Fresnel Diffraction From An Aperture

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The Fresnel Diffraction created by light passing through an aperture of variable size onto a screen at varying distance was modelled in both 1 and 2 dimensions. The intensity function required use of a simpson's integration method, which was subsequently compared to the trapezoid integration method to examine the accuracy. By varying the aperture parameters and distance to the screen, both far field and near field effects were observed.

THEORY

The intensity of light on a screen coordinate, (x, y), distance , z, from an aperture is given by:

$$I(x, y, z) = \epsilon_0 c E(x, y, z) E^*(x, y, z) \tag{1}$$

where ϵ_0 is the permittivity of free space, c the speed of light, E the electric field of the light at the screen coordinate (x, y, z) and E^* the electric field conjugate. Through an integral; summing the contributions of each aperture coordinate (x', y') across the range, the electric field, E can be calculated as:

$$E(x, y, z) = \frac{kE_0}{2\pi z} \int_{x_1'}^{x_2'} exp\frac{ik}{2z}(x - x')^2 dx' \int_{y_1'}^{y_2'} exp\frac{ik}{2z}(y - y')^2 dy' \tag{2}$$

where k is the wavenumber of the light, x_1' and x_2' the x width of the aperture and y_1' and y_2' the y width of the aperture. In order to compute the integral an approximation must be taken.

One method of approximating an integral of a function over an interval is to use the trapezoid rule. The area contained by the function is split into trapezia with equal width and summed:

$$\int_{x_0}^{x_N} f(x)dx \approx h[\frac{1}{2}[f(x_0) + f(x_N)] + \sum_{i=1}^{N-1} f(x_i)] \tag{3}$$

where x_0 and x_N are the starting and ending coordinates of the interval, N the number of trapezia the interval is divided into and h the width of the trapezia given by:

$$h = \frac{x_N - x_0}{N}. \tag{4}$$

This has a truncation error of order $\mathcal{O}(h^2)$. An improved method with lower truncation error of order $\mathcal{O}(h^5)$ is simpson's rule and is used to calculate the integrals in the subsequent investigation. The interval is split into a series of quadratic polynomials and summed:

$$\int_{x_0}^{x_N} f(x)dx \approx \frac{h}{3}[f(x_0) + f(x_N) + \sum_{i=odd} 4f(x_i) + \sum_{i=even} 2f(x_i)]. \tag{5}$$

The resultant diffraction pattern can be described as either near field or far field dependant upon the Fresnel number of the diffraction, described as:

$$F = \frac{a^2}{z\lambda} \tag{6}$$

where F is the Fresnel number, a the characteristic parameter (width) of the aperture, z the distance to the screen and λ the wavelength. Near field effects are well described by Fresnel diffraction and observable for Fresnel numbers $F \gg 1$. By altering the parameters of Eqn.6 to give Fresnel numbers $F \ll 1$, far field effects are observed, described by Fraunhofer diffraction.

RESULTS AND DISCUSSION

The effect on diffraction pattern by varying the distance of the aperture to the screen, z, was investigated in both 1 and 2 dimensions for light of wavelength 1×10^{-6}m. The aperture parameters were held constant at an x width of 2×10^{-5}m in 1 dimension and additionally a y width of 2×10^{-5} for the 2 dimensions.

