Numerical Analysis HW 1

Newton 2D Root Finding

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系級：資工 4 B

1. Result

- [3.01, 3.99], 不收斂

```
3.01 3.99
       |x_n                    y_n                    l2_norm
-------|-------------------------------------------------------
      0|3.0099999999999998     3.9800000000000004     ---
      1|-50.5076623376634473   -413.1415519480607372  420.5407581693443149
      2|-23.6875308495723367   -204.5953969882001218  210.2636873109113083
      3|-10.1321237002419799   -100.3533797075952236  105.1196805061452153
      4|-2.9212617003051378    -48.3052842610737656   52.5452259524339738
      5|3.9174236702134273     -22.5865620445693480   26.6124085728192306
      6|0.4764855145626998     -9.6600459162995485    13.3766540885765703
      7|-1.2969247876538099    -3.0568197704843900    6.8372201685176535
      8|-1.7231931256738471    0.3413036010235446     3.4247550487570373
      9|-4.0003473431796071    3.8166528494206560     4.1549348645482560
     10|-2.0040141611553706    -2.9773017378504907    7.0811838775415001
     11|-1.6845539069613589    0.3665591975263585     3.3590863057011560
     12|-4.3421115552294713    4.1070969937300150     4.5884894746197737
     13|-2.4930662177432854    -1.1897219295131620    5.6102815763298741
     14|-2.2742912847899923    1.5759558072030102     2.7743171798942181
     15|-3.6031074644226386    5.0104160320793572     3.6825628949837630
     16|-2.7337664899557499    2.2501925118064410     2.8938879749836293
     17|-3.8717744722928886    6.9520264518500525     4.8375928484742365
     18|-3.1556658548615677    4.1340837258566552     2.9075097177719793
     19|-2.2252687933188979    0.6356449237539112     3.6200431964529534
     20|-2.9362907365404474    3.2436696126698004     2.7032101253393841
```

- [7, 9], 收斂到 [1.80796, 7.88466]

```
7 9
       |x_n                    y_n                    l2_norm
-------|-------------------------------------------------------
      0|7.0000000000000000     9.0000000000000000     ---
      1|3.5525678294573644     8.8410852713178301     3.4510929372424086
      2|2.1717947972802980     8.1537480696669142     1.5423898972570986
      3|1.8323022260218305     7.9066806935885312     0.4198779516263537
      4|1.8080923010476697     7.8847962507359419     0.0326350931701248
      5|1.8079638758342076     7.8846611114968299     0.0001864286710798
      6|1.8079638721065365     7.8846611070555337     0.0000000057983311
1.80796 7.88466
```

• [1, 2], 收斂到 [-0.07894, 0.84835]

```
1 2
       |x_n                    y_n                    l2_norm
-------|-------------------------------------------------------
      0|0.9900000000000000     1.9900000000000000     ---
      1|0.0488184618035366     0.2440974779780751     1.9834309426456922
      2|-0.0918890511364101    0.8048668912481844     0.5781530412071253
      3|-0.0789991647954649    0.8480763892775687     0.0450911287265449
      4|-0.0789396246322419    0.8483522115275645     0.0002821753792047
      5|-0.0789396210314415    0.8483522217057046     0.0000000107963095
-0.0789396      0.848352
```

• [10, 10], 收斂到 [1.80796, 7.88466]

