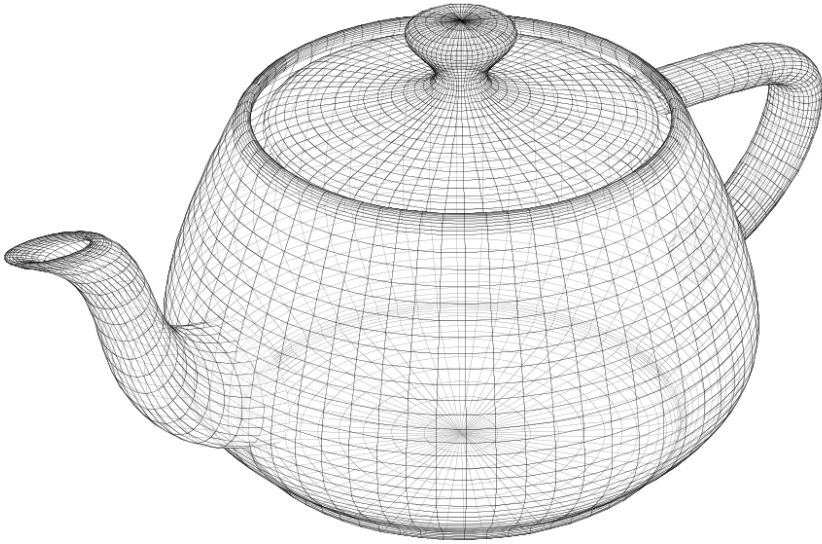


# Geometric Modeling: Polygonal Meshes



CS 418: Interactive Computer Graphics  
Professor Eric Shaffer

# Polygonal Meshes

In rendering, we typically will generate an image by simulating the reflection of light off surfaces

Rasterization engines most often use polygonal meshes to represent surfaces

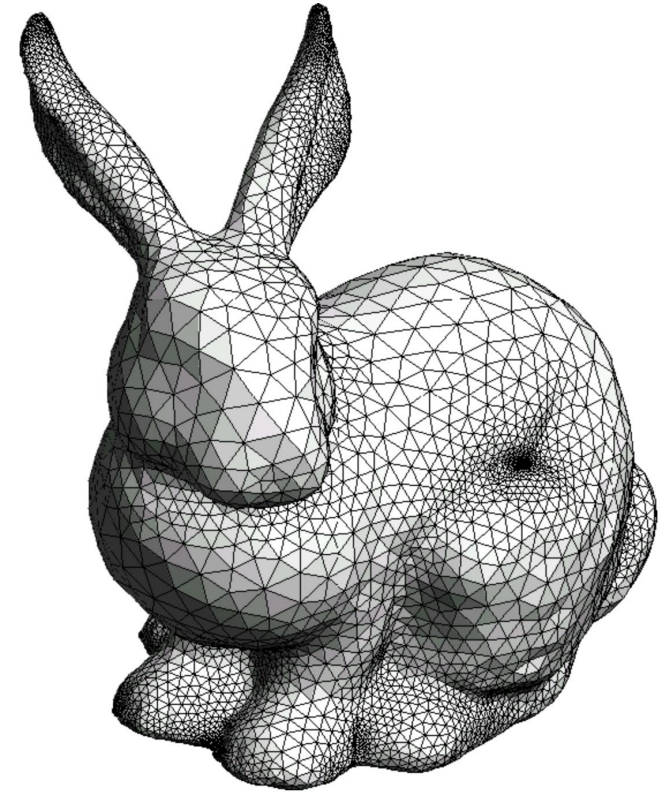
Modern GPUs are designed specifically to rasterize triangles

Many advantages to using triangles:

- Simplest 2D primitive...
- Any 2D polygon can be triangulated
- Can easily represent sharp surface features

Any disadvantages you can think of?

Why do we say 2D when  
we are rendering in 3D?

