

Linear Interpolation

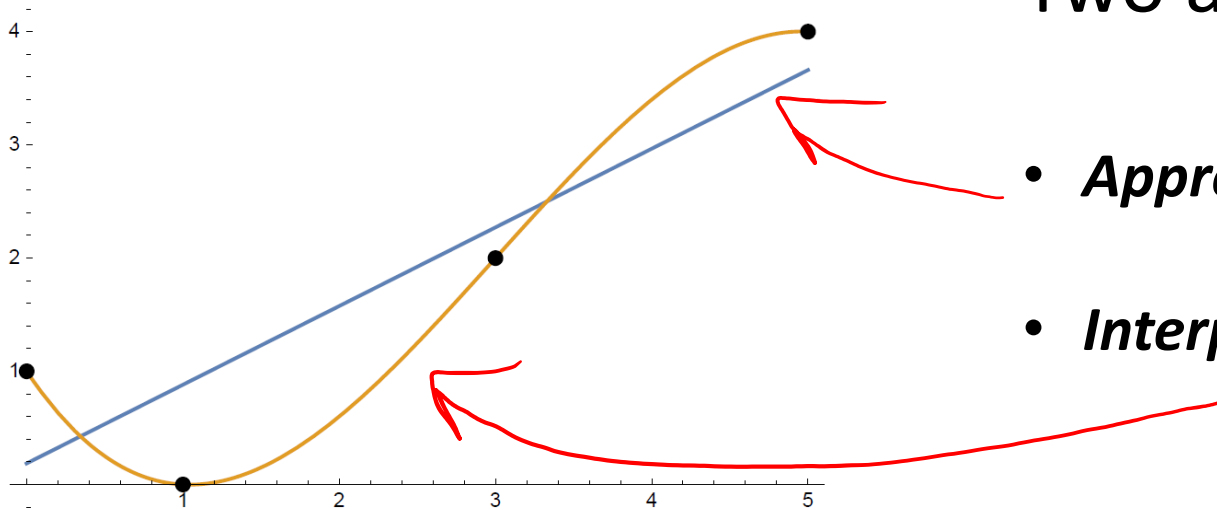


Interactive Computer Graphics
Professor Eric Shaffer

What is Interpolation?

Two approaches to data fitting

- **Approximation**...captures the behavior of the data
- **Interpolation**...matches the observed data exactly

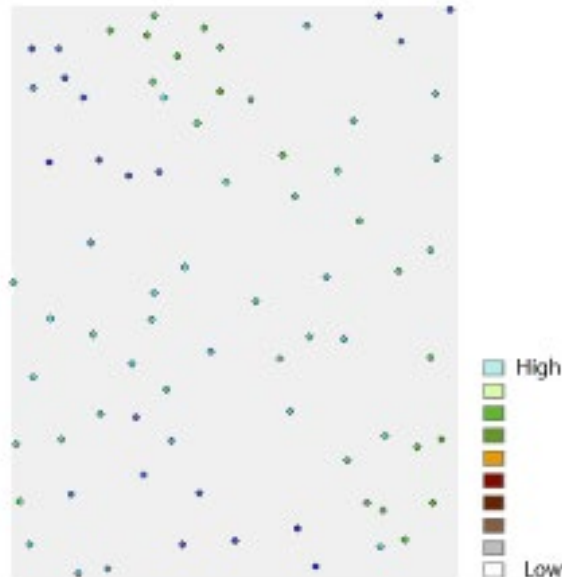


“Interpolation simply means fitting some function to given data so that the function has the same values as the given data.” – Professor Michael T. Heath.

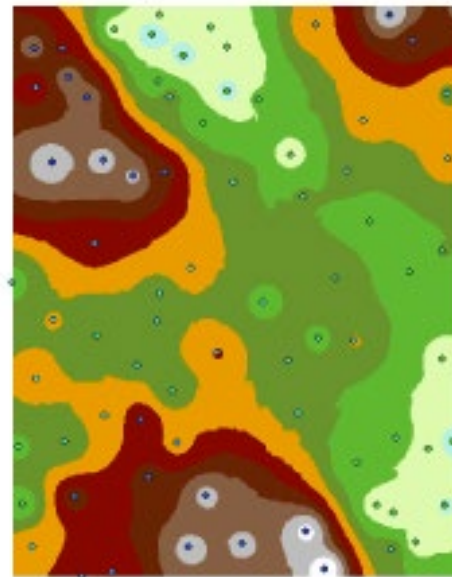
And Why Are We Data Fitting?

