Data Analysis with R for Social Scientists

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Contents

Intro

This course offers an accessible and easy introduction to one of the fastest growing statistical packages used in social science and data science more generally.

Please download the data used in the course here. To find more about me, have a look at my website. Also, feel free to watch me as I walk you through each lesson here.

Overview over the Course :

- **Week 1: Introduction to Seminar**
- **Week 2: Exploratory Data Analysis-I**
- **Week 3: Exploratory Data Analysis-II**
- **Week 4: DAGs**
- **Week 5: Linear Regression Theory I: Simple Linear Regression**
- **Week 6: Linear Regression Theory II: Multiple Linear Regression**
- **Week 7: Linear Regression Theory III: Diagnostics**
- **Week 8: Linear Regression - Application**
- **Week 9: Logistic Regression - Exercises**
- **Week 10: Mediation**
- **Week 11: Prediction - Theory**
- **Week 12: Prediction - Application**
- **Week 13: Other Estimators**
- **Week 14: Course Paper - Discussion - Outlook**

Chapter 1

Introduction to Seminar

1.1 Introduction

Welcome to this course! In this course, you will learn how to analyse data using multiple regression in R. The course is aimed at undergraduate students who have completed the course "Intro to R for Social Scientists.

1.2 Why should I take this course?

- What is regression?
- What do you use it for?

1.3 Objectives

-
-

1.4 What is not covered

-
-

1.5 Prerequisites

- Basic knowledge of how to use R (data cleaning, management etc.) (include hyperlink to "Intro to R course")
- Descriptive statistics
- Data visualization using ggplot()

1.6 Structure

1. Intro to seminar

BLOCK I - Pre-processing/ EDA

2. Exploratory data analysis (EDA) - I (ggplot; gtsummary; x)
3. EDA - II -> in class exercise

–> add visualization/ reporting here.

–> knitting

BLOCK II - Modelling/ Linear regression

4. DAGS
5. Linear Regression - theory I: Simple Linear Regression
6. Linear Regression - theory II: Multiple Linear Regression
7. Linear Regression - theory III: Diagnostics

Block III - Application

8. Linear Regression – application
9. Linear Regression - exercise

Block IV - Advanced uses

10. Mediation (?)
11. Prediction - Theory
12. Prediction - Application
13. Other estimators: Logistic/ Poisson/ Multilevel/ FE
14. Course paper/Discussion/ Outlook

Chapter 2

Exloratory Data Analysis - I

2.1 Objectives

- Remember how to load and explore datasets in R
- Conduct basic descriptive data analysis
- Undestand and visualize distribution of and relationship between variables

2.2 R functions covered this week

- `load()`
- `read_excel()`
- `str()`
- `glimpse()`
- `table()`
- `summary()`
- `mutate()`
- `case_when()`
- `ggplot()`
- `corr ()`

2.3 Why is EDA so important?

- Every regression analysis is based on proper EDA; EDA often used for "hypothesis generation" (i.e. finding things that could be interesting to study).

- `EDA` helps understand the data and issues in the data. The better we understand the data, the better we can "fine-tune" our regression model later.
- `EDA` helps prepping data for regression (i.e. "cleaning"; "pre-processing"). Small errors in the data can lead to completely wrong conclusions or even prevent the model from working altogether.

2.4 Importing data into R

The first step of any data analysis is getting data into R. To get started, we first need to follow some preparatory steps

1. In this course, we will use data on NBA players (Basketball). Usually, you need to first download the data and documentation and save them in a folder on your own computer. We have already done this, and provide the data for you here.
2. Install R and Rstudio. If you don't already have R installed, here is a link to how it is done (hyperlink)
3. In the folder which you will use for this class, create a new R project. You will see that all files appear in the bottom right window in R studio.

Now, Let's get started.

2.5 Import data into R

First, we need to install some packages.

```
library("tidyverse")
library("readxl")
```

Now, let's import the data.You can see in the folder that we have 2 csv files. We can use `read_delim()` or `read_csv` function. Note that you need other function for Stata datasets (`read_data` from the haven package). To load Rdata files, you use the `load()` function.

Make sure you name the correct sub-directory in case you saved the data a sub-folder of your project folder (which I have done).

```
# import data
nba_salaries <- read_csv("../datasets/nba/salaries_1985to2018.csv", show_col_types = FA
nba_players <- read_csv("../datasets/nba/players.csv", show_col_types = FALSE)
```

Great, the two dataframes should appear in your environment in the upper right side in R studio.

Let's take a quick look at these trend dataframe using str() function The str() function shows the number of rows (observations) and columns (variables). It also provides information on the name of each column, its type and an example of some of the values in each column.

```
str(nba_players)
```

```
## spc_tbl_ [4,685 x 25] (S3: spec_tbl_df/tbl_df/tbl/data.frame)
##  $ index      : num [1:4685] 0 1 2 3 4 5 6 7 8 9 ...
##  $ _id        : chr [1:4685] "abdelal01" "abdulza01" "abdulka01" "abdulma02" ...
##  $ birthDate  : chr [1:4685] "June 24, 1968" "April 7, 1946" "April 16, 1947" "March 9, 1969"
##  $ birthPlace : chr [1:4685] "Cairo, Egypt" "Brooklyn, New York" "New York, New York" "Gulfpor
##  $ career_AST : num [1:4685] 0.3 1.2 3.6 3.5 1.1 2.5 1.2 1 0.7 0.5 ...
##  $ career_FG% : chr [1:4685] "50.2" "42.8" "55.9" "44.2" ...
##  $ career_FG3%: chr [1:4685] "0.0" NA "5.6" "35.4" ...
##  $ career_FT% : chr [1:4685] "70.1" "72.8" "72.1" "90.5" ...
##  $ career_G   : num [1:4685] 256 505 1560 586 236 830 319 1 56 174 ...
##  $ career_PER : chr [1:4685] "13.0" "15.1" "24.6" "15.4" ...
##  $ career_PTS : num [1:4685] 5.7 9 24.6 14.6 7.8 18.1 5.6 0 9.5 5.3 ...
##  $ career_TRB : chr [1:4685] "3.3" "8.0" "11.2" "1.9" ...
##  $ career_WS  : num [1:4685] 4.8 17.5 273.4 25.2 3.5 ...
##  $ career_eFG%: chr [1:4685] "50.2" NA "55.9" "47.2" ...
##  $ college    : chr [1:4685] "Duke University" "Iowa State University" "University of Californ
##  $ draft_pick : chr [1:4685] "25th overall" "5th overall" "1st overall" "3rd overall" ...
##  $ draft_round: chr [1:4685] "1st round" "1st round" "1st round" "1st round" ...
##  $ draft_team : chr [1:4685] "Portland Trail Blazers" "Cincinnati Royals" "Milwaukee Bucks" "D
##  $ draft_year : chr [1:4685] "1990" "1968" "1969" "1990" ...
##  $ height     : chr [1:4685] "6-10" "6-9" "7-2" "6-1" ...
##  $ highSchool : chr [1:4685] "Bloomfield in Bloomfield, New Jersey" "John Jay in Brooklyn, New
##  $ name       : chr [1:4685] "Alaa Abdelnaby" "Zaid Abdul-Aziz" "Kareem Abdul-Jabbar" "Mahmoud
##  $ position   : chr [1:4685] "Power Forward" "Power Forward and Center" "Center" "Point Guard"
##  $ shoots     : chr [1:4685] "Right" "Right" "Right" "Right" ...
##  $ weight     : chr [1:4685] "240lb" "235lb" "225lb" "162lb" ...
##  - attr(*, "spec")=
##   .. cols(
##   ..   index = col_double(),
##   ..   `_id` = col_character(),
##   ..   birthDate = col_character(),
##   ..   birthPlace = col_character(),
##   ..   career_AST = col_double(),
##   ..   `career_FG%` = col_character(),
##   ..   `career_FG3%` = col_character(),
##   ..   `career_FT%` = col_character(),
```

```
##   ..   career_G = col_double(),
##   ..   career_PER = col_character(),
##   ..   career_PTS = col_double(),
##   ..   career_TRB = col_character(),
##   ..   career_WS = col_double(),
##   ..   `career_eFG%` = col_character(),
##   ..   college = col_character(),
##   ..   draft_pick = col_character(),
##   ..   draft_round = col_character(),
##   ..   draft_team = col_character(),
##   ..   draft_year = col_character(),
##   ..   height = col_character(),
##   ..   highSchool = col_character(),
##   ..   name = col_character(),
##   ..   position = col_character(),
##   ..   shoots = col_character(),
##   ..   weight = col_character()
##   .. )
##  - attr(*, "problems")=<externalptr>
```

```
str(nba_salaries)
```

```
## spc_tbl_ [14,163 x 8] (S3: spec_tbl_df/tbl_df/tbl/data.frame)
##  $ index       : num [1:14163] 0 1 2 3 4 5 6 7 8 9 ...
##  $ league      : chr [1:14163] "NBA" "NBA" "NBA" "NBA" ...
##  $ player_id   : chr [1:14163] "abdelal01" "abdelal01" "abdelal01" "abdelal01" ...
##  $ salary      : num [1:14163] 395000 494000 500000 805000 650000 1530000 2030000 2
##  $ season      : chr [1:14163] "1990-91" "1991-92" "1992-93" "1993-94" ...
##  $ season_end  : num [1:14163] 1991 1992 1993 1994 1995 ...
##  $ season_start: num [1:14163] 1990 1991 1992 1993 1994 ...
##  $ team        : chr [1:14163] "Portland Trail Blazers" "Portland Trail Blazers" "B
##  - attr(*, "spec")=
##   .. cols(
##   ..   index = col_double(),
##   ..   league = col_character(),
##   ..   player_id = col_character(),
##   ..   salary = col_double(),
##   ..   season = col_character(),
##   ..   season_end = col_double(),
##   ..   season_start = col_double(),
##   ..   team = col_character()
##   .. )
##  - attr(*, "problems")=<externalptr>
```

