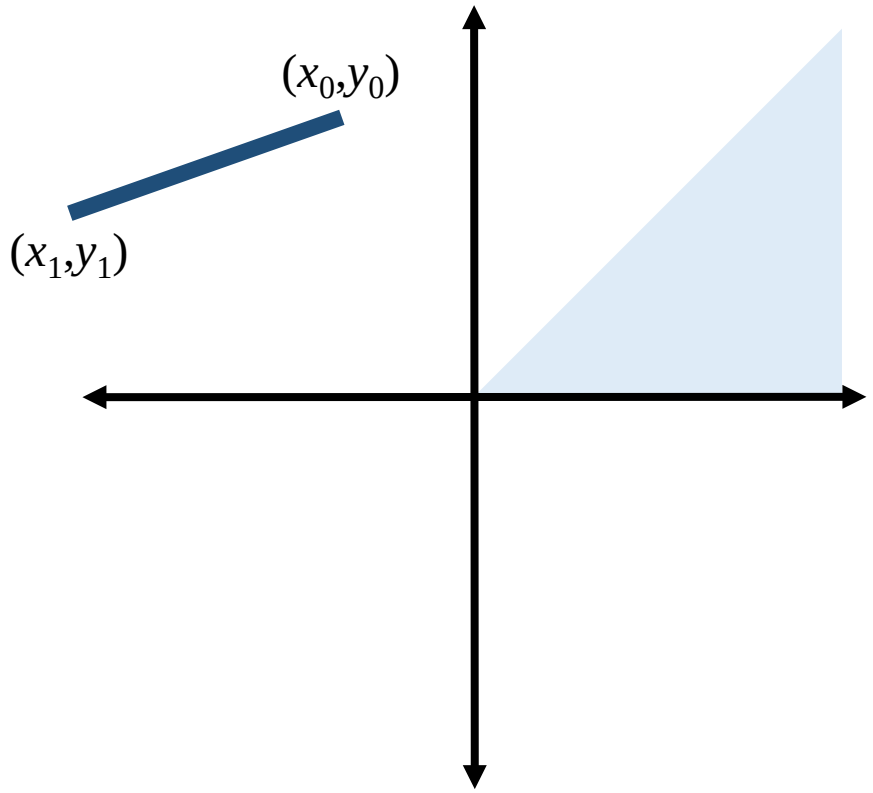
Need to rasterize lines between any two clipped screen points,
from (x_0,y_0) to (x_1,y_1)



Line Rasterization

Only rasterize lines from the origin to a point in the first octant

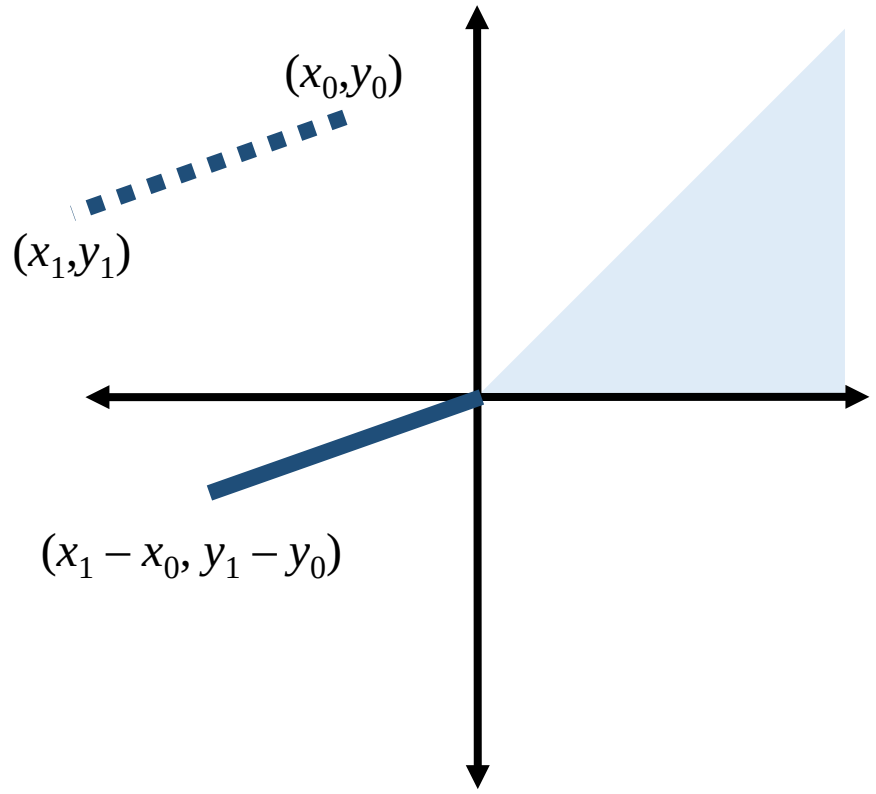
Need to rasterize lines between any two clipped screen points, from (x_0,y_0) to (x_1,y_1)



Line Rasterization

Only rasterize lines from the origin to a point in the first octant

Need to rasterize lines between any two clipped screen points, from (x_0,y_0) to (x_1,y_1)