- Often have empirical data...sampled values in some domain
- Would like to fill in unknown values in the domain
- Interpolation constructs a function that can fill in these unknown values

Sampled Elevations



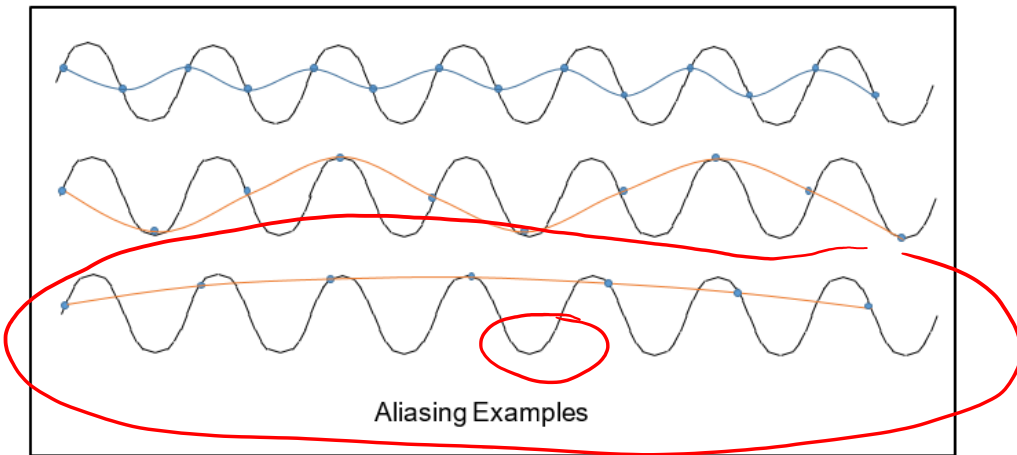
Interpolated Elevations



Why Linear Interpolation

- Simple....conceptually and computationally
- With no other information about underlying function...linear is fine

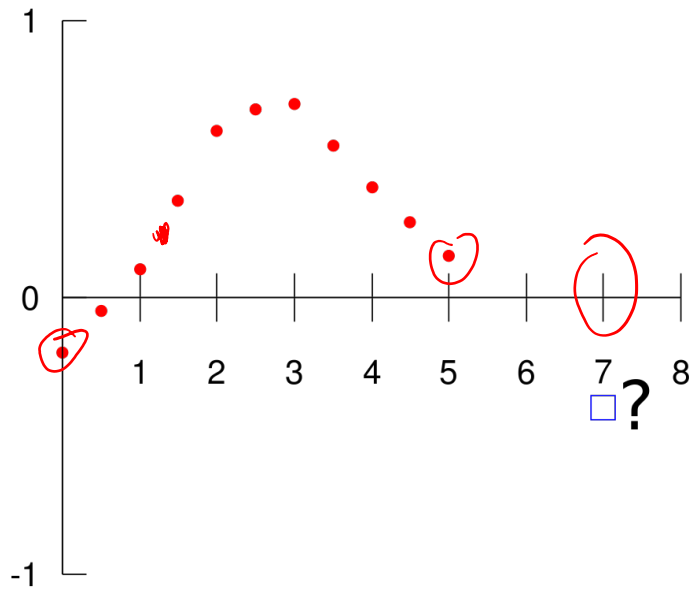
...but if you know the underlying function is not linear...fit with a non-linear function



