Stacked Denoising Autoencoders: Parallel Stochastic Gradient Descent and Parallel Genetic Algorithm

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Abstract

We implement stacked denoising autoencoders, a special neural network that is capable of learning powerful representations of high dimensional data. Learning for autoencoders is accomplished via stochastic gradient descent and we show experimental results when training autoencoders on a standard image classification dataset.

I. Introduction

Autoencoders are a method for performing representation learning, an unsupervised pretraining process during which a more useful representation of the input data is automatically determined. Representation learning is important in machine learning since "the performance of machine learning methods is heavily dependent on the choice of data representation (or features) in which they are applied" [1]. For many supervised classification tasks, the high dimensionality of the input data means that the classifier requires an enormous number of training examples in order to generalize well and not overfit. One solution is to use unsupervised pretraining to learn a good representation for the input data and during actual training, transform the input examples into an easier form for the classifier to learn. Autoencoders are one such representation learning tool.

An autoencoder is a neural network with a single hidden layer and where the output layer and the input layer have the same size. Suppose that the input $x \in \mathbb{R}^m$ (and the output as well) and suppose that the hidden layer has n nodes. Then we have a weight matrix $W \in \mathbb{R}^{m \times n}$ and bias vectors b and $b^{'}$ in $\mathbb{R}^m$ and $\mathbb{R}^n$, respectively. Let $s(x) = 1/(1 + e^{-x})$ be the sigmoid (logistic) transfer function. Then we have a neural network as shown in Fig. 1. When using an autoencoder to encode data, we calculate the vector $y = s(Wx+b)$; corresponding when we use an autoencoder to decode and reconstruct back the original input, we calculate $z = s(W^T x + b^{'})$. The weight matrix of the decoding stage is the transpose of weight matrix of the encoding stage in order to reduce the number of parameters to learn. We want to optimize W, b, and $b^{'}$ so that the reconstruction is as similar to the original input as possible with respect to some loss function. In this report, the loss function used is the least squares loss: $E(t, z) = \frac{1}{2} \|t - z\|_2^2$, where t is the original input. After an autoencoder is trained, its decoding stage is discarded and the encoding stage is used to transform the training input examples as a preprocessing step. We will refer to the trained encoding stage of the autoencoder as an "autoencoder layer".

Once an autoencoder layer has been trained, a second autoencoder can be trained using the output of the first autoencoder layer. This procedure can be repeated indefinitely and create stacked autoencoder layers of arbitrary depth. It is been shown that each subsequent trained layer learns a better representation of the output of the previous layer. Using deep neural networks such as stacked autoencoders to do representation learning is also called deep learning, a subfield of machine learning that has received much attention and breakthroughs lately.

For ordinary autoencoders, we usually want that $n < m$ so that the learned representation of the input exists in a lower dimensional space than the input. This is done to ensure that the autoencoder does not learn a trivial identity transformation. However, there also exists an autoencoder variant called *denoising autoencoders* that use a different reconstruction criterion to learn overcomplete representations [2]. In other words, even if $n > m$, a denoising autoencoder can still learn a good representation of the input. This is achieved by corrupting the input image and training the autoencoder to reconstruct the original uncorrupted image. By learning how to denoise, the autoencoder is forced to understand the true structure of input data and learn a good representation of it. Although the loss function $E(t, z)$ for neural networks in general is non-convex, past work has shown that stochastic gradient descent (SGD) is sufficient for most problems. In this report, we will consider training denoising autoencoders with SGD.

Lastly, we will examine training autoencoders with other optimization methods such as a genetic algorithm (GA). A GA is a biologically inspired black-box optimization algorithm is capable of optimizing arbitrary non-convex, non-differential objective functions. The motivations behind using GAs for training autoencoders are two fold: 1) GAs are a novel approach to optimization of deep neural networks with large number of parameters such as autoencoders. We would like to evaluate the performance of GAs and compare it to SGD. 2) GAs have some advantages over SGD such as being able to escape local optimas and easier to parallelize. However GAs also have drawbacks, the main one being its computational complexity over gradient descent. Unlike SGD, a GA must keep a population of individuals and must evaluate each individual for each training example. The computational complexity of a GA is $O(mnd)$ where d is the dimensionality of the individual and objective function, n is the population size, and m is the number of training examples; in comparison, the complexity of SGD is $O(md)$. The optimal population size depends on the problem being solved, but in most cases, $n = O(d)$.

We will not just implement a conventional GA (CGA), but also explore hybrid GAs that also make use of gradient information (HGA). The key idea is that with additional gradient information to intelligently update the population every generation, the population size can be kept constant. As a result, the complexity of HGA becomes the same as SGD: $O(md)$, while still retaining the advantages of being more scalable and capable of optimizing arbitrary objective functions.

The rest of this report is as follows: Background literature for autoencoders, representation and deep learning can be found in the related work section. The algorithm description section contains more details about how SGD, CGA, and HGA are implemented for autoencoders. The experiments section describes the performance of SGA, CGA, and HGA, as well as the results of training an autoencoder on a handwritten digit image dataset. In the discussion section, we will analyze our findings and report key findings.

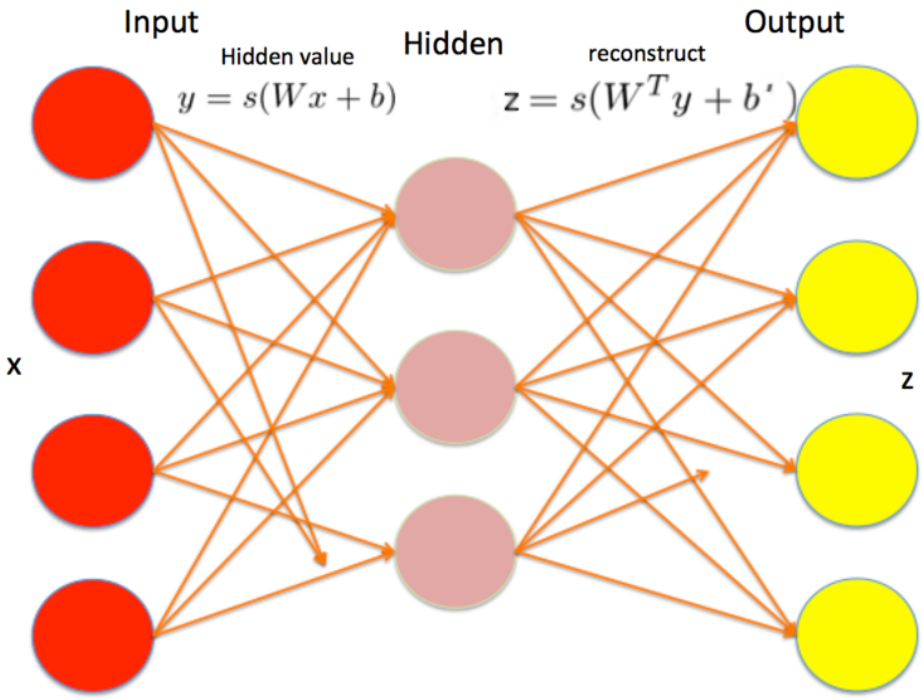


Fig. 1: Overview of an autoencoder and its encoding, decoding stages. The weight matrix of the decoding stage is the transpose of the weight matrix of the encoding stage.