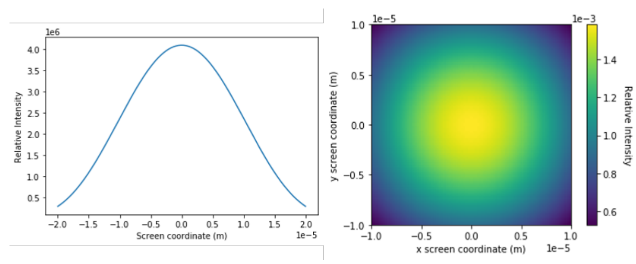

FIG. 1. z=5×10^{-4}m, Fresnel number=0.8

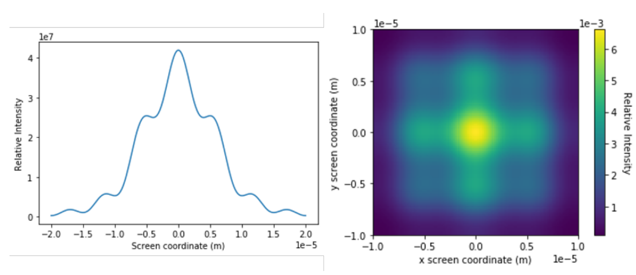

FIG. 2. z=1×10^{-4}m, Fresnel number=4

As can be seen in Figure 1. the Fresnel number is less than 1 and so the diffraction produced is that of a far field effect. The central peak spans a region on the screen greater than that of the aperture size. By taking increasingly smaller distances to the screen in Figures 2-6. the near field effect is observed by