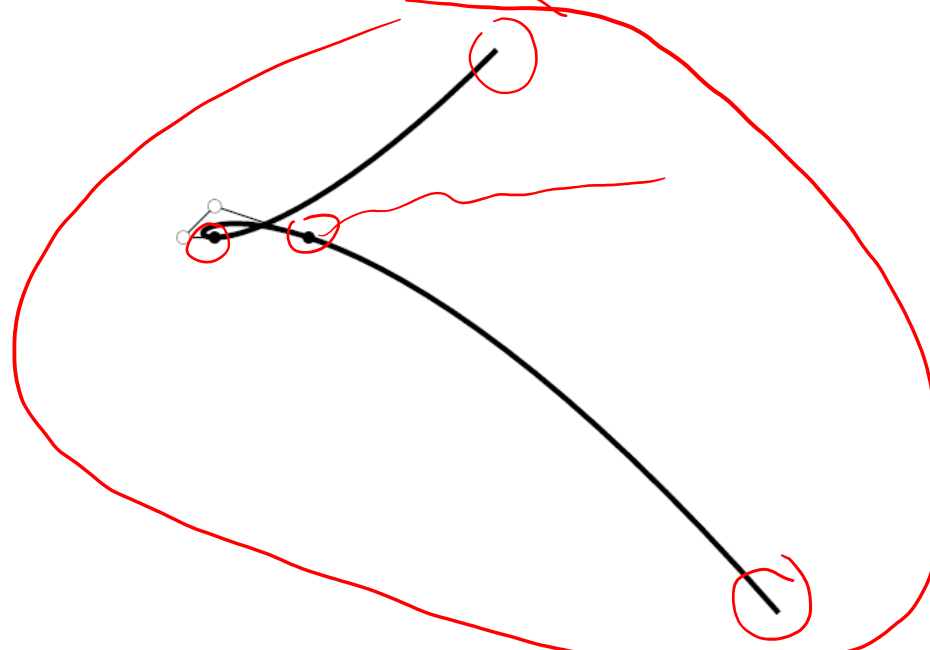
In signal processing and related disciplines, aliasing is an effect that causes different signals to become indistinguishable when sampled. It also often refers to the distortion or artifact that results when a signal reconstructed from samples is different from the original continuous signal. [Wikipedia](#)

Extrapolation

Inferring unknown values beyond the range of known values



Interpolative methods may perform especially poorly for extrapolation



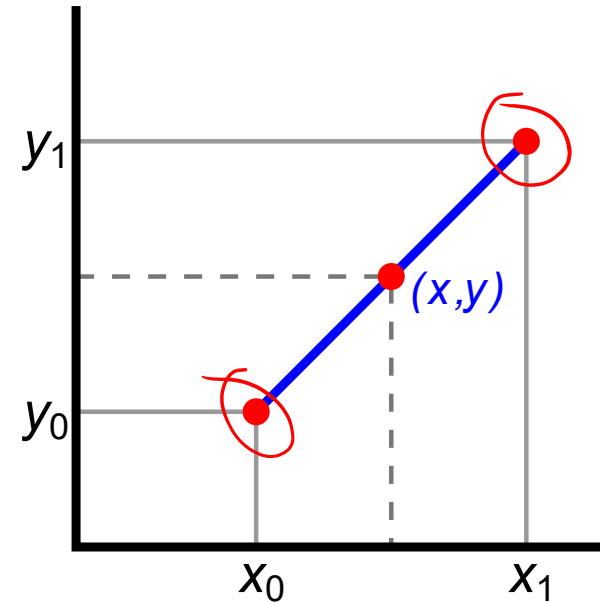
Linear Interpolation of Position

We have initial position of $p_0 = (x_0, y_0)$ and a final position of $p_1 = (x_1, y_1)$