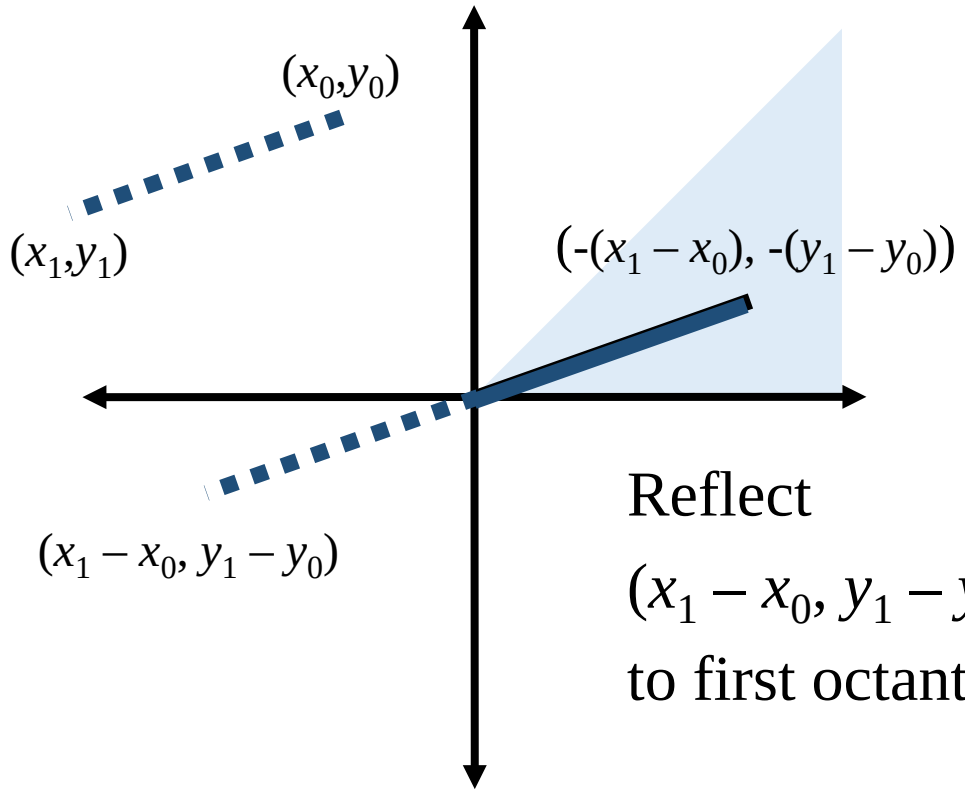


Translate (x_0,y_0) to origin

Line Rasterization

Only rasterize lines from the origin to a point in the first octant

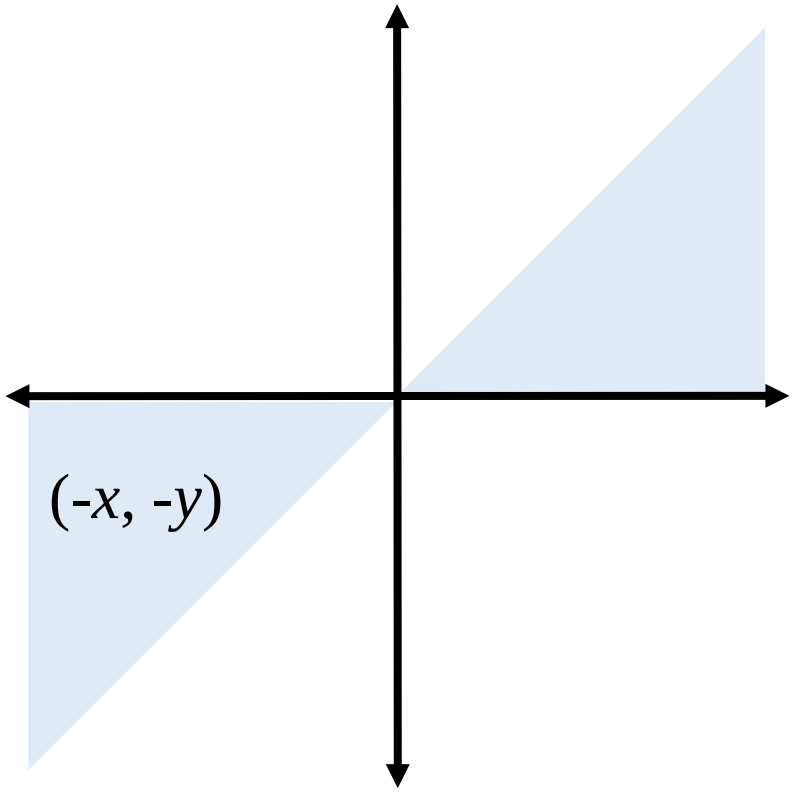
Need to rasterize lines between any two clipped screen points, from (x_0,y_0) to (x_1,y_1)