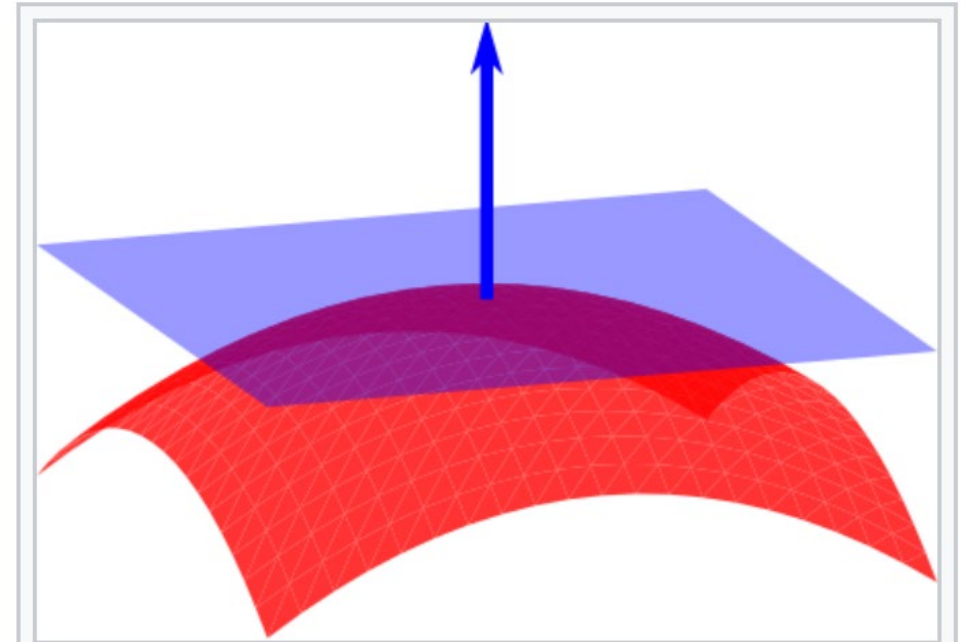
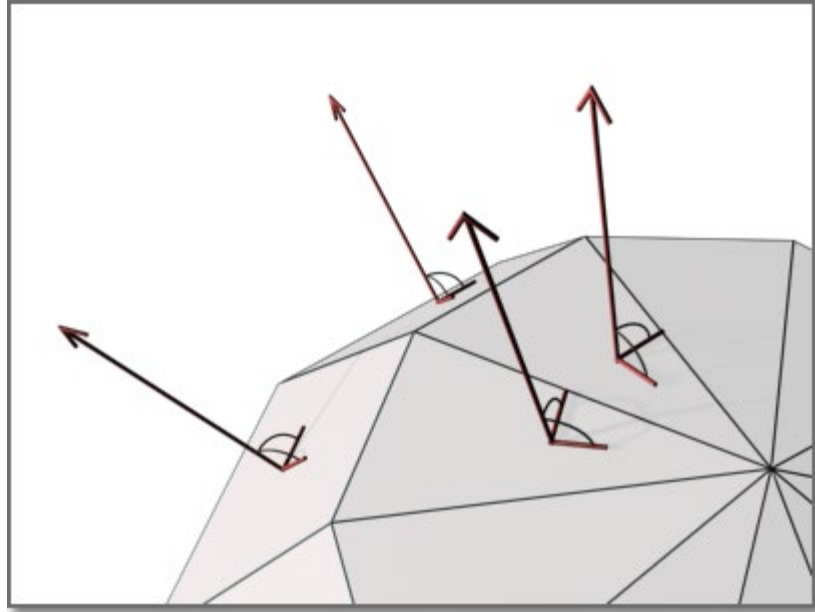


# Vocabulary: Surface Normals

A normal is a vector that is perpendicular to object

Each triangle in a surface mesh has outward facing normal

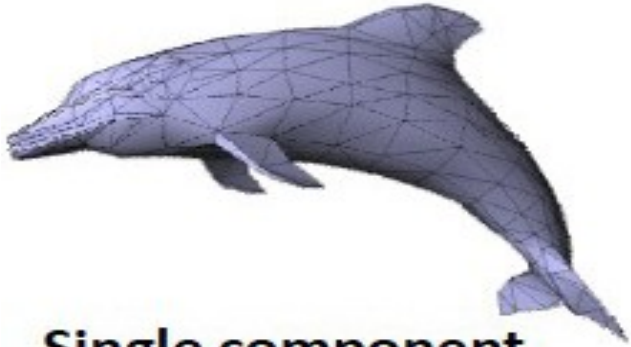
The normal is just the vector perpendicular to the triangle