Can generate intermediate positions using a parameterized linear function

$$P(t) = (1-t)p_0 + tp_1$$

$$x(t) = x_0(1-t) + x_1(t)$$



Linear Interpolation of Function Values

If we have function values sampled at points p_0 and p_1

$$f(p_0) = v_0 \quad \text{and} \quad f(p_1) = v_1$$

We can find $f(t) = (1-t) v_0 + t v_1$

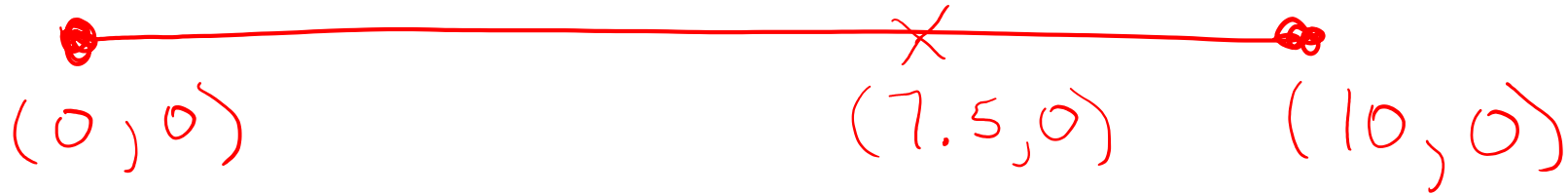
What if we are given a point p_i on the line and don't know t ?

$$t = \frac{\text{dist}(p_i, p_0)}{\text{dist}(p_1, p_0)}$$

Linear Interpolation of Function Values: Example

$$f(x) = 4$$

$$f(x) = 20$$



$$t = \frac{(7.5 - 0)}{(10 - 0)} = 0.75$$

$$\begin{aligned} f(7.5) &\approx (1 - 0.75)4 + (0.75)20 \\ &\approx 1 + 15 = \boxed{16} \end{aligned}$$

Bilinear Interpolation

Assume we know a function value at the four points

$$Q_{11} = (x_1, y_1), Q_{12} = (x_1, y_2),$$

$$Q_{21} = (x_2, y_1), Q_{22} = (x_2, y_2)$$

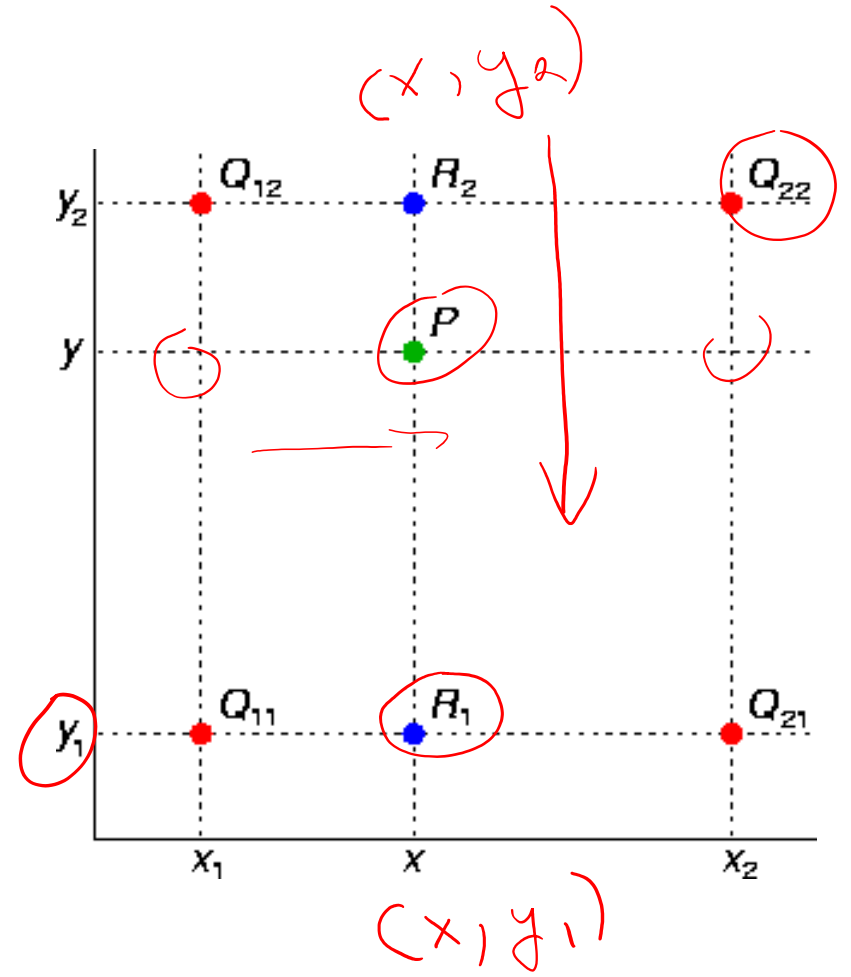
We first do linear interpolation in the x-direction

- Find function values at R1 and R2

Then in the y direction

- Interpolate between R1 and R2 to find value at P

Order in which you
interpolate (which axis goes
1st) does not matter.



