1. "unusual" unique subsets of 5 letters (of 7) : $\frac{7!}{3!3} = \boxed{210}$

"unsal" (order doesn't matter)

Different strings $= \frac{7!}{3!} = \boxed{840}$

2. $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = 78 \times 6 \times 6 \times 11 \times 4 = \boxed{123552}$

$\binom{13}{2}$ ⇓ number; $\binom{4}{2}\binom{4}{2}$ ⇓ suits → two pairs; $\binom{11}{1}$ ⇓ fifth card

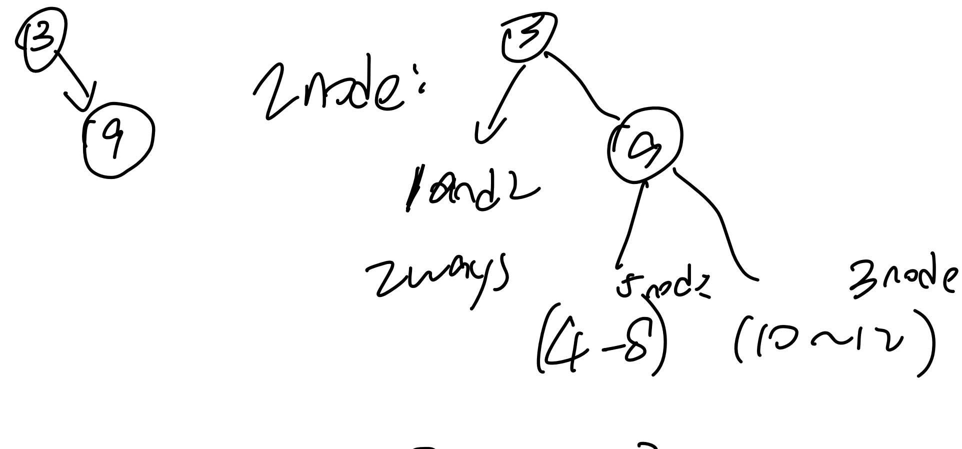
3. ① song played for TC

16 stars, 6 bars $\binom{16+6-1}{6-1} = \binom{21}{5}$

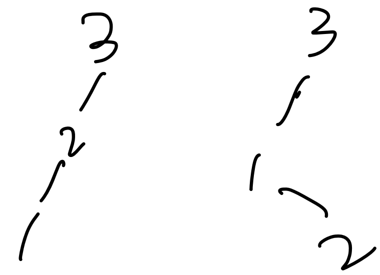
1 song for TC: 15 stars, 6 bars $\binom{15+6-1}{6-1} = \binom{20}{5}$

$\binom{21}{5} + \binom{20}{5} = 20349 + 15504 = \boxed{35853}$

4.



2node:

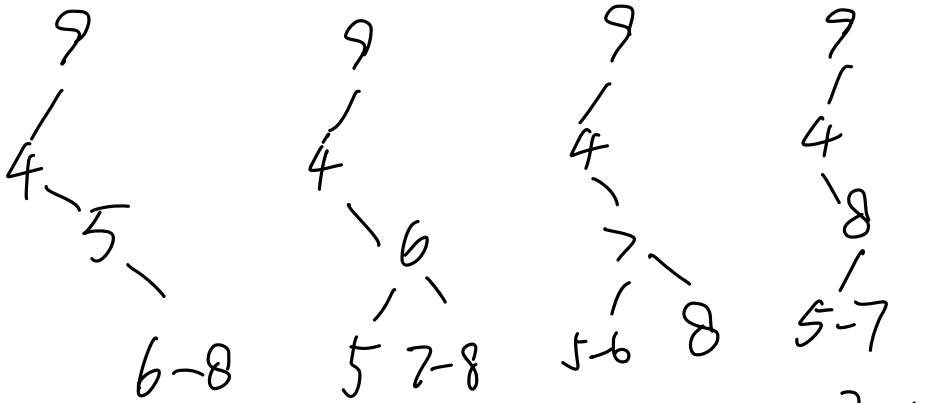


3 node:

5 ways

4 node:

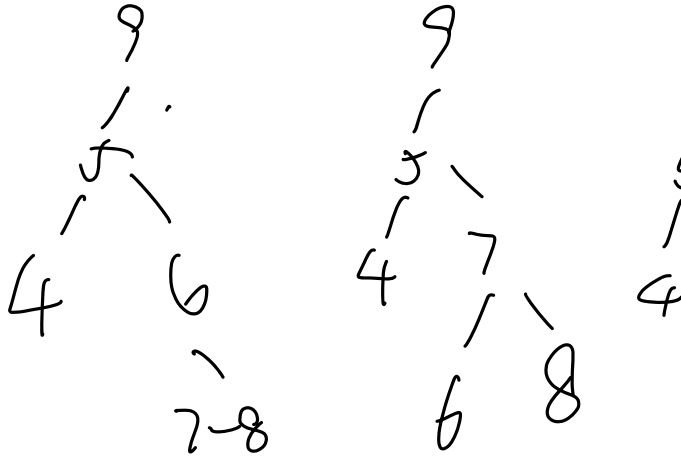
14 ways



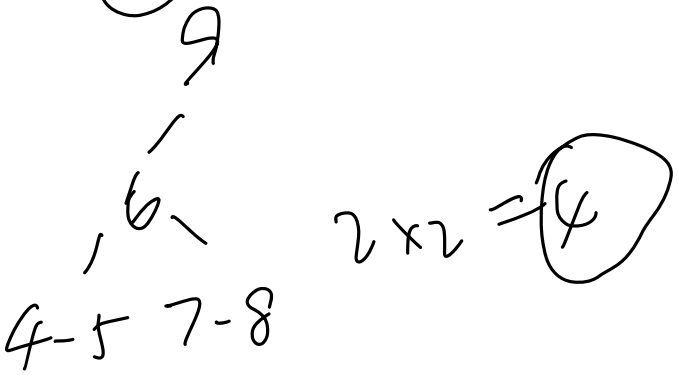
3 node ⑤ + 2 node ② + 2 node ② + 3 node ⑤

= 14

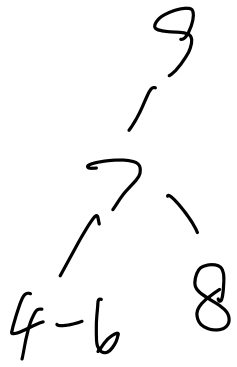
5 node



⑤ + ① + ② = 5



② × ② = ④



3node = 5

4node = 14

(14+5+4 +5+14) x5x2 = 420

5. Identical nurse ⇒ indistinguishable box
10 friends all get vaccinated

3 nurse:

118
127
136
145
226
235
244
334

(8)

4 nurse:

1117
1126
1135
1144
1225
1234
1333
2224
2233

(9)

9 × 8 = (17)