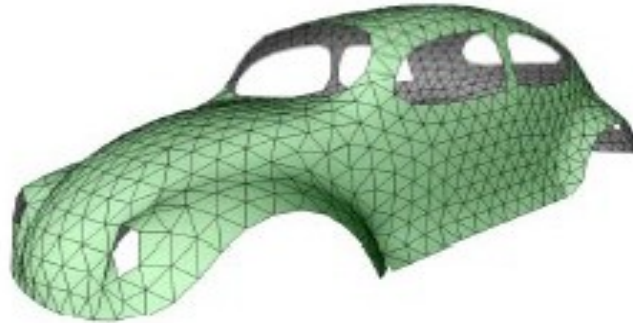
A normal to a surface at a point is the same as a normal to the tangent plane to the surface at the same point.

Courtesy Wikipedia

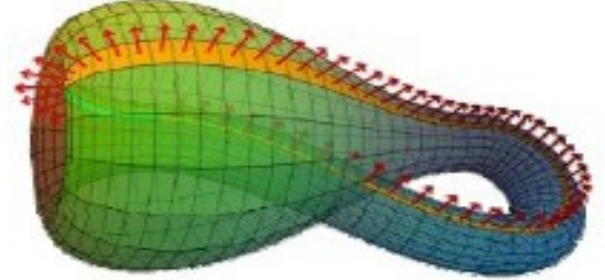
# Vocabulary: Surface Mesh Properties



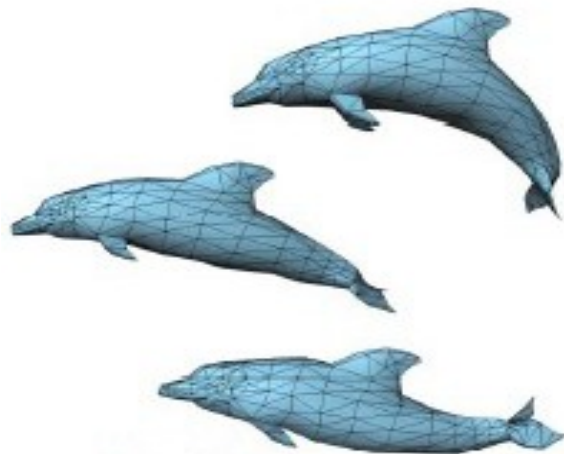
**Single component,  
closed, triangular,  
orientable manifold**



