THE UNIVERSITY *of* EDINBURGH
School of Engineering

# Fluid Mechanics 2 (SCEE08003) laboratory briefing note

# Guidance on dealing effectively with uncertainties in experiments

**Terminology:** The terms “errors”, “experimental errors”, “measurement errors” are widely used. In the note, we prefer “uncertainties”, “experimental uncertainties”; “measurement uncertainties”. Although these terms are usually taken to mean the same thing, the word “error” can give a sense that a mistake has been made, or that the number is wrong. This is not the intended meaning.

## 1. Introduction

“Dealing with uncertainty” is a key aspect of being a successful professional engineer. In school and university, we become accustomed to tackling and solving problems in which all necessary data is given, often to a precision that suggests that it is known rather exactly! Intuitively, we know that the real world is not like this… so we need to develop strategies to cope with this and do excellent engineering design dealing with the uncertainties. The sort of questions that you will more and more often be asking yourself are things like:

How accurately do I need to know X?
*e.g.* I need to select a pump. How accurately do I need to estimate the energy losses in my pipe network?

How does my uncertainty in some input Y feed through to an uncertainty in my final result?
*e.g.* I estimate my flow rate by using a stopwatch to measure how long it takes to gather 1 litre of water. If I can measure the time to $\pm$ 0.5 s, what is the effect of this uncertainty on the value of the discharge coefficient that I calculate using the flow rate and other inputs?

Are my measured data in line with the prediction equation?
The data will never match exactly with a prediction, so here, you need to know the uncertainty associated with each data point. You can then judge whether the measurements and predictions agree, to within the uncertainty bounds.

What are the most significant contributors to the uncertainty in my final output?
If you know which is the main source of your uncertainty in your final result, and you need to reduce this uncertainty, then you know upon which measurement to focus.

These guidance notes should give you a box of tools with which to approach your management of uncertainties that arise in experimentation; in measurement; data gathering and in the analysis (*i.e.* combination) of quantities, all of which have some inherent uncertainty associated with them. Understanding and estimating the scale of uncertainties in the lab setting is essential if you are to be able to draw the most robust and therefore useful conclusions.

## 2. Systematic and random errors

**Systematic errors** come from inherent defects or uncertainties in the equipment or method, and will be repeatable. Thus, systematic errors are not reduced by averaging.

> Example 1: A manometer scale was calibrated in Pa for oil as the manometer fluid, but water was used. Thus, all measurements are going to be wrong, but all by the same, repeatable factor (= $\rho_{water} / \rho_{oil}$).
>
> Example 2: The diameter d of a pipe is known to be 10 mm ± 1 mm. This introduces an uncertainty into any quantities that depend upon d, but all data is affected in the same way (d is not actually varying between 9 mm and 11 mm!)

**Random errors** are due to uncertainties in measurement which cannot be controlled or removed. Random errors may be reduced by averaging results of repeats of nominally identical tests (or other statistical processing).

> Example 1: A small sphere falls at constant speed through a column of oil. Its speed is determined by measurement of the time taken to fall 1.00 m. There is an inherent uncertainty in the measurement of the time. Nominally identical repeats of the experiment will produce different values. If many tests are done and the results averaged, the uncertainty should be reduced.
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> Example 2: A manometer is used to measure the pressure in a liquid flow in a pipe. The manometer height can be read to ± 2 mm due to the meniscus moving a little, and uncertainty in visually interpolating between points on the measurement scale that are 5 mm apart.

## 3. Estimating measurement uncertainty (or "measurement error")

Often, when measuring a quantity on a meter or instrument, we read to the nearest scale division and estimate one further digit. This involves a reading error, and the accepted rule for estimating this error is to use +/- 0.5 times the smallest scale division.

The zero of the scale may also be subject to a setting error of the same magnitude. In such cases, we should add them to get an overall **measurement error estimate of ± 1 scale division.**

Red for danger! This is the minimum measurement error we would *normally* use, BUT please think carefully before "diving in" with this – there are many cases where this would be too optimistic an estimate and it is essential that you "sanity check" your estimation of uncertainty.