We see that most variables/ columns, already are in the type which we want. R automatically picked up on the format. Text variables are “chr” for “character”;

Numeric variables are "num". It is very important that you understand the types of columns/ variables that R recognized. If you need a refresher, go here [hyperlink].

2.6 Merge datasets

We see that "nba_salaries" contains the salaries of players for various seasons. "players" contains many career statistics about players, for example, how many points they scored on average per game, across their whole career.

Now we want to link both datasets. We can do this using the `merge()` function. An alternative would be `join()`.

```
data_nba <- merge(nba_players, nba_salaries, by.x = c("_id"), by.y=c("player_id"))
rm(nba_players, nba_salaries)
```

2.7 Clean dataset

We can use the select() function to kick out columns and the filter() function to kick out rows. First, we need to look at the codebook and the questionnaire to understand the what each variable refers to (see .txt file "data_description").

When using the select function, we will also rename the variables to make them more intuitive.

First, let's filter out Let's filter all years between 2012-2022.

Afterwards, we use the save() function to store the data as a .Rdata file and the write_excel() function to store the reduced dataset. This way we can simply load that one next time and save time.

```
data_nba <- data_nba %>%
        select(everything(), -league, -highSchool) %>%
        filter(season_start>=1998)
save(data_nba, file ="../datasets/nba/data_nba.RData")
```

2.8 change variables

Let's do some data cleaning. First, we want to calculate each players' age at the beginning of each season. Currently, we only have the date of birth and the year for each season.

Second, we want recode the "position" variable. Some players played multiple position, so that info is messy. We want to create varies dummy variables for each position.

```
# let's calculate age for every season
class(data_nba$birthDate) # nice, it is already a date variable
```

```
## [1] "character"
```

```
library(lubridate)
data_nba <- data_nba %>%
  mutate(year_of_birth = year(mdy(birthDate)),
         age = season_start - year_of_birth)


# let's clean the position

table(data_nba$position)
```

```
##
##                                                                 Center
##                                                                   1204
##                                              Center and Power Forward
##                                                                    686
##                          Center and Power Forward and Small Forward
##                                                                      3
##                          Center and Small Forward and Power Forward
##                                                                     30
##                                                            Point Guard
##                                                                   1162
## Point Guard and Power Forward and Small Forward and Shooting Guard
##                                                                      5
##                                         Point Guard and Shooting Guard
##                                                                    573
##                     Point Guard and Shooting Guard and Small Forward
##                                                                     13
##                                       Point Guard and Small Forward
##                                                                      3
##                     Point Guard and Small Forward and Shooting Guard
##                                                                     21
##                                                       Power Forward
##                                                                    667
##                                              Power Forward and Center
##                                                                    884
```

```
##                         Power Forward and Center and Small Forward
##                                                                  51
##                                  Power Forward and Shooting Guard
##                                                                  14
##                Power Forward and Shooting Guard and Small Forward
##                                                                  31
##                                   Power Forward and Small Forward
##                                                                 418
##                         Power Forward and Small Forward and Center
##                                                                   3
##                Power Forward and Small Forward and Shooting Guard
##                                                                  11
##                                                       Shooting Guard
##                                                                 735
##                                    Shooting Guard and Point Guard
##                                                                 581
##                  Shooting Guard and Point Guard and Small Forward
##                                                                  17
##                Shooting Guard and Power Forward and Small Forward
##                                                                  25
##                                  Shooting Guard and Small Forward
##                                                                 592
##                  Shooting Guard and Small Forward and Point Guard
##                                                                  73
##                Shooting Guard and Small Forward and Power Forward
##                                                                  25
##                                                     Small Forward
##                                                                 620
##                                          Small Forward and Center
##                                                                   4
##                         Small Forward and Center and Power Forward
##                                                                  72
##                  Small Forward and Point Guard and Shooting Guard
##                                                                  19
##                                   Small Forward and Power Forward
##                                                                 425
##                         Small Forward and Power Forward and Center
##                                                                  49
##                Small Forward and Power Forward and Shooting Guard
##                                                                  33
##                                  Small Forward and Shooting Guard
##                                                                 588
##                  Small Forward and Shooting Guard and Point Guard
##                                                                  22
##                Small Forward and Shooting Guard and Power Forward
##                                                                  69
```

```
data_nba <- data_nba %>%
  mutate(
    position_center =
      case_when(position = str_detect(position,"Center") ~ 1,
                TRUE ~ 0),
    position_sf =
      case_when(position = str_detect(position,"Small Forward") ~ 1,
                TRUE ~ 0),
    position_pf =
      case_when(position = str_detect(position,"Power Forward") ~ 1,
                TRUE ~ 0),
    position_sg =
      case_when(position = str_detect(position,"Shooting Guard") ~ 1,
                TRUE ~ 0),
    position_pg =
      case_when(position = str_detect(position,"Point Guard") ~ 1,
                TRUE ~ 0))



data_nba <- data_nba %>%
  select("_id", name, age, weight, height, birthPlace, everything(), -position, -birthl
save(data_nba, file ="../datasets/nba/data_nba.RData")
```

Now, we want to create a variable that gives us the number of seasons (i.e.years) that each player played. Since the dataset is organized in seasons, each row is one season. Counting the rows per player gives us the years they played.

```
data_nba <- data_nba %>%
  group_by("_id") %>%
  mutate(seasons_played = n()) %>%
  ungroup()
```