II. Related Work

There are several papers that we would like to mention developing the topics related to the present work. Yoshua Bengio, Aaron Courville, and Pascal Vincent review representation learning and why it is important, single layer and deep models, autoencoders, as well as other related architectures for deep learning [1]. For further elaboration on stacked autoencoders (and more specifically denoising autoencoders) we refer the reader to [2]. Hinton et al.

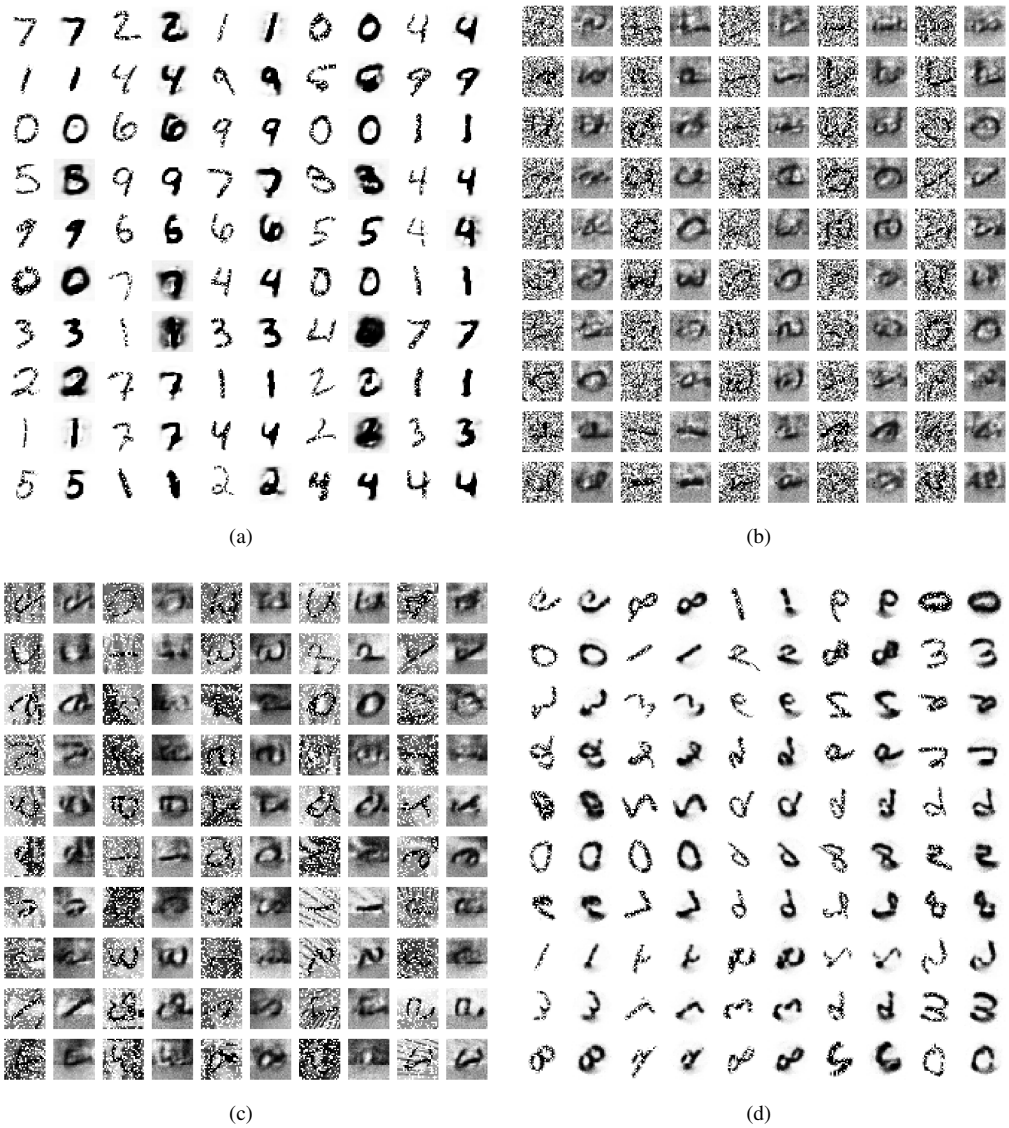


Fig. 2: Reconstructions of corrupted digits from the (a) MNIST dataset (b) bg-rand dataset, (c) bg-img dataset, and (d) rot dataset

describe in [3] how to successfully train deep multilayer networks, like the stacked autoencoder we use in this paper. The theory and methods behind the backpropagation algorithm, a variant of SGD designed for neural networks, that we use is classic and can be found in [4], [5]. Future plans include using a genetic algorithm (GA) to train the autoencoder instead of backpropagation. A survey of GAs and how they perform optimization can be found in [6]. Cantú-Paz discusses parallel GAs, which will be essential to efficiently training our autoencoder [7].

III. Algorithm Description

A. Stochastic Gradient Descent

We start with a random weight matrix W and random biases b and $b^{'}$. We take a given input x, and feed it forward through the network and compute the error between the target output t and the actual output z. Often we use the squared loss error $E(t,z) = \frac{1}{2}\left\|t-z\right\|_2^2$ to determine the difference between the two. In the case of an autoencoder, the target output is the same as the input. If the error is not satisfactory, we can adjust the weight matrix and that biases in order to attempt to learn a better representation of the data. A common method of updating

the weight and biases is via backpropagation [4]; when applied to training inputs one at a time, it is also known as stochastic gradient descent (SGD). We will first consider the update for the weights and biases from the last hidden layer to the output layer with a squared loss error function and derive the updates. We use as an example a simple three layer neural network (input, one hidden and output layer). Some notation is given in Table I.

Symbol	Meaning
E	Error as computed at the output layer
x_j	Node j in the input layer
y_j	Node j in the hidden layer
z_j	Node j in the output layer
n_j	$\sum_{i=1}^{n} W_{ij}x_i + b_j$
t_j	Target output at node j
W_{ij}^H	Weight i, j from input to hidden layer
W_{ij}^O	Weight i, j from hidden to output layer
$s(x_j)$	$1/(1+e^{-x_j})$
$b_j^{\{H,O\}}$	Biases for hidden and output layer

TABLE I: Table giving notation for the derivation of updates.

