Perspective Projection
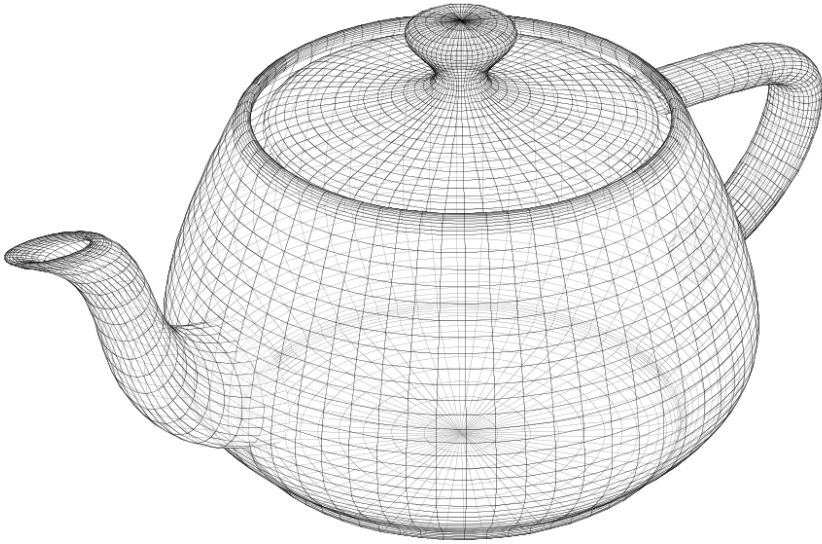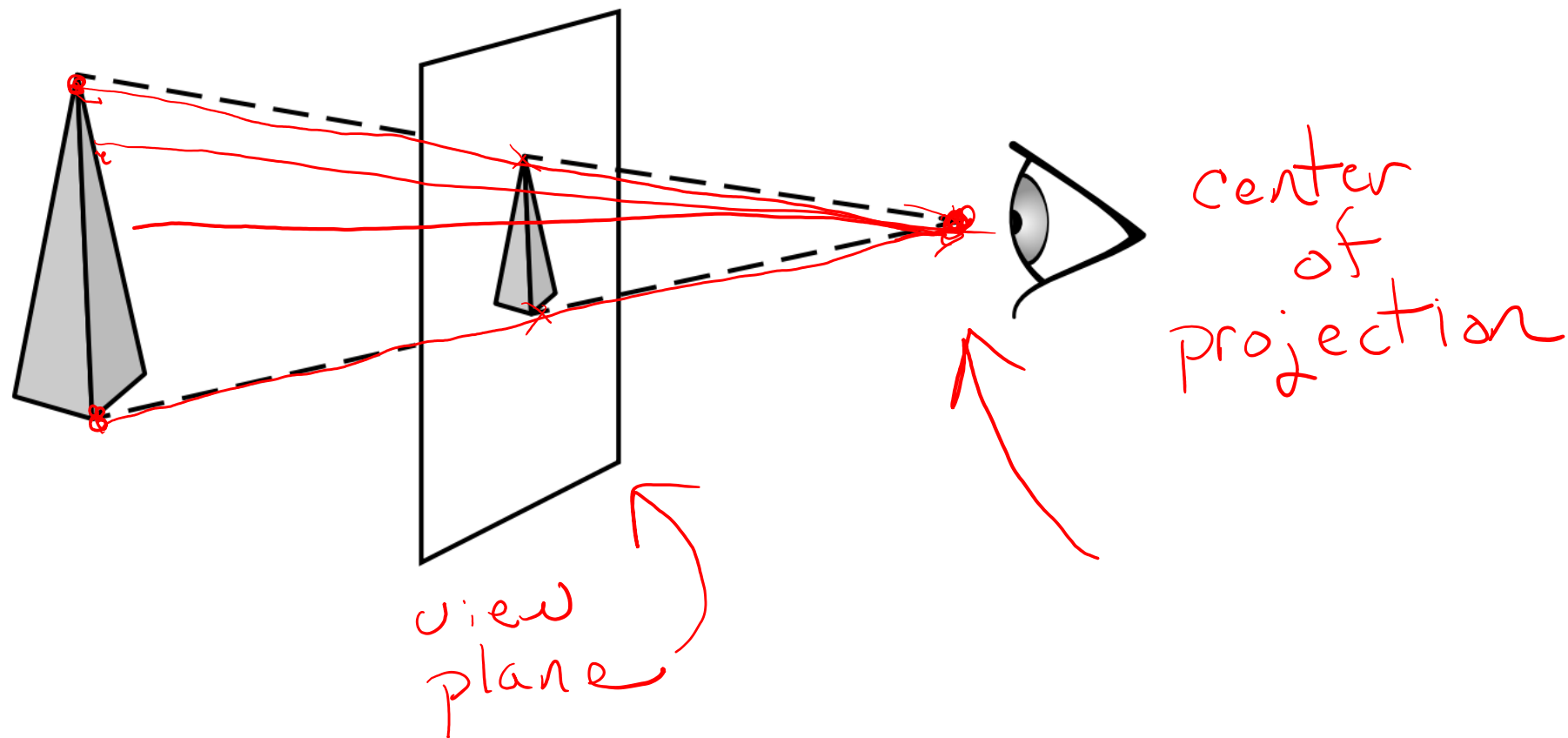
CS 418: Interactive Computer Graphics