Almost done. When we browse the dataset, we recognize that the height and weight variables are stored as characters. Let's convert them to numeric, so we can use them in operations.

```
str(data_nba$weight)
```

```
##  chr [1:9728] "162lb" "223lb" "223lb" "223lb" "223lb" "223lb" "223lb" ...
```

```
str(data_nba$height)
```

```
##  chr [1:9728] "6-1" "6-6" "6-6" "6-6" "6-6" "6-6" "6-6" "6-6" "6-6" "6-6" ...
```

```
data_nba <- data_nba %>%
  mutate(weight = str_replace(weight, "lb", ""),
         weight = as.numeric(weight),
         height = str_replace(height, "-", "."),
         height = as.numeric(height))

str(data_nba)
```

```
## tibble [9,728 x 36] (S3: tbl_df/tbl/data.frame)
##  $ _id            : chr [1:9728] "abdulma02" "abdulta01" "abdulta01" "abdulta01" ...
##  $ name           : chr [1:9728] "Mahmoud Abdul-Rauf" "Tariq Abdul-Wahad" "Tariq Abdul-Wahad"
##  $ age            : num [1:9728] 31 24 25 26 27 28 29 30 31 32 ...
##  $ weight         : num [1:9728] 162 223 223 223 223 223 223 223 223 223 ...
##  $ height         : num [1:9728] 6.1 6.6 6.6 6.6 6.6 6.6 6.6 6.6 6.6 6.6 ...
##  $ birthPlace     : chr [1:9728] "Gulfport, Mississippi" "Maisons Alfort, France" "Maisons Alf
##  $ index.x        : num [1:9728] 3 4 4 4 4 4 4 4 4 4 ...
##  $ career_AST     : num [1:9728] 3.5 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 ...
##  $ career_FG%     : chr [1:9728] "44.2" "41.7" "41.7" "41.7" ...
##  $ career_FG3%    : chr [1:9728] "35.4" "23.7" "23.7" "23.7" ...
##  $ career_FT%     : chr [1:9728] "90.5" "70.3" "70.3" "70.3" ...
##  $ career_G       : num [1:9728] 586 236 236 236 236 236 236 236 236 236 ...
##  $ career_PER     : chr [1:9728] "15.4" "11.4" "11.4" "11.4" ...
##  $ career_PTS     : num [1:9728] 14.6 7.8 7.8 7.8 7.8 7.8 7.8 7.8 7.8 7.8 ...
##  $ career_TRB     : chr [1:9728] "1.9" "3.3" "3.3" "3.3" ...
##  $ career_WS      : num [1:9728] 25.2 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 ...
##  $ career_eFG%    : chr [1:9728] "47.2" "42.2" "42.2" "42.2" ...
##  $ college        : chr [1:9728] "Louisiana State University" "University of Michigan, San Jos
##  $ draft_pick     : chr [1:9728] "3rd overall" "11th overall" "11th overall" "11th overall" ..
##  $ draft_round    : chr [1:9728] "1st round" "1st round" "1st round" "1st round" ...
##  $ draft_team     : chr [1:9728] "Denver Nuggets" "Sacramento Kings" "Sacramento Kings" "Sacra
##  $ draft_year     : chr [1:9728] "1990" "1997" "1997" "1997" ...
##  $ shoots         : chr [1:9728] "Right" "Right" "Right" "Right" ...
##  $ index.y        : num [1:9728] 17 19 20 21 22 23 24 25 26 27 ...
##  $ salary         : num [1:9728] 798500 1411000 1594920 4500000 5062500 ...
##  $ season         : chr [1:9728] "2000-01" "1998-99" "1999-00" "2000-01" ...
##  $ season_end     : num [1:9728] 2001 1999 2000 2001 2002 ...
##  $ season_start   : num [1:9728] 2000 1998 1999 2000 2001 ...
##  $ team           : chr [1:9728] "Vancouver Grizzlies" "Sacramento Kings" "Denver Nuggets" "De
##  $ position_center: num [1:9728] 0 0 0 0 0 0 0 0 0 0 ...
##  $ position_sf    : num [1:9728] 0 0 0 0 0 0 0 0 0 0 ...
##  $ position_pf    : num [1:9728] 0 0 0 0 0 0 0 0 0 0 ...
##  $ position_sg    : num [1:9728] 0 1 1 1 1 1 1 1 1 1 ...
##  $ position_pg    : num [1:9728] 1 0 0 0 0 0 0 0 0 0 ...
##  $ "_id"          : chr [1:9728] "_id" "_id" "_id" "_id" ...
##  $ seasons_played : int [1:9728] 9728 9728 9728 9728 9728 9728 9728 9728 9728 9728 ...
```

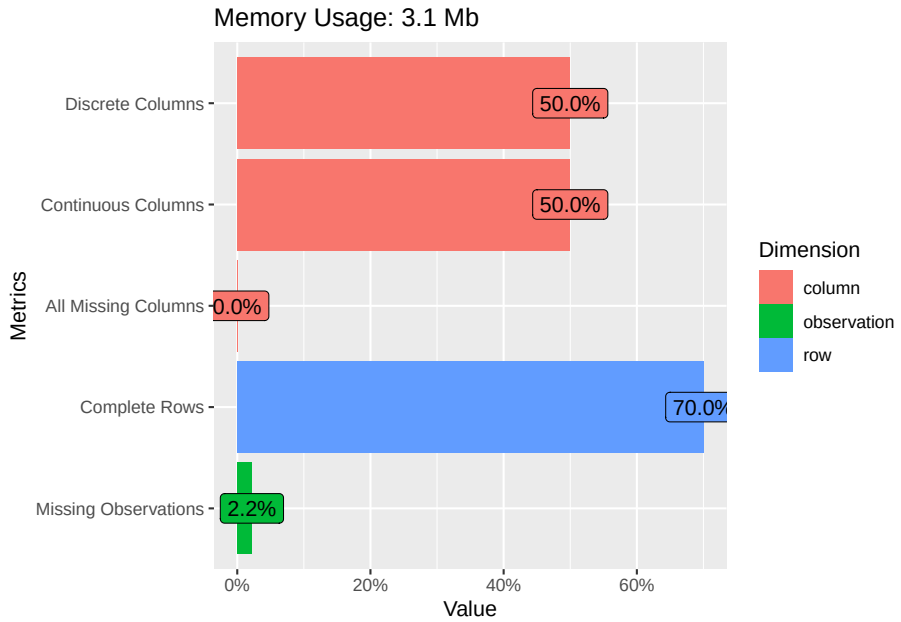
2.9 Explore the whole dataset

Now, let's explore some packages which help us to explore the dataframe as a whole. Let's start with the DataExplorer package. It is nice to get an overview of variables and the "missingness" of data.