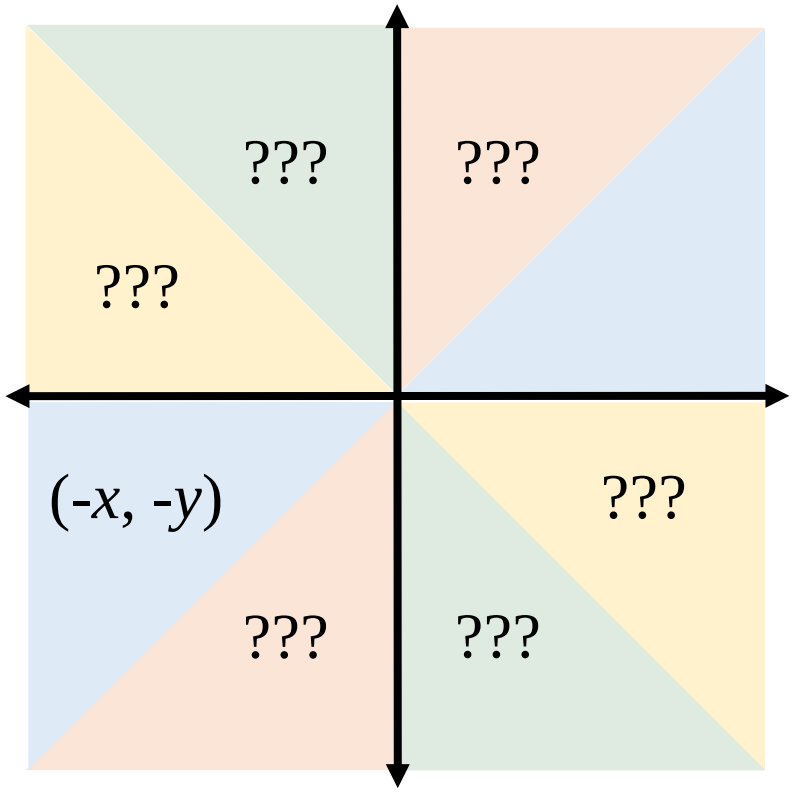
Translate (x_0,y_0) to origin

Reflect $(x_1 - x_0, y_1 - y_0)$ to first octant

Line Rasterization

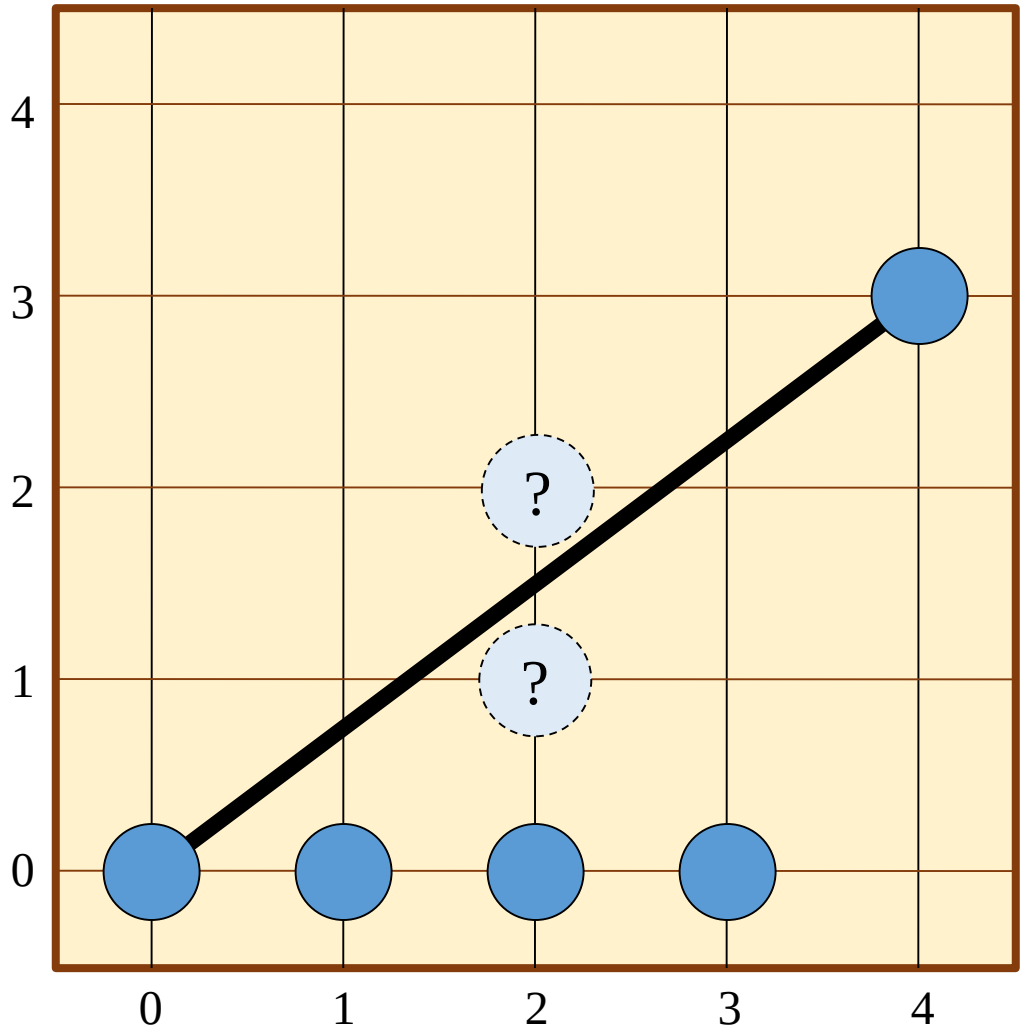


Line Rasterization



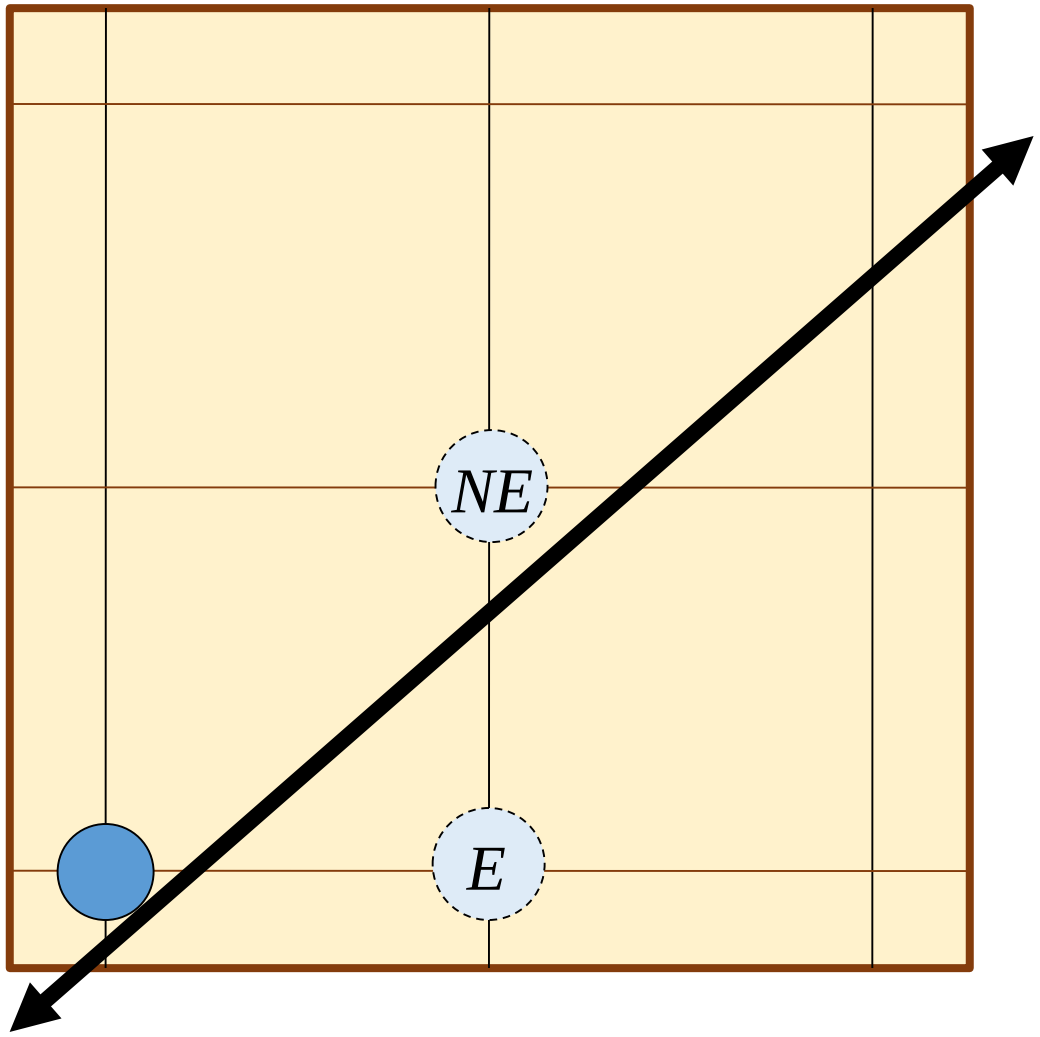
Line Rasterization

- How to rasterize a line from (0,0) to (4,3)
- Pixel (0,0) and (4,3) easy
- One pixel for each integer x-coordinate
- Pixel's y-coordinate closest to line
- If line equal distance between two pixels, pick on arbitrarily but consistently



Midpoint Algorithm

- Which pixel should be plotted next?
  - East?
  - Northeast?



