1. Which of the following sets are linearly independent and which are linearly dependent?
   (a) $\{1,\, t,\, t^2,\, t^3,\, t^4,\, t^5,\, t^6\}$
   (b) $\{1+t,\, t^2,\, t^2-t,\, 1-t^2\}$
   (c) $\{\sin t,\, \cos t,\, \sin t - \cos t,\, 2\sin t + \cos t,\, 2\sin t - \cos t\}$
   (d) $\{1-2t^2,\, t-3t^3,\, 3t^2+4t^3,\, t^3\}$
   (e) $\{e^t,\, e^{2t}-e^t,\, e^{3t}-e^{2t}, e^{2t}\}$
   (f) $\{1-2t^2,\, t-3t^3,\, 2t^2-4t^4,\, 4t^3\}$

2. For each of the following differential equations, write down the differential operator, L, that will allow the equation to be expressed in the form $L[x(t)] = 0$.
   (a)
   $$\frac{\mathrm{d}x}{\mathrm{d}t} - kx = 0$$
   (b)
   $$\frac{\mathrm{d}^3x}{\mathrm{d}t^3} + (\sin t)\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4t^2x = 0$$
   (c)
   $$\frac{\mathrm{d}}{\mathrm{d}t}\left(t^2\frac{\mathrm{d}x}{\mathrm{d}t}\right) = t\frac{\mathrm{d}}{\mathrm{d}t}(xt)$$
   (d)
   $$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{t}\frac{\mathrm{d}}{\mathrm{d}t}(t^2x)\right) = xt.$$

3. Determine which members of the given sets are solutions of the differential equation. Hence write down the general solution of the differential equation
   (a)
   $$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - p^2x = 0\ \left\{e^{pt},\, e^{-pt},\, \cos pt,\, \sin pt\right\}$$
   (b)
   $$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} = 0\ \left\{\cos 2t,\, \sin 2t,\, e^{-2t},\, e^{2t},\, t^2,\, t,\, 1\right\}$$
   (c)
   $$\frac{\mathrm{d}^3x}{\mathrm{d}t^3} - \frac{\mathrm{d}^2x}{\mathrm{d}t^2} - \frac{\mathrm{d}x}{\mathrm{d}t} + x = 0\ \left\{e^t,\, e^{-t},\, e^{2t},\, e^{-2t},\, te^t,\, te^{-t},\, te^{2t},\, te^{-2t}\right\}$$

4. Solve the following initial value problems:
   (a)
   $$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 2\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = 0,\ x(0) = 1,\ \left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} = 0.$$

(b)
$$\frac{d^3x}{dt^3} - 6\frac{d^2x}{dt^2} + 11\frac{dx}{dt} - 6x = 0,\ x(0) = 1,\ \left.\frac{dx}{dt}\right|_{t=0} = 0,\ \left.\frac{d^2x}{dt^2}\right|_{t=0} = 1.$$

(c)
$$\frac{d^3x}{dt^3} + 6\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 8x = 0,\ x(1) = 1,\ \left.\frac{dx}{dt}\right|_{t=1} = 1,\ \left.\frac{d^2x}{dt^2}\right|_{t=1} = 0.$$

5. A initial value problem (IVP) comprises

$$-\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - 8x = 0$$

together with the initial conditions $x(0) = -1$ and $\dot{x}(0) = 0$

(a) Show that the complementary function is

$$x(t) = e^{-2t}\left(A\cos 2t + B\sin 2t\right).$$

(b) Given that the general solution to the ODE is

$$x(t) = e^{-2t}\left(A\cos 2t + B\sin 2t\right),$$

use the boundary conditions, $x(0) = 1$ and $\dot{x}(0) = 0$, find the particular solution to the given IVP.