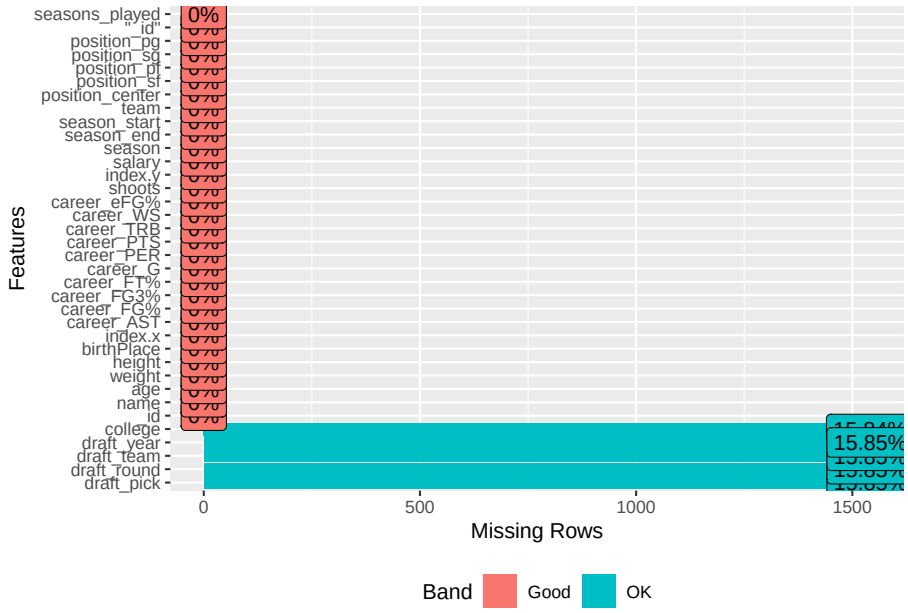
```
library(DataExplorer)

# overview of types of variables and missingness
introduce(data_nba)
```

```
## # A tibble: 1 x 9
##    rows columns discrete_columns continuous_columns all_missing_columns
##   <int>   <int>            <int>              <int>               <int>
## 1  9728      36               18                 18                   0
## # i 4 more variables: total_missing_values <int>, complete_rows <int>,
## #   total_observations <int>, memory_usage <dbl>
```

```
# plots the info from above
plot_intro(data_nba)
```

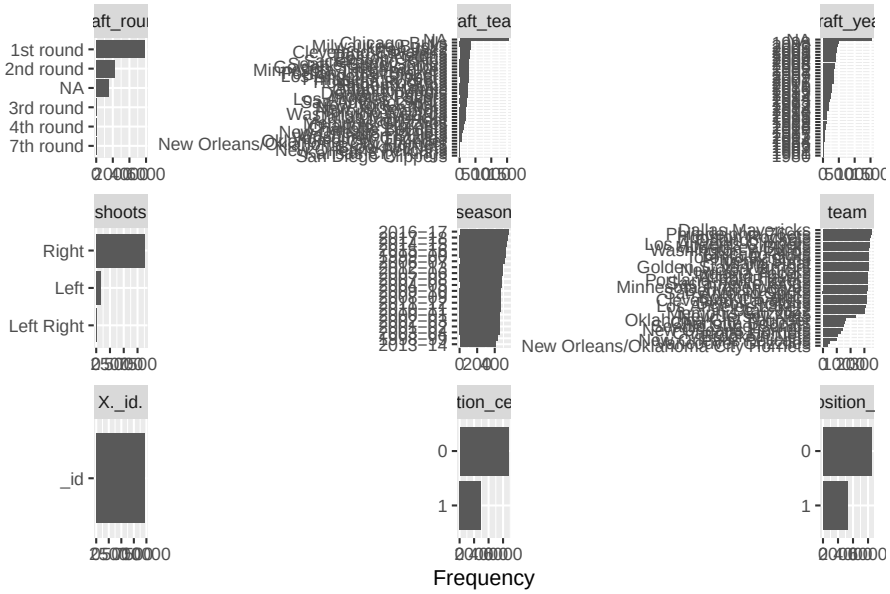


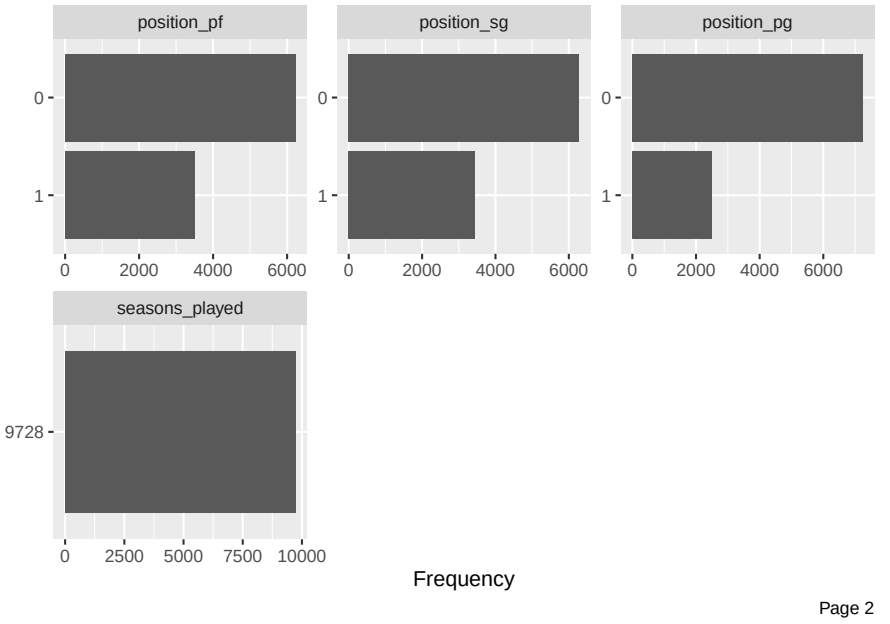

```
#plots percentages missing across variables
plot_missing(data_nba)
```




```
# plots frequencies across variables
plot_bar(data_nba)
```

```
## 11 columns ignored with more than 50 categories.
## X_id: 1794 categories
## name: 1790 categories
## birthPlace: 849 categories
## career_FG.: 333 categories
## career_FG3.: 297 categories
## career_FT.: 413 categories
## career_PER: 274 categories
## career_TRB: 113 categories
## career_eFG.: 310 categories
## college: 380 categories
## draft_pick: 67 categories
```





Now, let's try gtSummary for summary tables. A summary table is always a useful start once you have identified the type of variables you are interested in.

Let's assume we are interested in age, seasons played, career points, salary, position, and right or left handed (i.e. "shoots")

```
library(gtsummary)

data_nba %>%
  select(age, seasons_played, shoots, career_PTS, salary, contains("position")) %>%
         tbl_summary(
           statistic = all_continuous() ~ c("{mean} ({min}, {max})"))
```

```
## Table printed with `knitr::kable()`, not {gt}. Learn why at
## https://www.danieldsjoberg.com/gtsummary/articles/rmarkdown.html
## To suppress this message, include `message = FALSE` in code chunk header.
```

Characteristic	**N = 9,728**
age	27.0 (18.0, 42.0)
seasons_played	
9728	9,728 (100%)
shoots	
Left	856 (8.8%)
Left Right	7 (<0.1%)
Right	8,865 (91%)
career_PTS	8.9 (0.0, 30.1)
salary	4,072,633 (2,706, 34,682,550)
position_center	2,986 (31%)
position_sf	3,222 (33%)
position_pf	3,501 (36%)
position_sg	3,447 (35%)
position_pg	2,489 (26%)

Now, let's look at a correlation matrix between all numeric variables in the dataset.