> Example 1: The classic example is when using a stop-watch. The reading may be to 0.01 s, but the actual uncertainty comes from the user's selection of exactly when to start and stop the stopwatch. With good anticipation, we might imagine ± 0.2 s, but this applies to the starting and the stopping, so we would estimate uncertainty at ± 0.4 s.

Example 2: Imagine a manometer, with scale divisions at 2 mm, whose level was fluctuating by ± 5mm. It would be *wrong* to apply the "± half a scale division" rule. Instead, you should estimate the uncertainty. Imagine making many measurements, or having others make the same measurement. How much scatter would you see?

## 4. Recording errors

We can quote errors in raw form *i.e.* **absolute error**, or as **relative error**, or as a **percentage error**. The following example illustrates the definitions involved.

*Example*: A ruler is calibrated in mm. If we measure the length L of a steel block as 32.3 mm, the reading error is 1 mm. In a table of results, this should be recorded as

$L = 32.3 \pm 1\text{mm}$

Here, the **absolute error** in L is $\delta L = 1\text{mm}$,
the **relative error** in L is $(\delta L/L) = 0.03$, and
the **percentage error** in L is $(\delta L/L)\times 100 = 3\%$.

The **relative error** is useful when calculating errors in quantities which combine measured values in a formula (see section 6). The relative error is sometimes also called the **fractional error** because it is the error expressed as a fraction of the quantity itself.

The **percentage error** conveys the clearest impression of the magnitude of the error when discussing results. Avoid using percentage error for quantities like efficiency which are themselves percentages.

Also, beware of using large percentages, which can be confusing or ambiguous. An example might be a gauge pressure measurement that was fluctuating wildly between 50 Pa and 0 Pa. This would be recorded correctly as $p = 25 \pm 25$ Pa. But although quoting the percentage error as 100% may be strictly correct, what might "a 100% error" be taken to mean?

Estimating errors in this way is a crude process and a guideline only. **Avoid excessive precision** when quoting error bounds. The precision to which the quantity and its associated uncertainty are quoted should align:

✗ $L = 1.46 \pm 0.02043$ m
*uncertainty is estimate only; much too much unjustified (meaningless) precision*
✗ $L = 1.5 \pm 0.02$ m
*quantity has been rounded unnecessarily (if we believe the uncertainty estimate)*
✓ $L = 1.46 \pm 0.02$ m
*quantity rounded to an extent consistent with size of associated uncertainty.*

In general, always try to quote measured values **rounded** to be consistent with the size of the error:

✗ $L = 3.46275 \pm 0.2$ m
✓ $L = 3.5 \pm 0.2$ m

## 5. Digital Meters

Digital meters look more accurate than they are. Don't be fooled into thinking that they are as accurate as the number of digits on the display suggests. Usually they will have errors due to:

- the uncertainty of $\pm 1$ in the final digit;
- an instrument error quoted by the manufacturer as a % of the reading, related to the quality of the instrument;
- a zero reading error; and
- any error due to random fluctuations in the measured quantity.

*Example:* A typical digital voltmeter with a four-digit display, set to 2V DC range is reading 0.356 V.

| | | | |
|---|---|---|---|
| Final digit uncertainty | | = | 0.001 V |
| Instrument error from handbook | | = | 0.2% of reading |
| | | = | 0.0007 V |
| Estimate zero error | | = | 0.002 V |
| Random error | | = | 0 V |
| Total error | $\delta V$ | = | 0.004 V |

The measurement should therefore be recorded as $V = 0.356 \pm 0.004$ V

## 6. Uncertainty estimates in derived quantities (combining errors)

It is often necessary to calculate results from a formula which combines several measured quantities. We need to estimate the error bounds for the final result. We do this by applying the calculus of small changes. If a quantity $y$ is related to $x_1, x_2, \ldots x_n$ by an expression $y=f(x_1, x_2, x_3, \ldots x_n)$ then the change in $y$ which results from changes in the $x$'s can be expressed as:

$$\delta y = \frac{\partial f}{\partial x_1}\delta x_1 + \frac{\partial f}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f}{\partial x_n}\delta x_n$$

We use this for estimating the error in $y$ arising from the combination of the errors in the $x$ quantities. In applying this formulation, we usually assume that the signs of the errors are such as to add up to give the worst possible result, rather than cancel out.