The derivative of the output error E with respect to an output matrix weight W_{ij}^O is as follows.

$$\begin{aligned}\frac{\partial E}{\partial W_{ij}^O} &= \frac{\partial E}{\partial z_j}\frac{\partial z_j}{\partial W_{ij}^O} \\ &= (z_j - t_j)\frac{\partial s(n_j)}{\partial x_j}\frac{\partial x_j}{\partial W_{ij}^O} \\ &= (z_j - t_j)s(n_j)(1-s(n_j))x_i \\ &= (z_j - t_j)z_j(1-z_j)x_i\end{aligned} \tag{1}$$

Now that we have the gradient for the error associated to a single training example, we can compute the updates.

$$\begin{aligned}\delta_j^O &= (z_j - t_j)z_j(1-z_j) \\ W_{ij}^O &\leftarrow W_{ij}^O - \eta\delta_j^O x_i \\ b_j^O &\leftarrow b_j^O - \eta\delta_j^O\end{aligned} \tag{2}$$

The computation of the gradient for the weight matrix between hidden layers is similarly easy to compute.

$$\begin{aligned}\frac{\partial E}{\partial W_{ij}^H} &= \frac{\partial E}{\partial y_j}\frac{\partial y_j}{\partial W_{ij}^H} \\ &= \left(\sum_{k=1}^{m}\frac{\partial E}{\partial z_k}\frac{\partial z_k}{\partial n_k}\frac{\partial n_k}{\partial y_j}\right)\frac{\partial y_j}{\partial n_j}\frac{\partial n_j}{\partial W_{ij}^H} \\ &= \left(\sum_{k=1}^{m}(z_k - t_k)(1-z_k)z_k W_{jk}^O\right)y_j(1-y_j)x_i\end{aligned} \tag{3}$$

And then using the computed gradient we can define the updates to be used for the hidden layers

$$\begin{aligned}\delta_j^H &= \left(\sum_{k=1}^{m}(z_k - t_k)(1-z_k)z_k W_{jk}^O\right)y_j(1-y_j) \\ W_{ij}^H &\leftarrow W_{ij}^H - \eta\delta_j^H x_i \\ b_j^H &\leftarrow b_j^H - \eta\delta_j^H\end{aligned} \tag{4}$$

In general, for a neural network we may have different output error functions and these will result in different update rules. We will also give the updates for the cross-entropy error function with softmax activation in the final layer. The cross entropy error function is given by $E(x,t) = -\sum_{i=1}^{n}(t_i \ln z_i + (1-t_i)\ln(1-z_i))$ and the softmax function is given by $\sigma(x_j) = e^{x_j}/(\sum_k e^{x_k})$. Following the same procedure as above for computing the gradient and the updates, we find that for hidden/output layer

$$\begin{aligned} \frac{\partial E}{\partial W_{ij}^O} &= (z_j - t_j) y_i \\ \delta_j^O &= (z_j - t_j) \\ W_{ij}^O &\leftarrow W_{ij}^O - \eta \delta_j^O x_i \\ b_j^O &\leftarrow b_j^O - \eta \delta_j^O. \end{aligned} \tag{5}$$

Note also that we find that the updates for the hidden layer is the same as in the squared error loss function with sigmoid activation. A general overview of the backpropagation algorithm is given by by Algorithm 1.

The algorithm and derivations for the autoencoder are a slight variation on the above derivations for a more general neural network. The weight matrix of the output layer (decoding stage) is the transpose of the weight matrix of the hidden layer (encoding stage). Thus $z = s(W^O(W^H x + b) + b^{'})$, $(W^H)^T = W^O$, and $W_{ij}^H = W_{ji}^O$. For training denoising autoencoders in particular, $z = s(W^O(W^H x_{\text{corr}} + b) + b^{'})$, where x_{corr} is a randomly corrupted version of the original input data x_{orig} and the loss function is defined as $E(x_{\text{orig}}, z)$. In order words, we are trying to learn an autoencoder takes a corrupted input and tries to reconstruct the original uncorrupted version. Once we have trained a single autoencoder layer, we can stack another autoencoder layer on top of the first one for further training. This second autoencoder takes the corrupted output of the hidden layer (encoding stage) of the first autoencoder as input and is trained to minimize the loss function $E(x_{\text{orig}}, z)$.

Algorithm 1 Backpropagation

Initialize the weights and biases randomly
for iter $= 1, 2, 3...$ **do**
  for all Examples x in training set (randomize) **do**
    $z \leftarrow$ Feedforward x
    Compute output layer δ_j^O
    $W_{ij} \leftarrow W_{ij} - \eta \delta_j^O x_i$
    $b_j \leftarrow b_j - \eta \delta_j^O$
    for all Layers in reverse order **do**
      Compute hidden layer delta δ_k^H
      $W_{ij}^H \leftarrow W_{ij}^H - \eta \delta_j^H x_i$
      $b_j \leftarrow b_j - \eta \delta_j^H$
    end for
  end for
end for

After using backpropagation (or a genetic algorithm) to train each of the autoencoder layers, we can then attach an output layer to the autoencoder to be used for classification. At this point we use supervised learning to train the output layer and *fine-tune* the autoencoder layers to produce a classifier based on the autoencoder. When pretraining the autoencoder we train one layer at a time using backpropagation, but during the fine-tuning step we train the entire network via backpropagation, one layer at a time per training image.

B. Genetic Algorithm

As mentioned in the introduction, a genetic algorithm (GA) is a biologically inspired black-box optimization algorithm is capable of optimizing arbitrary non-convex, non-differential objective functions. The GA works by iteratively improving upon a population of candidate solution vectors (also know as individuals) via genetic operations such as selection, mutation, and crossover. The goal is to maximize the fitness of each individual, which is determined by evaluating the individual with some objective function. In the case of autoencoders, the objective function is the reconstruction loss function defined earlier, the individual is a real valued vector that represents the weights W and biases $b_j^{\{H,O\}}$, and the fitness of an individual is simply $1/E(t, z)$. Algorithm 4 shows the pseudocode of a simple conventional genetic algorithm, which we will refer to as CGA.

We will explain step by step what each bolded term means and how it affects individuals in the population:

- Power Scaling: All individuals in the population are ranked in ascending order according to their fitness. Each individual is assigned a scaled fitness is is equal to R^γ where R is the position of the individual

Algorithm 2 Conventional Genetic Algorithm (CGA)

Initialize N individuals randomly by uniformly sampling values from $[-r, r]$
for iter $= 1, 2, 3...M$, where M is number of training examples **do**
  Evaluate each individual with objective function and assign fitness
  Scale fitness of all individuals, typically with **power scaling** with parameter γ.
  Select existing individuals via **roulette selection**
  for Every two individuals a and b selected **do**
    Create copies a' and b'
    Perform **uniform mutation** on a' and b' with mutation rate mr and mutation amount ma
    Perform **uniform crossover** using a' and b' with crossover rate cr
  end for
  Replace worst αN $(0 < \alpha < 1)$ individuals in population with newly created individuals
end for

within the ranking and γ is a parameter to be tuned. Higher values for γ will create a scaled fitness distribution that favor individuals with higher actual fitness. Thus γ will affect the selection of individuals later and determine how elitist the selection of individuals will be (in other words, how much more likely an individual of higher fitness will be selected).

- Roulette Selection: Also known as fitness proportionate selection, roulette selection randomly samples an individual from the population with proportion to its scaled fitness. There are many other selection methods such as tournament selection and truncation selection. However, our preliminary experimental results suggest that roulette selection is most appropriate for training weights of autoencoders. The purpose of this operation is to select individuals with good fitness to create the next generation's population.
- Uniform Cauchy Mutation: Each element of the individual (a vector of real numbers) is mutated with probability mr. For each element, we generate a random number between 0 and 1. If this random number is less than mr, we sample value from a zero-meaned Cauchy distribution with standard deviation ma and add that value to the element. The purpose of this operation is to randomly perturb existing individuals and possibly create new ones whose fitness is slightly better.
- Uniform Crossover: Each element of the individual is swapped with the same corresponding element of another individual with probability cr. If crossover is performed on two individuals with good fitness, there is a chance that the elements (also known as genes) of the individuals that are responsible for their fitnesses might be combined into a new individual.