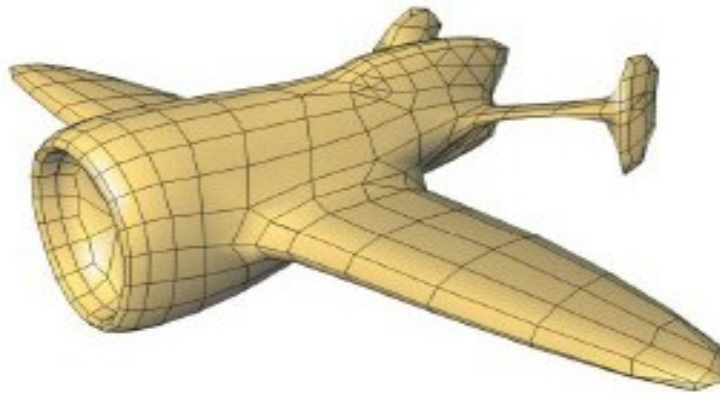
**With boundaries**



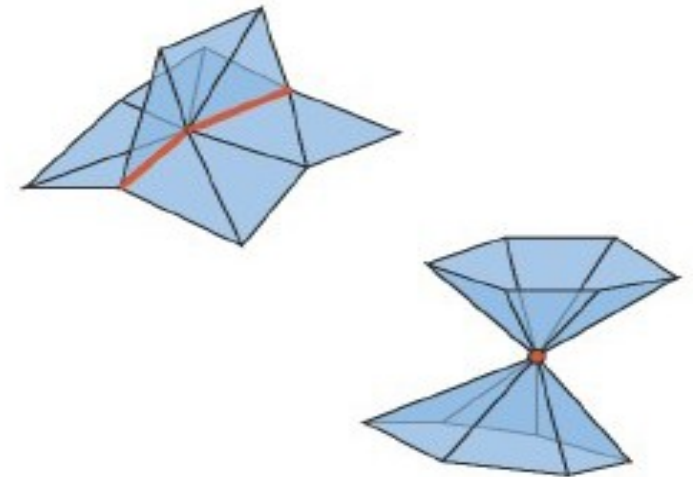
**Not orientable**



**Multiple components**



**Not only triangles**

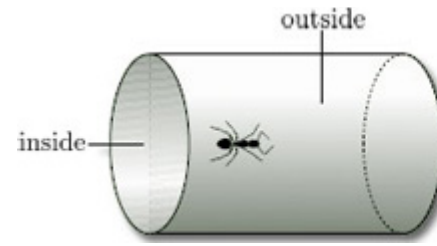


**Non manifold**





# Orientability



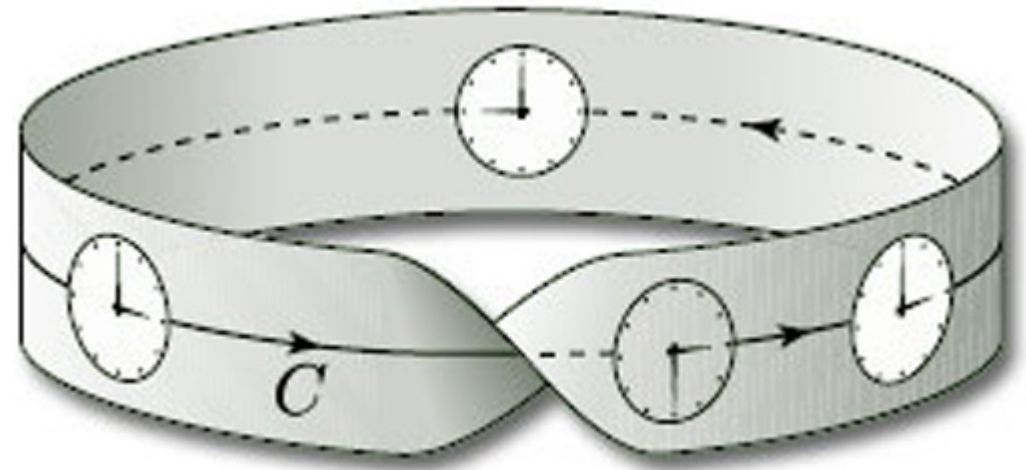
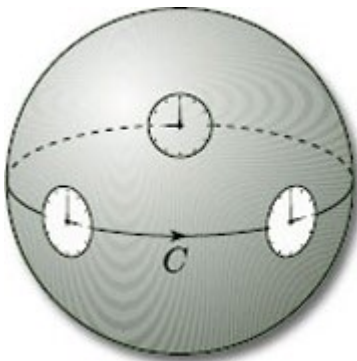
Not that relevant to this course....but it is interesting....

Imagine sliding a clock face around a surface.

On an orientable surface the clock face will return to starting point and appear the same. (Sphere)

On a non-orientable surface it will be a mirror image of the original clock face (Möbius Strip)