Professor Eric Shaffer

ILLINOIS

Perspective Projection

center of projection

view plane

ILLINOIS

Perspective

Perception relies on shape constancy
- Real world objects do not resize
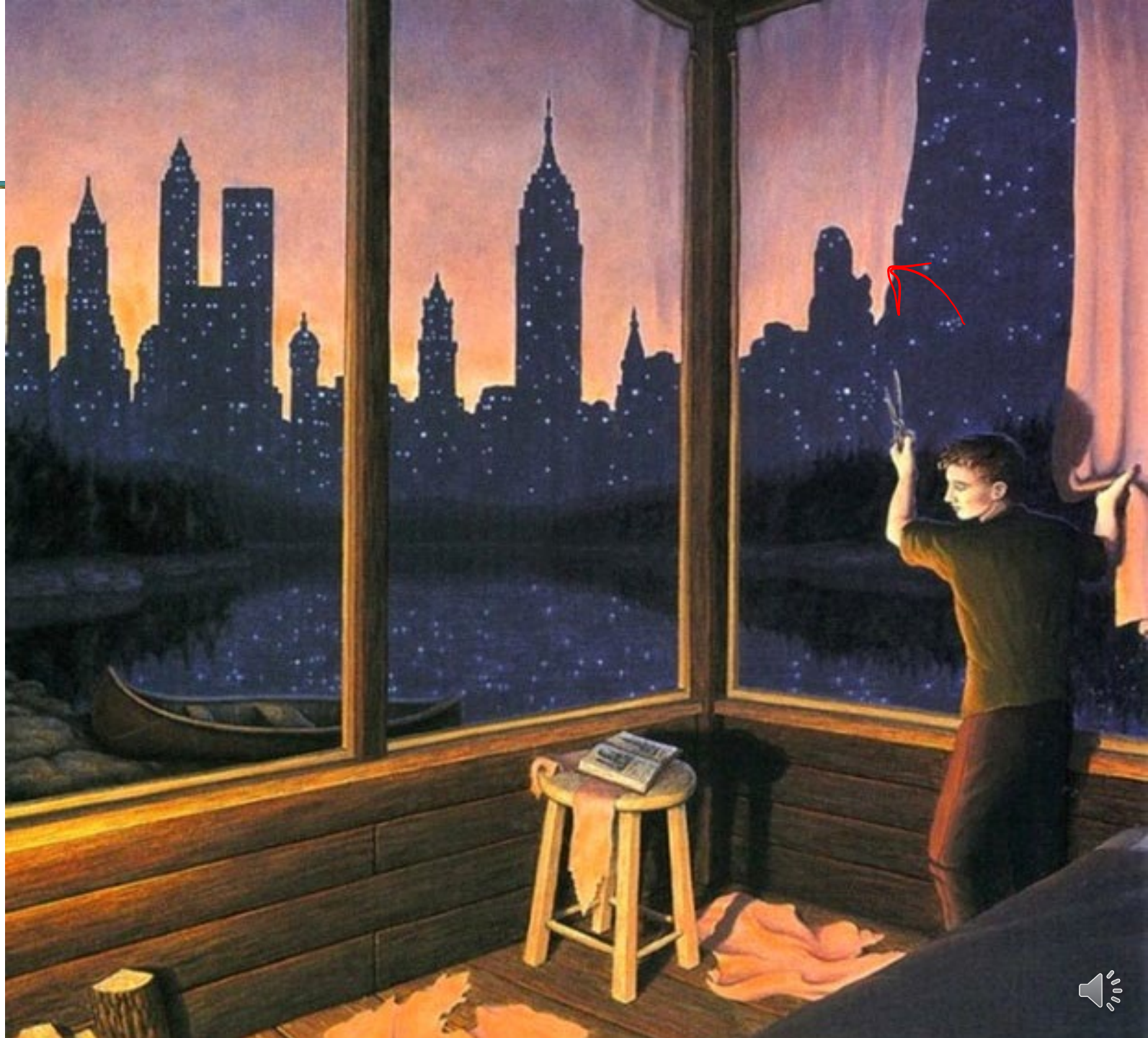- Change in size due to depth
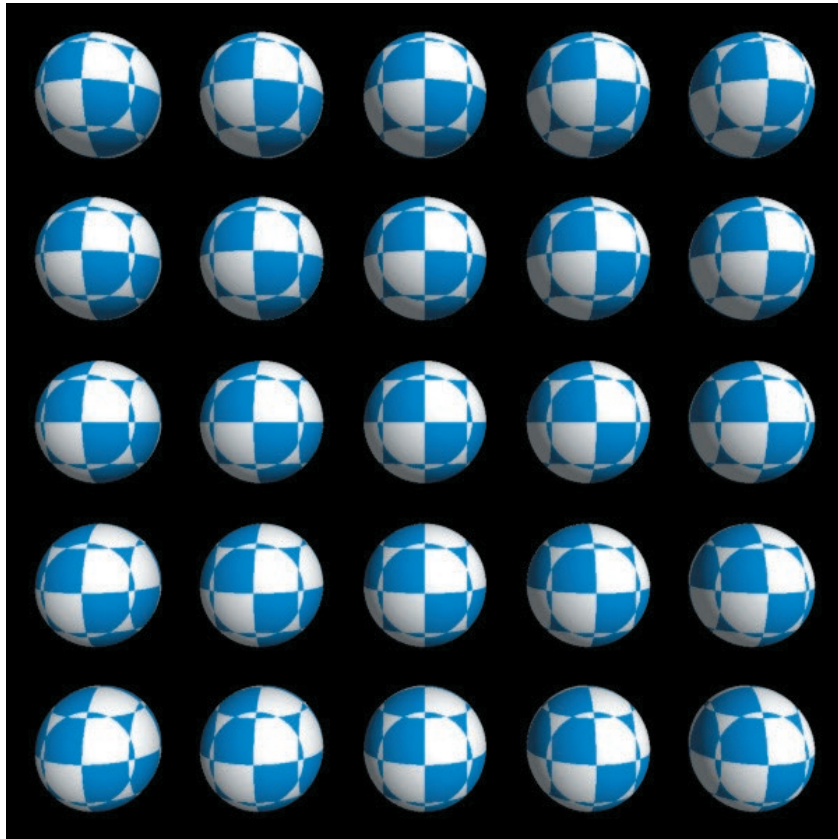
Closer objects larger