Midpoint Algorithm

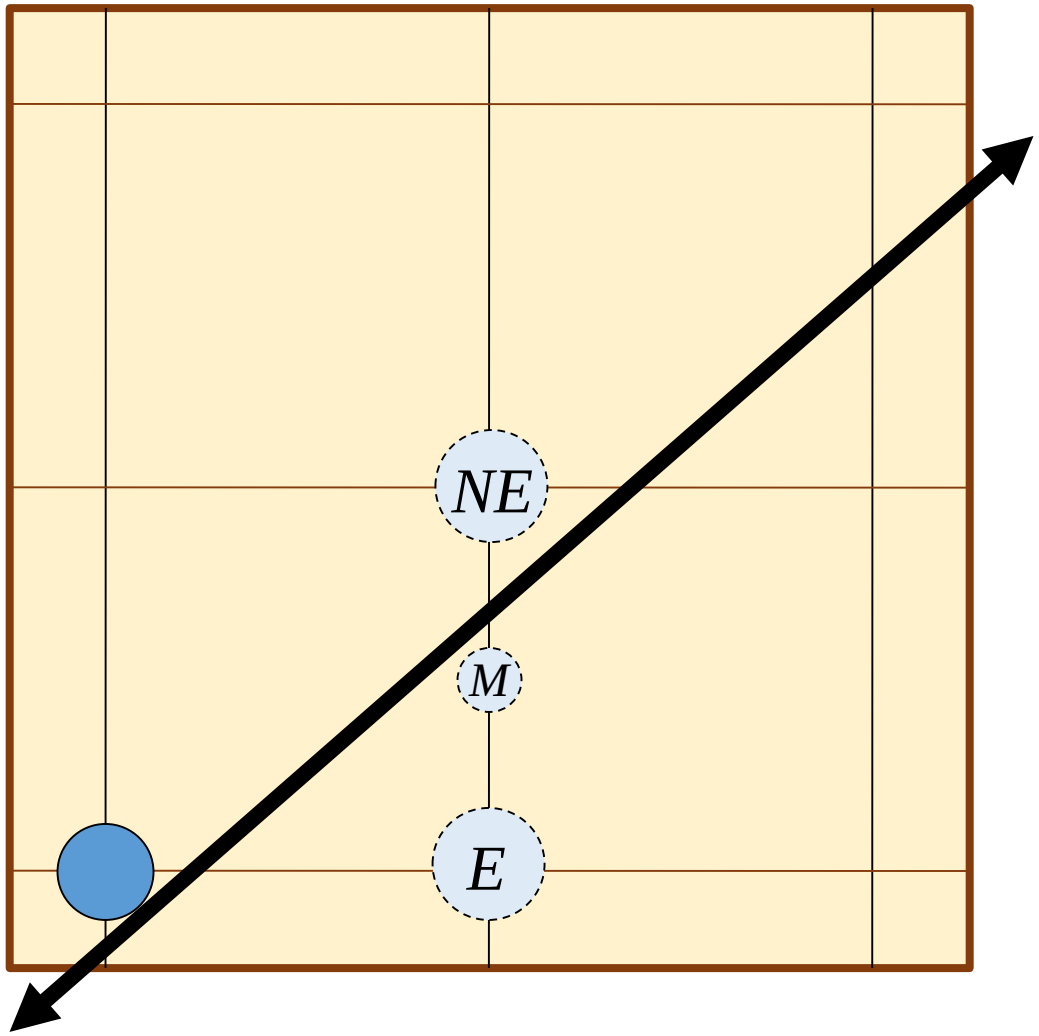
- Which pixel should be plotted next?
  - East?
  - Northeast?
- Line equation

$$y = mx + b$$

$$m = (y_1 - y_0)/(x_1 - x_0)$$

$$b = y_0 - mx_0$$

$$f(x,y) = mx + b - y$$



Midpoint Algorithm

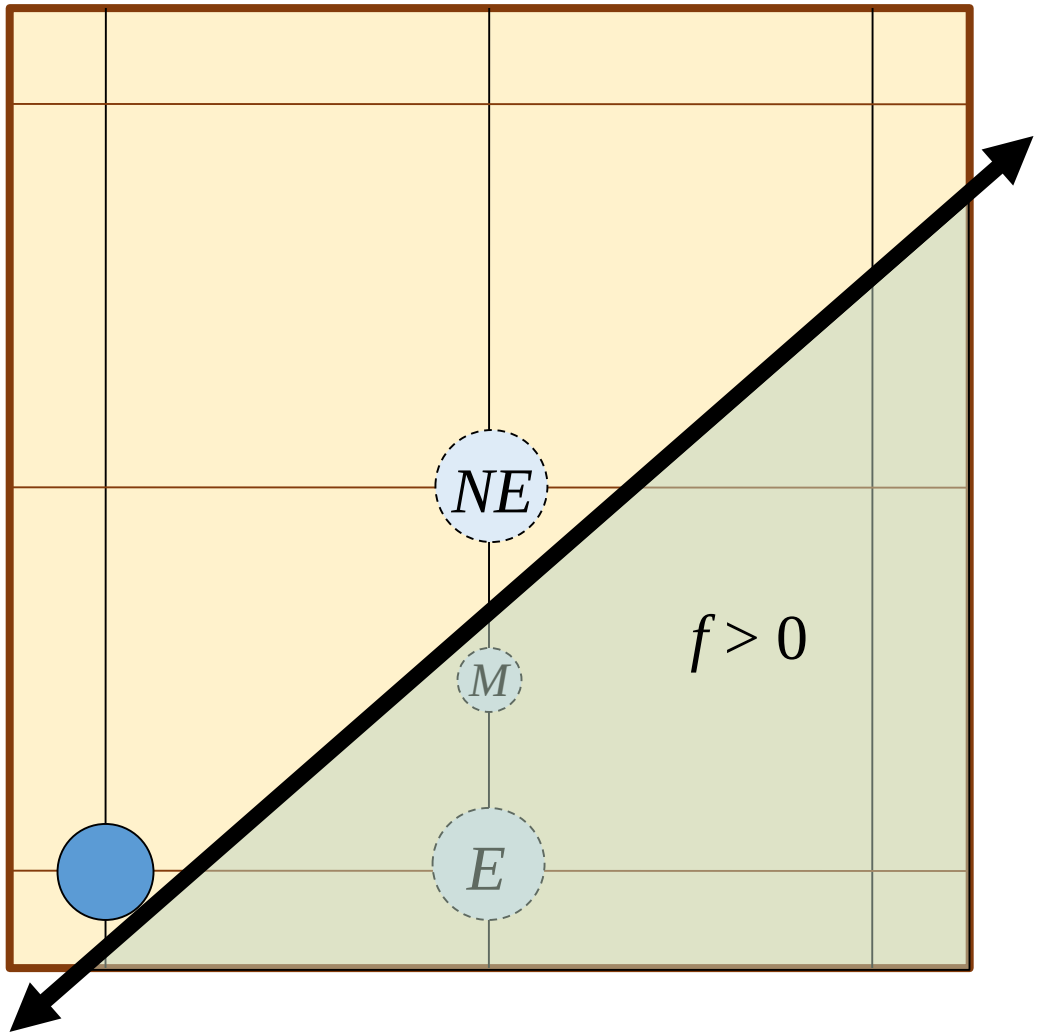
- Which pixel should be plotted next?
  - East?
  - Northeast?
- Line equation