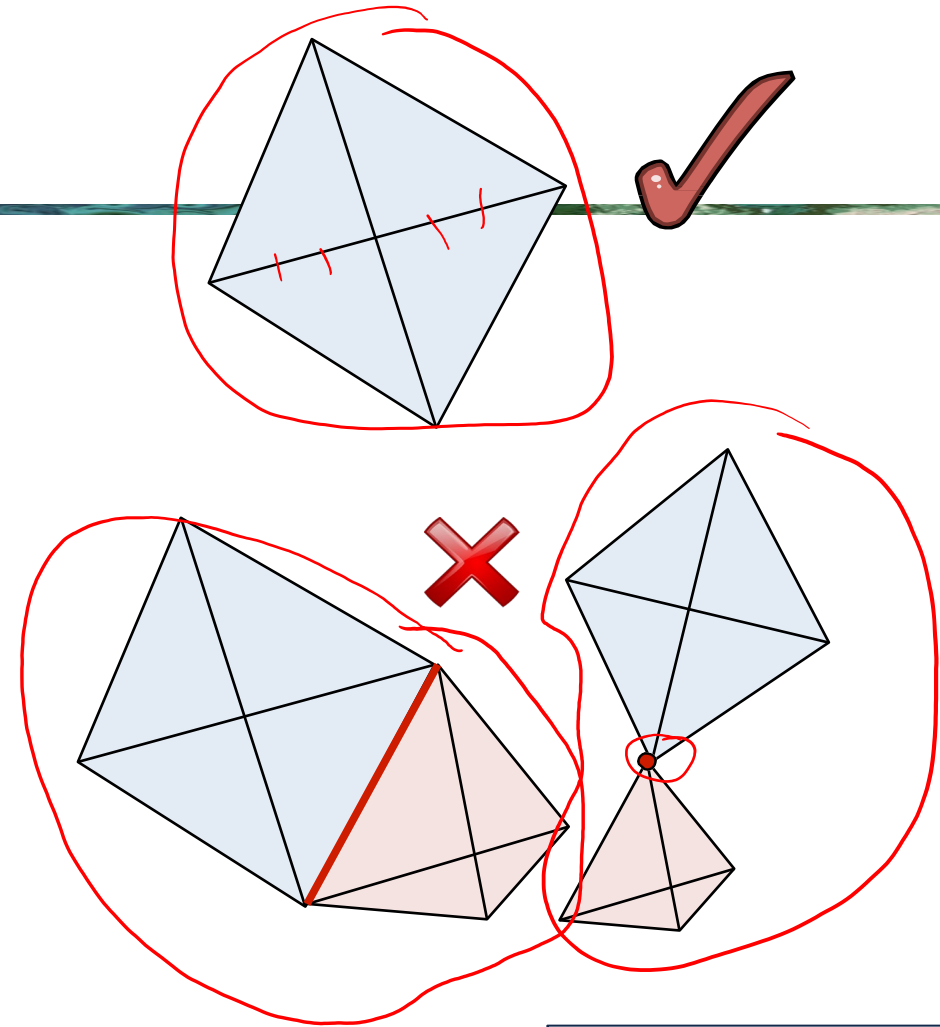
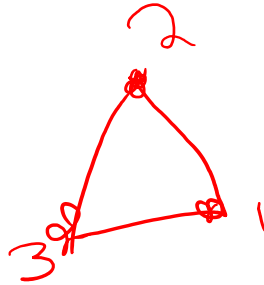
...there is no consistent definition of clockwise on a non-orientable surface.



