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Midterm

1.

(a) $(4),(9)$: Linear Program with new constraints $\begin{pmatrix} (4): x_1 \le 1,\ x_2 \ge 4 \\ (9): x_1 \ge 2,\ x_2 \ge 4 \end{pmatrix}$ has no feasible solution.

$(3),(7)$: Find an integer solution set.

(8): $Z = 8$ is smaller than $(3)\ Z = 13$ $\Rightarrow \zeta \le \zeta_{cur\_lower}$

(b) $\max Z = x_1 + 4x_2$ $\Rightarrow$ objective function

Decision variables: x_1, x_2.

s.t.

$$\left.\begin{aligned} 5x_1 + 8x_2 &\le 40 \\ -2x_1 + 3x_2 &\le 9 \\ x_1 &\ge 2 \\ x_2 &\le 3 \end{aligned}\right\}\ \text{constraints}$$

$x_1, x_2 \ge 0$

2.

$x_1 + x_2 - M \cdot y_1 \le 5$

$x_3 - x_4 - M \cdot y_2 \le 4$

$y_1 + y_2 = 1$

$x_j \ge 0$, for $j = 1, 2$

$y_i = 0$ or 1, for $i = 1, 2$

M: large positive number

y_i: whether or not $\underline{\sum_j a_{ij} x_j \le b_i}$ is used, for $i = 1, 2$.

$\Rightarrow x_1 + x_2 \le 5$

$x_3 - x_4 \le 4$

2.(a)

$$\min \sum_{i\in V}\sum_{j\in V} c_{ij}x_{ij}$$

s.t.

$$\sum_{i\in\{V\setminus j\}} x_{ij}=1,\ \forall j\in V$$

① $$\sum_{j\in\{V\setminus i\}} x_{ij}=1,\ \forall i\in V$$

$$u_i-u_j+N\cdot x_{ij}\le N-1,\ \forall i,j\in\{V\setminus N\} \text{ and } i\neq j$$

$$u_i\ge 0,\ \forall i\in\{V\setminus N\}$$

$$x_{ij} \text{ binary},\ \forall i,j\in V$$

$V=\{1,2,3,4\}$ the set of all cities $\left(\begin{matrix} A=1 & C=3\\ B=2 & D=4\end{matrix}\right)$

c_{ij}: the cost needed to go from city i to j

x_{ij} $\begin{cases}1, & \text{if the salesman goes from city } i \text{ to } j\\ 0, & \text{otherwise.}\end{cases}$

$$C=\begin{bmatrix} - & 10 & 12 & 20\\ 10 & - & 17 & 15\\ 12 & 17 & - & 11\\ 20 & 15 & 11 & -\end{bmatrix}$$

② $D\to B\to A\to C\to D$

③ the final cost is 48 #

(b)

$$\min \sum_{(i,j)\in U} c_{ij}x_{ij}$$

s.t. $$\sum_{i\in\{V\setminus j\}} x_{ij}=1,\ \forall (i,j)\in U$$

① $$\sum_{i\in\{V\setminus i\}} x_{ij}=1,\ \forall (i,j)\in U$$

$$u_i-u_j+N\cdot x_{ij}\le N-1,\ \forall i,j\in\{V\setminus 1\} \text{ and } i\neq j \text{ and } (i,j)\in U$$

$$u_i\ge 0,\ \forall i\in\{V\setminus 1\} \text{ and } (i,j)\in U$$

$$x_{ij} \text{ binary},\ \forall i,j\in V \text{ and } (i,j)\in U$$

$V=\{1,2,3,4,5,6,7\}$: the set of all cities ($a=1, b=2, \cdots, g=7$)

c_{ij}: the cost needed to go from city i to j

x_{ij} $\begin{cases}1, & \text{if the salesman goes from city } i \text{ to } j\\ 0, & \text{otherwise}\end{cases}$

② $a\to b\to d\to f\to g\to e\to c\to a$

③ the final cost is 63

$$C=\begin{bmatrix} - & 12 & 10 & - & - & - & 12\\ 12 & - & 8 & 12 & - & - & -\\ 10 & 8 & - & 11 & 3 & - & 9\\ - & 12 & 11 & - & 11 & 10 & -\\ - & - & 3 & 11 & - & 6 & 7\\ - & - & - & 10 & 6 & - & 9\\ 12 & - & 9 & - & 7 & 9 & -\end{bmatrix}$$

$U=\{(i,j) \text{ where } c_{ij} \text{ exists}\}$

4.