Farther objects smaller

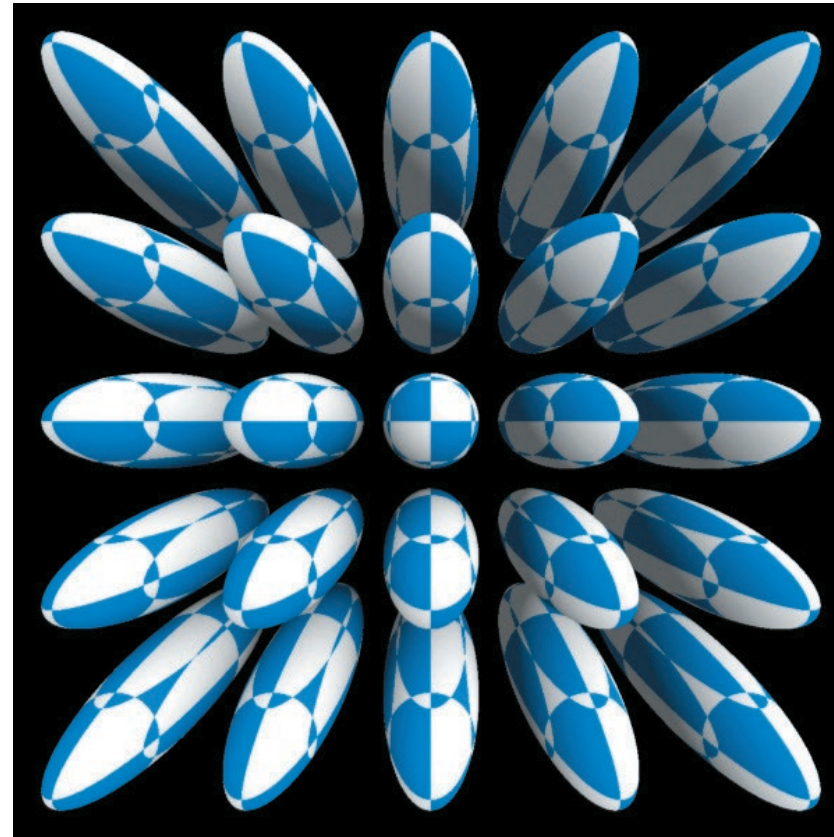Gustave Caillebotte - Paris Street, Rainy Day Art Institute of Chicago

A Change Of Scenery
by Rob Gonsalves
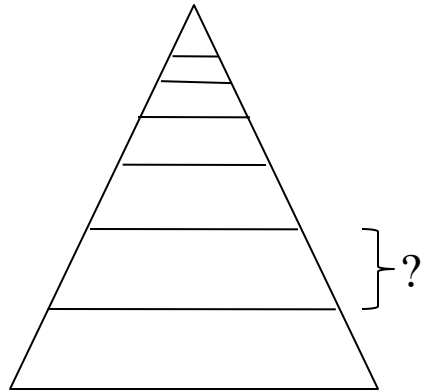
Perspective Distortion
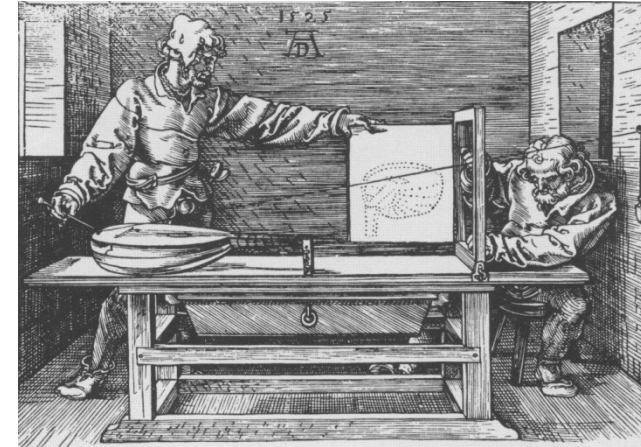

Orthographic

Perspective
Should the spheres look like this?
Why don't our eyes see like this?
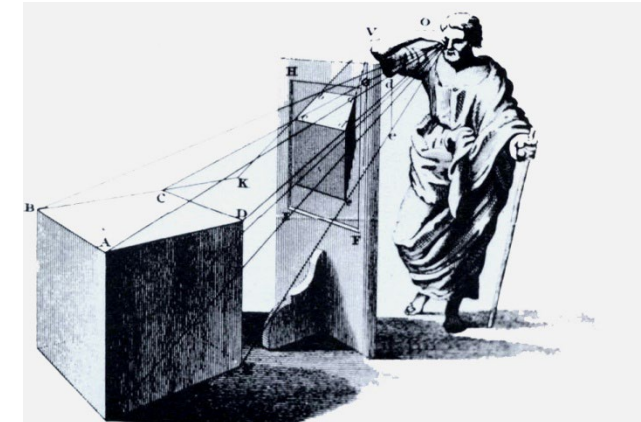
Perspective Projection

- Brain depends on shape constancy
  - Real world objects do not resize
  - Change in size due to depth
- Closer objects larger
- Farther objects smaller
- How large, how small?

