

# TREVR: Tree-based Reverse Ray Tracing in Gasoline

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## ABSTRACT

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## 1 INTRODUCTION

Radiation is arguably the most important physical phenomena to the field of astrophysics. Almost all of the information we receive from outer space comes in the form of photons we detect on or around earth. Understanding the process of radiative transfer (RT) is key in interpreting this information, as the photons are affected by the medium they travel through on the way to our telescopes and detectors. Interactions between photons and the medium along their paths do not only affect the photons themselves, but the matter as well. These interactions with baryons exchange energy and momentum, and also affect the excitation and ionization states of said baryons, thus determining the chemical and thermodynamic properties of the matter. This in turn makes radiation a driving factor in many of the physical processes we study.

It may then come as a surprise to some that RT is treated rather poorly in most astrophysical simulations, usually as some imposed uniform background. This is not because of carelessness or lack of effort, but because RT is inherently a complicated problem. The full RT problem depends on 3 spatial dimensions, 2 angular dimensions, frequency and time, and has a characteristic speed of  $c$ , the speed of light. Also, unlike a force at a distance problem such as gravity, RT depends on the intervening material. Because of this complexity, a naive numerical solution for RT would scale with the number resolution elements  $N$  like  $\mathcal{O}(N^2)$ . This makes it impractical to run such an RT method alongside modern gravity and hydrodynamics codes that scale like  $\mathcal{O}(N \log N)$ .

When it comes to solving RT problem, we can separate methods into two different categories by looking at the classical RT equation (Mihalas & Mihalas 1984),

$$\left[ \frac{1}{c} \frac{\partial}{\partial t} + \hat{\mathbf{n}} \cdot \nabla \right] I(\mathbf{x}, \hat{\mathbf{n}}, t, \nu) = \epsilon(\mathbf{x}, \hat{\mathbf{n}}, t, \nu) - \alpha(\mathbf{x}, \hat{\mathbf{n}}, t, \nu) I(\mathbf{x}, \hat{\mathbf{n}}, t, \nu), \quad (1)$$

where  $I$ ,  $\epsilon$  and  $\alpha$  are the intensity, emissivity and extinction coefficient respectively which all depend on position  $\mathbf{x}$ , unit direction of light propagation  $\hat{\mathbf{n}}$ , time  $t$  and frequency  $\nu$ . The

difference comes from how different methods treat the speed of light  $c$ . For methods that use a finite  $c$ , which is often a reduced speed of light, the partial time derivative remains in equation 1 and the radiation field is advected or “evolved”. Methods that solve the RT equation in this way, which we will call evolutionary methods, include moment methods like OTVET (Gnedin & Abel 2001) and RAMESE-RT (Rosdahl & Teyssier 2015) as well as photon packet propagation methods like TRAPHIC (Pawlik & Schaye 2008), SPHRAY (Altay et al. 2008) and SimpleX2 (Paardekooper et al. 2010). On the other hand, in limit where  $c$  is taken to be infinite, the partial time derivative in equation 1 goes to zero and the radiation field can be computed instantaneously. Methods that solve the RT equation in this way, which we will refer to as instantaneous methods, include forward ray tracers such as C<sup>2</sup>Ray (Mellema et al. 2006), Moray (Wise & Abel 2011) and Fervent (Baczynski et al. 2015) as well as reverse ray tracers such as TreeCol (Clark et al. 2012) and URCHIN (Altay & Theuns 2013).

All of these methods require simplifications to the full RT problem to make them feasible, thus they will all have areas of strength and weakness. These strengths and weaknesses define what types of problems they can solve. This makes comparing these methods on an equal footing somewhat confusing, as some authors state how their method scales with resolution elements in the context of the problem their RT method is meant to solve. For example, Clark et al. (2012) state that their reverse ray tracer TreeCol scales with resolution elements like  $\mathcal{O}(N \log N)$ . The specific problem they are trying to solve is the calculation of column densities via RT from background radiation, and thus the number of sources of radiation they simulate is some small fixed number. They treat the number of sources as a constant factor in their scaling function, and so what is an  $\mathcal{O}(N^2)$  method in the general case looks like a much more feasible method at first glance. For this reason we will be very careful with scaling statements, using the number of radiation sources  $N_{\text{source}}$  and sinks  $N_{\text{sink}}$  where appropriate to make the limitations and possible uses of a given method clear.

