

# A Simple Physics Engine

Interactive Computer Graphics  
Professor Eric Shaffer

# Newtonian Physics

- We will animate particles (aka point masses)
  - Position is changed by velocity
  - Velocity is changed by acceleration
  - Forces alter acceleration
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- Our physics engine will integrate to compute
    - Position
    - Velocity
  - We set the acceleration by applying forces

# Force and Mass and Acceleration

- How do we update acceleration when force is applied?
- To find the acceleration due to a force we have

$$\underline{\ddot{\mathbf{p}}} = \frac{1}{m} \mathbf{f}$$

- So we need to know the inverse mass of the particle
  - You can model infinite mass objects by setting this value to 0
- For the MP, you can use a uniform mass of 1
  - Or make the masses different if you want...

# Force: Gravity

- Law of Universal Gravitation

$$f = G \frac{m_1 m_2}{r^2}$$

- G is a universal constant
- $m_i$  is the mass of an object
- $r$  is the distance between object centers
- if we care only about gravity of the Earth
  - $m_1$  and  $r$  are constants
  - $r$  is about 6400 km on Earth
- We simplify to  $f = mg$ 
  - $g$  is about  $10\text{ms}^{-2}$

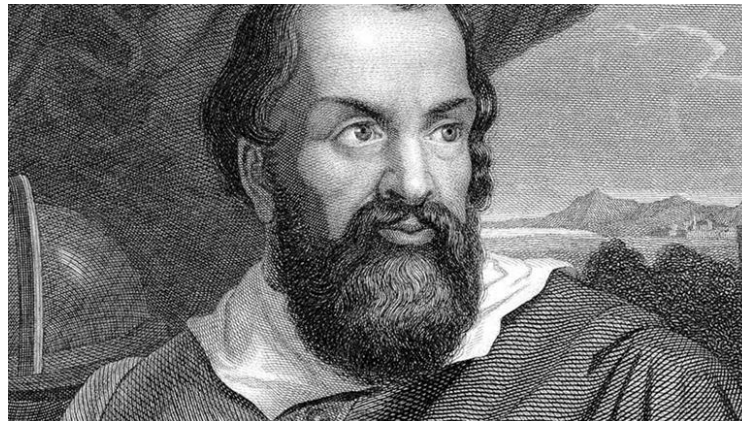
# Acceleration due to Gravity

- If we consider acceleration due to gravity we have

$$\ddot{p} = \frac{1}{m}(mg) = g$$

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- So acceleration due to gravity is independent of mass



# Acceleration due to Gravity

- In your MP the magnitude and direction of acceleration would be

$$\mathbf{g} = \langle 0, -g, 0 \rangle$$

- For gaming,  $10\text{ms}^{-2}$  tends to look boring
  - Shooters often use  $15\text{ms}^{-2}$
  - Driving games often use  $20\text{ms}^{-2}$
  - Some tune  $g$  object-by-object

# Force: Drag

- Drag dampens velocity
  - Caused by friction with the medium the object moves through
- Even neglecting drag, you need to dampen velocity
  - Otherwise numerical errors likely drive it higher than it should be
- A velocity update with drag can be implemented as

$$\dot{\mathbf{p}}_{new} = \dot{\mathbf{p}} d^t$$

- important to incorporate time so drag changes if the frame rate varies
  - for the MP, have all objects have the same drag, calculate once per frame
- What range should  $d$  be in?  $(0, 1)$

# The Integrator

- The position update can found using Euler's Method:

$$\underline{\mathbf{P}_{new} = \mathbf{P}_{old} + \dot{\mathbf{P}}t}$$

- This is a pretty inaccurate approximation of analytical integration
  - formula gets more inaccurate as acceleration gets larger
    - why?
  - In general we can characterize Euler method error as  $O(t)$
  - ...good enough for the MP
- The velocity update is computed using Euler integration as well

$$\underline{\dot{\mathbf{p}}_{new} = \dot{\mathbf{p}}d^t + \ddot{\mathbf{p}}t}$$



# The Integrator

- You should ideally use actual time for  $t$ 
  - or some scaled version of it
- In JavaScript, Date.now() returns current time in ms
  - so keep a previous time variable
  - each frame find out how much time has elapsed
- ...or you could use some uniform timestep you like