① Denote the set of all cities by V and let $|U|$ denote the number of cities in any subset $U \subset V$

$\sum_{i \in U, j \in U} x_{ij} \le |U|-1$, for all U with $2 \le |U| \le |V|-2$

a. eliminates the subtour solution

$|V| = 5$ and $2 \le |U| \le |V|-2 = 3$

if $|U| = 2$; $x_{23} + x_{32} \le |U|-1 = 1$

b. allows the tour solution

S_{12543}: Tour $1 \to 2 \to 5 \to 4 \to 3$

$S_{12543} > |V|-2 = 3$, tour solution is not eliminated by the constraints.

② $u_i - u_j + N \cdot x_{ij} \le N-1$, $\forall i,j \in \{V \backslash N\}$ and $i \neq j$

$\underset{\text{home city}}{N}$

$u_i \ge 0$, $\forall i \in \{V \backslash N\}$

a. eliminates the subtour solution

Assume $2 \to 3 \to 2$ forms a subtour S_{23}

$u_2 - u_3 + 5x_{23} \le 5-1 = 4$

$u_3 - u_2 + 5x_{32} \le 4$

By summing the above equations together:

$5 \cdot 2 \le (5-1) \cdot 2$ $\Rightarrow$ contradictory

S_{23} will never be formed with these constraints in place $\Rightarrow$ S_{23} is eliminated.

b. allows the tour solution

Assume $4 \to 3 \to 1 \to 2$ forms a subtour S_{4312}

$$u_4 - u_3 + 5x_{43} \le 5-1 = 4$$
$$u_3 - u_1 + 5x_{31} \le 4$$
$$+) \quad u_1 - u_2 + 5x_{12} \le 4$$

$u_4 - u_2 + 5\cdot 3 \le 4\cdot 3 \Rightarrow$ there exist u_4, u_2 to satisfy $u_4 - u_2 \le -3$

5. (a)

① u_i is fixed at non-negative multiplier

② We assume that the constraints of the original problem (P) have been partitioned into several sets so that (P) is relatively easy to solve if the constraint set $\left(\sum_{j=1}^{N} x_{ij} = 1, i=1,\dots,N\right)$ is removed.

(b) $$Z_d(u) = \min \sum_{i=1}^{N}\sum_{j=1}^{N}\{d_{ij} + u_i\}x_{ij} - \sum_{i=1}^{N} u_i$$

s.t.

$$x_{ij} \le x_{jj}, \quad i,j = 1,\dots,N$$

$$\sum_{j=1}^{N} x_{jj} \le K$$

$$x_{ij} \in \{0,1\}, \quad i,j = 1,\dots,N$$

where $u^{k+1} = \max\{0, u^k - t_k(b - Ax^k)\}$

$t_k = \dfrac{\lambda_k (Z_D(u^k) - Z^*)}{\sum_{i=1}^{m}(b_i - \sum_{j=1}^{n} a_{ij}x_j^k)^2}$, λ_k is chosen to be a scalar between 0 and 2

Z^* is initially set to 0 and later updated by a solution from $Z_D(u^k)$ that is also feasible to original problem.

(c) if $1 - \sum_{j=1}^{N} x_{ij}^k > 0$

$\Rightarrow$ we try to make $u_i^{k+1} < 0$

$\Rightarrow$ we want $t_k > 0$

if $1 - \sum_{j=1}^{N} x_{ij}^k < 0$

$\Rightarrow$ we try to make $u_i^{k+1} > 0 \Rightarrow$ We want $t_k > 0$

$\therefore t_k$ will be positive #