Instantaneous methods come in the form of ray tracers. Ray tracers are the most simple, natural way to go about solving the RT problem. Forward ray tracers trace many

rays outward from sources of radiation, similarly to the actual phenomena, in hope that they will intersect resolution elements for which the radiation field will be computed. Each source needs to compute a number of rays comparable to the number of resolution elements to ensure accuracy, meaning forward ray tracers scale with number of resolution elements like  $\mathcal{O}(N_{\text{source}}N_{\text{sink}})$ . This scaling limits forward ray tracers to problems with few sources to avoid  $\mathcal{O}(N^2)$  like scaling. This also rules out the inclusion of scattering in the method as scatterings are treated as re-emission events and thus all sinks would have to be treated as sources as well.

Recently there has been some focus on reverse ray tracing methods (Clark et al. 2012; Altay & Theuns 2013). Reverse ray tracers trace rays from the sink particle directly to the sources. This way of tracing the rays has a couple of benefits over forward ray tracing. Firstly, tracing from the sinks guarantees the density distribution is well sampled as apposed to forward ray tracing where one would have to increase the number of rays per sink to guarantee accuracy. Put simply, radiation is computed exactly where it is needed. This is especially advantageous in smoothed particle hydrodynamics (SPH) simulations, as low density regions are represented by few SPH particles, and thus extra work is not done to resolve said regions. Another benefit is that sub time steps can be used. However, just performing a naive reverse ray trace does not negate  $\mathcal{O}(N_{\text{source}}N_{\text{sink}})$  scaling with resolution elements, and so the inability to model many sources remains the most significant barrier current instantaneous methods face when trying to solve the general RT problem.

Evolutionary methods do not suffer from the linear dependence on number of sources. The main benefit of evolutionary methods is that they have no dependence on the number of sources, and just scale like  $\mathcal{O}(N)$  with the number of resolution elements, allowing them to handle large numbers of sources and scattering. Moment methods are limited to the optically thick diffusive limit. They also lack sharp directionality, resulting in poor shadows behind optically thick objects. This also makes moment methods reliant on the partitioning of space into uniform grids. If implemented in a smooth particle hydrodynamics (SPH) like scheme, the lack of resolution elements in less dense regions would only exacerbate the directionality problem.

Photon packet propagation methods, specifically TRAPHIC (Pawlik & Schaye 2008), perform better in the optically thin regime. TRAPHIC introduces virtual particles (ViPs) to propagate their photon packets in less dense, optically thin regions lacking in SPH particles. They also preserve directionality quite well, however Monte Carlo aspects of how they propagate their photon packets introduce significant noise into their computed radiation field. Monte Carlo resampling is shown to reduce this noise but is quite expensive and deteriorates the initially sharp shadows. Both of these methods scale linearly with resolution elements as mentioned before, but are also forced to operate on every resolution element. In moment methods the radiation field for every grid cell needs to be computed, and in photon packet propagation methods the photon packets hop particle to particle. In the case of TRAPHIC, their  $N$  is even greater than the number of SPH particles including the addition of ViPs. Regardless, TRAPHIC is arguably the best general RT method due to its ability to handle both the optically thick and thin regimes with feasible scaling.

We hope from this introduction to the state of the art in RT methods it is apparent that there is room for improvement. Although promising work has been done with reverse ray tracers like TreeCol, a general implementation of one has yet to be published. There is also the problem of scaling with sources in instantaneous methods. If this could be solved, instantaneous ray tracers could compete with the feasibility and improve upon the accuracy of evolutionary codes like TRAPHIC.