Next we will look at HGA, a hybrid GA that combines genetic operators with backpropagation to keep the population size small and remain computationally tractable. The pseudocode for HGA is shown in Algorithm 3 and is based off recent work done by David and Greental [8] on applying GAs to learn stacked autoencoders. The main difference between our HGA and the algorithm described in [8] is that we use roulette selection instead of truncation selection and uniform Cauchy mutation instead of mutating weights to zero.

Algorithm 3 Hybrid Genetic Algorithm (HGA)

Initialize N individuals randomly by uniformly sampling values from $[-r, r]$
for iter $= 1, 2, 3...M$, where M is number of training examples **do**
  Evaluate each individual with objective function and assign fitness
  Scale fitness of all individuals, typically with **power scaling** with parameter γ
  Perform backpropagation on top β individuals with highest fitness
  Select existing individuals via **roulette selection**
  for Every two individuals a and b selected **do**
    Create copies a' and b'
    Perform **uniform mutation** on a' and b' with mutation rate mr and mutation amount ma
    Perform **uniform crossover** using a' and b' with crossover rate cr
  end for
  Replace worst αN $(0 < \alpha < 1)$ individuals in population with newly created individuals
end for

The key idea of performing backpropagation is that it assists the mutation operator in moving the individuals towards regions of high fitness. While the mutation operator perturbs individuals randomly, backpropagation will

always follow the gradient. This distinction is significant in high dimensional spaces, as the mutation operator will require a large population size to work optimally, while backpropagation does not. Instead, the mutation operator serves a secondary role of helping the individuals to escape from a local optima through random perturbations. Preliminary results show that compared to CGA with the same population size, HGA performs significantly better. Furthermore, the performance of HGA does not degrade significantly with smaller population sizes, even when using a extremely small population size of 2.

IV. Experimental Results

For the following experiments, we train our autoencoder over the MNIST handwritten digit dataset. The MNIST dataset is composed of 60000 training images and 10000 testing images Each image is in greyscale, is 28 by 28 pixels in size, and has a corresponding label ranging from 0 to 9. Thus, the input vector for our autoencoder has 784 dimensions. We also make use of the denoising criterion mentioned in [2], and for each training image, randomly corrupt it by setting each pixel to zero with probability 0.25.

A. Stochastic Gradient Descent

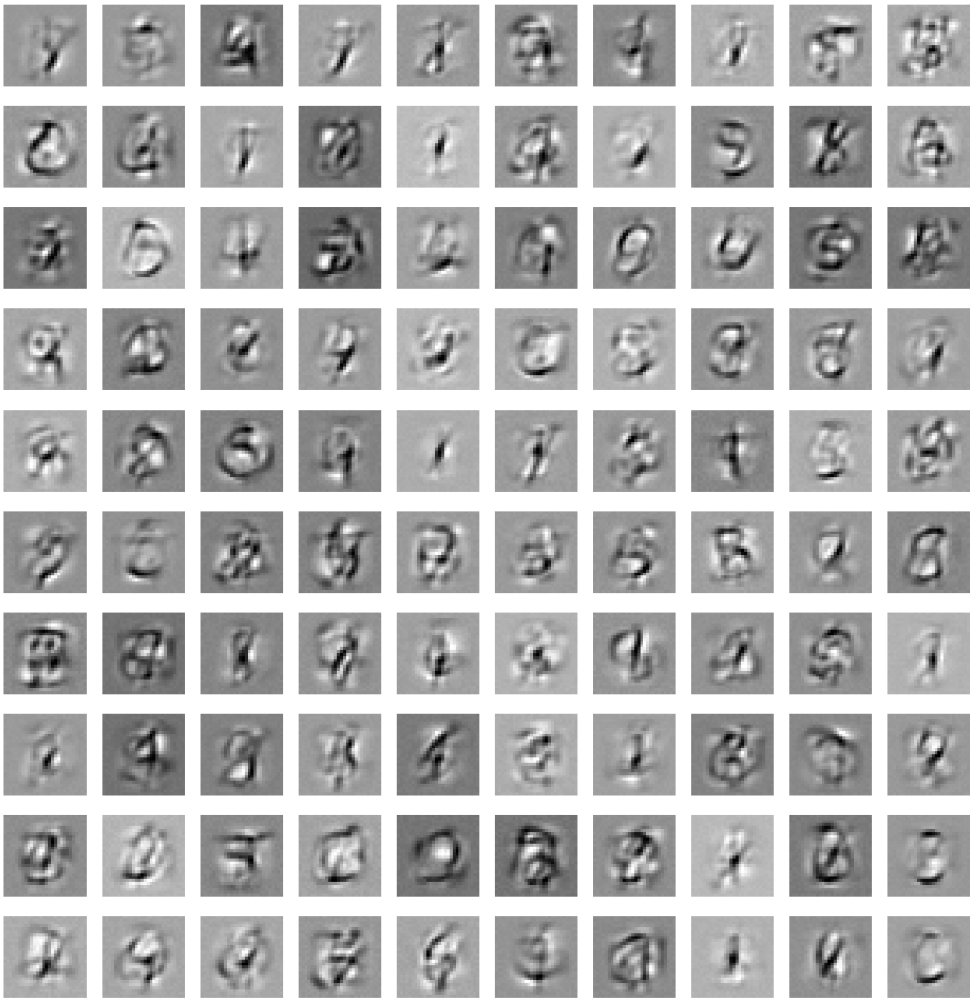


Fig. 3: Visualization of the filters of the first 100 hidden nodes in an denoising autoencoder trained over all 60000 images.

We first analyze how the number of threads affects the rate at which reconstruction error decreases for SGD. We train a single autoencoder layer with 500 hidden nodes for 15 iterations over 5000 training images. An iteration involves going through all training images and for each image, use SGD to update the weight matrix. We initialize

the weights of the autoencoder randomly by sampling from the interval $[-1/\sqrt{\text{FANIN}}, 1/\sqrt{\text{FANIN}}]$. Fig. 4 shows the relationship between reconstruction error, total time elapsed, and the number of threads used. Regardless of the number of threads, the reconstruction error decreases sharply in the first few iterations before flattening out to around the same value after 15 iterations. The rate at which error decreases is significantly faster for four and eight threads when compared to just using one. Nonetheless, the speedup is not linear and is due to two reasons: 1) Possible cache conflicts as each thread read and writes to different locations in the weight matrix. 2) All the steps for backpropagation/SGD, described in Algorithm 1, must be done sequentially. Parallelization can only be done within each step and incurs an overhead cost.

