Your solution to the following question is to be submitted to the online dropbox on Learn by **15:00 on Thursday the 24th of November 2022**. This is coursework number 3 of 3. To pass the coursework component of Engineering Mathematics 2A you must obtain an average mark over the three courseworks and the STACK quizzes of more than 40%. Failing to do so will lead to a *Forced Fail* for the course, even if you pass the exam and obtain an average mark of more than 40%.

Your solution should be handwritten on paper and must not be longer than **7**. You can either use the engineering calculation sheets that can be downloaded from LEARN or standard A4 paper. For the latter you should clearly write your name and student number on the top of the page and leave at least 3cm of space on the right hand side for marking. The solution needs to be **scanned and uploaded** to the **ETO online dropbox**.

40% of the marks for this coursework will be awarded based on the clarity and logical progression of the solution, details of the marking rubric is given at the end of the handout on laying out mathematical calculations which is also available on LEARN. The remaining 60% of the marks will be awarded based on the mathematical accuracy of the solution.

Coursework preparation and submission:

- Use the calculation sheets or a close approximation of it (top margin with date and your exam number and a delimited space on the right hand side for marking).
- Prepare a handwritten solution on paper following the layout guidelines given on Learn.
- This solution must not be longer than **7 pages**: The Flow mark will be reduced by one grade for each page beyond the limit.
- Scan your solution and upload it to the **ETO online dropbox**.
- Check that your solution uploaded correctly.

1. Consider the non-periodic function $f(t)$ on the interval $[0, 2]$ given by

$$f(t) = \begin{cases} 0.5 + t - 0.5t^2, & 0 \leq t < 1, \\ 1, & 1 \leq t \leq 2. \end{cases}$$

   (a) Sketch the odd periodic extension of the function $f(t)$ and calculate the half-range sine Fourier series of $f(t)$. [8]

   (b) You realise that the convergence of the odd half-range Fourier series is not very good and you try to find a faster converging Fourier series. The first thing you try is the even half-range Fourier series. [6]

   Sketch the even half-range extension of the function $f(t)$. Derive the formula of the even half-range extension over the interval from -2 to 2. Discuss the convergence order of the even half-range extension.

   You don't have to calculate the Fourier series for the even half-range Fourier series. No marks are allocated to calculating Fourier series coefficients.

   (c) Since you are still not happy with the convergence, you want to find a similar function which has a Fourier series with faster convergence. [6]

   Use a linear transformation so that you can find an odd extension for the resulting function that has a Fourier series that converges at least as $1/n^3$; explain why this is the case. Sketch the transformed function with its odd extension that has a Fourier series that converges at least as $1/n^3$.

   You don't have to calculate the Fourier series for the shifted and extended function. No marks are allocated to calculating Fourier series coefficients.

Formula sheet

Useful integrals

$$
\begin{aligned}
\int x \sin ax \, \mathrm{d}x &= \frac{1}{a^2}\big(\sin ax - ax \cos ax\big) \\
\int x \cos ax \, \mathrm{d}x &= \frac{1}{a^2}\big(\cos ax + ax \sin ax\big) \\
\int x^2 \sin ax \, \mathrm{d}x &= \frac{2x}{a^2} \sin ax - \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \cos ax \\
\int x^2 \cos ax \, \mathrm{d}x &= \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \sin ax
\end{aligned}
$$