Bilinear Interpolation

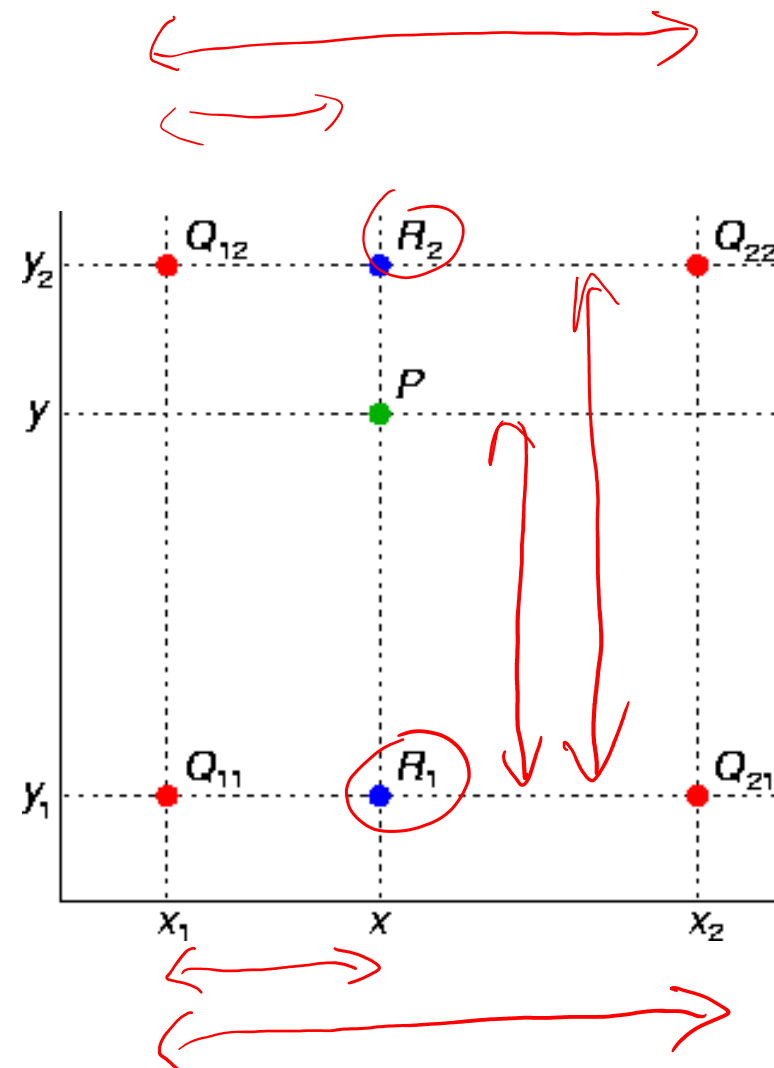
(1-H)