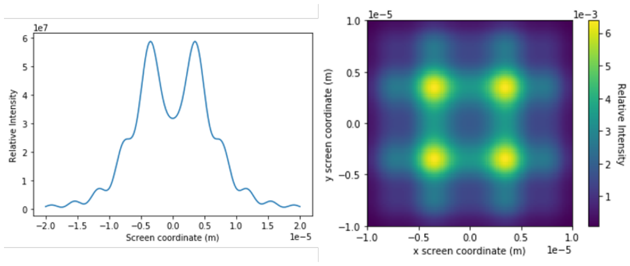

FIG. 3. z=7 × 10^{-5}m, Fresnel number=6

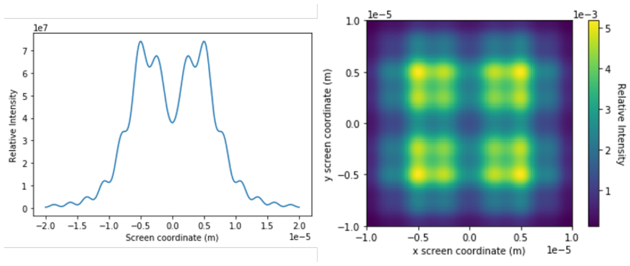

FIG. 4. z=5 × 10^{-5}m, Fresnel number=8

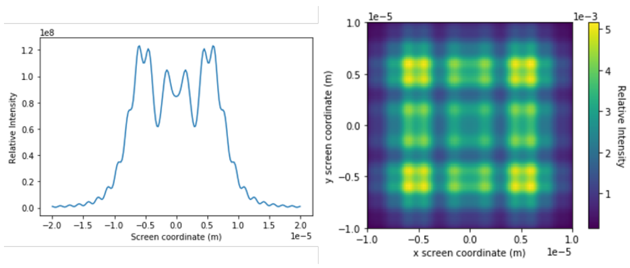

FIG. 5. z=3 × 10^{-5}m, Fresnel number=13

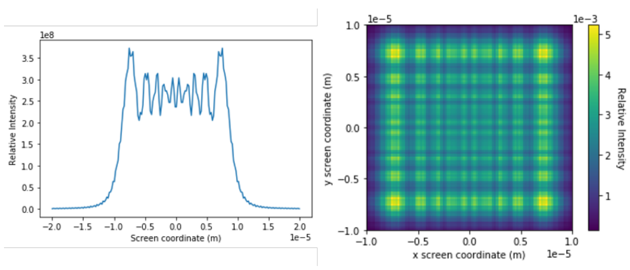

FIG. 6. z=1 × 10^{-5}m, Fresnel number=40

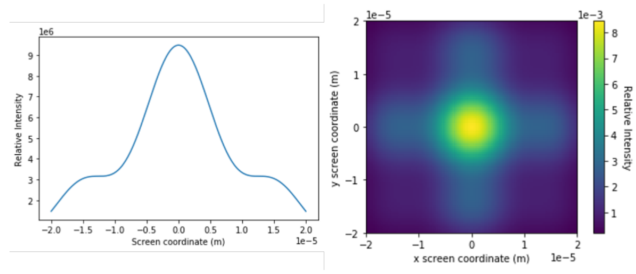

FIG. 7. width=4 × 10^{-5}m, Fresnel number=3.2

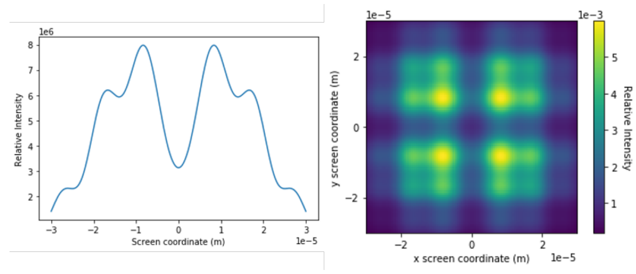

FIG. 8. width=6 × 10^{-5}m, Fresnel number=7.2

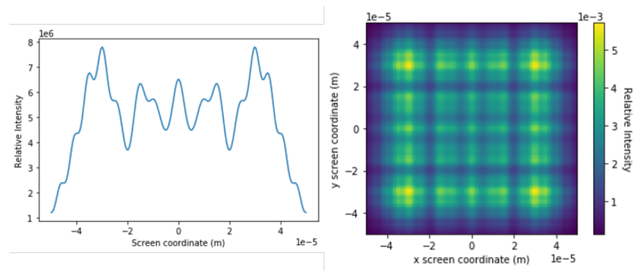

FIG. 9. width=1 × 10^{-4}m, Fresnel number=20

the characteristic gradual splitting of the smooth central peak into a set of thinner peaks. The number of peaks increases alongside the Fresnel number, with maxima in intensity found at the edges in 1 dimension, or corners in 2 dimensions. Furthermore for near field diffraction it can be seen that in contrast to the far field, the region of the diffraction pattern aligns with the size of the aperture. A subsequent investigation was carried out to determine whether this would hold true for light of wavelength 1×10^{-6}m passing through varying aperture sizes at a distance z=5 × 10^{-4}m from the screen.

As can be seen in Figures 1. and 7-9. increasing the aperture width increases the Fresnel number, as expected by Eqn 6. leading to a change in far field to near field diffraction patterns. Furthermore, the investigation confirmed through Figures 7-9. the previous observation that the near field diffraction patterns span a range corresponding to that of the aperture size. Further evidence of this effect was found by taking the near field diffraction pattern of a rectangular aperture, x width 4×10^{-5}m, y width 2×10^{-5}m at distance 1×10^{-5}m in Fig 10. and observing that the resultant rectangular diffraction pattern corresponded to the size of the aperture.

A further investigation sought to examine whether the rectangular aperture could be appropriately parameterised to give near field diffraction in 1 dimension and far field diffraction in a 2nd dimension simultaneously. Light of wavelength 1×10^{-6}m was passed through an aperture of x width 8×10^{-5}m, y width 2×10^{-5}m at a screen distance 5×10^{-4}m.

As can be seen from Fig 11. the resultant diffraction pattern was both near field in x and far field in y. Once more the near field diffraction in x corresponded to the width of the aperture whilst the far field diffraction in y exceeded the aperture width.

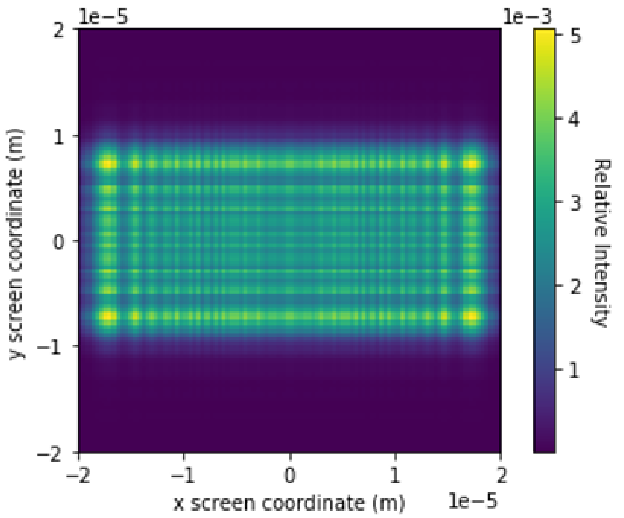

FIG. 10. x width=4×10^{-5}m, y width=2×10^{-5}m, x Fresnel number=160, y Fresnel number=40

