

HW 3-4

$$\max Z = -x_1 + 3x_2 - 3x_3$$

s.t.

$$s_1 = 7 - 3x_1 + x_2 + 2x_3$$

$$s_2 = 3 + 2x_1 + 4x_2 - 4x_3$$

$$s_3 = 4 - x_1 + 2x_3$$

$$s_4 = 8 + 2x_1 - 2x_2 - x_3 \Rightarrow x_2 \leq 4 \Rightarrow x_2 = 4 + x_1 - \frac{1}{2}x_3 - \frac{1}{2}s_4$$

$$s_5 = 5 - 3x_1$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

pivot on x_2

$$(x_1, x_2, x_3, s_1, s_2, s_3, s_4, s_5) = (0, 0, 0, 7, 3, 4, 8, 5)$$

$$\Rightarrow \max Z = 12 + 2x_1 - \frac{9}{2}x_3 - \frac{3}{2}s_4$$

$$s.t. \quad s_1 = 11 - 2x_1 + \frac{3}{2}x_3 - \frac{1}{2}s_4 \Rightarrow x_1 \leq \frac{11}{2}$$

$$s_2 = 19 + 6x_1 - 6x_3 - 2s_4$$

$$s_3 = 4 - x_1 + 2x_3$$

$$\Rightarrow x_1 \leq 4$$

$$x_2 = 4 + x_1 - \frac{1}{2}x_3 - \frac{1}{2}s_4$$

$$s_5 = 5 - 3x_1$$

$$x_1, x_2, x_3, s_1, s_2, s_3, s_4, s_5 \geq 0$$

$$\Rightarrow \underline{x_1 \leq \frac{5}{3}} \Rightarrow x_1 = \frac{5}{3} - \frac{1}{3}s_5$$

$$\Rightarrow \max Z = \frac{46}{3} - \frac{2}{3}s_5 - \frac{9}{2}x_3 - \frac{3}{2}s_4$$

$$s.t. \quad s_1 = \frac{23}{3} + \frac{2}{3}s_5 + \frac{3}{2}x_3 - \frac{1}{2}s_4$$

$$s_2 = 29 - 2s_5 - 6x_3 - 2s_4$$

$$s_3 = \frac{7}{3} + \frac{1}{3}s_5 + 2x_3$$

$$x_2 = \frac{17}{3} - \frac{1}{3}s_5 - \frac{1}{2}x_3 - \frac{1}{2}s_4$$

$$x_1 = \frac{5}{3} - \frac{1}{3}s_5$$

$$x_1, x_2, x_3, s_1, s_2, s_3, s_4, s_5 \geq 0$$

$$\therefore \max Z = \frac{46}{3}$$

$$(x_1, x_2, x_3, s_1, s_2, s_3, s_4, s_5) = \left(\frac{5}{3}, \frac{17}{3}, 0, \frac{23}{3}, 29, \frac{7}{3}, 0, 0\right)$$

HW3-5

$$\max \{-x_0\}, \max z = 2x_1 + x_2$$

pivot on x_0

\Rightarrow s.t.

$$s_1 = x_0 + 10 - x_1 - x_2$$

$$s_2 = x_0 - 2 - x_1 + x_2 \Rightarrow x_0 = 2 + s_2 + x_1 - x_2$$

$$x_0, x_1, x_2, s_1, s_2 \geq 0$$

$$\Rightarrow \max \{-2 - s_2 - x_1 + x_2\}, \max z = 2x_1 + x_2$$

pivot on x_2

s.t.

$$s_1 = 12 + s_2 - 2x_2$$

$$x_0 = 2 + s_2 + x_1 - x_2$$

$$\Rightarrow x_2 = 2 - x_0 + x_1 + s_2$$

$$x_0, x_1, x_2, s_1, s_2 \geq 0$$

$$\Rightarrow \max \{-x_0\}, \max z = 2 - x_0 + 3x_1 + s_2$$

$$\text{s.t. } s_1 = 8 - x_0 - 2x_1 - s_2$$

$$x_2 = 2 + x_0 + x_1 + s_2$$

$$x_0, x_1, x_2, s_1, s_2 \geq 0$$

$$\therefore \max \{-x_0\} = 0$$

set $x_0 = 0$

$$\max z = 2 + 3x_1 + s_2$$

s.t.

$$s_1 = 8 - 2x_1 - s_2$$

$$x_2 = 2 + x_1 + s_2$$

$$x_1, x_2, s_1, s_2 \geq 0$$

pivot on x_1

$$\Rightarrow x_1 = 4 - \frac{1}{2}s_1 - \frac{1}{2}s_2$$

$$(x_1, x_2, s_1, s_2) = (0, 2, 8, 0)$$

$$\Rightarrow \max z = 14 - \frac{3}{2}s_1 - \frac{5}{2}s_2$$

$$\text{s.t. } x_1 = 4 - \frac{1}{2}s_1 - \frac{1}{2}s_2$$

$$x_2 = 6 - \frac{1}{2}s_1 + \frac{1}{2}s_2$$

$$x_1, x_2, s_1, s_2 \geq 0$$

$$\therefore \max z = 14$$

$$(x_1, x_2, s_1, s_2)$$

$$= (4, 6, 0, 0)$$