$$y = mx + b$$

$$m = (y_1 - y_0)/(x_1 - x_0)$$

$$b = y_0 - mx_0$$

$$f(x,y) = mx + b - y$$



Midpoint Algorithm

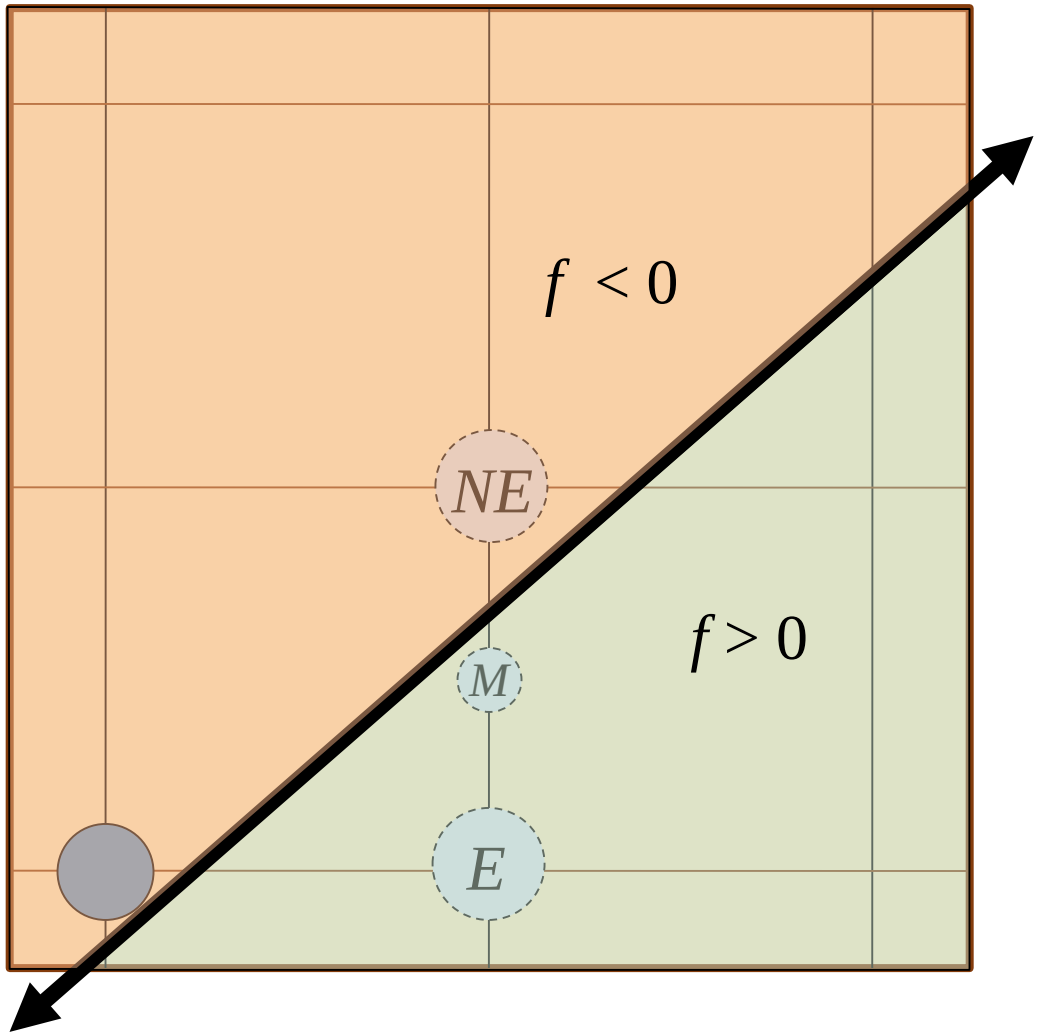
- Which pixel should be plotted next?
  - East?
  - Northeast?
- Line equation