$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}),$$

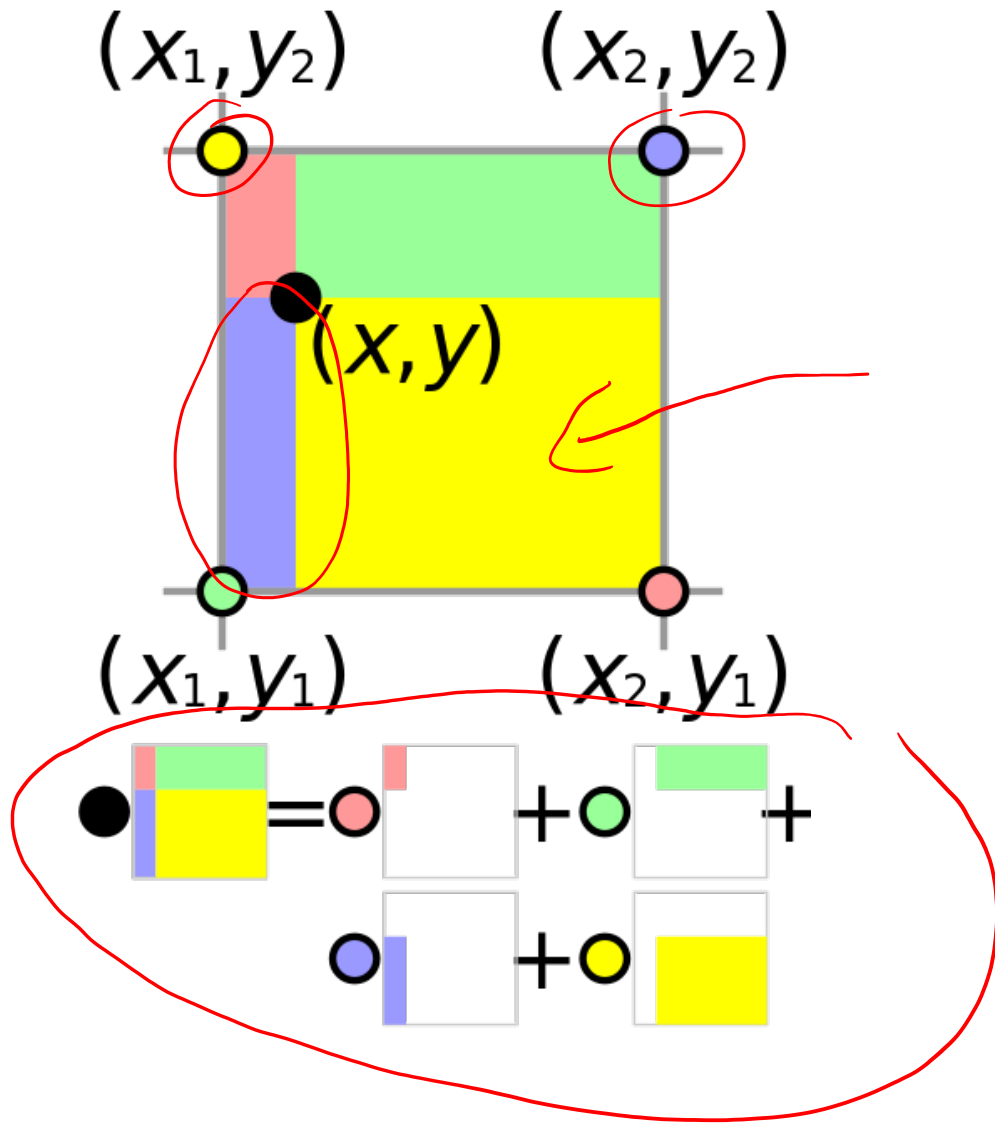
$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}).$$

