

Numerical Analysis HW 4

Linear System Direct Methods

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系級：資工 4 B

1. Test data

```
11, 8
1.0000000000000000 2.0000000000000000 4.0000000000000000 8.0000000000000000 16.0000000000000000 32.0000000000000000 64.0000000000000000 128.0000000000000000
1.0000000000000000 2.2000000000000000 4.8400000000000001 10.6480000000000003 23.4256000000000010 51.5363200000000075 113.3799040000000067 249.4357888000000182
1.0000000000000000 2.4000000000000000 5.7600000000000000 13.8240000000000000 33.1775999999999998 79.6262399999999996 191.1029759999999984 458.6471423999999950
1.0000000000000000 2.6000000000000000 6.7600000000000001 17.5760000000000004 45.6976000000000016 118.8137600000000045 308.9157760000000108 803.1810176000000359
1.0000000000000000 2.8000000000000000 7.8399999999999999 21.9519999999999995 61.465599999999981 172.1036799999999940 481.8903039999999787 1349.2928511999999404
1.0000000000000000 3.0000000000000000 9.0000000000000000 27.0000000000000000 81.0000000000000000 243.0000000000000000 729.0000000000000000 2187.0000000000000000
1.0000000000000000 3.2000000000000000 10.2400000000000002 32.7680000000000008 104.8576000000000033 335.5443200000000141 1073.7418240000000406 3435.9738368000001664
1.0000000000000000 3.4000000000000000 11.5600000000000002 39.3040000000000009 133.6336000000000058 454.3542400000000232 1544.8044160000000856 5252.3350144000003274
1.0000000000000000 3.6000000000000000 12.9600000000000001 46.6560000000000006 167.9616000000000033 604.6617600000000186 2176.7823360000000669 7836.4164096000002225
1.0000000000000000 3.8000000000000000 14.4400000000000000 54.8719999999999993 208.5135999999999968 792.3516799999999874 3010.9363839999999180 11441.5582591999995518
1.0000000000000000 4.0000000000000000 16.0000000000000000 64.0000000000000000 256.0000000000000000 1024.0000000000000000 4096.0000000000000000 16384.0000000000000000
11, 1
255.0000000000000000
456.4656128000000258
785.5379583999999980
1304.5441536000000527
2098.344435199999225
3280.0000000000000000
4997.325580800002172
7440.3912704000004942
10850.038105600004201
15527.471923199995217
21845.0000000000000000
```

2. 3 results and precisions

[NEW] Gaussian elimination	[NEW] QR decomposition	[ORIGINAL] QR decomposition
<pre>[NEW] gaussian elimination: ----- solution ----- 8, 1 -1.973663077385359 8.347332055607017 -6.701760866148485 5.440490562093165 -0.521068135405708 1.309622115544814 0.965313578881938 1.001650213219384 ----- 2-norm ----- 12.011317883668086 ----- inf-norm ----- 7.701760866148485 -----</pre>	<pre>[NEW] QR decomposition: ----- solution ----- 8, 1 -16.144492058901559 43.274344362968158 -43.222781909942945 26.444668354084637 -7.698241304300296 2.767031654767264 0.802430997653689 1.009381400149063 ----- 2-norm ----- 69.014203689953803 ----- inf-norm ----- 44.222781909942945 -----</pre>	<pre>[ORIGINAL] QR decomposition: ----- solution ----- 8, 1 1.000000217114967 0.999999464041379 1.000000561346167 0.999999676588960 1.000000110716489 0.999999977472877 1.000000002523056 0.99999999879968 ----- 2-norm ----- 0.000000875709292 ----- inf-norm ----- 0.000000561346167 -----</pre>