Albrecht Durer woodcut c. 1525,
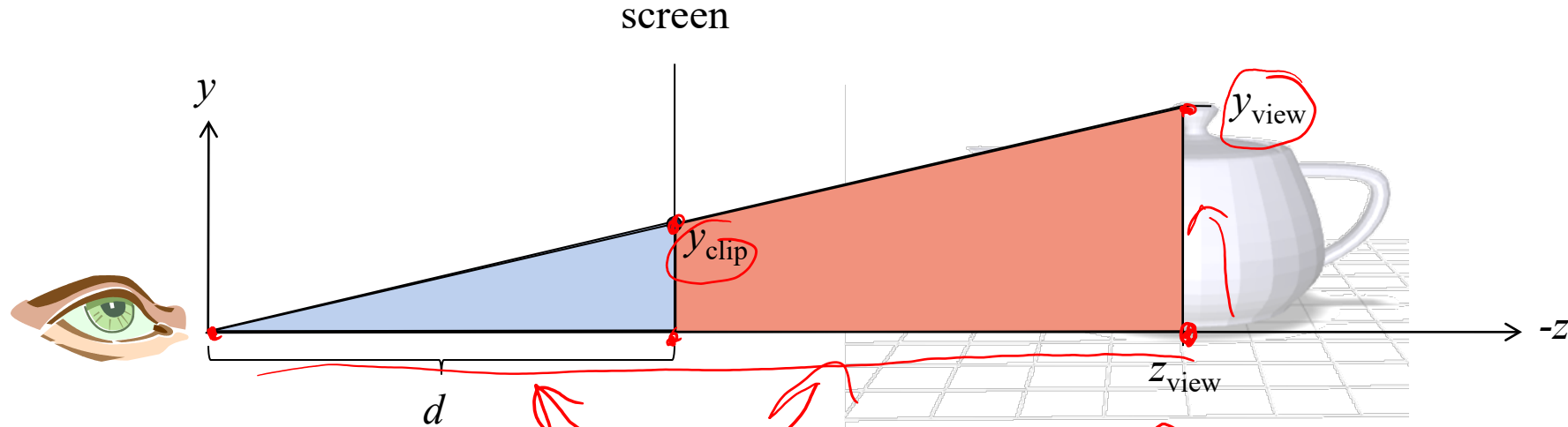swiped from Marc Levoy's CS48N notes c. 2007

More Durer, swiped from Fredo Durand's Art of Depiction

The visual cone, from B. Taylor, *New Principles of Linear Perspective* (1715).