Example 1. The error in measuring the volume of a cylinder from its diameter and length.

Here, $V = \frac{\pi D^2 L}{4} \quad \Rightarrow \quad \delta V = \frac{\pi D L}{2}\delta D + \frac{\pi D^2}{4}\delta L$

Dividing through by V, $\quad \Rightarrow \quad \frac{\delta V}{V} = 2\frac{\delta D}{D} + \frac{\delta L}{L}$

We can recognise the terms $\delta V/V$, $\delta D/D$ and $\delta L/L$ as the relative errors in V, D and L respectively. Thus, if we have estimates of the relative errors in D and L, we can very easily combine these to get an estimate of the relative error in the combined quantity, V.

This will be the case when the formula is in the form of *products* of terms which may be raised to powers, as shown by the following example which is useful to remember.

Example 2. Suppose a quantity F is related by the formula

$$F = \frac{L^a M^b}{T^c} = L^a M^b T^{-c}$$

Then by applying the partial derivatives, the result for the relative error in *F* is shown to be

$$\frac{\delta F}{F} = \pm\left[a\frac{\delta L}{L} + b\frac{\delta M}{M} + c\frac{\delta T}{T}\right]$$

Note that relative errors always **add** in combination.

Example 3: Sums and Differences.
For calculations which involve sums or differences, the **absolute error** in the sum or difference is the sum of the absolute errors in the individual terms.

$$S = A + B \quad \Rightarrow \quad \delta S = \delta A + \delta B$$
$$D = A - B \quad \Rightarrow \quad \delta D = \delta A + \delta B$$

The last case follows because we assume the worst case when estimating errors. A very important consequence of the last case is that **the error in the difference of two large, near-equal quantities can be very large indeed**. We see that from the relative error in *D* which is

$$\frac{\delta D}{D} = \frac{\delta A + \delta B}{A - B}$$

An example where this arises is in calculating the energy balance in a thermodynamic system. *D* may then be the net energy flow into a system and you may be trying to establish that it is zero. The error limits on *D* may be large as a result of the errors in determining the various energy inflows and outflows.

*Harder Examples:*
The complexity of formulae or procedures for calculating results may make it difficult to deduce the size of the errors in the results - for example, suppose it was necessary to invert a matrix of values to get results. How would the errors in the results be related to the errors in the matrix elements? A good approach is often to do sample calculations with your data values increased, then reduced by error magnitudes as a means of getting some guidance on the size of the errors in your calculated values.

## 7. Repeated Experiments

When seeking an accurate measurement of a quantity with a random error associated with it, making a number of repeated measurements and taking the mean value can be a useful approach. Typically, making *N* repeated measurements of a quantity with an associated random error (of roughly Gaussian distribution) should reduce the magnitude of the random error bound by a factor of $\sqrt{N}$.

Strictly, the number of nominally-identical repeats should be reasonably large for this approximation to be appropriate. For our pragmatic error estimation strategy, $N \geq 10$ would be reasonable.

*Example:* A flow rate is to be established by measuring the time taken for a known volume of water to collect in a measuring cylinder. Both time and volume measurements have associated (random) reading errors.

A single measurement gave a value of *eg* 0.1468 litres/s, but with an estimated error bound of 0.01 litres/s, so the result was recorded as

**flow rate** = $0.15 \pm 0.01$ litres/s.

The test was carried out a total of 10 times, so the estimated error bound was reduced by a factor of $\sqrt{10} \approx 3$ and the result was

**flow rate =** $0.147 \pm 0.003$ litres/s.