In this paper we introduce TREVR (Tree-based Reverse Ray Tracing), a novel method that hopes to address all of the issues presented in the previous paragraph. TREVR is a reverse ray tracer based on the Barnes & Hut (1986) tree gravity solver. TREVR is implemented in the SPH code Gasoline (Wadsley et al. 2004), but we would like to note that the basic algorithm is not SPH or Gasoline specific, TREVR only requires that the simulation volume be divided hierarchically in space.

A main feature of TREVR is that sources can be merged via an opening angle criteria similarly to the Barnes and Hut gravity method. This means that TREVR scales with sources like  $\mathcal{O}(\log N_{\text{source}})$ , alleviating the linear scaling with sources characteristic of other instantaneous RT methods. Being a reverse ray tracer enables us to easily merge sources, as sources interact with a single sink at a time. This is the first time source merging has been implemented in a reverse ray tracer, and thus TREVR is the first general reverse ray tracer that can feasibly operate on all sources in a simulation and not just a small, fixed amount of background sources as in TreeCol and URCHIN.

TREVR traces rays per each sink because it is a reverse ray tracer, this adds a linear dependence on the number of sinks. This coupled with source merging means TREVR can solve the optically thin case  $\mathcal{O}(N_{\text{sink}} \log N_{\text{source}})$  operations. The fact that TREVR depends on a tree-data structure allows it to trace rays through an optically thick medium by traversing the sub-tree between the sink and source which is at most  $\log N$  tree cells. This results in scaling with resolution elements in the optically thick case that goes like  $\mathcal{O}(N_{\text{sink}} \log N_{\text{source}} \log N)$ . This scaling in both the optically thin and thick case makes TREVR a feasible method as it scales similarly to modern Gravity and Hydro solvers.

In the coming sections we will further explain the general method in section 2 and its implementation in section 3. Tests of scaling with resolution elements and the method's accuracy as a function of opening angle and refinement criteria will be presented in section 4. At the end of section 4 we will do our best to compare TREVR to other methods in common RT tests such as the Strömgren sphere test and dense clump tests used in the Iliev et al. (2006) RT method comparison project. Finally, in section 5 we will discuss the method's merits and drawbacks to conclude where TREVR fits in with other RT methods and what scientific problems it can solve.

## 2 METHOD

## 3 IMPLEMENTATION

Temporarily commented out

#### 4 CODE TESTS

#### 5 DISCUSSION AND CONCLUSION

#### ACKNOWLEDGEMENTS

#### REFERENCES

- Altay G., Theuns T., 2013, [MNRAS](#), **434**, 748
- Altay G., Croft R. A. C., Pelupessy I., 2008, [MNRAS](#), **386**, 1931
- Baczynski C., Glover S. C. O., Klessen R. S., 2015, [MNRAS](#), **454**, 380
- Barnes J., Hut P., 1986, [Nature](#), **324**, 446
- Clark P. C., Glover S. C. O., Klessen R. S., 2012, [MNRAS](#), **420**, 745
- Gnedin N. Y., Abel T., 2001, [New Astron.](#), **6**, 437
- Iliev I. T., et al., 2006, [MNRAS](#), **371**, 1057
- Mellema G., Iliev I. T., Alvarez M. A., Shapiro P. R., 2006, [New Astron.](#), **11**, 374
- Mihalas D., Mihalas B. W., 1984, Foundations of radiation hydrodynamics
- Paardekooper J.-P., Kruip C. J. H., Icke V., 2010, [A&A](#), **515**, A79
- Pawlik A. H., Schaye J., 2008, [MNRAS](#), **389**, 651
- Rosdahl J., Teyssier R., 2015, [MNRAS](#), **449**, 4380
- Wadsley J. W., Stadel J., Quinn T., 2004, [New Astron.](#), **9**, 137
- Wise J. H., Abel T., 2011, [MNRAS](#), **414**, 3458

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