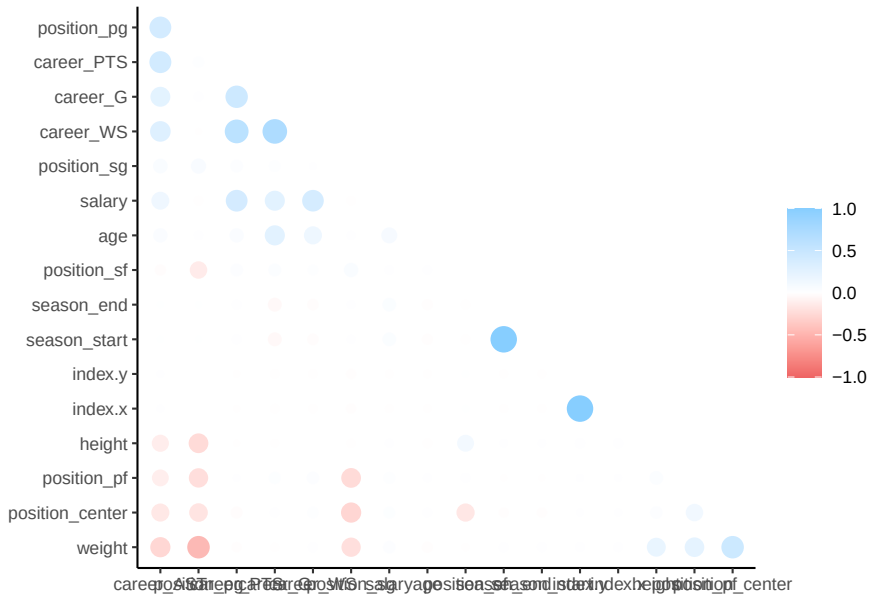
```
library("corrr")
library("corrplot")

data_nba_numeric <- data_nba %>%
  select(where(is.numeric)) %>%
  na.omit()

# Find constant columns
constant_columns <- sapply(data_nba_numeric, function(x) length(unique(x)) == 1)

# Remove constant columns
data_nba_numeric <- data_nba_numeric[, !constant_columns]
# as number
corrmatrix <- data_nba_numeric %>%
```

```
  correlate() %>%    # Create correlation data frame (cor_df)
  rearrange() %>%  # rearrange by correlations
  shave()

fashion(corrmatrix)
```

```
##              term career_AST position_pg career_PTS career_G career_WS
## 1       career_AST
## 2      position_pg        .61
## 3       career_PTS        .61         .08
## 4         career_G        .47         .04        .65
## 5        career_WS        .52        -.01        .76       .83
## 6      position_sg        .19         .22        .12       .09        .01
## 7           salary        .35        -.04        .60       .48        .59
## 8             age        .18         .03        .18       .49        .36
## 9      position_sf       -.08        -.33        .12       .13        .07
## 10      season_end        .01         .01        .04      -.17       -.09
## 11    season_start        .01         .01        .04      -.17       -.09
## 12         index.y        .01         .00       -.01      -.04       -.03
## 13         index.x        .01         .00       -.01      -.04       -.03
## 14          height       -.31        -.46       -.01      -.02       -.00
## 15     position_pf       -.29        -.44        .01       .11        .12
## 16 position_center       -.36        -.39       -.10       .04        .08
## 17          weight       -.49        -.66       -.06      -.03        .05
##    position_sg salary  age position_sf season_end season_start index.y index.x
## 1
## 2
## 3
## 4
## 5
## 6
## 7         -.03
## 8          .03    .25
## 9          .18    .03  .03
## 10         .02    .16 -.07        -.04
## 11         .02    .16 -.07        -.04       1.00
## 12        -.05   -.02 -.02         .01       -.03        -.03
## 13        -.06   -.02 -.02         .01       -.03        -.03    1.00
## 14        -.03    .03 -.04         .30        .02         .02     .05     .05
## 15        -.46    .09  .03         .04       -.00        -.00     .01     .01
## 16        -.49    .10  .04        -.37       -.06        -.06     .01     .02
## 17        -.43    .12 -.07        -.01        .04         .04     .04     .04
##    height position_pf position_center weight
## 1
## 2
```

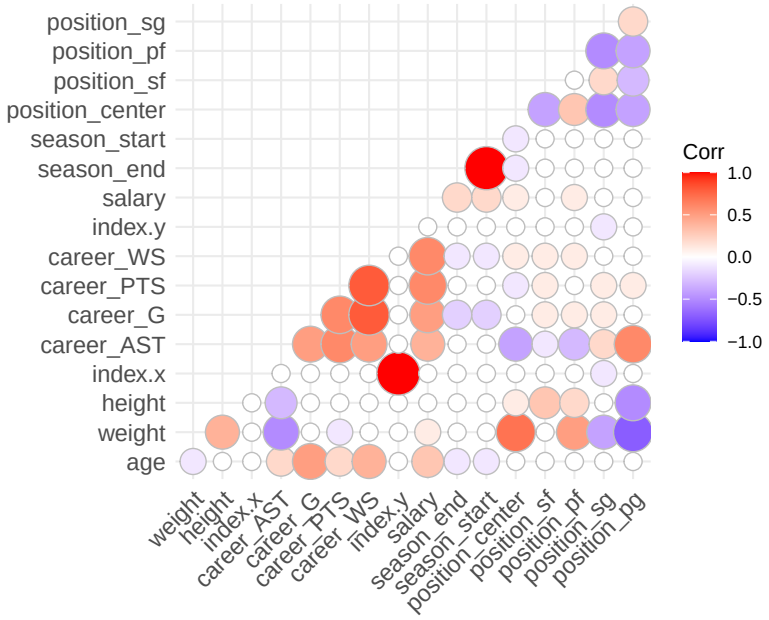
```
## 3
## 4
## 5
## 6
## 7
## 8
## 9
## 10
## 11
## 12
## 13
## 14
## 15    .15
## 16    .12         .33
## 17    .41         .45             .66
```

```
# as plot
rplot(corrmatrix)
```




```
# or...ggplot approach
library("ggcorrplot")

corrmatrix <- round(cor(data_nba_numeric), 1)
```

```
ggcorrplot(corrmatrix,
           method = "circle",
           type="lower")
```



This is interesting for a first look. For example, it seems that the weight is strongly correlated with whatever position you play. Centers are heavy, point guards are light weights. We also see that most performance metrics ("career_…") are correlated with each other and also with salary. Good players seem to be good in many things, and good players seem to be paid more.

2.10 explore individual variables

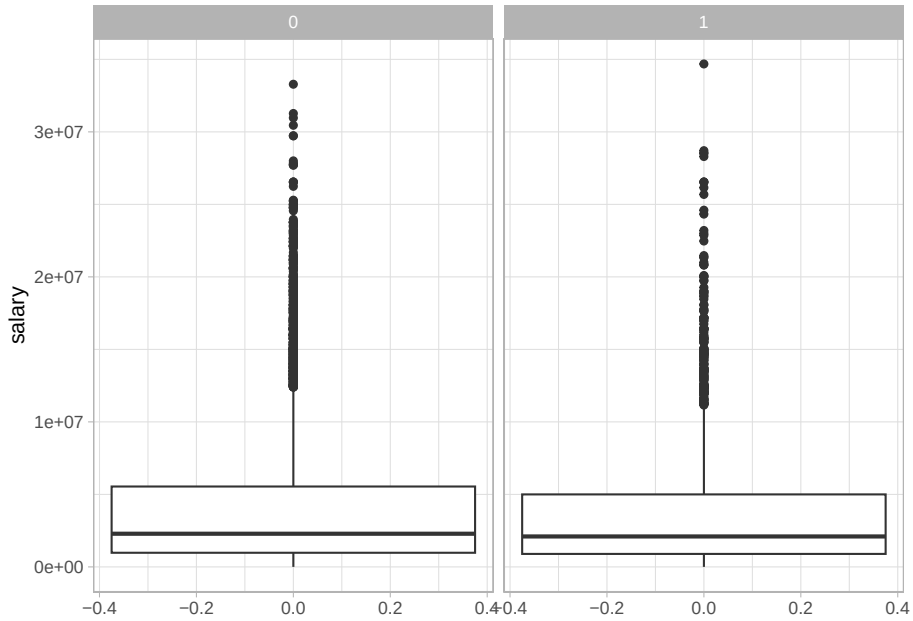
Now, that we have a feeling for the whole dataset, we want to explore individual variables. To keep it focused, we want to further explore the question of whether players that score more on average are also paid more.