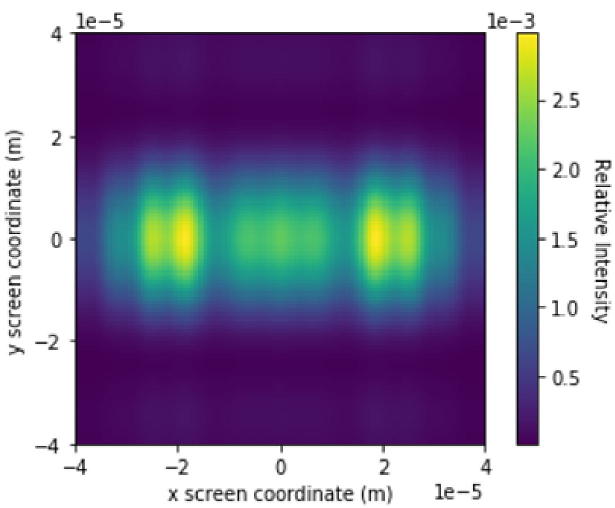

FIG. 11. x width=8×10^{-5}m, y width=2×10^{-5}m, x Fresnel number=12.8, y Fresnel number=0.8

The accuracy of simpson's integration is dependant upon the number of intervals, N, taken. An investigation sought to examine the effect of changing the number of intervals for light of wavelength 1×10^{-6}m passing through an aperture of width 2×10^{-5}m at a distance 1×10^{-5}m to the screen.

As can be seen in Fig 12. the accuracy of simpson's integration for low intervals is poor. Increasing the number of intervals to a moderate amount such as 100 as in Fig 13. leads to an an accurate approximation for the central peaks, but a retention of error on the region beyond the central peaks. Increasing the number further still in Fig 14. and Fig 15. leads to a decline in the error beyond the central peaks and thus an improvement on the accuracy of the approximation.

A further and final investigation sought to examine the accuracy of the simpsons integration in comparison to the trapezoid integration. Light of wavelength 1×10^{-6}m was passed through an aperture of width 2×10^{-5}m at a distance 1×10^{-5}m to the screen with varying integration intervals.