# Surface Mesh Properties

## Manifold:

1. Every edge connects exactly two faces
2. Vertex neighborhood is “disk-like”



**Orientable:** Consistent normals

**Watertight:** Orientable + Manifold

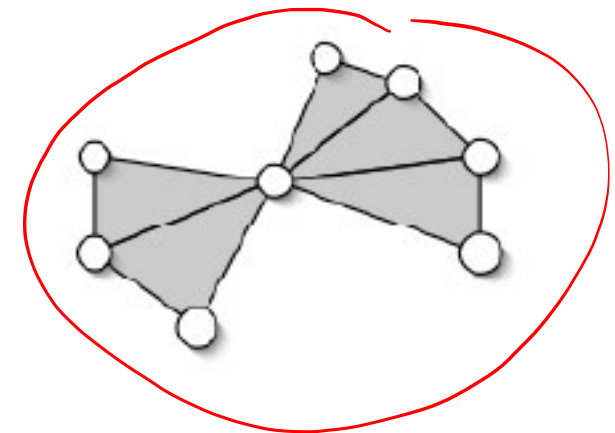
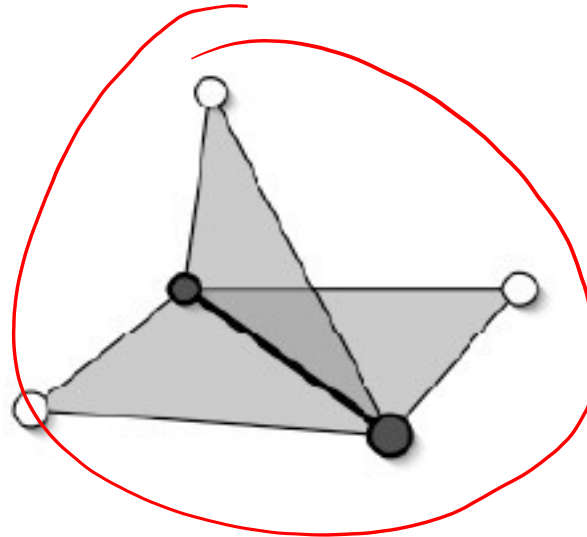
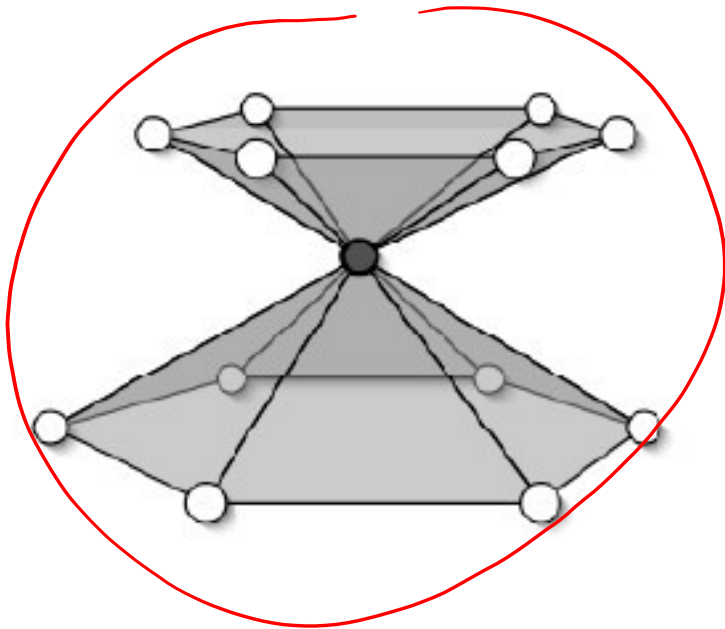
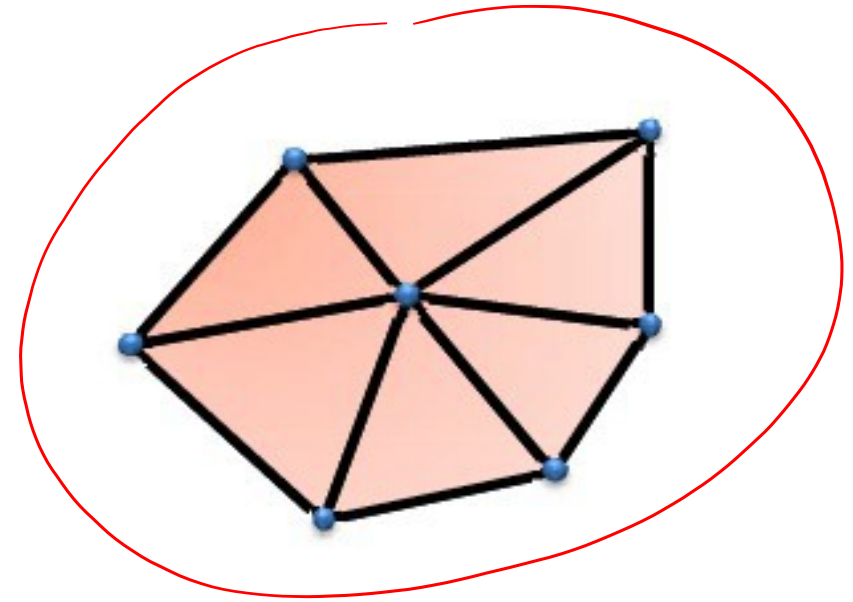
**Boundary:** Some edges bound only one face

**Ordered:** Vertices in CCW order when viewed from normal

The blue and pink meshes above are tetrahedra, like 4-sided pyramids

# 2-Manifold Mesh Examples

Disk-shaped neighborhoods



non-manifolds



# Genus

Genus of orientable surfaces



genus 0



genus 1



genus 2



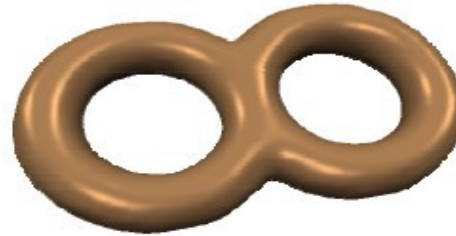
genus 3



Genus 0



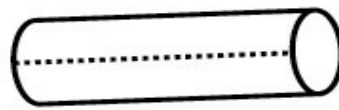
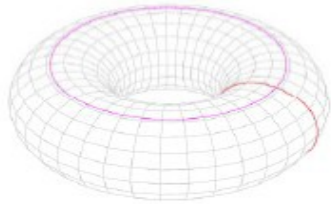
Genus 1



Genus 2



Genus ?