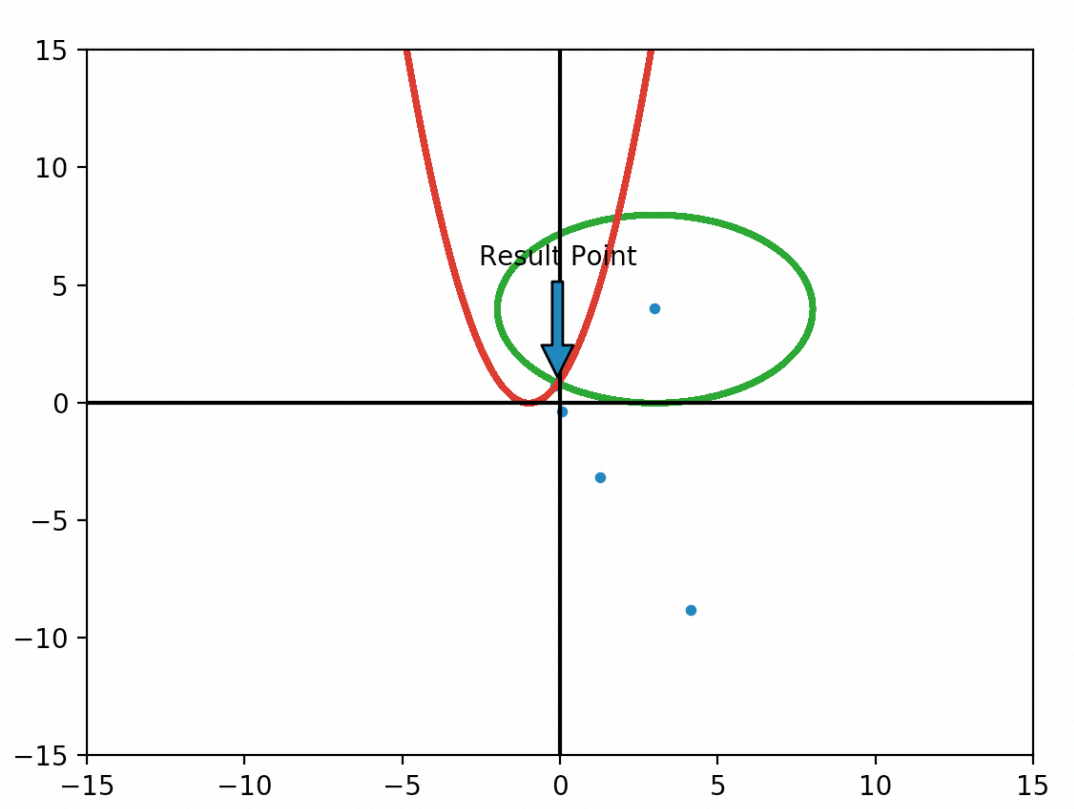
```
10 10
       |x_n                    y_n                    l2_norm
-------|-------------------------------------------------------
      0|10.000000000000000     10.000000000000000     ---
      1|4.9320046893317704     9.5041031652989449     5.0921989493362974
      2|2.6819468697658579     8.4939725603888885     2.4663990006415557
      3|1.9254103391933342     7.9856781305685471     0.9114333488945295
      4|1.8108135559481249     7.8875404235715925     0.1508755522434618
      5|1.8079656884907045     7.8846631973920216     0.0040483057620989
      6|1.8079638721072990     7.8846611070565178     0.0000027692510168
      7|1.8079638721065365     7.8846611070555337     0.0000000000012451
1.80796 7.88466
```

