

HW4

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HW4-1

1. What is the optimal solution?

x1: 3.000000

x2: 1.500000

Zs = 21.000000

2. What are the shadow prices and reduced costs?

reduced costs:

[0.0, 0.0]

shadow prices:

[0.75, 0.5, 0.0, 0.0]

3. What will be the objective function at the optimal state?

$Z = 21 - 0.75 s_1 - 0.5 s_2$

HW4-2

HW4-2

$$\min Z = 2x_1 + 3x_2 + 4x_3$$

$$\begin{aligned} \text{s.t.} \quad & x_1 - x_2 + x_3 \geq 10 \\ & x_1 - 2x_2 + 3x_3 \geq 6 \\ & 3x_1 - 4x_2 + 5x_3 \geq 15 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Dual simplex

$$\Rightarrow \max \{-Z\} = -20 - 2s_1 - 5x_2 - 2x_3$$

$$\begin{aligned} \text{s.t.} \quad & x_1 = 10 + s_1 + x_2 - x_3 \\ & s_2 = 4 + s_1 - x_2 + 2x_3 \\ & s_3 = 15 + 3s_1 - x_2 + 2x_3 \\ & x_1, x_2, x_3, s_1, s_2, s_3 \geq 0 \\ & (s_1, x_2, x_3) = (0, 0, 0) \text{ is feasible} \end{aligned}$$

$$\max \{-Z\} = -2x_1 - 3x_2 - 4x_3$$

$$\begin{aligned} \text{s.t.} \quad & s_1 = -10 + x_1 - x_2 + x_3 \Rightarrow \text{pivot } s_1, \\ & s_2 = -6 + x_1 - 2x_2 + 3x_3 \\ & s_3 = -15 + 3x_1 - 4x_2 + 5x_3 \\ & x_1, x_2, x_3, s_1, s_2, s_3 \geq 0 \end{aligned}$$

Optimal solution:

$$\begin{aligned} & x_1 = 10 \\ & x_2 = 0 \\ & x_3 = 0 \\ & s_1 = 0 \\ & s_2 = 4 \\ & s_3 = 15 \\ & Z_s = \min \{Z\} = 20 \end{aligned}$$

Q: Is it appropriate to use the (Primal) Simplex Method? Can you use the 2-Phase Method?

It's not appropriate to use the (Primal) Simplex Method. If we change $\min\{z\}$ to $\max\{-z\}$, we will get all $b_i < 0$. To tackle this, we need to use the 2-phase Method.

Q: Would the Dual Simplex Method work?

As the handwritten sheet above, Dual Simplex Method is feasible.

Q: What if you convert the problem into its dual problem?

$$\begin{aligned}
 \max \{-z\} &= -2x_1 - 3x_2 - 4x_3 \\
 \text{s.t. } & -x_1 + x_2 - x_3 \leq -10 \\
 & -x_1 + 2x_2 - 3x_3 \leq -6 \\
 & -3x_1 + 4x_2 - 5x_3 \leq -15 \\
 & x_1, x_2, x_3 \geq 0
 \end{aligned}$$

$$\Rightarrow \begin{aligned}
 & y_1(-x_1 + x_2 - x_3) + y_2(-x_1 + 2x_2 - 3x_3) \\
 & + y_3(-3x_1 + 4x_2 - 5x_3) \leq -10y_1 - 6y_2 - 15y_3
 \end{aligned}$$

$$\Rightarrow \begin{aligned}
 & (-y_1 - y_2 - 3y_3)x_1 + (y_1 + 2y_2 + 4y_3)x_2 \\
 & + (-y_1 - 3y_2 - 5y_3)x_3 \leq -10y_1 - 6y_2 - 15y_3
 \end{aligned}$$

$$\Rightarrow \begin{aligned}
 \min z_d &= -10y_1 - 6y_2 - 15y_3 \\
 \text{s.t. } & -y_1 - y_2 - 3y_3 \geq -2 \\
 & y_1 + 2y_2 + 4y_3 \geq -3 \\
 & -y_1 - 3y_2 - 5y_3 \geq -4 \\
 & y_1, y_2, y_3 \geq 0
 \end{aligned}$$

Q: Can you use the Simplex on the Dual problem?

(see next page)

$$\max\{-z\} = 10y_1 + 6y_2 + 15y_3$$

s.t.

$$s_1 = 2 - y_1 - y_2 - 3y_3$$

$$s_2 = 3 + y_1 + 2y_2 + 4y_3$$

$$s_3 = 4 - y_1 - 3y_2 - 5y_3$$

$$s_1, s_2, s_3, y_1, y_2, y_3 \geq 0$$

\therefore all $b_i > 0$

\therefore we can use simplex method to solve it

$$\Rightarrow \max\{-z\} = 20 - 10s_1 - 4y_2 - 15y_3$$

$$\therefore \min z_d = 20 \quad \#$$

$$y_1 = 2 - s_1 - y_2 - 3y_3$$

$$s_2 = 5 - s_1 + y_2 + y_3$$

$$s_3 = 2 + s_1 - 2y_2 - 2y_3$$

$$y_1, y_2, y_3, s_1, s_2, s_3 \geq 0$$