ILLINOIS

Perspective

screen

y

y_{view}

y_{clip}

z_{view}

-z

d

Eye is at origin (0,0,0)
Screen is distance d from the eye.
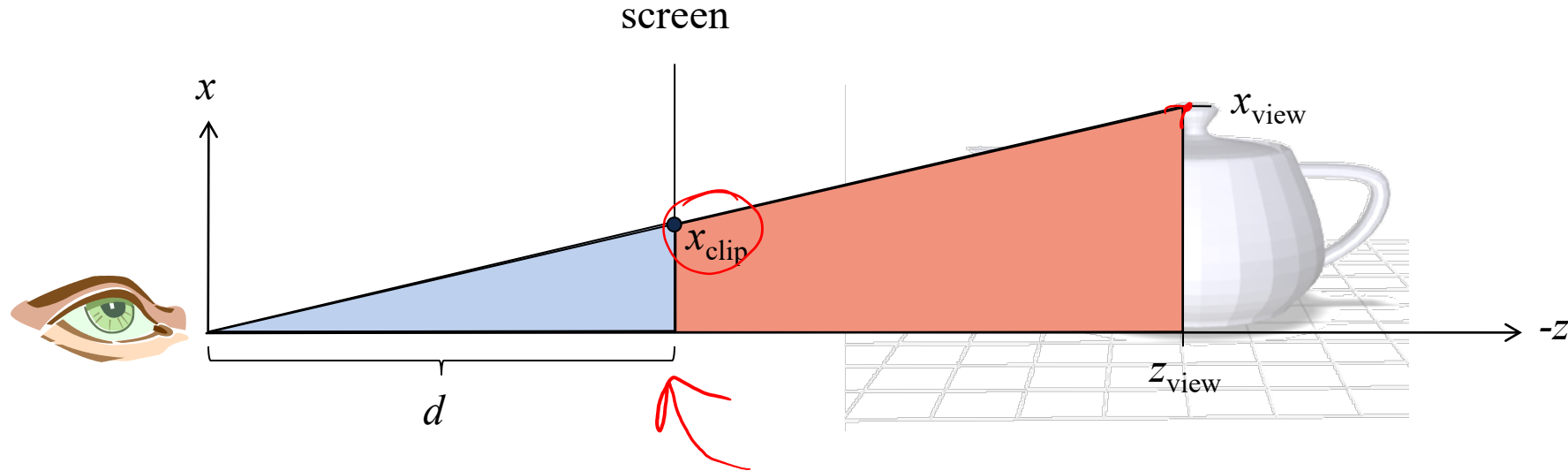Looking down negative z-axis.

The two triangles are *similar*
(two angles are obviously congruent)

This means corresponding sides
are in the same proportions

$$\frac{y_{\text{clip}}}{d} = \frac{y_{\text{view}}}{-z_{\text{view}}}$$

$$y_{\text{clip}} = d\,\frac{y_{\text{view}}}{-z_{\text{view}}} = \frac{y_{\text{view}}}{-z_{\text{view}}/d}$$

Perspective

screen

x

x_{view}

x_{clip}

$-z$

d

z_{view}

Same process derives the projection for the x coordinate.

What is z_{clip}?