Maybe roles are clearly divided on the team. Maybe really good passers are highly paid because they give great passes to people who then score. Or maybe teams don't care about passers and just pay more to people who score more.

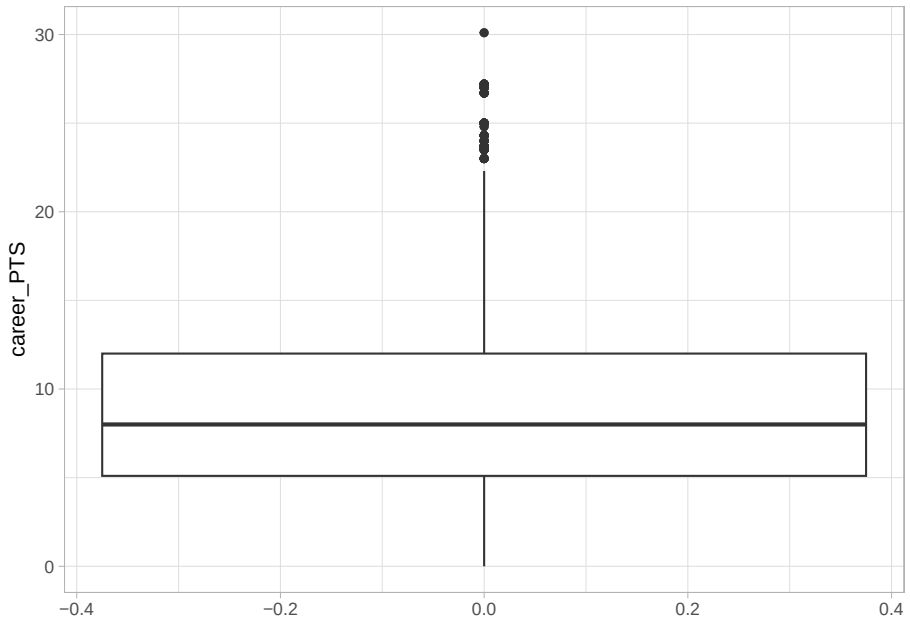
So, now, let's look at salary and average number of points scored by game. Also, we want to know whether point guards (short people who pass a lot) are paid less then other positions (who score more).

First, let's look how the two continuous variables are distributed:

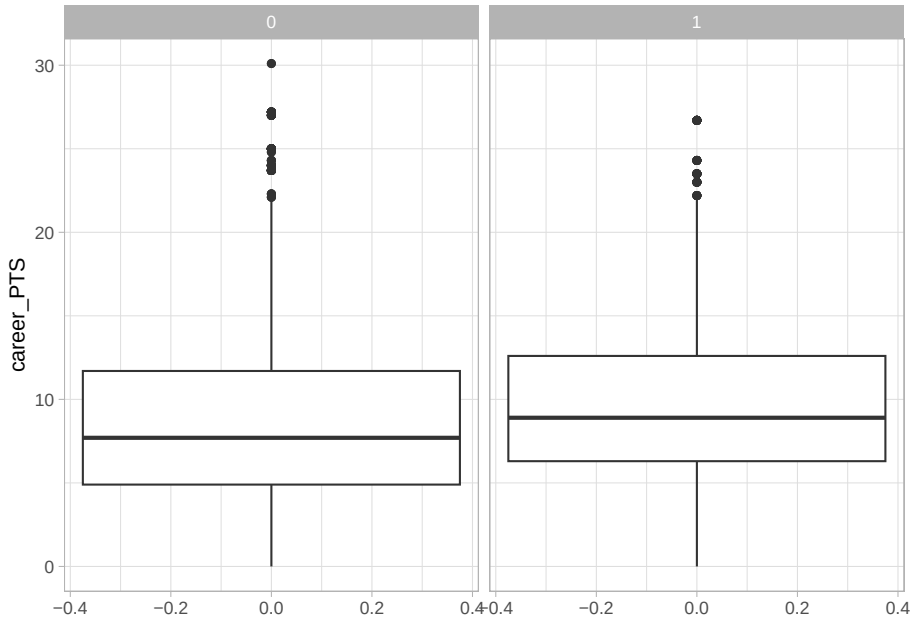
```
data_nba %>% ggplot() +
geom_boxplot(aes(y=salary)) +
  facet_wrap(~position_pg) +
  theme_light()
```




```
data_nba %>% ggplot() +
geom_boxplot(aes(y=career_PTS)) +
  theme_light()
```




```
data_nba %>% ggplot() +
geom_boxplot(aes(y=career_PTS)) +
  facet_wrap(~position_pg) +
  theme_light()
```



Let's look at the relationship between salary and position as well as the relationship between position and points.

```
str(data_nba$position_pg)
```

```
##  num [1:9728] 1 0 0 0 0 0 0 0 0 0 ...
```

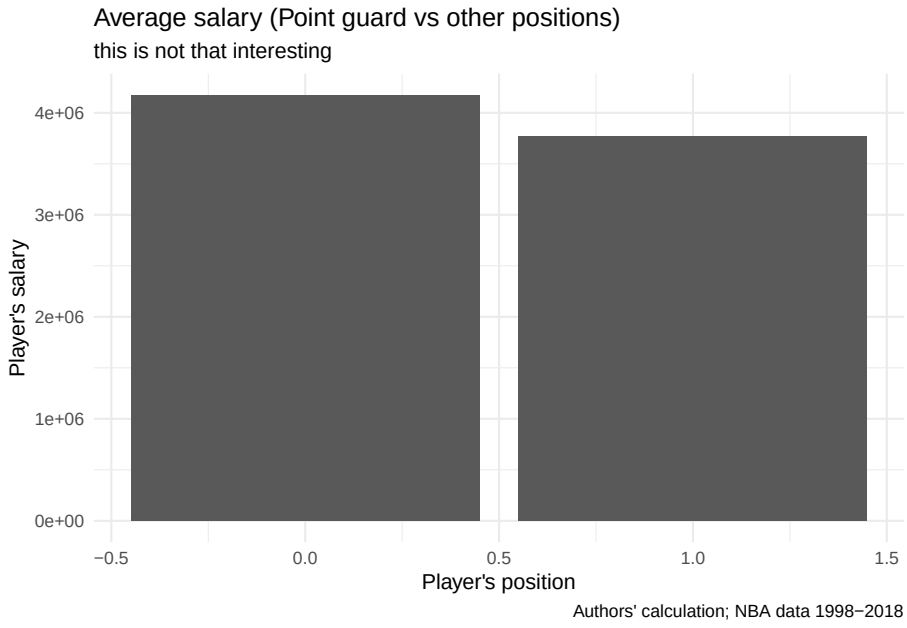
```
str(data_nba$salary)
```

```
##  num [1:9728] 798500 1411000 1594920 4500000 5062500 ...
```