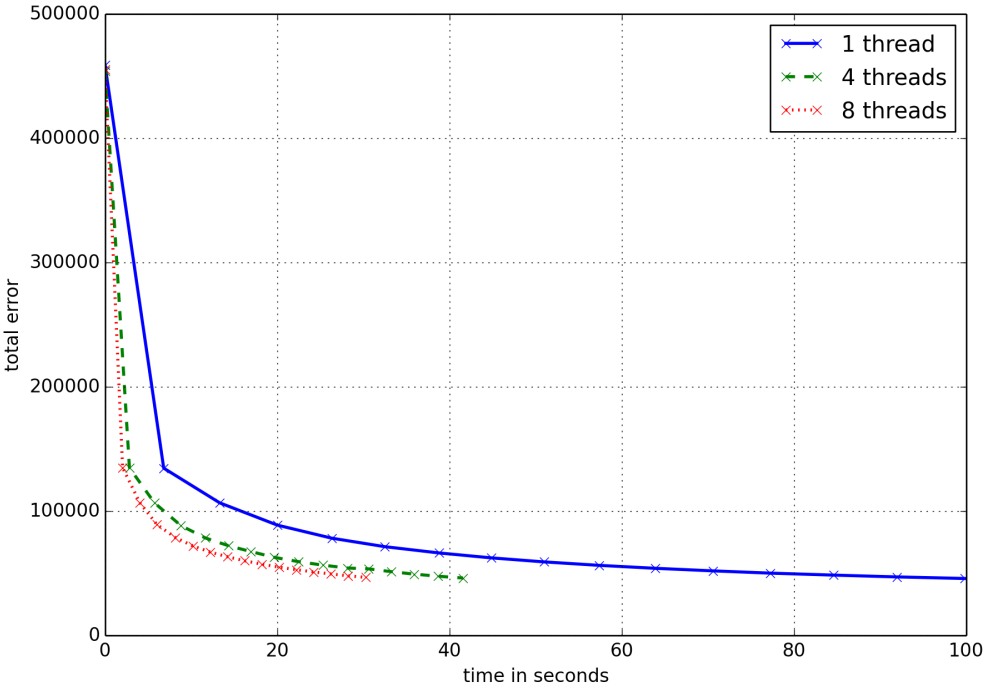


Fig. 4: Performance results on a single autoencoder layer with 500 hidden nodes and trained for 15 iterations. Plot shows time elapsed versus total reconstruction error over 5000 images for 1, 4, and 8 threads.

We note that the steps of backpropagation can only be done in sequence; thus we can only parallelize the operations done within each step. The three major operations which benefit from parallelization are computing the matrix-vector products $W^H x$ and $W^O y$, computing δ_j^O and δ_j^H, and updating the entries of the weight matrices with the gradient. For performance reasons we don't store W^O separately; instead we access W^H with transposed indexes when decoding, calculating δ_j^O, and applying the gradient update.

The most expensive parts of the backpropagation algorithm are computing the forward activations of the network and updating the weight matrices for the network. Computing the forward propagation requires performing a matrix-vector multiplication at each layer of the network. The size of that matrix depends on the input and output sizes of that layer. Thus if we have a network with N layers and the sizes of each of those layers is n_i, then We have N matrix-vector multiplications of size $n_{i-1} \times n_i$. Updating the weights also has complexity based on the size of the matrix since it requires updating all the entries of the matrix at each iteration.

To improve the performance of these two expensive steps, we parallelized them first using OpenMP and then later parallelized the matrix-vector products using OpenBLAS. Parallelization of these two steps is fairly straight forward and we give some results on scaling in Figs. 5, 6, and 7.

In Fig. 5 we give the performance of training the autoencoder using SGD for different numbers of hidden nodes and different numbers of threads using our own parallel implementation. In Fig. 6 we give the performance of training the autoencoder using SGD for different numbers of hidden nodes and different numbers of threads using OpenBLAS for parallelization of the matrix-vector multiplies and matrix-transpose-vector multiplies. The weight updates are not able to be done in OpenBLAS, and so we continue to do that using OpenMP. We consider the relative performance of these two methods in Fig. 7.

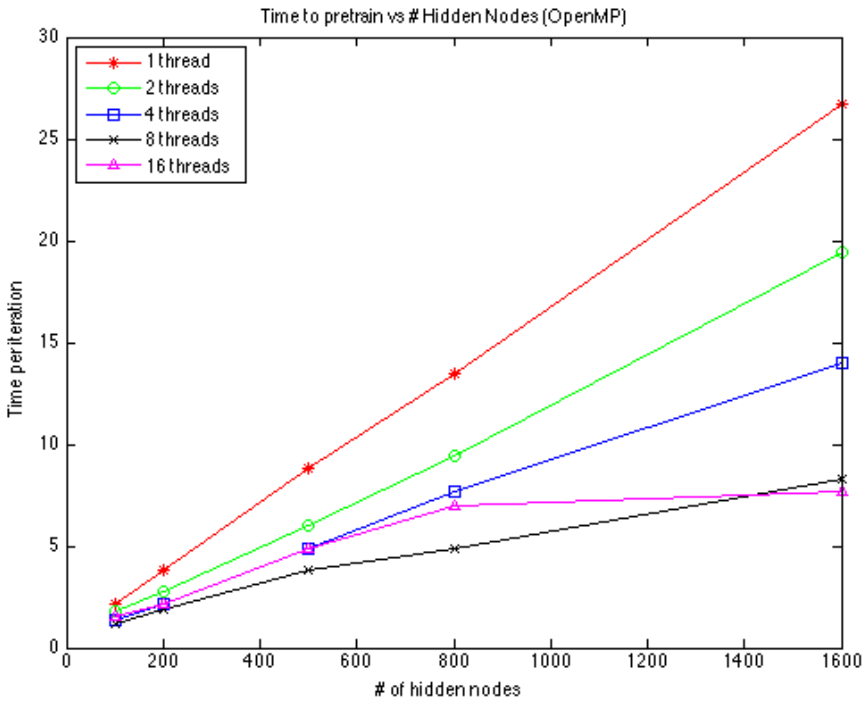


Fig. 5: Time per iteration versus the number of threads and hidden nodes. We use our own parallel implementation using OpenMP and we use 5000 training images.

We note that Figs. 5 and 6 show similar scaling, though in most cases the OpenBLAS version outperforms our own implementation. However, for small numbers of nodes, OpenBLAS does not perform well with many threads.

We generally get increasing amounts of speedup with increasing number of threads, but with a large number of threads we do not see improvement until the problem size increases and becomes large enough. We do not achieve linear scaling (i.e. twice as many threads does not result in the algorithm running twice as fast), since not all parts of the algorithm are parallelizable. That said, our algorithm does continue to get faster with increasing number of threads.

B. Visualization of Trained Weights and Reconstructed Images

Next, in Fig. 3, we visualize the filters that are learned by training an autoencoder layer with 500 hidden nodes over all 60000 training images. The filter for each hidden node is a row vector of the weight matrix and indicates which aspects of the input the hidden unit is sensitive to. Since each row in the weight matrix is the same dimensionality as the input, we can visualize it as a 28 by 28 pixel image. The filters are not identical to the input images, but do show some similarity to them. In Fig. 2, we visualize the reconstructed digits when given noisy test digits as input. The reconstructed outputs for most of the input images are easily recognizable as digits, which indicates that the autoencoder is indeed denoising and learning a good representation of the images.