$= -d$

$$x_{clip} = \frac{x_{view}}{-z_{view}/d}$$

Perspective

screen

x

x_{view}

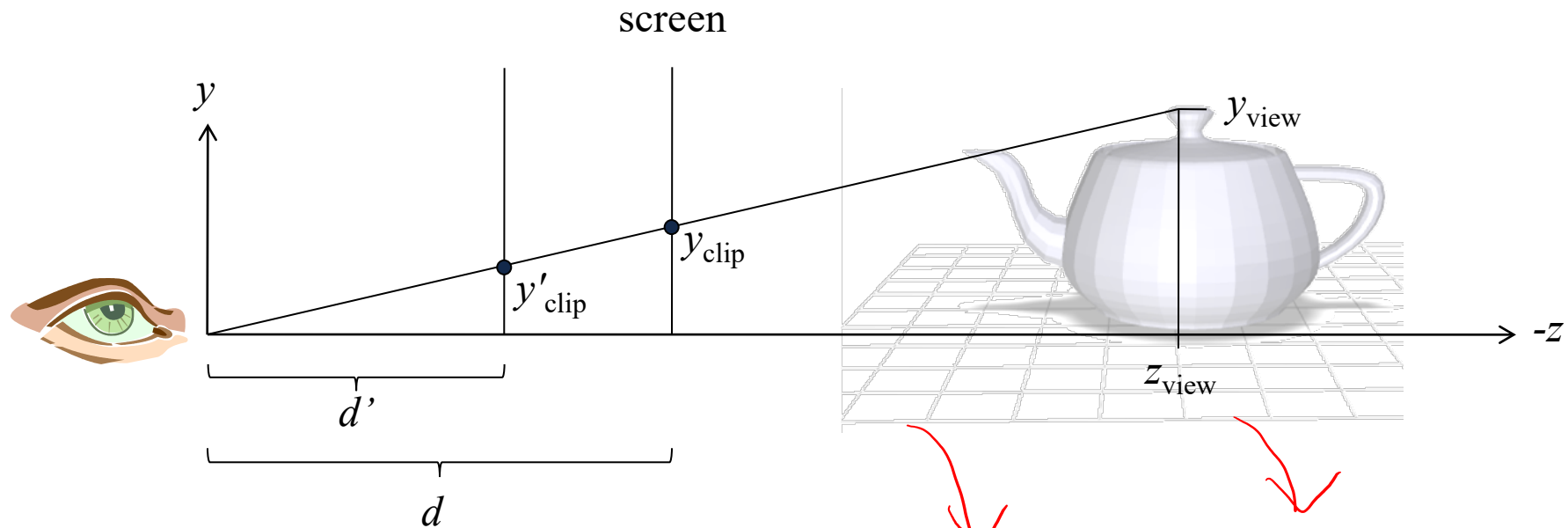x_{clip}

d

z_{view}

$-z$

x_{view}
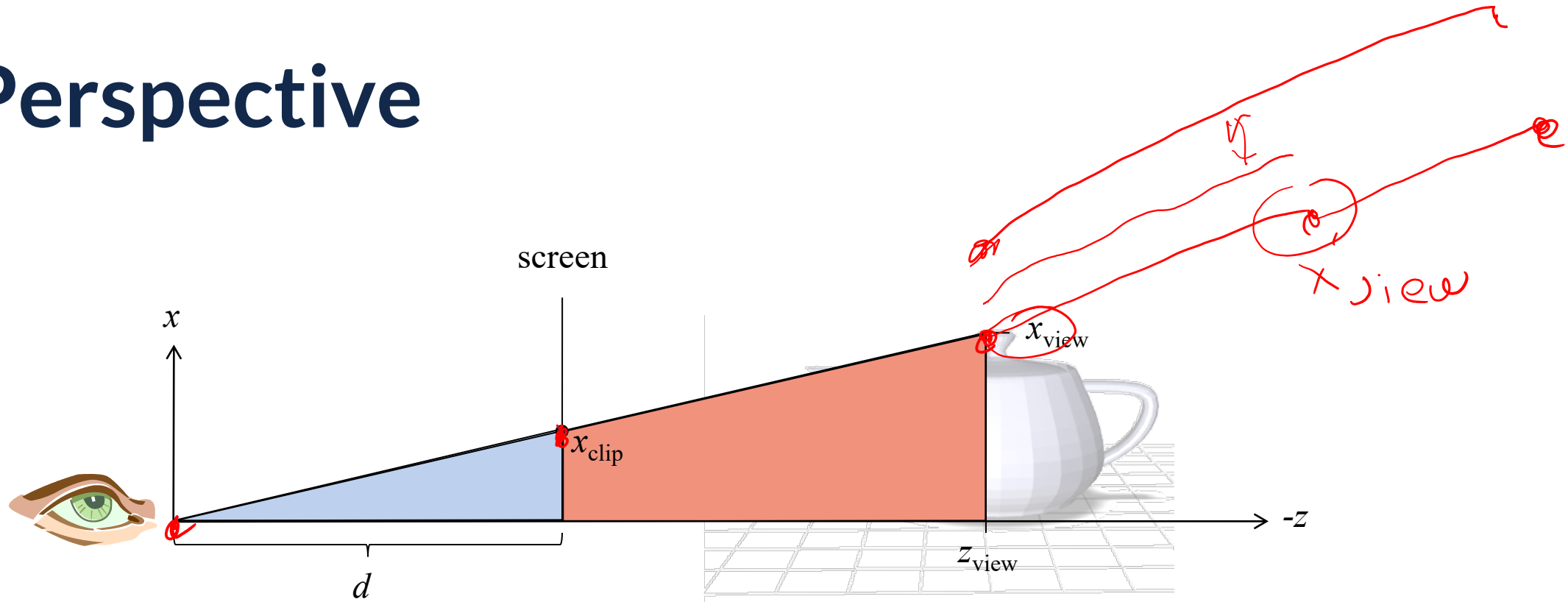
1. This transformation is not invertible
2. It does preserve lines (except when?)
3. It is not an affine transformation
   (it does not preserve ratios of distances)
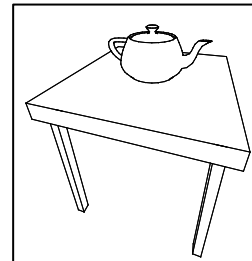
Distance

