Analysis of Algorithms
CS 375, Fall 2022
Smaller Assignment 4
Due **BY THE BEGINNING OF CLASS** Monday, November 21

- For this Smaller Assignment, as long as you submit answers **on time, before the deadline,** with a demonstrable, strong effort to solve the problems and thorough explanations of your answers—including, if you were not able to solve an exercise, what progress you made, what was left unsolved, and what made it hard to solve—if you do not receive full credit, you will be able to revise your answer after receiving feedback, to bring your grade up to full credit!

- For this Smaller Assignment, the standard file naming conventions apply: Please submit your typewritten answers in a PDF file named

  `CS375_SA4_<userid>.pdf`

  where `<userid>` is replaced by your Colby userid (your full userid, including class year) and submit it to your `SubmittedWork` folder in your Google drive space for this course.

- *A general note for CS375:* As always, please present answers cleanly and **explain them clearly and thoroughly**, giving all details needed to make your answers easy to understand; typed-up (rather than handwritten) answers are especially appreciated. Graders may not award full credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

  In general in CS375, unless explicitly specified otherwise, answers without explanations may not receive full credit. Please feel free to ask me any questions about explanations that might come up!

Exercises

1. Using the *unwinding* method from class, solve the following recurrence relation and give the Θ class of the solution.

   $T(n) = 3T(n-1)$ for $n > 1$; $T(1) = 4$.

   Show your work in doing the "unwinding." To show a full understanding of the unwinding method, please be sure to show a k'th step—a step at some representative k'th step in the unwinding, in terms of k rather than some specific number, as demonstrated in the lecture of November 11 and shown in lecture notes—which shows the pattern upon which your solution is based.

2. Solve the following recurrence **showing your work, using the recursion tree method** and give the Θ class of the solution.

   $T(n) = 3T(n-1)$ for $n > 1$; $T(1) = 4$.

   Please show your work by including a table of the following form. The table should include one row for each level in the tree; be sure to show a representative k'th level to illustrate the relevant pattern, similar to the k'th step in the unwinding method.

level	number of nodes on level	total work
0	number of nodes on level 0	total work on level 0
1	number of nodes on level 1	total work on level 1
2	number of nodes on level 2	total work on level 2
...	...	...
k	number of nodes on level k	total work on level k
...	...	...
$\langle$*last level*$\rangle$	number of nodes on $\langle$*last level*$\rangle$	total work on $\langle$*last level*$\rangle$

Then, explain how you use information from your table to arrive at a final answer. Recall that you can use high-level explanations for Θ analysis; you don't need to give witnesses for the values of the relevant constants.

NOTES: You might want to explicitly include the second-to-last level in your table, as well as the last level, because the last level is sometimes so different from the remainder of the table that it does not fit into the pattern. Also, you are also welcome to include a picture of the recursion tree as part of your answer if you'd like, but it is not required for the exercise.

3. Give a Θ bound for the following recurrence. *Be sure to use the Master Theorem for this exercise.* As always, be sure to give a brief explanation of your answer—here, that will include the values for each relevant variable in the Master Theorem, what case of the Theorem you are applying, and a brief explanation of how you know what case to apply. (A full explanation could take no more than 3–4 sentences.)

   $T(n) = 4T(n/2) + n^2$, $T(1) = 1$.

4. Give a Θ bound for the following recurrence, using any of the three methods introduced in class (unwinding, recursion tree, Master method).

   $T(n) = 3T(n/2) + n \lg n$, $T(1) = 1$

   As always, be sure to give a brief explanation (with appropriate details) of your answer.