

企望 = 甲 A10729m12 詳抄

例 6.4

$$\text{依 } E(X_i) = \mu \quad V(X_i) = \sigma^2 = E(X_i^2) - \mu^2$$

$$\text{則 } E(\bar{x}) = \mu \quad V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left[\frac{\sum_{i=1}^n (X_i - \bar{x})}{n}\right] = \frac{1}{n} \left(\sum_{i=1}^n X_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \sigma^2 - \frac{\sigma^2}{n}$$

$$= \frac{n-1}{n} \sigma^2 \Rightarrow E(\hat{\theta}_1) \text{ 不滿足 } \sigma^2 \text{ 之 不偏估計量 } \Rightarrow \text{ 偏誤估計量.}$$

$$E(\hat{\theta}_2) = E\left[\frac{\sum_{i=1}^n (X_i - \bar{x})^2}{n-1}\right] = \frac{1}{n-1} E\left[\sum_{i=1}^n (X_i - \bar{x})^2\right]$$

$$= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \left(\frac{n\sigma^2}{n-1} + \frac{n\mu^2}{n-1} - \frac{\sigma^2}{n-1} - \frac{n\mu^2}{n-1}\right)$$

$$= \frac{n\sigma^2}{n-1} - \frac{\sigma^2}{n-1} = \sigma^2 \Rightarrow E(\hat{\theta}_2) \text{ 滿足 } \sigma^2 \text{ 之 不偏估計量 } \Rightarrow \text{ 不偏估計量.}$$