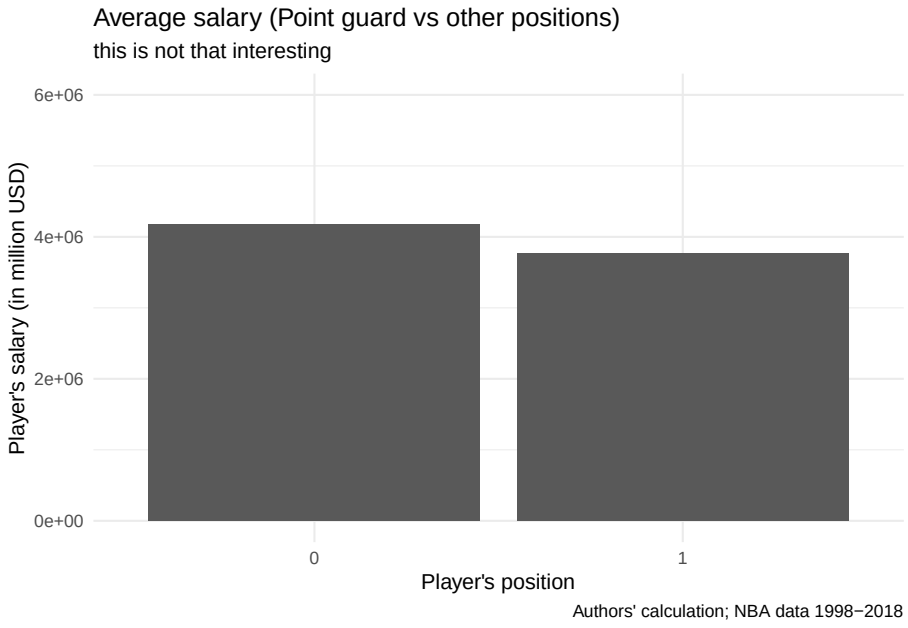
```
str(data_nba$career_pts)
```

```
##  NULL
```

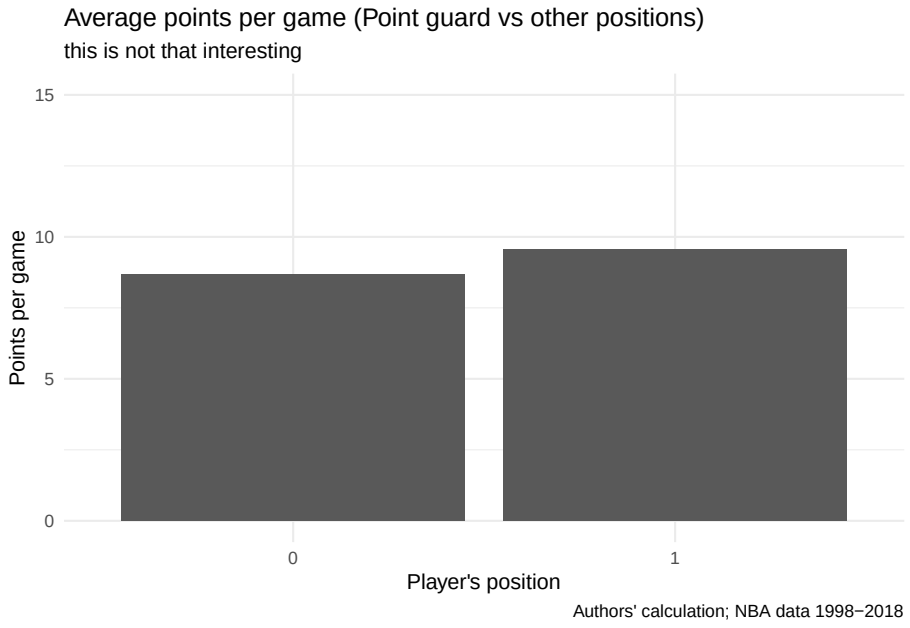
```
data_nba %>%
  ggplot() +
  geom_bar(aes(x = position_pg,
               y = salary),
           position = "dodge",
           stat = "summary",
           fun = "mean") +
  #xlim(0, 10000000) +
  labs(title = "Average salary (Point guard vs other positions)",
       subtitle = "this is not that interesting",
       caption = "Authors' calculation; NBA data 1998-2018") +
  xlab("Player's position") +
  ylab("Player's salary") +
  theme_minimal()
```




```
## alternatively:

data_nba %>%
  group_by(position_pg) %>%
  summarize(mean_salary = mean(salary, na.rm=T)) %>%
  ggplot() +
  geom_bar(aes(x = as.factor(position_pg),
               y = mean_salary),
           stat = "identity") +
  ylim(0, 6000000) +
  labs(title = "Average salary (Point guard vs other positions)",
       subtitle = "this is not that interesting",
       caption = "Authors' calculation; NBA data 1998-2018") +
  xlab("Player's position") +
  ylab("Player's salary (in million USD)") +
  theme_minimal()
```




```
# Now the relationship between position and points:

data_nba %>%
  group_by(position_pg) %>%
  summarize(mean_points = mean(career_PTS, na.rm=T)) %>%
  ggplot() +
  geom_bar(aes(x = as.factor(position_pg),
               y = mean_points),
           stat = "identity") +
  ylim(0,15) +
  labs(title = "Average points per game (Point guard vs other positions)",
       subtitle = "this is not that interesting",
       caption = "Authors' calculation; NBA data 1998-2018") +
  xlab("Player's position") +
  ylab("Points per game") +
  theme_minimal()
```

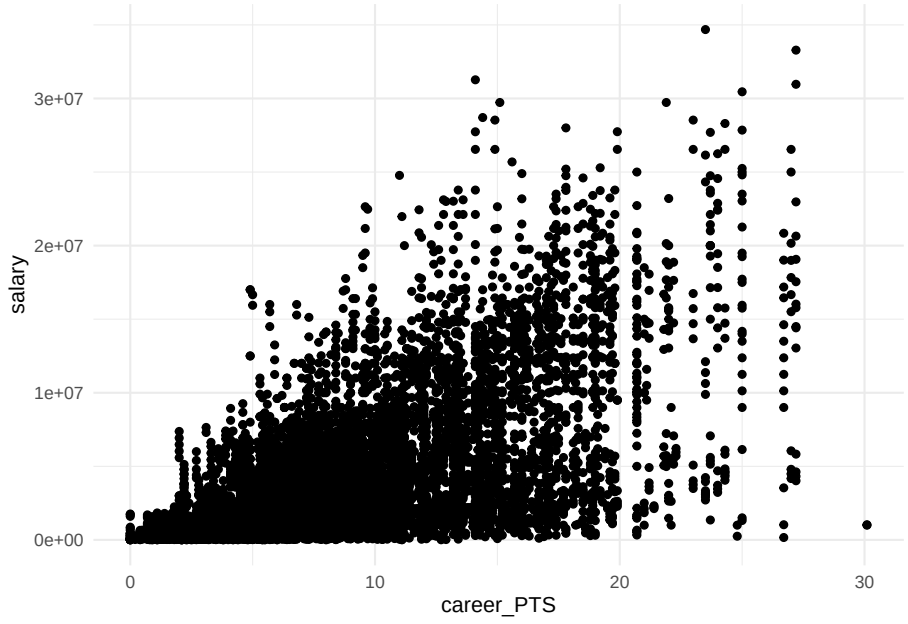


So, it seems that point gards are paid a little less even though they make a few more points on average. Interesting puzzle to explore.

