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FAHRT: Fast Adaptive Hierarchical Radia tive Transfer in Gasoline

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ABSTRACT

We present FAHRT (Fast Adaptive Hierarchical Radiative Transfer), a novel algorithm for computing the radiation field in astrophysical simulations. FAHRT prioritizes the ability to deal with a large number of sources and computational speed over accuracy, while keeping equilibrium behaviour correct. The algorithm is based on a *tree* data structure similar to many gravity solvers. This allows for computation of radiative transfer in $O(N_{\rm sink} \log N_{\rm source})$ time without absorption and $O(N_{\rm sink} \log N_{\rm source} \log N)$ time with absorption. Its tree based nature also allows it to scale well with number of processors and be highly tunable in both speed and accuracy. A main feature of FAHRT is the implementation of a refinement criteria based on the worst case optical depth of a tree cell, allowing us to save cost whilst being confident in the accuracy of our solution. The algorithm is also weakly dependent on the energy band of radiation and the number of bands used, allowing for the radiation fields of multiple bands to be computed on the fly with a negligible increase in computational cost. We provide a suite of tests demonstrating the algorithm's ability to accurately compute fluxes, ionization fronts and shadows. We also analyze the algorithm's computational complexity, in how it scales with the number of sources (star particles) and sinks (gas particles). We also examine how the aforementioned refinement criterion's value affects speed and accuracy. Finally, we will discuss strengths and shortcomings of this algorithm and how they constrain the niche of problems it can handle.

Key words: radiative transfer – methods: numerical – galaxies

1 INTRODUCTION

2 METHOD

We now present the FAHRT algorithm. FAHRT prioritizes the ability to deal with a large number of sources over high accuracy, though we still insist equilibrium behaviour be correct. In order to accomplish this, we start by making some simplifying assumptions to the radiative transfer equation

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \sigma_\nu)(I_\nu - S_\nu), \tag{1}$$

where α_ν and σ_ν are the specific absorption and scattering coefficients respectively, S_ν is the combined source function for absorption and scattering, and I_ν is the specific intensity.

We can simplify FAHRT to only include absorption. This still allows us to treat scattering, as it can be expressed as an absorption followed by an emission. For now though, we only account for absorption by treating star particles as emitters and gas particles as absorbers. In the future one could implement scattering by simply treating gas particles as emitters *and* absorbers (more discussion on this in section 4). This reduces equation 1 to

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu, \tag{2}$$

where τ_ν is absorption-only optical depth and is defined as

$$d\tau_\nu = \alpha_\nu ds = \rho \kappa_\nu ds, \tag{3}$$

where ρ is density and κ_ν is specific opacity due to absorption.

It is useful to consider only absorption, as many astrophysical simulations only model a single or few sources. This causes the emission coefficient to be zero at most points. In our case we only consider star particles to be emitters, meaning we have a non emitting medium along a ray. This reduces equation 2 by setting the source function to 0 and setting I_ν to be the initial intensity from a single star particle $I_\nu(0)$

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu}. \tag{4}$$

This now allows us to turn the initial integral over all sources to a sum of diminished contributions from each star particle.

We compute the radiation field as a flux magnitude,

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as this is the quantity most useful to applications of our algorithm. When expressed as a flux 4 becomes,

$$F_\nu = \frac{L_\nu}{4\pi s^2} e^{-\tau_\nu}, \tag{5}$$

where L_ν is the specific luminosity of the star particle and s is the distance between the star particle and the absorbing gas particle. We can then sum this equation over all contributing star particles to find the flux at the gas particle in question. This summation is what FAHRT computes for all gas-type SPH particles in a simulation, thus approximating the radiation field.

Please note that FAHRT has been implemented in the Smoothed Particle Hydrodynamics (SPH) code GASOLINE (Wadsley et al. 2004). However, FAHRT is *not* specific to GASOLINE or SPH. We only require that the simulation volume can be hierarchically partitioned in space.

2.1 The Optically Thin Regime

In the absence of absorbing material, the optical depth is zero and equation 5 becomes just $L/4\pi s^2$. This problem is almost identical to gravity, and so we use the same tree-based technique as gravity to solve it. The tree-based gravity solver of Barnes & Hut (1986) has become commonplace in astrophysical simulations (Hubber et al. 2011; Wadsley et al. 2004; Springel et al. 2001; Vine & Sigurdsson 1998; Benz 1988). Like the Barnes & Hut algorithm our optically thin method should scale with resolution elements like $O(N \log N)$. In our case the N factor represents the number of gas particles ($N_{\rm sink}$), and the $\log N$ factor represents the number of emitting particles ($N_{\rm source}$). This means that for the optically thin case we should see scaling with number of sinks and sources go as $O(N_{\rm sink} \log N_{\rm source})$.

In our GASOLINE implementation we use a binary tree. During the tree build process we can compute useful average properties of tree cells such as total luminosity, centre of luminosity, average density and average opacity (the latter two are used in the optically thick regime). Computing the radiation field is accomplished by traversing the tree structure. Receiving gas particles which live in the leaf nodes of the tree are looped over. An opening angle criterion, just as in gravity, is used to decide on how the gas particles interact with the emitters.

