1. Solve the Boundary Value Problem

$$\frac{\mathrm{d}^4 x}{\mathrm{d}t^4} + 16x = 0$$

given that

$$x(0) = 0 \quad \left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} = 0, \; x\left(\frac{\pi}{\sqrt{2}}\right) = 1, \text{ and } \left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=\frac{\pi}{\sqrt{2}}} = 0.$$

2. A boundary value problem (BVP) comprises

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - \frac{\mathrm{d}x}{\mathrm{d}t} - 2x = 0$$

together with the boundary conditions $x(0) = 1$ and $x(1) = 1$

(a) Show that the complementary function is

$$x(t) = Ae^{-t} + Be^{2t}.$$

(b) Given that the general solution to the ODE is

$$x(t) = Ae^{-t} + Be^{2t}$$

use the boundary conditions, $x(0) = 1$ and $x(1) = 1$, find the particular solution to the given BVP.

3. Find the general solutions to the following inhomogeneous, constant coefficient, linear differential equations.

(a)

$$\frac{d^2 s}{dt^2} - \frac{ds}{dt} - 2s = 4t^2$$

(b)

$$\frac{d^2 x}{dt^2} - \frac{dx}{dt} - 2x = e^{3t}$$

(c)

$$\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = \sin 2x$$

4. Solve

$$\frac{d^2 u}{dt^2} - 6\frac{du}{dt} + 25u = 2\sin\frac{t}{2} - \cos\frac{t}{2}.$$

5. An initial value problem (IVP) comprises

$$-\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - 4\frac{\mathrm{d}x}{\mathrm{d}t} - 8x = \cos 2t$$

together with the initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$

(a) Show that the complementary function is

$$x(t) = e^{-2t}\left(A\cos 2t + B\sin 2t\right).$$

(b) Show that the particular integral is

$$x(t) = -\frac{2\sin 2t + \cos 2t}{20}$$

(c) Given that the general solution to the ODE is

$$x(t) = e^{-2t}\left(A\cos 2t + B\sin 2t\right) - \frac{2\sin 2t + \cos 2t}{20},$$

use the boundary conditions, $x(0) = 1$ and $\dot{x}(0) = 0$, find the particular solution to the given IVP.

(d) by considering the behaviour of the solution as $t \to \infty$ sketch the phase portrait of the general solution to the ODE