To further demonstrate the reconstruction capabilities of the autoencoder, we trained the autoencoder on corrupted or otherwise altered digits. In particular, we test the reconstruction on *bg-rand*, which is generated from the MNIST dataset, excpt that a random background is added. In *bg-img*, each image is given a background randomly selected from one of twenty images downloaded form the internet. In *rot*, the digits are simply rotated by some random angle. These alterations to the images make the classification task more difficult. Indeed the digits are very difficult to identify, but the autoencoder creates an easier to identify representation, even to the human eye. We show also in Fig. 2 the reconstructed images form the *bg-img* and *rot* datasets. Note that the images in the *bg-rand* and *bg-img* datasets are rotated, but they are all rotated in the same way.

Finally we evaluate the classification accuracy of a deep neural network that has multiple stacked denoising autoencoders. We train 3 stacked autoencoder layers, each with 1000 hidden units, and using noise levels 0.1, 0.2,

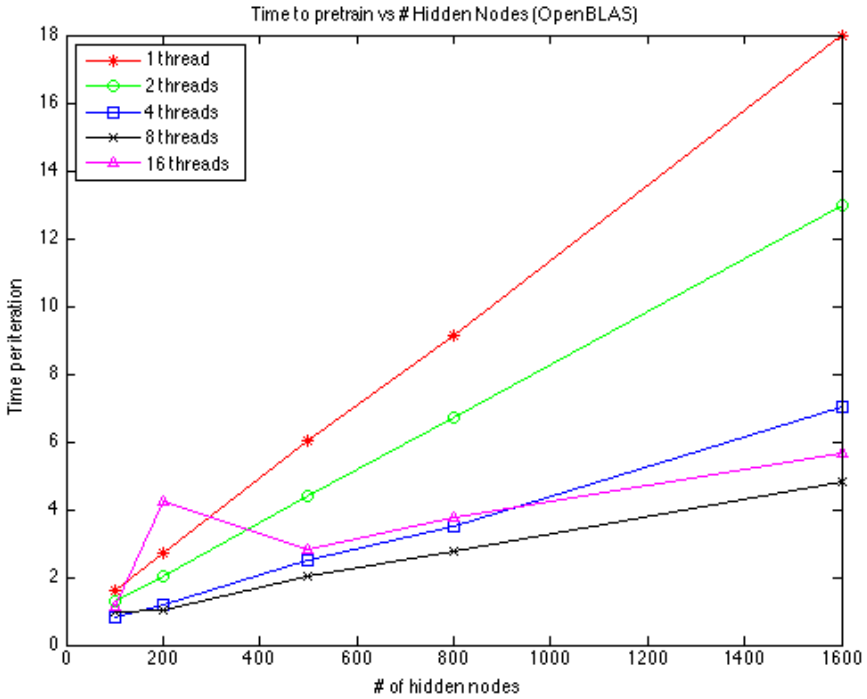


Fig. 6: Time per iteration versus the number of threads and hidden nodes. We parallelize with OpenBLAS and we use 5000 training images.

and 0.3 respectively. Each layer is trained for 15 iterations with a learning rate of 0.001. After the unsupervised pretraining, a conventional feedforward network with 1000 input units, 500 hidden units and 10 outputs is connected to the hidden units of the last autoencoder layer. This conventional network is then trained for 30 iterations (learning rate 0.1) in a supervised manner, where the target t is the indicator vector representation of the training label. Our final classification accuracy is 98.04%. In comparison, the accuracy achieved with a SVM with RBF kernel is 98.60% [2].

C. Representation Learning for Supervised Classification

Recall that one of the main reasons for using an autoencoder is to determine a more useful representation of the data for other tasks, for example in a classification task. To this end, we constructed and trained (15 iterations) an autoencoder with just a single layer and 1000 hidden units and used it to create a more useful representation of the digits in the MNIST dataset. After this more useful representation is constructed, we can then use the output from the autoencoder as input to another type of classification algorithm. Since the autoencoder produces a better representation of the data, we expect that given the encoded data, the other classification algorithms should perform better. The results of these experiments is given in Table. II.

To test this,we used liblinear to attempt to train a model and then predict on a test set for both the original and encoded datasets. With the original data liblinear gives an accuracy of 91.68% on the test set when using the default parameters. However, when the encoded data from the trained autoencoder gives an accuracy of 97.07%. This is a nontrivial improvement in the classification accuracy. Thus, the autoencoder has created a better representation of the data which made it easier for liblinear to classify. This verifies that the autoencoder is doing what it is expected to do.

Similarly, we performed the same experiment as above, except in this case we used libsvm with an RBF kernel and all the default parameters. Without encoding the data first, we get an accuracy of 94.46%, but using the encoded data gives a prediction accuracy of 95.48%. As above, the encoded data allows libsvm better classify the data.

Using logistic regression to perform the classification, we experienced similar results. Again we use liblinear with all default options except selecting logistic regression. Using the original MNIST data, this algorithm achieved

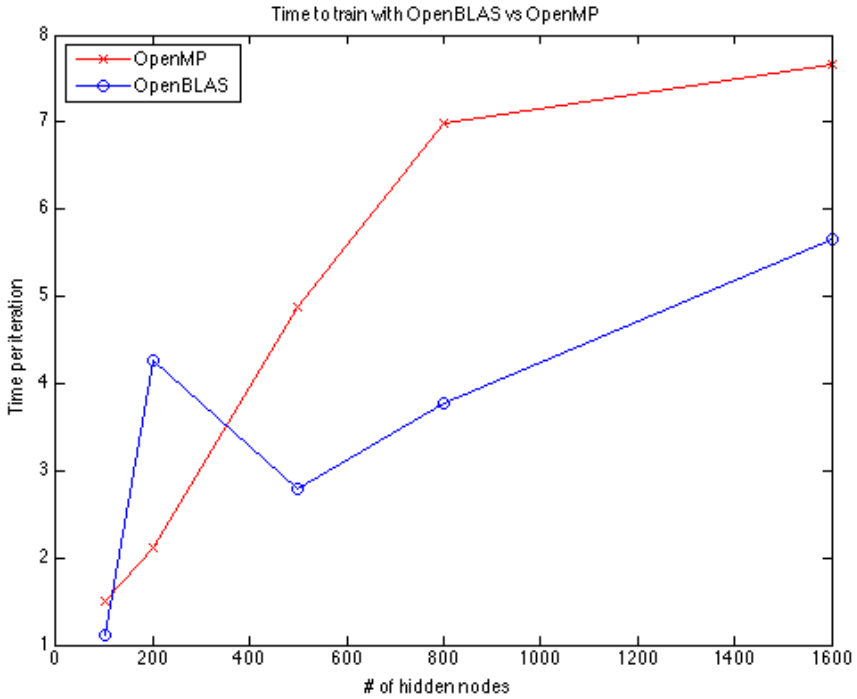


Fig. 7: Time per iteration versus parallelization technique and hidden nodes. We use 16 threads here.

an accuracy of 91.82% while with the encoded data we achieved an accuracy of 96.86%.