• [3, 4], 原本是不收斂, 但在點 [3, 4] 時我會先讓他偏移 0.01, 使程式不會遇到「除以 0」的情況

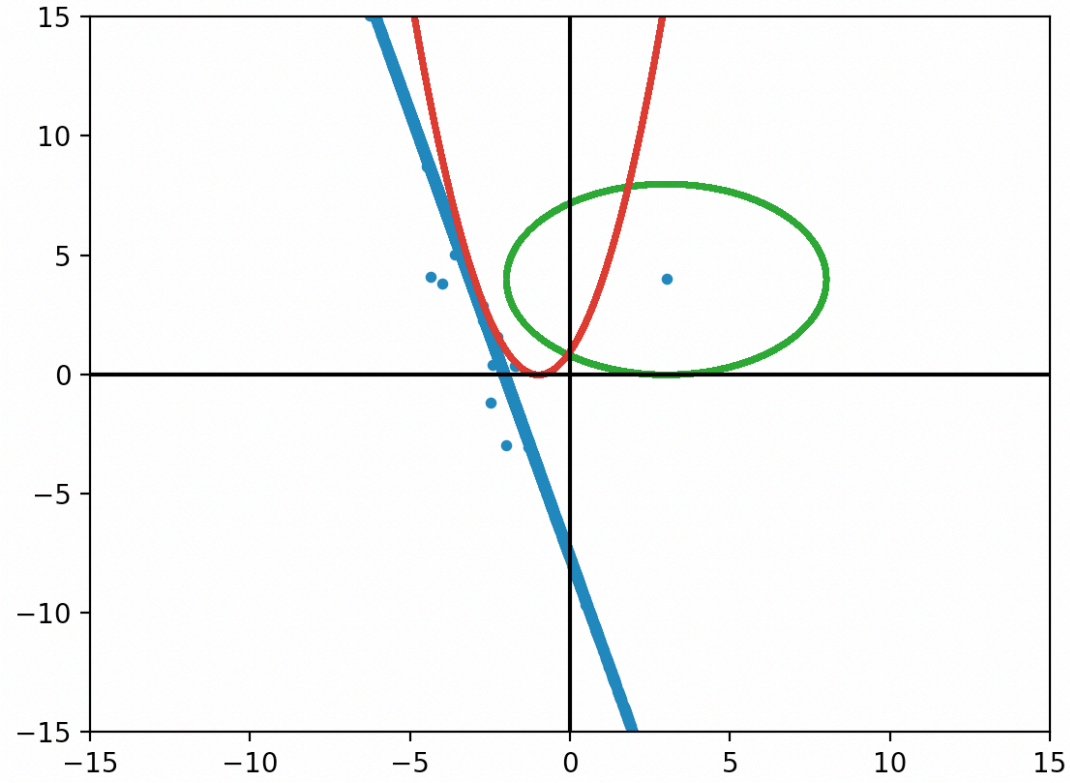
```
3 4
       |x_n                    y_n                    l2_norm
-------|-------------------------------------------------------
      0|2.9900000000000002     3.9900000000000002     ---
      1|-91.2095359628790021   -735.7222969837745268  745.6861503907884980
      2|-44.0768546647071915   -365.8742385507296717  372.8392092753228440
      3|-20.4379149204168478   -180.9669357808257359  186.4122047771419659
      4|-8.4412120448916355    -88.5492431865944809   93.1930833502436542
      5|-1.8697005286859012    -42.4283845980280390   46.5866757834701843
      6|10.0777655551794005    -20.0250561295925245   25.3899797615068010
      7|4.1418050766288035     -8.7974673568689035    12.7001723001030236
      8|1.2640470323237341     -3.1555823969886267    6.3334317129093458
      9|0.0497599340212942     -0.3724972380273677    3.0364545376366459
     10|-0.1402058454538355    0.7031589908201601     1.0923018447435116
     11|-0.0788571636741381    0.8447404641583469     0.1543015694953175
     12|-0.0789404565472603    0.8483506756476664     0.0036111722058534
     13|-0.0789396210313412    0.8483522217051913     0.0000017573789351
     14|-0.0789396210314414    0.8483522217057049     0.0000000000005232
-0.0789396      0.848352
```