3. Further more degree

```
[NEW] gaussian elimination:
----- solution -----
9, 1
6.736297111737250
-16.110457851557403
22.995962848378777
-14.922878881589272
8.102137512273625
-0.999544819474626
1.347172015805712
0.965996980283696
1.001439055189169

----- 2-norm -----
33.430428495043088

----- inf-norm -----
21.995962848378777

[NEW] QR decomposition:
----- solution -----
9, 1
599.662826018215924
-1650.659924398349858
1975.273192994590772
-1334.566035917564477
560.328793922009709
-147.518407059498259
25.422170831281196
-1.274315231143035
1.091852995952413

----- 2-norm -----
3017.180306563379418

----- inf-norm -----
1974.273192994590772

[ORIGINAL] QR decomposition:
----- solution -----
9, 1
1.000001513314467
0.999995943978058
1.000004713048326
0.999996897814905
1.000001265479582
0.99999672284802
1.000000052628078
0.99999995206893
1.000000000189592

----- 2-norm -----
0.000007231143762

----- inf-norm -----
0.000004713048326
```

```
[NEW] gaussian elimination:
----- solution -----
10, 1
1475.416101124574425
-4442.487393480646460
5896.743997668666452
-4519.746026709959551
2208.974227650898683
-711.446856931074763
152.901542887778731
-19.640150769320766
2.622171586891993
0.943803509426302

----- 2-norm -----
9084.143240967336169

----- inf-norm -----
5895.743997668666452

[NEW] QR decomposition:
----- solution -----
10, 1
-2266.158201209053459
5532.207327897120194
-5595.133626507879853
2950.621498728189636
-785.497607270621870
45.098818483227681
35.801903139003180
-9.642039768270829
2.292097549260665
0.939813825363897

----- 2-norm -----
8739.169792127227993

----- inf-norm -----
5596.133626507879853

[ORIGINAL] QR decomposition:
----- solution -----
10, 1
1.000684354201022
0.997934183855796
1.002745576065247
0.997891154191427
1.001031747988594
0.999666516166337
1.000071222672186
0.999990306337235
1.000000763081600
0.99999973523617

----- 2-norm -----
0.004231103660620

----- inf-norm -----
0.002745576065247
```

```
[NEW] gaussian elimination:
----- solution -----
11, 1
40680.448739027473493
-113908.351637315383414
137505.624726514361100
-92760.246997220456251
37536.591323733628087
-8805.280679250254252
880.167754547578738
96.365626951928760
-38.868132508716165
5.645461175386926
0.801254708550348

----- 2-norm -----
208876.957903113216162

----- inf-norm -----
137504.624726514361100

[NEW] QR decomposition:
----- solution -----
11, 1
-53.468955036551144
391.616230729336394
-185.310633312902354
-770.641496083552852
1283.133590081753255
-930.000786543129720
390.357917329833185
-99.064097558261480
16.660131188789606
-0.373683391265170
1.051896351865309

----- 2-norm -----
1859.617892339997070

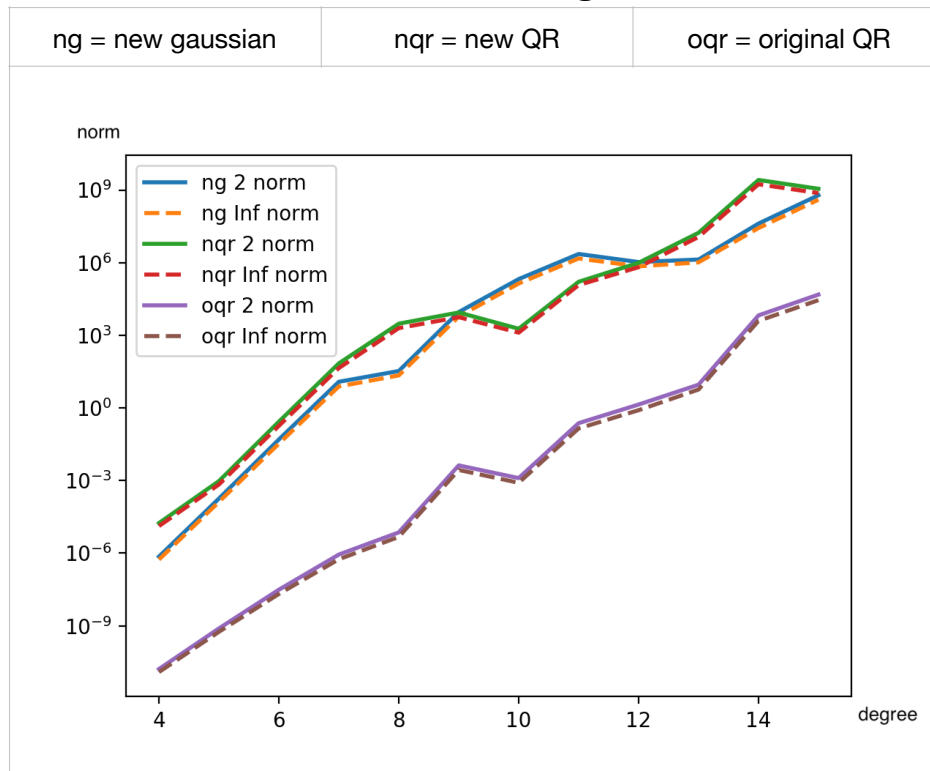
----- inf-norm -----
1282.133590081753255

[ORIGINAL] QR decomposition:
----- solution -----
11, 1
0.999814228125565
1.000581766966939
0.999190761724861
1.000658562466538
0.999652679671408
1.000124069961781
0.999969589529569
1.000005051613095
0.999999455575557
1.000000034383778
0.999999999033393

----- 2-norm -----
0.001264322142728

----- inf-norm -----
0.000809238275139
```

4. Error curve & meaningful result discuss



若假定標準為 $2\text{ norm} < 1$ ，在 new system 的情況下，無論是 Gaussian Elimination 或是 QR decomposition 皆在 $\text{degree} > 6$ 之後便無參考價值，而在 original system 中，QR decomposition 在 $\text{degree} > 11$ ($\text{degree} = 12, 2\text{-norm} = 1.37876965$) 之後無參考價值。