Dimensional Analysis with Linear Algebra

Jasper Day

Hydrodynamics is fully defined by these factors:

Mass density ρ, speed v, pressure p, viscosity μ, and the acceleration due to gravity g.

Take for example the capillary effect:

Symbol	Description	Base Dimensions
h	Distance water is drawn into the tube	L
d	Diameter of the tube	L
σ	Surface tension of the water	MT^{-2}
ρ	Mass density of water	ML^{-3}
g	Acceleration due to gravity	LT^{-2}

h is some function of the other three quantities:

$$h = f(d, \sigma, \rho, g)$$

Then

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & -2 \end{bmatrix}$$

The null space of $\mathbf{A}$ is linear combinations of the vector $(-2, 1, -1, 1)$

Therefore

$$h = d \cdot g\left(\frac{\sigma g}{d^2 p}\right)$$