screen

y_{view}

$-z$

z_{view}

y_{clip}

y'_{clip}

d'
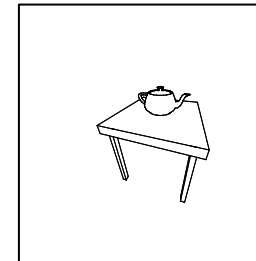
d

y_{clip} = d y_{view}/(-z_{view})

y'$_{clip}$ = d' y_{view}/(-z_{view}) = (d'/d) y_{clip}

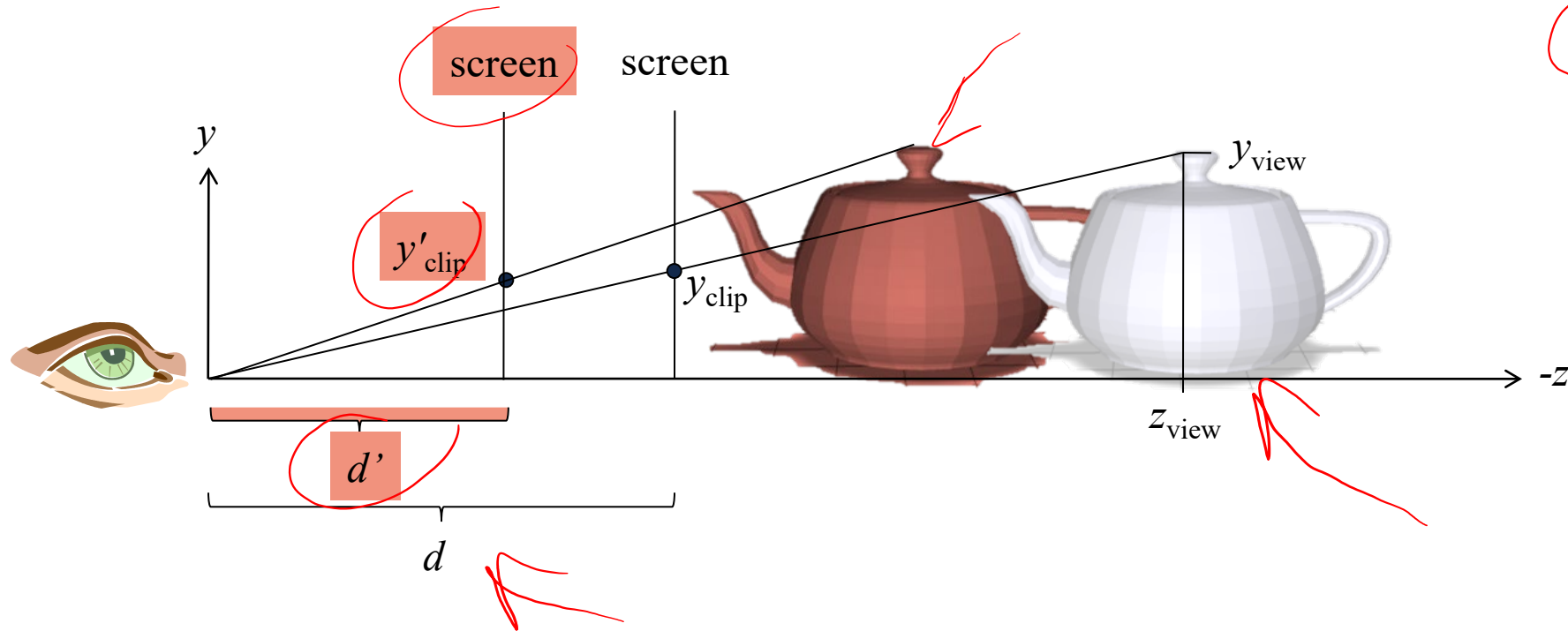Changing parameter d just changes
scale of projection

Using d

Using d'

Perspective Distortion and Distance

To change degree of perspective distortion, need to change distance from eye to scene,

…by moving scene closer or farther to eye,

… along z axis in viewing coordinates