$$f(x, y) \approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2)$$

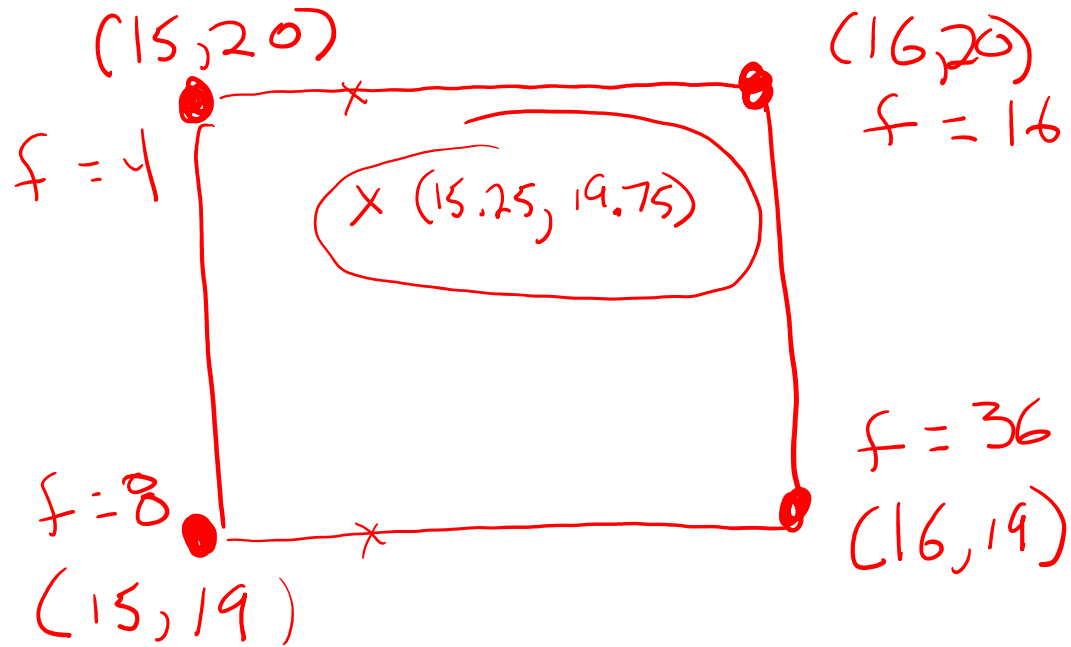
from Wikipedia



Bilinear Interpolation



Bilinear Interpolation: Example

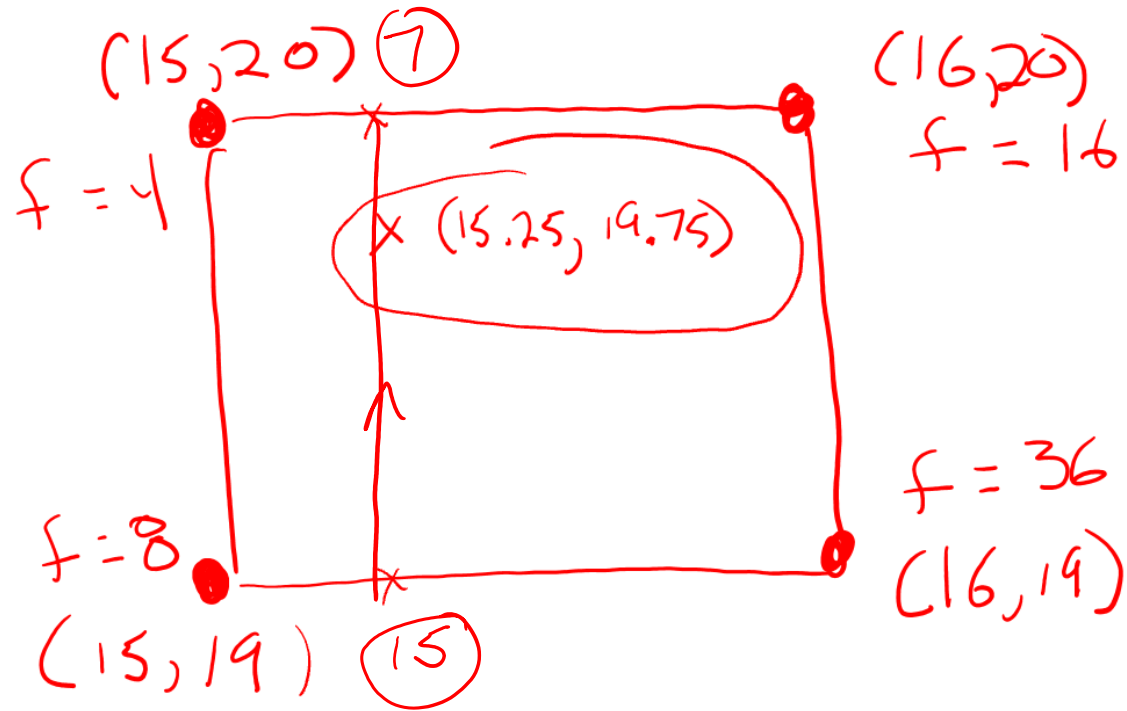


$$t = \frac{15.25 - 15}{16 - 15} = 0.25$$

$$f(15.25, 20) \approx (1 - 0.25)4 + 0.25(16) \\ \approx 3 + 4 = 7$$

$$f(15.25, 19) \approx (1 - 0.25)8 + 0.25(36) \\ = 6 + 9 = 15$$

Bilinear Interpolation: Example



$$\begin{aligned} f(15.25, 19.75) &\approx \\ (1 - 0.75) \cdot 15 + (0.75) \cdot 7 \\ &= \frac{15}{4} + \frac{21}{4} = \frac{36}{4} \\ &= \boxed{9} \end{aligned}$$

