HW4-2

$$\min z = 2x_1 + 3x_2 + 4x_3$$

s.t.
$$x_1 - x_2 + x_3 \geq 10$$
$$x_1 - 2x_2 + 3x_3 \geq 6$$
$$3x_1 - 4x_2 + 5x_3 \geq 15$$
$$x_1, x_2, x_3 \geq 0$$

$$\max\{-z\} = -2x_1 - 3x_2 - 4x_3$$

s.t. $s_1 = -10 + x_1 - x_2 + x_3$ ⇒ pivot s_1, $\min\{\frac{-2}{-1}, \frac{-4}{-1}\}$ ↓
$$s_2 = -6 + x_1 - 2x_2 + 3x_3$$
$$s_3 = -15 + 3x_1 - 4x_2 + 5x_3$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Dual simplex

$$\max\{-z\} = -20 - 2s_1 - 5x_2 - 2x_3$$

s.t.
$$x_1 = 10 + s_1 + x_2 - x_3$$
$$s_2 = 4 + s_1 - x_2 + 2x_3$$
$$s_3 = 15 + 3s_1 - x_2 + 2x_3$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$
$(s_1, x_2, x_3) = (0,0,0)$ is feasible

optimal solution:
$$x_1 = 10$$
$$x_2 = 0$$
$$x_3 = 0$$
$$s_1 = 0$$
$$s_2 = 4$$
$$s_3 = 15$$
$$z_S = \min\{z\} = 20$$ #

$$\max\{-z\} = -2x_1 - 3x_2 - 4x_3$$

s.t.
$$-x_1 + x_2 - x_3 \leq -10$$
$$-x_1 + 2x_2 - 3x_3 \leq -6$$
$$-3x_1 + 4x_2 - 5x_3 \leq -15$$
$$x_1, x_2, x_3 \geq 0$$

$$y_1(-x_1 + x_2 - x_3) + y_2(-x_1 + 2x_2 - 3x_3) + y_3(-3x_1 + 4x_2 - 5x_3) \leq -10y_1 - 6y_2 - 15y_3$$

$$\Rightarrow (-y_1 - y_2 - 3y_3)x_1 + (y_1 + 2y_2 + 4y_3)x_2 + (-y_1 - 3y_2 - 5y_3)x_3 \leq -10y_1 - 6y_2 - 15y_3$$

$$\min z_d = -10y_1 - 6y_2 - 15y_3$$

s.t.
$$-y_1 - y_2 - 3y_3 \geq -2$$
$$y_1 + 2y_2 + 4y_3 \geq -3$$
$$-y_1 - 3y_2 - 5y_3 \geq -4$$
$$y_1, y_2, y_3 \geq 0$$