Note that we already have used early versions of this algorithm to investigate the effects of ionizing feedback on gas cooling in galaxies (Kannan et al. 2014).

2.2 The Optically Thick Regime

In the presence of absorbing material along the ray, we need to compute the optical depth along said ray. To do this we traverse the tree from the interacting nodes to their common parent node to build up the optical depth along the ray. This is possible because the tree is partitioned in space, thus all intervening material should be contained in the sub-tree we traverse. Using the average properties computed in the tree build we can compute the optical depth of a piece of the ray using the geometry of the cell and ray, and the average density and opacity

$$\tau_i = \bar{\rho}_i \bar{\kappa}_i s_i. \tag{6}$$

The total optical depth is then summed up during the tree walk,

$$\tau = \sum_i \tau_i, \tag{7}$$

giving us everything needed to evaluate equation 5. Because of this extra tree walk another $\log N$ factor can be added to the scaling equation and so we would expect the scaling with resolution elements to now look like $O(N_{\rm sink} \log N_{\rm source} \log N)$. This algorithm is depicted in the left half of figure 1

Since we are calculating the radiation field at the receiving cell we are doing a process similar to a reverse ray trace, like URCHIN (Altay & Theuns 2013). One advantage of reverse ray tracing is that rays are associated with with sinks rather than the source. Dense regions near the sink and source are therefore automatically well sampled and radiative transfer is computed exactly where it needs to be. Another benefit is that the simulation can make use of sub time steps. The problem with this simple method is that as the tree is traversed upwards the volume elements become larger and accuracy can be lost. That makes this algorithm very efficient at computing the radiation field for uniform density and opacity distributions, but highly inaccurate when dealing with sharp density and opacity gradients along the ray. To handle these situations a method of refinement is needed.

2.3 Refinement

Refinement is a straightforward addition to the algorithm. At a point in the tree walk where the average properties of the cell would be considered, we check to see if the current cell passes some refinement criteria. If the cell passes the criteria to refine, rather than using the average properties we recursively check the cell's children until the criteria fails building a better resolved section of the ray. This addition to the algorithm is depicted in the right half of figure 1

Difficulty comes in choosing a refinement criteria that is both accurate and efficient. Ideally, the criteria should be true when an average optical depth in a region may not be accurate to the true distribution, such as a clumpy medium where the average opacity is much higher than the "effective" opacity (Hegmann & Kegel 2003; Városi & Dwek 1999).

Our choice of refinement criteria is based on optical depth, and is unique to the FAHRT algorithm. Consider two rays through a large cell (see figure 2, note that this description is simplified to 2D). These rays represent what the case would be if the properties of the children were used instead of the parent cell. We can calculate the minimum and maximum absorption coefficients $\alpha_{\rm min}$ and $\alpha_{\rm max}$, via their average density and opacity values computed during the tree build. This multiplied by the intersection l, gives us the minimum and maximum optical depths, $\tau_{\rm min}$ and $\tau_{\rm max}$. We can then test the following refinement criteria

$$\tau_{\rm refine} < \tau_{\rm max} - \tau_{\rm min}, \tag{8}$$

and refine if it is true. The fractional error in flux for a chosen value of $\tau_{\rm refine}$ is

$$\text{FractionalError} = \frac{F_1 - F_2}{F_1} \leq 1 - e^{(-\tau_{\rm max} - \tau_{\rm min})} < \tau_{\rm refine}, \tag{9}$$

for small τ, making the refinement criteria a convenient choice of parameter for guaranteeing accuracy. This criteria is conservative, as it assumes the worst case difference in

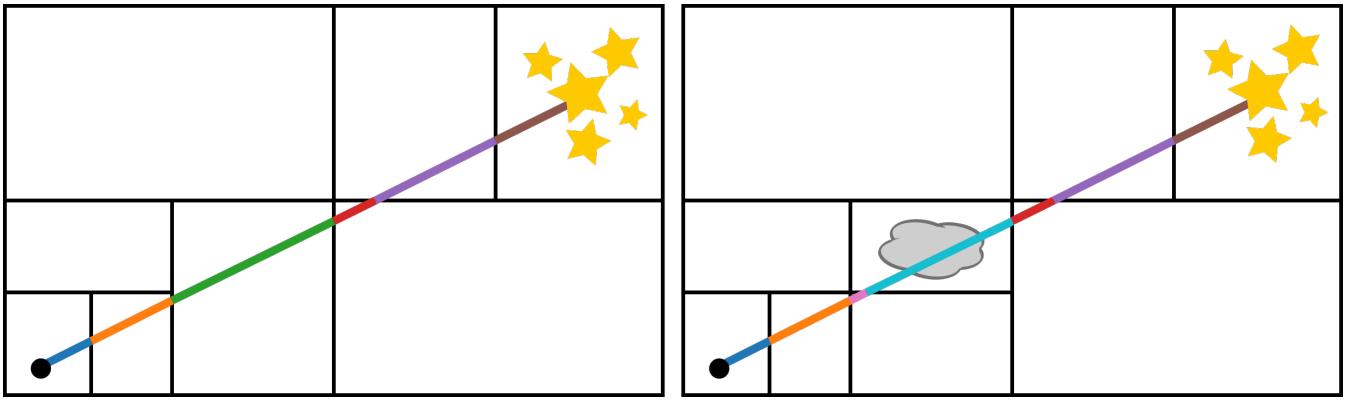

Figure 1. Depiction of the FAHRT algorithm with and without the need for refinement (left and right respectively.)