We performed the same experiments on the *bg-rand*, *bg-img* and *rot* datasets as well. Across the board we see similar results. The encoding does not have much of an effect on the ability of kernel SVM to classify the data, but for linear SVM and logistic regresssion, using the autoencoder always results in the classifier improving it's accuracy.

Dataset	Data	Linear SVM	Kernel SVM (RBF)	Logistic Regression
MNIST	Original	91.68%	94.46%	91.82%
	Encoded	97.07%	95.48%	96.86%
mnist-bg-rand	Original	58.975%	83.875%	65.5917%
	Encoded	81.675%	83.6583%	83.825%
mnist-bg-img	Original	69.25%	76.4667%	71.6333%
	Encoded	78.4333%	72.8417%	78.1333%
mnist-rot	Original	11.204%	13.328%	11.868%
	Encoded	18.428%	14.378%	18.78%

TABLE II: Summary of the results of running different classification algorithms on the raw MNIST data and on the output from a trained autoencoder. We see in all cases that using the encoded data produces a better result.

D. Genetic Algorithms

Hyperparameter	Value
Mutation Rate mr	0.0001
Mutation Amount ma	$0.1r$
Crossover Rate mr	0.5
Population Size N	2
Replacement Fraction α	0.5
Initial Value Range r	$1/\sqrt{\text{FANIN}}$
Power Scaling Parameter γ	1.0

TABLE III: List of hyperparameters for CGA and HGA. FANIN is input dimensionality of autoencoder layer.

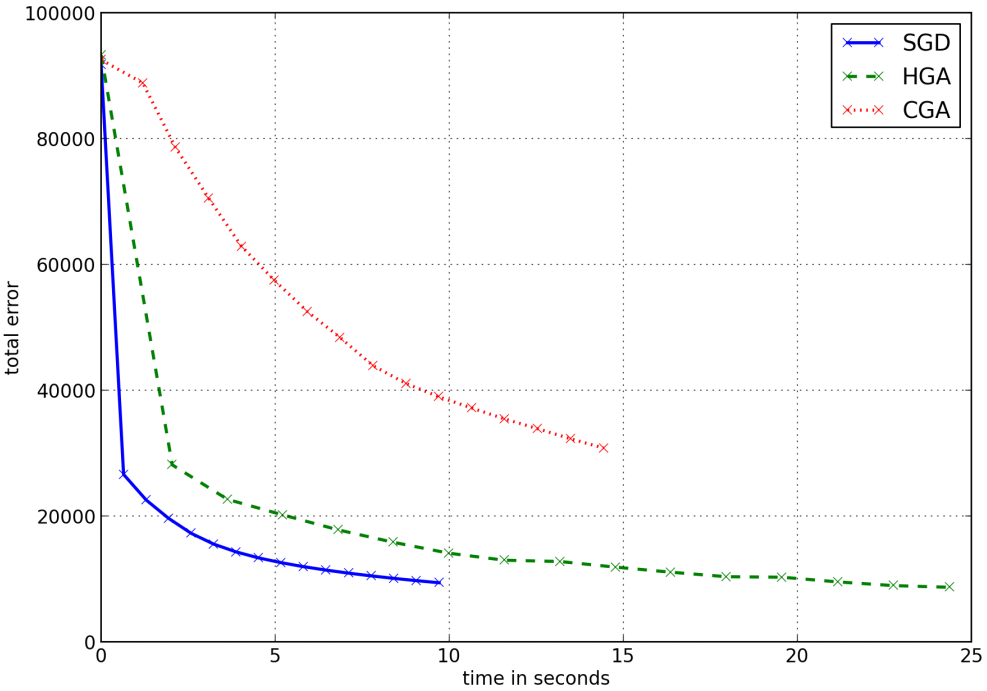


Fig. 8: Comparison of the performance of SGD, HGA, and CGA. SGD is fastest, while HGA achieves the lowest reconstruction error. HGA and CGA uses a default population size of 2 and all three algorithms cycle through 1000 training images for 15 iterations.

For all of the following experiments, we train a single autoencoder layer with 1000 hidden units and cycle through 1000 training digits for 15 iterations. The default hyperparameters for CGA and HGA are listed in Table. III.

In Fig. 8, we compare the performance of stochastic gradient descent (SGD), our previously mentioned hybrid GA (HGA), which uses backpropagation to update the best individuals in the population, and a conventional GA (CGA), which does not use any gradient information. SGD is the fastest by roughly a factor of two when compared to HGA, while CGA is somewhere in between. However, HGA is able to achieve the lowest reconstruction error (8709 vs SGD's 9427). CGA performed the worst out of all 3 algorithms, having a reconstruction error that is three times larger than that of the other algorithms. The hyperparameters for HGA and CGA are hand tuned and not necessary optimal. However, we believe that with properly tuned hyperparameters, HGA might be competitive with SGD for both final reconstruction error and performance time.

Next in Fig. 9, we examine the scalability of HGA. HGA shows roughly 3x speedup when using 4 threads, 5x speedup when using 8 threads, and 6x speedup when using 16 threads. The performance improvement versus the number of threads is sublinear and can be attributed to two main causes: 1) Cache conflicts when performing mutation and crossover due to multiple threads writing and reading different memory locations, 2) The sequential nature of backpropagation, which HGA utilizes to help optimize the weights. However, we do see that HGA does seem to moderately better scalability than our parallel implementation of SGD. This is attributable to the fact that the mutation and crossover operators are very straightforward to parallelize and scale in performance very well.

In Fig. 10, the performance of HGA versus the number of threads and number of hidden units in autoencoder layers is visualized for both when HGA has a population size of 50 and its default population size of 2. As we can see, the HGA's performance scales linearly with the number of hidden units and that this relationship holds even when the population size is increased to 50. Interestingly, the performance when using 8 or 16 threads is not distinguishable for small numbers of hidden units. This is probably due to the overhead that result from creating additional threads.

Finally, in Fig. 11, we plot the performance of CGA and HGA for population sizes of 2, 4, 8, 16, and 32. All other hyperparameters remain the same for both HGA and CGA. As we can see, larger populations does lead to a noticeable improvement in the final reconstruction error, but at a cost of increased running time that is linear to the population size. For each fixed population size, HGA performs significantly better than CGA, with a final