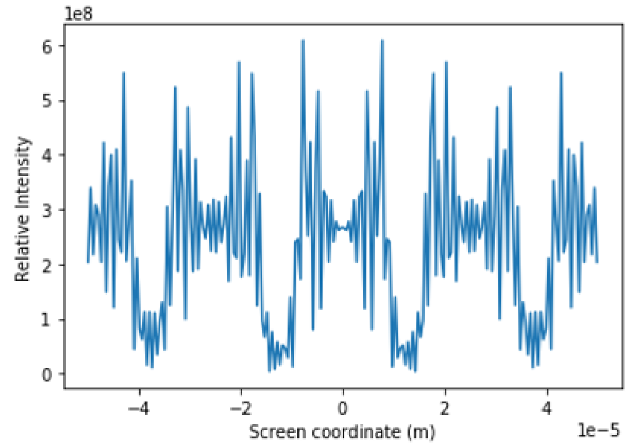

FIG. 12. N=50

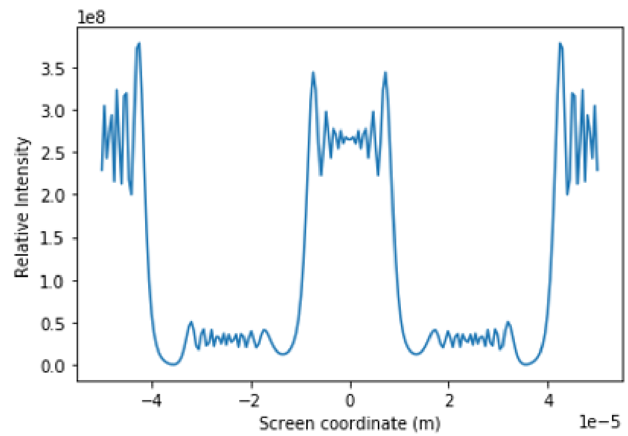

FIG. 13. N=100

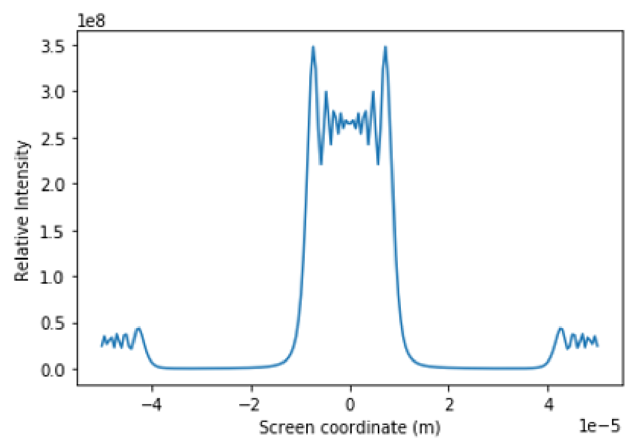

FIG. 14. N=200

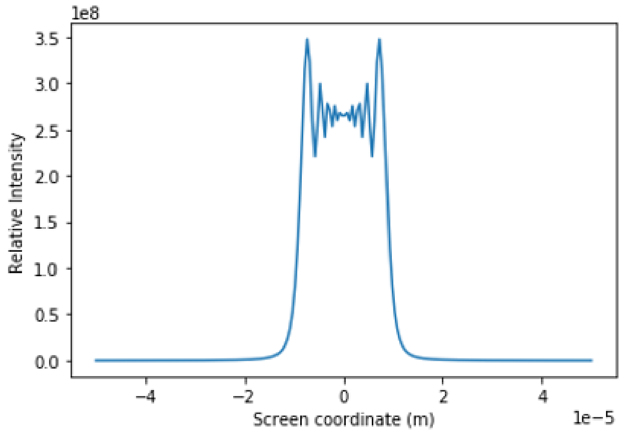

FIG. 15. N=1000

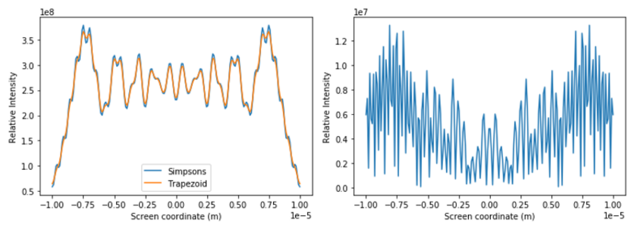

FIG. 16. N=1000

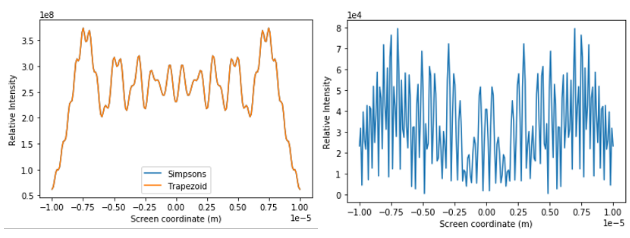

FIG. 17. N=1000

As can be seen in Fig 16. the two integration methods produced similar diffraction patterns as expected. The main deviation for the two methods, as can be seen from the plot of the differences, come from the outer peaks of the diffraction pattern. Increasing the number of intervals as in Fig 17. shows clearly that the difference between the two methods decreases significantly in magnitude. A final observation is that the simpson's integration produced a more accurate approximation than the trapezoid integration at the lower interval number in Fig 16. This is particularly noticeable at the aforementioned outer fringes. The accurate approximation at N=1000 in Fig 17. more closely aligns with the simpson's approximation at N=100 in Fig 16. than the trapezoid approximation. Thus it can be confirmed that the simpsons integration converges to the true value of the integral faster than the trapezoid integration.