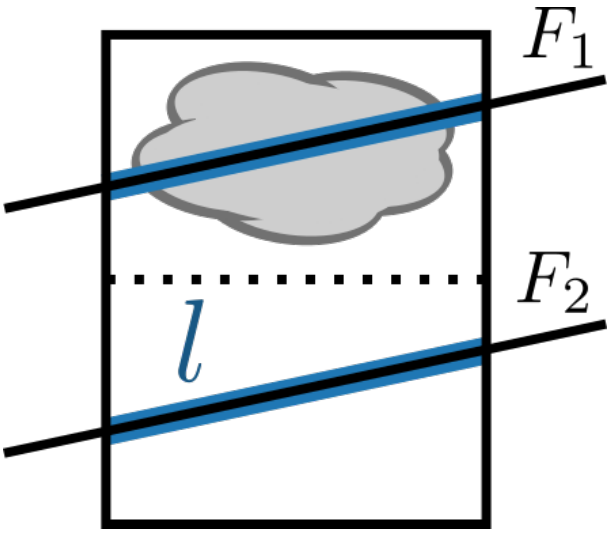

Figure 2. refine.pdf

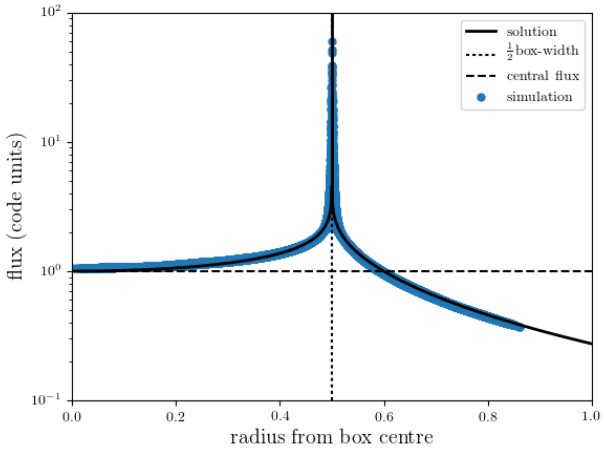

Figure 3. The distribution of flux that particles receive due to the cosmological background sources when distributed in a spherical shell on at the edge of the simulation box. Note that the value of the flux at the centre can be easily scaled by simply scaling L, the luminosity of all sources on the sphere. The important property is the near constant flux at small radii. In this example, we have used 1024 background sources. The number of sources determines the width of the peak.

optical depth. We suspect there is room for improvement in terms of efficiency.

If very high accuracy is required, sub-leaf node refinement is possible. If a leaf was reached during refinement and still passes the refinement criteria, the individual particles in the leaf can be considered. A ray tracing scheme through the cell similar to SPHRay (Altay et al. 2008) can be performed. The machinery to do this is implemented in FAHRT.

2.4 Cosmological Background Radiation

In order to treat cosmological simulations properly we must account for the radiation coming from the rest of the universe outside of the simulation volume. Most current codes apply a constant UV field to the entire box, essentially the lowest order approximation possible. Some specialized codes like URCHIN (Altay & Theuns 2013) do a reverse ray trace to the edge of the box, where the background flux is assumed to be coming from. Others, such as TRAPHIC (Pawlik & Schaye 2008) allow their ray trace to be periodic. We believe that this periodic treatment is problematic for reasons we will explain at the end of this subsection.

Instead, we have implemented a method involving "background sources". "Background" particles are distributed in a spiral pattern on the surface of a sphere at the very edge of the simulation volume (or at a large distance if required) and the number of sources can be varied to match the required angular resolution of the background. Finding the flux at the centre of a sphere of sources is a problem akin to Newton's Shell Theorem. However, because the intensity does not cancel like force, the solution differs and is as follows:

$$F(r) = \frac{L}{8\pi R} \ln\left(\frac{R+r}{R-r}\right), \tag{10}$$

where L is the total luminosity of the emitting shell, R is the radius of the sphere and r is the radius the flux is being computed at. The shape of the function can be seen if Figure 2.4 where we have plotted the flux as a function of radius for a homogeneous, optically thin test volume.

We note that due to the logarithm in equation 10, the flux is nearly constant at small radii. Since most cosmological zoom in simulations only consider gas at a fairly small radius, this setup of background sources is an acceptable method to provide a cosmological background flux. A benefit of this method is that we can use all of the existing machinery described in the methods section, and only have to add temporary background star particles as the source of the background radiation. This way, there is no need to create periodic copies of the simulation volume. **explain <-this here**

3 CODE TESTS

4 DISCUSSION AND CONCLUSION

ACKNOWLEDGEMENTS

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