# Euler Characteristic

For a closed (no boundary), manifold, connected surface mesh:

$$V - E + F = 2(1 - G)$$

$$V - E + F = 2$$

$V$  = number of vertices

$E$  = number of edges

$F$  = number of faces

$G$  = genus (number of holes in the surface)

A **2-manifold** is a surface (locally like a plane)



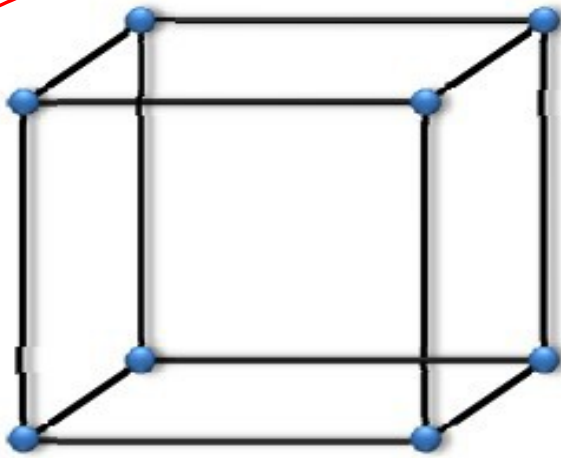
Leonhard Euler  
1707 - 1783

Can you think of  
anything else named  
for him?

# Euler Characteristic for Closed 2-Manifold Polygonal Meshes

$$V + F - E = \chi$$

Euler characteristic

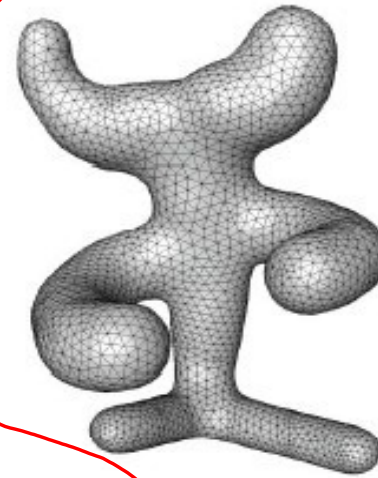


$$V = 8$$

$$E = 12$$

$$F = 6$$

$$\chi = 8 + 6 - 12 = 2$$



$$V = 3890$$

$$E = 11664$$

$$F = 7776$$

$$\chi = 2$$



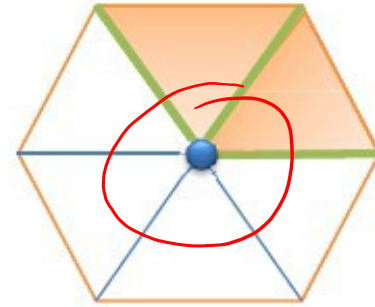
# ...and if they are triangle meshes

- *Triangle mesh statistics*

$$E \approx 3V$$

$$F \approx 2V$$

- Avg. valence  $\approx 6$   
*Show using Euler Formula*



Aside from being totally interesting on their own, these formulas are useful for computing memory usage

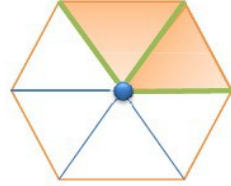


# ...and if they are triangle meshes

- Triangle mesh statistics

$$E \approx 3V$$

$$F \approx 2V$$



- Avg. valence  $\approx 6$   
Show using Euler Formula



$$V - E + F = 2$$

$$V - \frac{3F}{2} + F = 2$$

$$V - F/2 = 2$$

$$V = F/2 + 2$$

$$2V = F + 4$$

$$\boxed{2V \approx F}$$

