

Week 6 A10727 0012 陳柏瑋

9.

$$\begin{aligned} (1) \quad S &= \int \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \int \frac{\sum X_i^2 - n\bar{X}^2}{n-1} \\ &= \int \frac{1284 - 6 \times 14.33^2}{5} = \int 10.30 \end{aligned}$$

$$(2) \quad 1 - \alpha = 0.9 \quad \frac{\alpha}{2} = 0.05 \quad n-1 = 5$$

$$\chi^2_{\frac{\alpha}{2}}(n-1) = \chi^2_{0.05}(5) = 11.07$$

$$\chi^2_{1-\alpha}(n-1) = \chi^2_{0.95}(5) = 1.15$$

$$\left( \int \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}, \int \frac{(n-1)S^2}{\chi^2_{1-\alpha}(n-1)} \right) = \left( \int \frac{5 \times 10.38}{11.07}, \int \frac{5 \times 10.38}{1.15} \right) = (2.17, 0.72)$$

20. (1)

$$V = \frac{\left( \frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right)^2}{\frac{\left( \frac{S_1^2}{n_1} \right)^2}{(n_1-1)} + \frac{\left( \frac{S_2^2}{n_2} \right)^2}{(n_2-1)}}$$

$$\Rightarrow \begin{aligned} n_1 &= 9, \bar{x} = 7.67, S_1 = 9.27 \\ n_2 &= 9, \bar{y} = 6.78, S_2 = 21.15 \end{aligned}$$

$$V = \frac{\left( \frac{9.27^2}{9} + \frac{21.15^2}{9} \right)^2}{\frac{\left( \frac{9.27^2}{9} \right)^2}{8} + \frac{\left( \frac{21.15^2}{9} \right)^2}{8}} = 10.96 \approx 11$$

$$\bar{x} - \bar{y} \pm t_{\frac{\alpha}{2}}(V) \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = (7.67 - 6.78) \pm t_{0.05}(11) \sqrt{\frac{9.27^2}{9} + \frac{21.15^2}{9}}$$

$$(2) \quad \left( \int \frac{8 \times 9.27^2}{\chi^2_{0.05}(8)}, \int \frac{8 \times 9.27^2}{\chi^2_{0.95}(8)} \right) = \left( \int \frac{687.46}{15.51}, \int \frac{687.46}{2.73} \right) = (0.89 \pm 16.95) = (6.66, 15.87)$$

$$(3) \quad \left( \frac{S_1^2}{S_2^2} \times \frac{1}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)}, \frac{S_1^2}{S_2^2} \times \frac{1}{1 - F_{\frac{\alpha}{2}}(n_1-1, n_2-1)} \right)$$

$$= \left( \frac{9.27^2}{21.15^2} \times \frac{1}{3.44}, \frac{9.27^2}{21.15^2} \times \frac{1}{\frac{1}{3.44}} \right) = (0.06, 0.66)$$