

You Are the Rendering Engine

1. Projection

Imagine we construct a digital model of a scene with a single triangle.

In our world coordinate system, the vertices of the triangle are at:

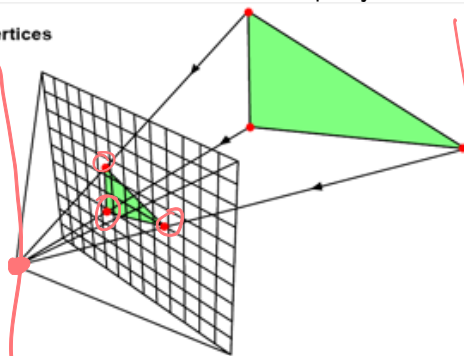
$v_1=(-4, 4, -4)$, $v_2=(4, 4, -4)$, $v_3=(4, -4, -8)$

We place the eyepoint at $(0,0,0)$ and will use perspective projection.

What are the coordinates when projected onto the image plane $z = -1$?

$(x_c, y_c, z_c) \rightarrow$ coordinates after projection

1) Project vertices

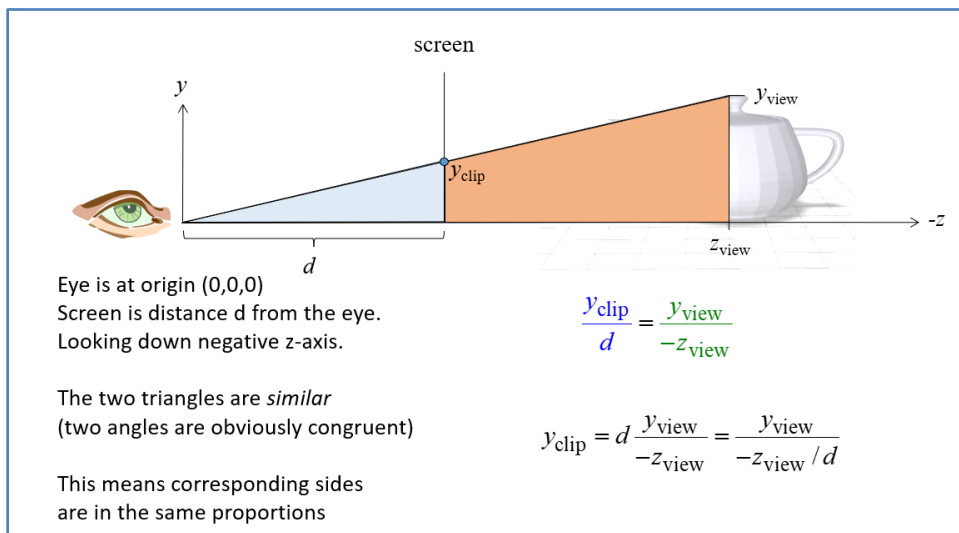


$$\begin{aligned} v_3 \quad x_c &= \frac{4}{8/1} = 1/2 \\ y_c &= \frac{-4}{8/1} = -1/2 \\ z_c &= -1 \end{aligned}$$

$$\begin{aligned} v_1 \quad x_c &= \frac{-4}{4/1} = -1 \\ y_c &= \frac{4}{4/1} = 1 \\ z_c &= -1 \end{aligned}$$

$$\begin{aligned} v_2 \quad x_c &= \frac{4}{4/1} = 1 \\ y_c &= \frac{4}{4/1} = 1 \\ z_c &= -1 \end{aligned}$$

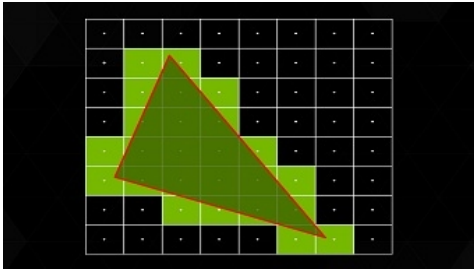
Below, you can see how to compute a perspective projection for the y coordinate of a vertex. Projecting an x coordinate is done similarly. The z coordinate in this case will project to $z=-1$, since that is the location of the image plane.



y_{clip} is
y coordinate
in clip
coordinates
... after
projection

normalized device coordinates \sim clip coordinates

2. Rasterization



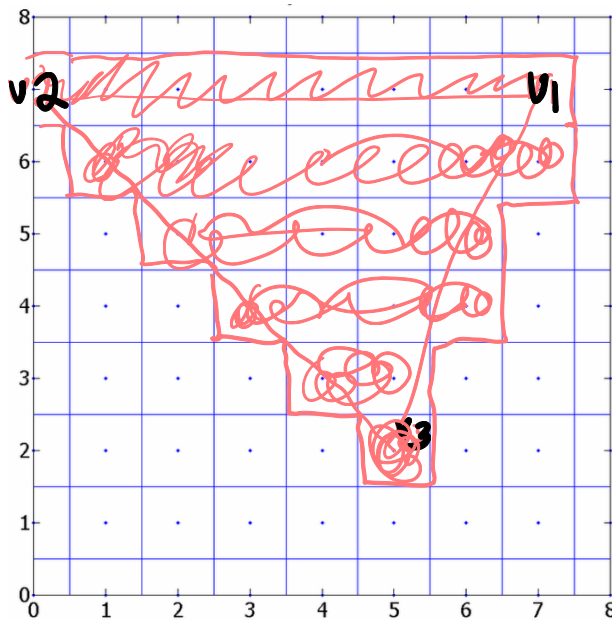
We want to transform from normalized device coords (or window coords) to viewport coords

$$\begin{bmatrix} \frac{w-1}{2} & 0 & \frac{w-1}{2} \\ 0 & \frac{h-1}{2} & \frac{h-1}{2} \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Here } w=h=8 \quad \rightarrow \quad \begin{bmatrix} 7/2 & 0 & 7/2 \\ 0 & 7/2 & 7/2 \\ 0 & 0 & 1 \end{bmatrix}$$

The raster (set of pixels) we are generating is an 8x8 grid of pixels with the bottom left corner at $(-1, -1)$ and the top right corner at $(1, 1)$ in normalized device coordinates.

We want to generate a list of pixels to light up that corresponds to projection of the triangle on the view plane. To refer to the pixels, we will use **viewport coordinates (also called screen space coordinates)**. These are 2D coordinates where (m, n) indicates the pixel in row m and column n of the raster. Our viewport coordinates will range from $(0, 0)$ to $(7, 7)$.

The bottom left pixel has viewport coordinates $(0, 0)$. Which pixels will be lit up for the triangle from question 1? You can find this by drawing on the grid below and coloring in any pixels the triangle crosses.



First point is $(x, z) = (1, 1) = (1, 1, 1)$ in

$$\begin{bmatrix} 7/2 & 0 & 7/2 \\ 0 & 7/2 & 7/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{2D homogeneous coords}$$

$$\rightarrow (7, 7, 1)$$

$$v_2 \rightarrow (0, 7, 1)$$

$$v_3 \rightarrow (5\frac{1}{4}, 2\frac{1}{4}, 1)$$