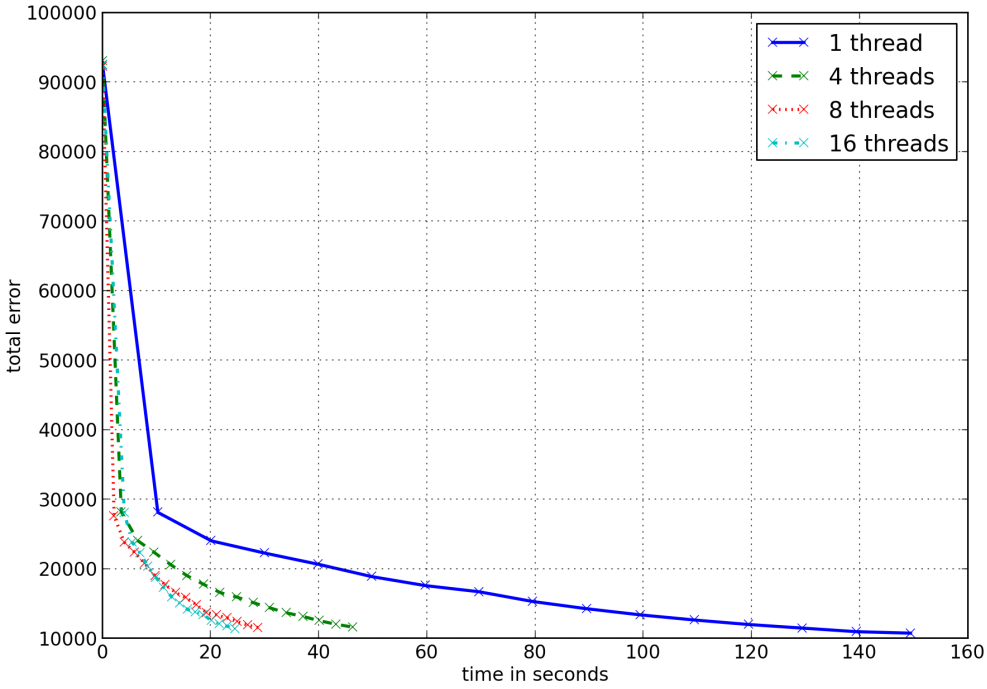
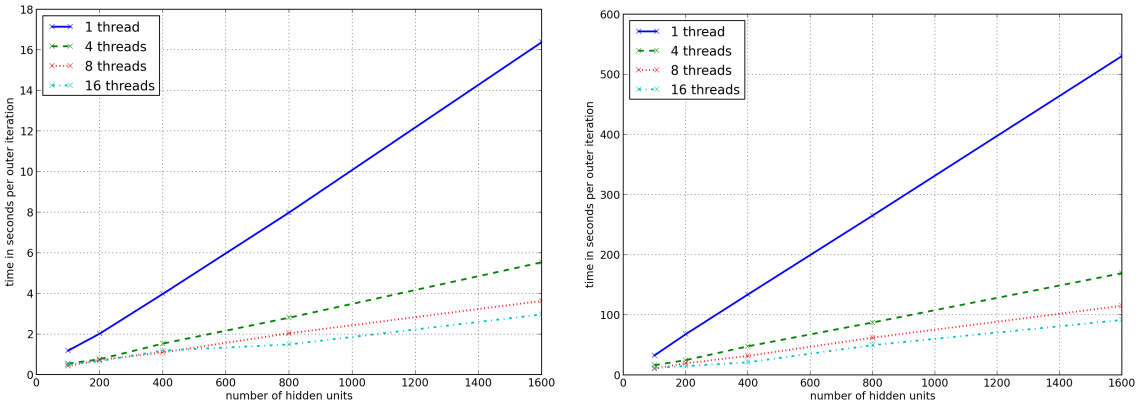


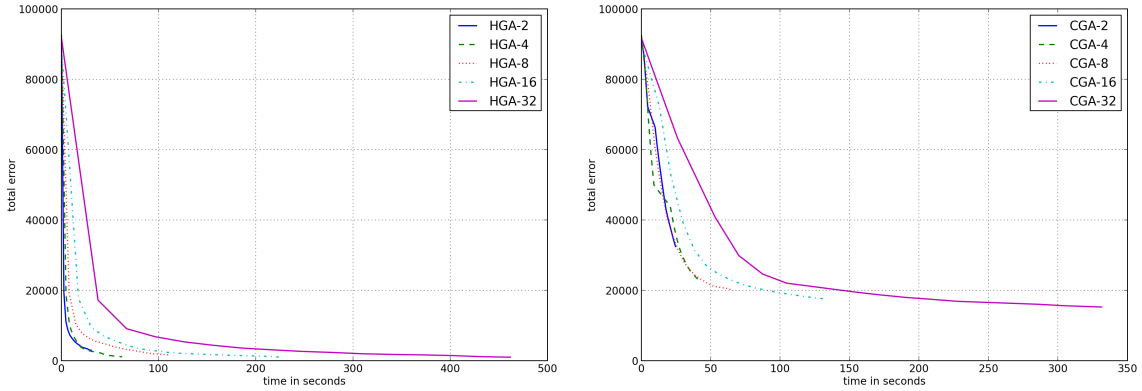
Fig. 9: Performance of HGA for 1, 4, 8, and 16 threads with a population size of 2 and 1000 training images.



(a) (b)

Fig. 10: Comparison of performance versus number of threads and number of hidden units in autoencoder layer for HGA with population size (a) 2 and (b) 50.

construction error is roughly 10 times lower. However for a given population size, HGA does take 40% longer to finish; this is attributable to the fact that CGA does not perform backpropagation, which takes additional time.



(a) (b)

Fig. 11: Comparison of performance for CGA and HGA with population sizes of 2, 4, 8, 16, and 32. HGA performs significantly better than CGA in terms of final reconstruction

V. Future Work

For future work, we plan on using a genetic algorithm (GA) to train the autoencoder's weight matrix and biases. A GA is a black-box optimization algorithm that iteratively improves upon a population of candidate solution vectors until the global optima of the objective function is reached [6]. It uses operators such as mutation and crossover, which are inspired by biological evolution. GAs are superior when compared to SGD in training autoencoders in two aspects: 1) Since the loss function is highly nonconvex, SGD will always converge to a local minima, while GA are capable of eventually reaching the global optima. 2) SGD is not trivial to parallelize; as seen by our experimental results, SGD does not achieve linear speedup. On the other hand, GAs are much simplier to parallelize in one of two following ways [7]: a) Each individual in a population can be evaluated in parallel. b) The mutation and crossover operators operate on each element of a solution vector independently and thus are embarrassing parallel. An overview of how genetic algorithms work is given by Algorithm 4. For autoencoders, each individual in the population is some particular weight matrix and its two associated bias vectors. The objective function would be the loss function described earlier and the fitness is how small the error outputted by the loss function is.

We also plan on evaluating the performance of our autoencoder on additional harder image datasets mentioned in [2], such as *bg-rand, bg-img-rot*, which contain images with noise and rotation.

VI. Conclusion

We have implemented stacked denoising autoencoders and shown that it achieves accuracy comparable to state of the art classifiers like a SVM with RBF kernel. We also shown that our autoencoder layers are learning good representations and are capable of denoising and reconstructing the input with little error. These learned representations improve the ability of other classification algorithms to correctly classify the data. We have shown that SGD scales well with increasing number of nodes and with increasing number of threads. We have shown also that a genetic algorithm for training the autoencoder scales well in both of these regards.

Algorithm 4 Genetic Algorithm

Initialize N individuals randomly
for iter $= 1, 2, 3...$ **do**
  Evaluate each individual with objective function and assign fitness.
  Create αN $(0 < \alpha < 1)$ new individuals by selecting good individuals from population and applying mutation and crossover operators to them.
  Replace worst αN individuals in population with newly created individuals.
end for

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