$$y = mx + b$$

$$m = (y_1 - y_0)/(x_1 - x_0)$$

$$b = y_0 - mx_0$$

$$f(x,y) = mx + b - y$$



Midpoint Algorithm

- Which pixel should be plotted next?
  - East?
  - Northeast?
- Line equation

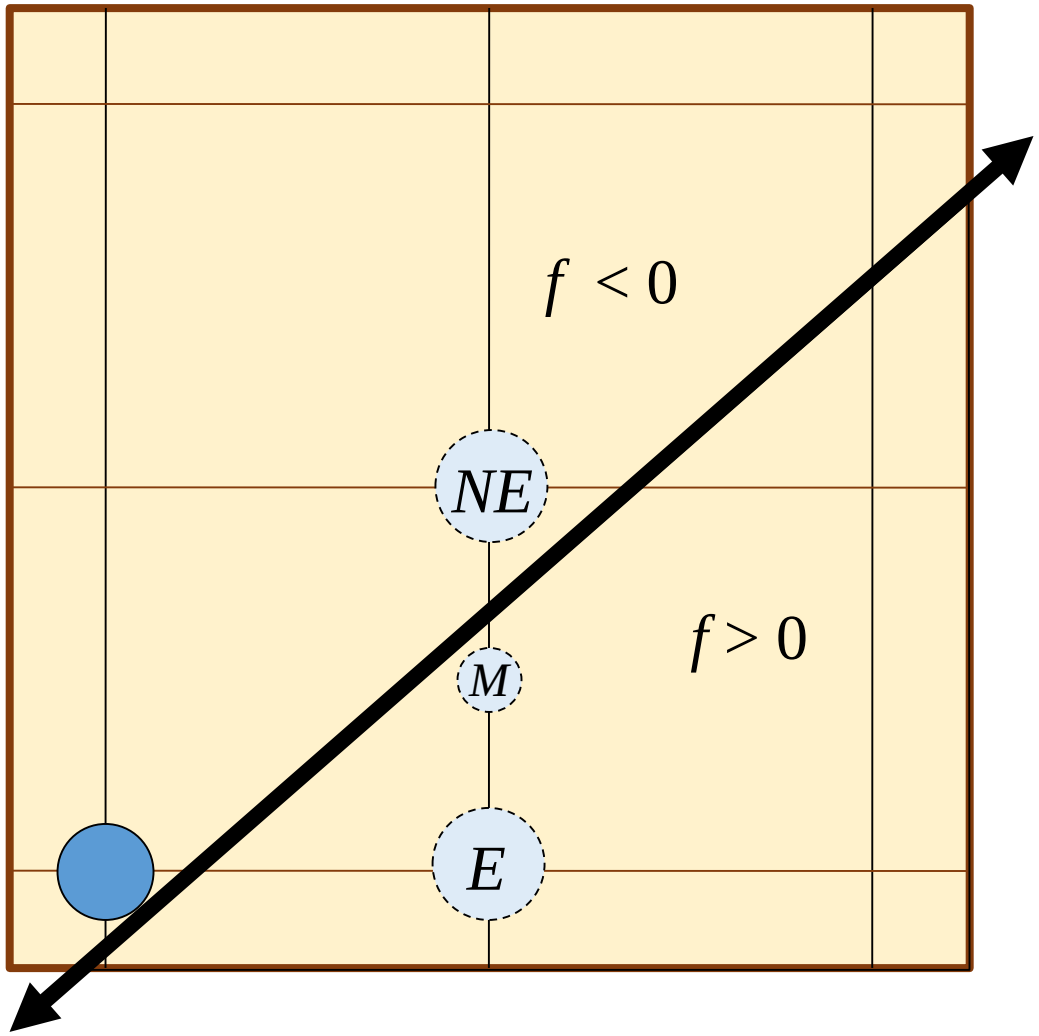
$$y = mx + b$$

$$m = (y_1 - y_0)/(x_1 - x_0)$$

$$b = y_0 - mx_0$$

$$f(x,y) = mx + b - y$$

- $f(M) \geq 0 \rightarrow NE$
- $f(M) < 0 \rightarrow E$



Pixel Increments

$f(x,y) = mx + b - y$

$M = P + (1,½)$

$f(M) = f(x+1,y+½)$

$= m(x+1) + b - (y+½)$

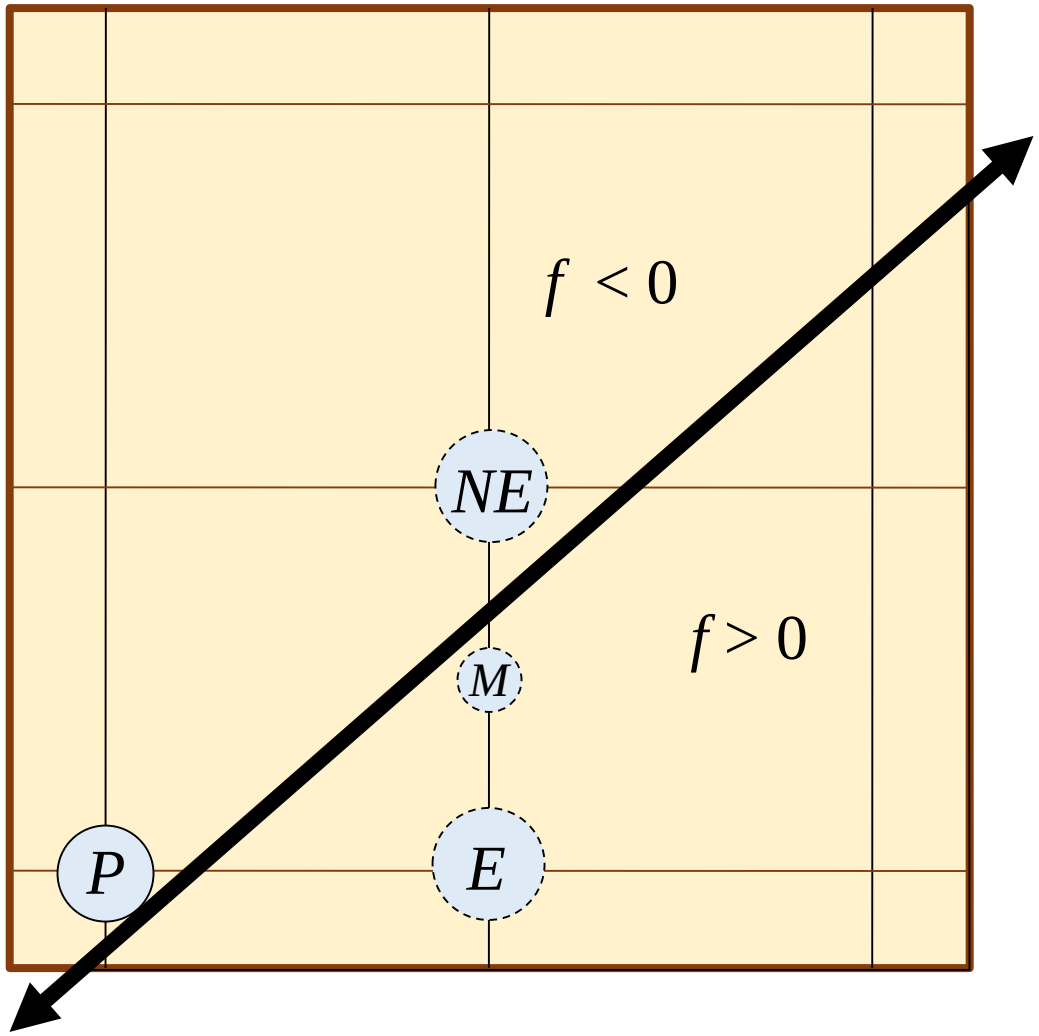
$= mx + m + b - y - ½$

$= mx + b - y + m - ½$

$= f(P) + m - ½$

$f(0,0) = b$

$= 0$ if line starts at origin