$$t = \frac{(19.75 - 19)}{(20 - 19)} = 0.75$$

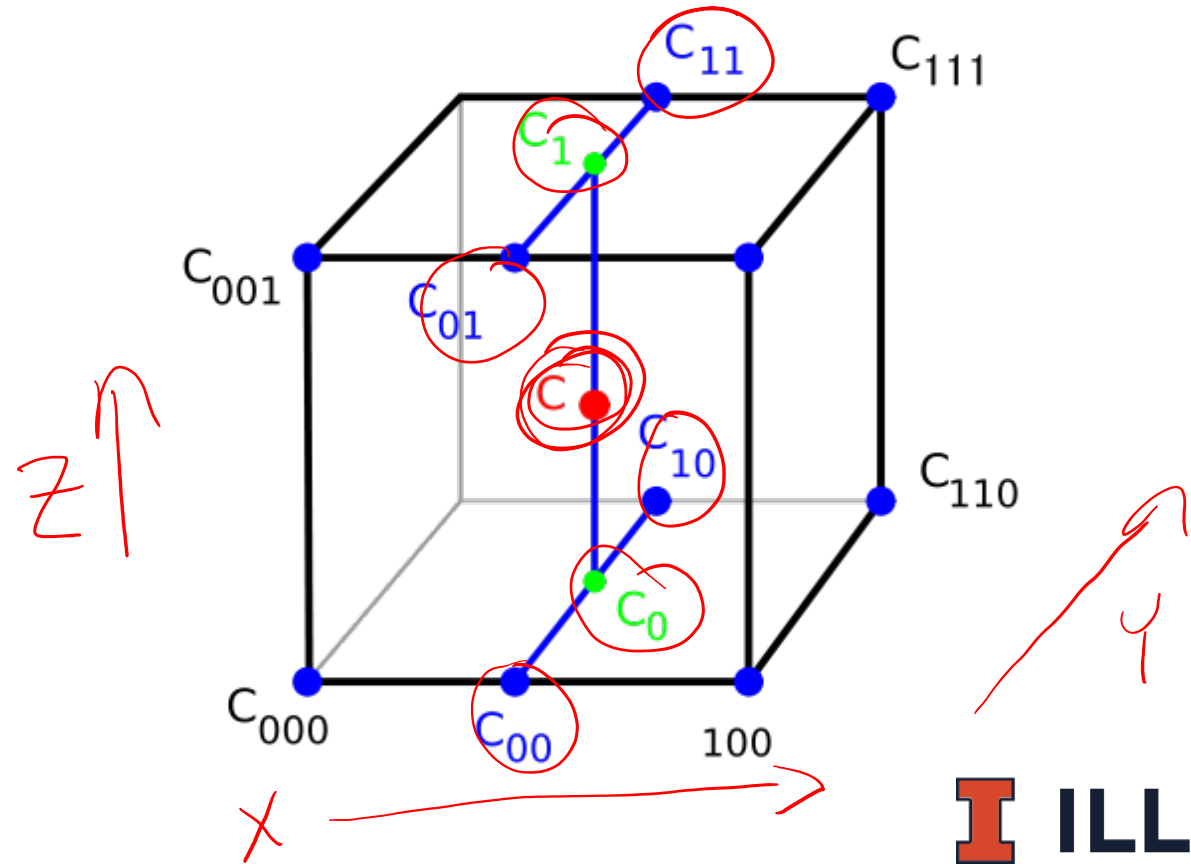
Trilinear Interpolation

First interpolate in x to find c_{00} , c_{01} , c_{10} , and c_{11}

Then in y to find C_0 and C_1

And then in z to find C

Order in which you
interpolate (which axis goes
1st, 2nd, 3rd) does not matter.



Trilinear Interpolation