2. Graphs of points

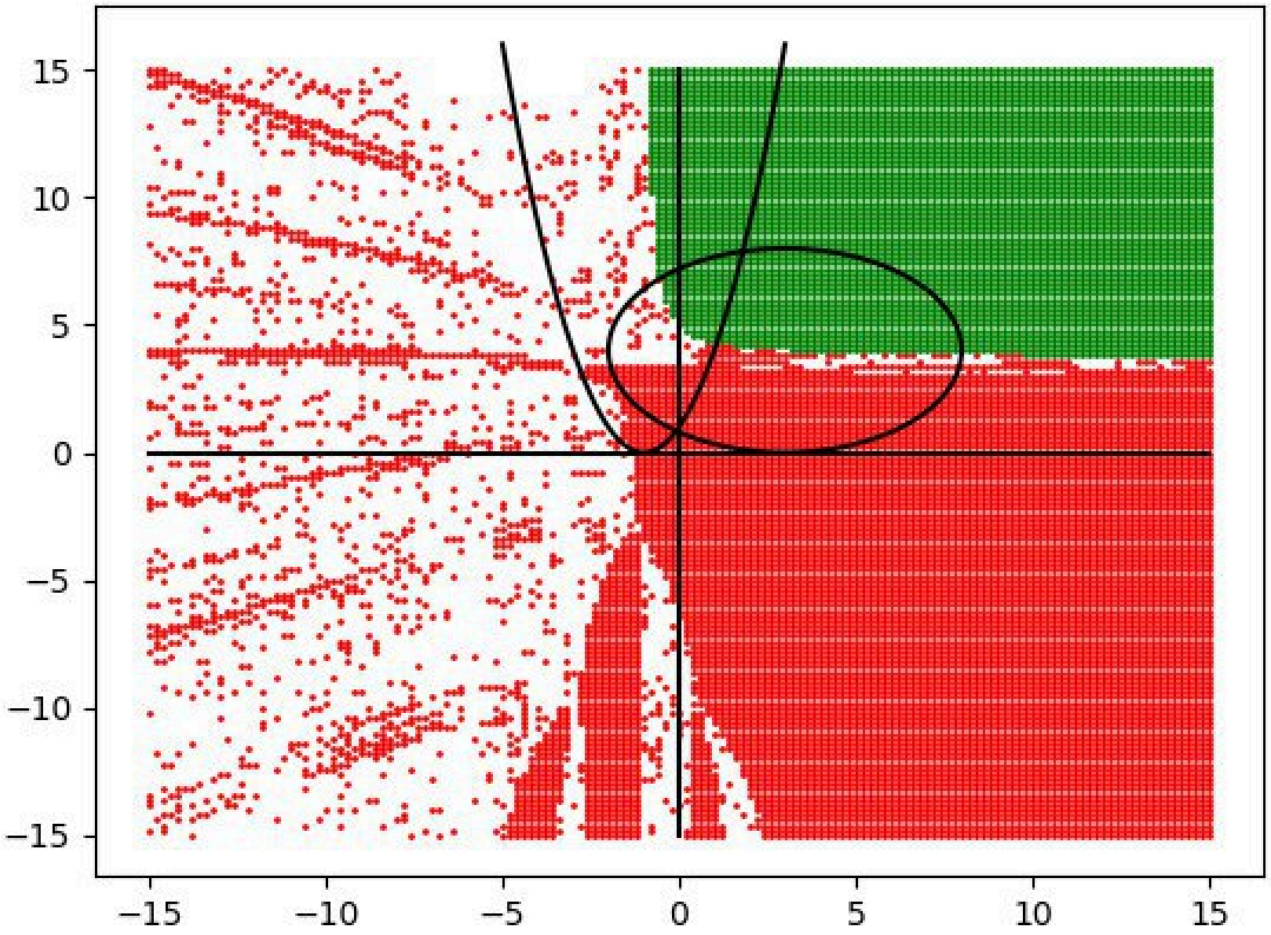
- Initial points: [3, 4]



- Initial points: [3.01, 3.99]



3. Diverge region



綠色的點會收斂到 [1.80796, 7.88466]，紅色的點會收斂到 [-0.07894, 0.84835]，未填色區域則表示發散。

發散區域猜測：

(1) 當 x < 0 時則有高機率發散

(2) 觀察 y < 0 區域發現與以 x 軸鏡像拋物線區域會發散

(3) 觀察左側區域發現有橫向拋物線的區域會收斂

4. 遇到問題

本次作業我遇到的問題有兩個，一是微分微錯導致結果有誤，二是對於發散原因不太清楚，且題目所要求之點 [3.01, 3.99] 也會發散，很懷疑是不是自己程式寫錯了，因此而花了很多時間把圖形畫出來。