Midpoint Increments

$f(M) = f(P) + m - ½$

$f(M_E) = f(x+2,y+½)$

$= m(x+2) + b - (y+½)$

$= f(P) + 2m - ½$

$= f(M) + m$

$f(M_{NE}) = f(x+2,y+1½)$

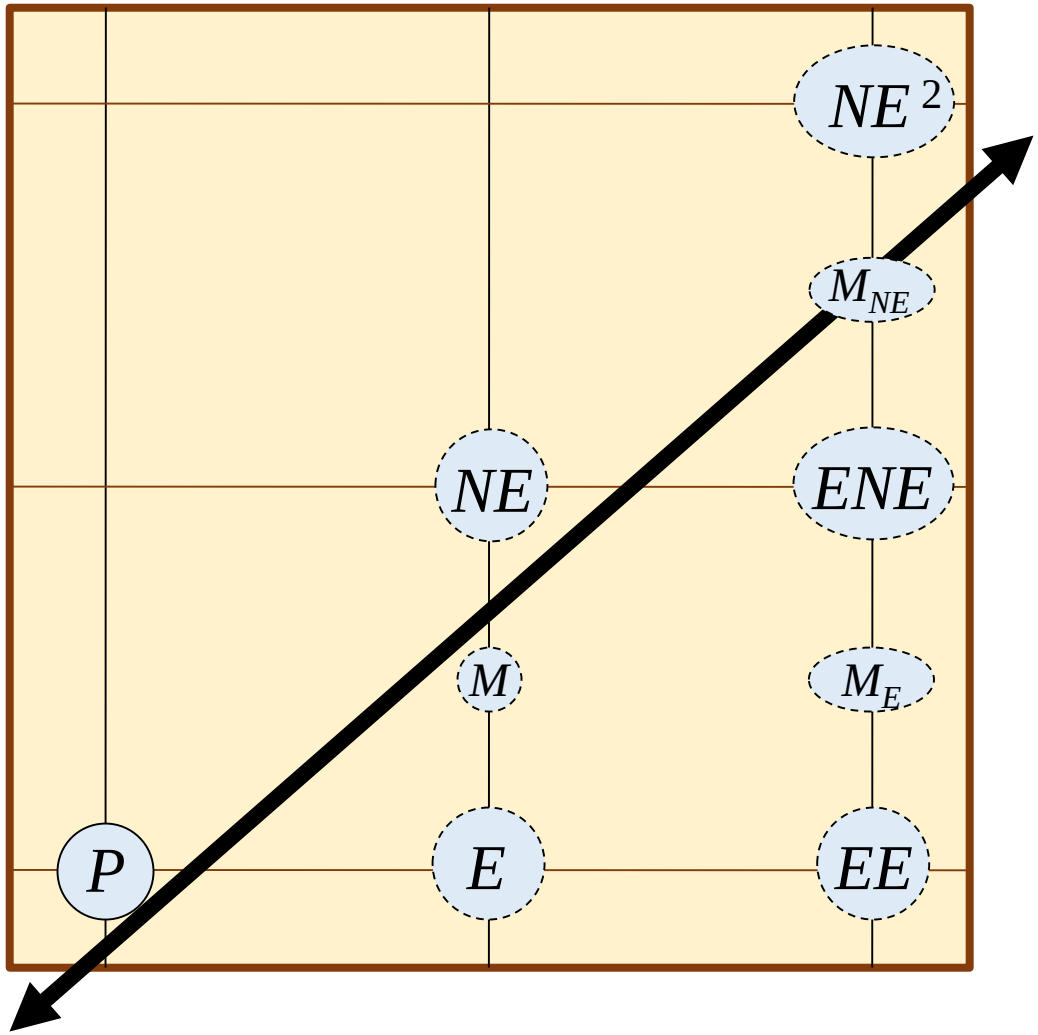
$= m(x+2) + b - (y+1½)$

$= f(P) + 2m - 1½$

$= f(M) + m - 1$

$f(1, ½) = m + b - ½$

$= m - ½$ if line starts at origin



Integer Math

$f(M_E) = f(M) + m$

$f(M_{NE}) = f(M) + m - 1$

$f(1, ½) = m + b - ½$

$b = 0$

$m = (y_1 - y_0)/(x_1 - x_0)$

$= \Delta y/\Delta x$

$\Delta x\, f(M_E) = \Delta x\, f(M) + \Delta y$

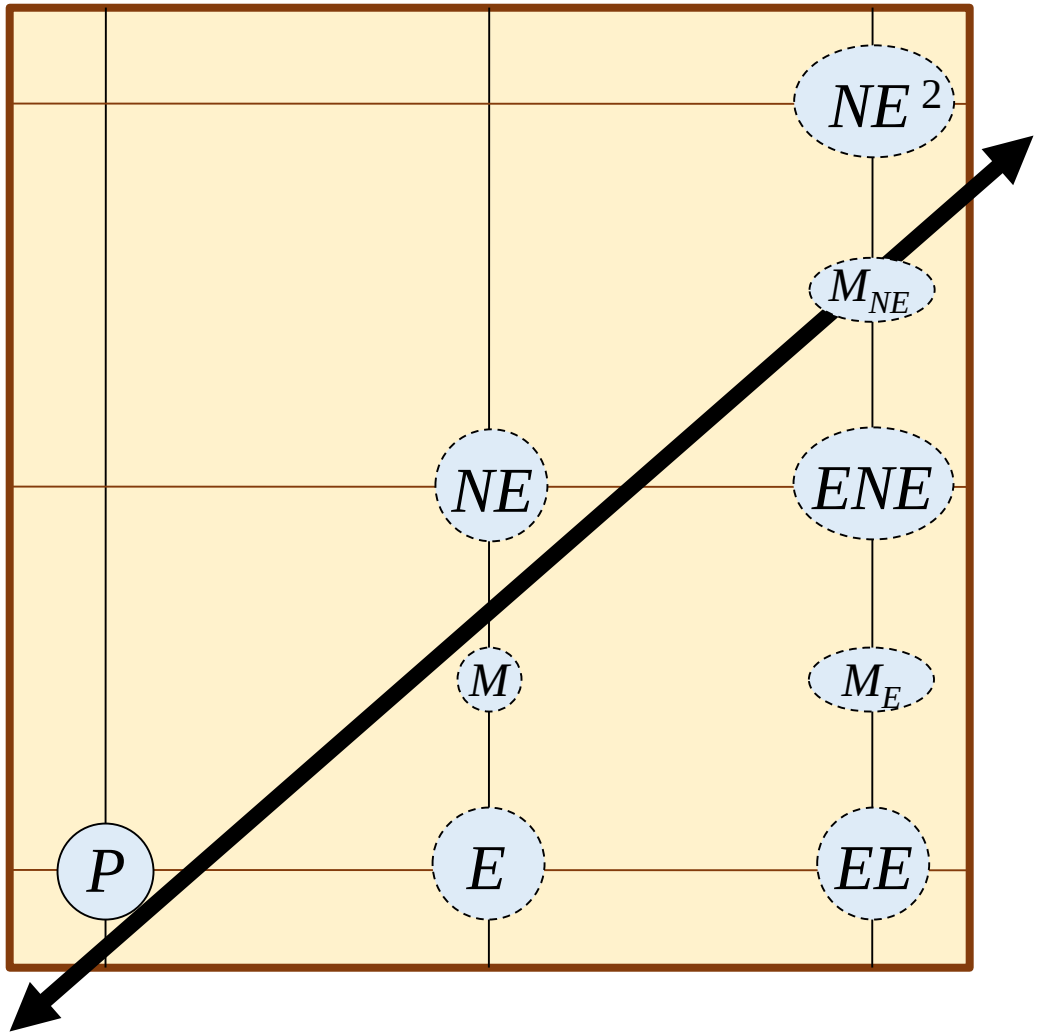
$\Delta x\, f(M_{NE}) = \Delta x\, f(M) + \Delta y - \Delta x$

$\Delta x\, f(1, ½) = \Delta y - ½\, \Delta x$

$2\Delta x f(M_E) = 2\Delta x\, f(M) + 2\Delta y$

$2\Delta x f(M_{NE}) = 2\Delta x\, f(M) + 2\Delta y - 2\Delta x$

$2\Delta x f(1, ½) = 2\Delta y - \Delta x$



Integer Math

$2\Delta x f(M_E) = 2\Delta x\, f(M) + 2\Delta y$

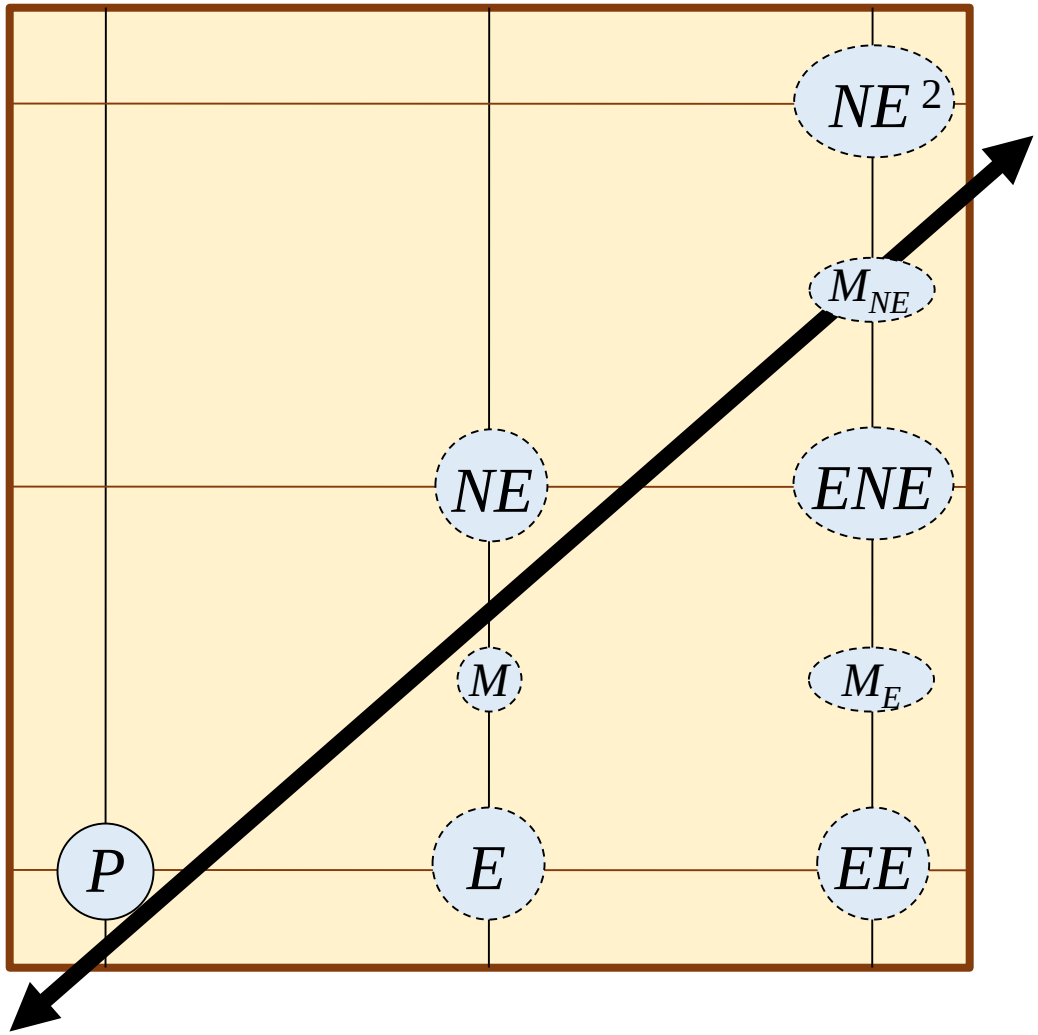
$2\Delta x f(M_{NE}) = 2\Delta x\, f(M) + 2\Delta y - 2\Delta x$

$2\Delta x f(1, ½) = 2\Delta y - \Delta x$

$F(M_E) = F(M) + 2\Delta y$

$F(M_{NE}) = F(M) + 2\Delta y - 2\Delta x$

$F(1,½) = 2\Delta y - \Delta x$



Integer Math

$F(M_E) = F(M) + 2\Delta y$

$F(M_{NE}) = F(M) + 2\Delta y - 2\Delta x$

$F(1,½) = 2\Delta y - \Delta x$

The Bresenham Line Algorithm

```
line(int x0,int y0,int x1,int y1)
{
  int dx = x1 - x0;
  int dy = y1 - y0;
  int F = 2*dy - dx;
  int dFE = 2*dy;
  int dFNE = 2*dy - 2*dx;
  int y = y0;
  for (int x = x0, x < x1; x++) {
    plot(x,y);
    if (F < 0) {
      F += dFE;
    } else {
      F += dFNE; y++;
    }
  }
}
```