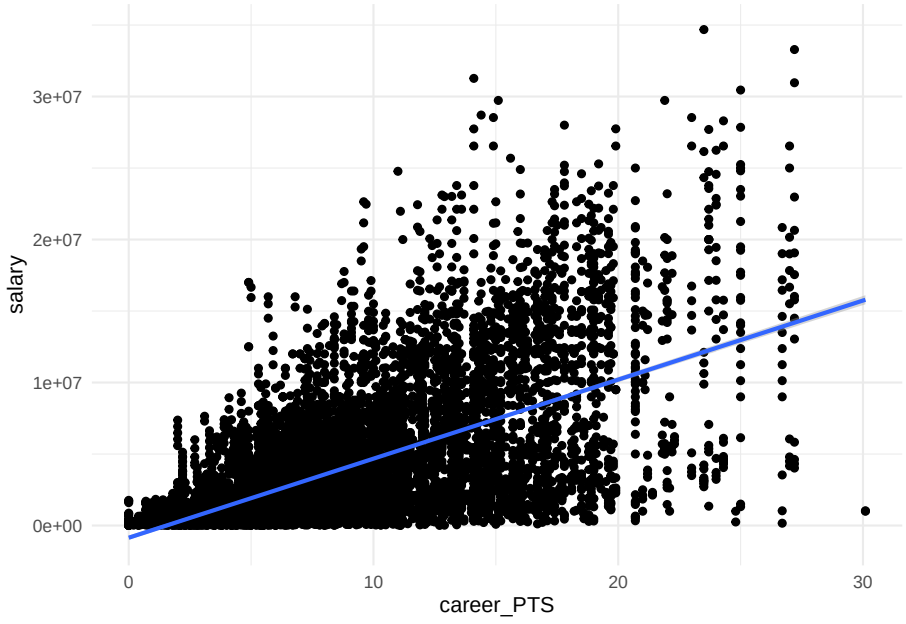
```
# find example for using histogramm()
# find example for first just creating cross tabs as perecntages tbl_cross(); summariz
```

Now, let's explore the relationship which is at the heart of our analysis from now on: salary and average points.

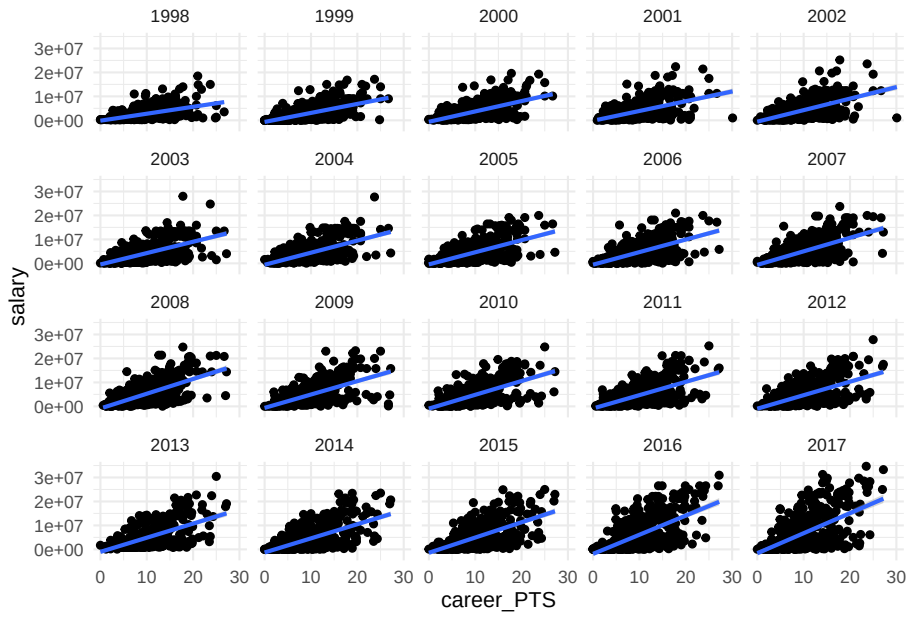
```
# simple scatterplot
data_nba %>% ggplot() +
geom_point(aes(y=salary, x=career_PTS)) +
  theme_minimal()
```



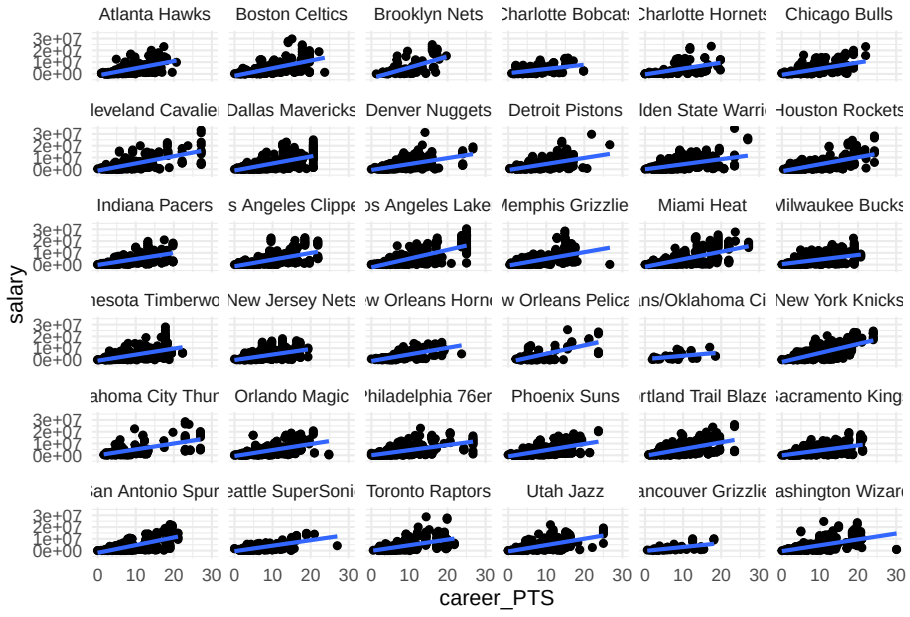

```
# Now, let's add a line
data_nba %>%
  ggplot(aes(y=salary, x=career_PTS)) +
    geom_point() +
     stat_smooth(method = "lm") +
       theme_minimal()
```




```
# let's look the relatiship separate for every year
data_nba %>%
  ggplot(aes(y=salary, x=career_PTS)) +
    geom_point() +
     stat_smooth(method = "lm") +
        facet_wrap(~season_start) +
       theme_minimal()
```

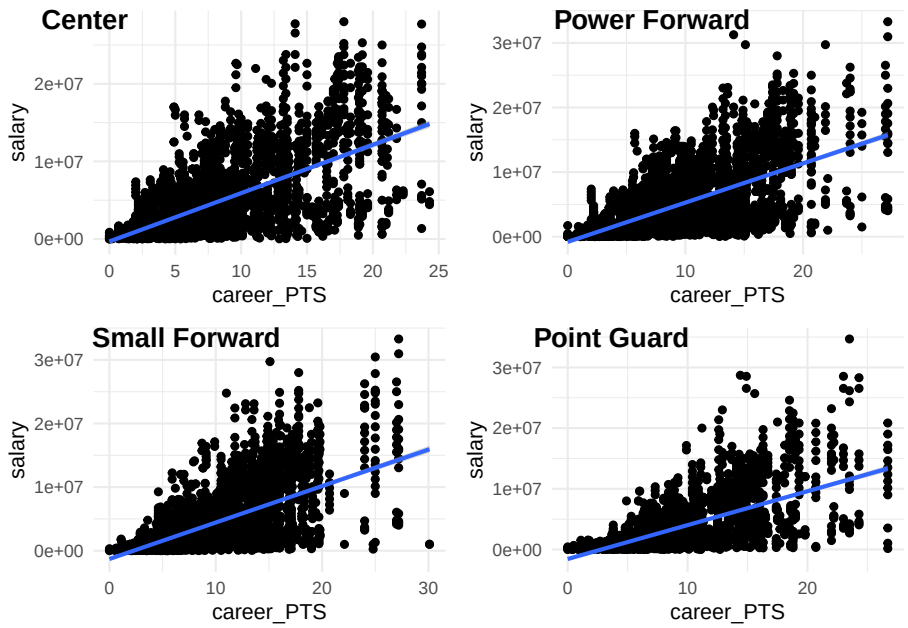



```
# let's look the relatiship for separate teams
data_nba %>%
  ggplot(aes(y=salary, x=career_PTS)) +
    geom_point() +
     stat_smooth(method = "lm") +
        facet_wrap(~team) +
       theme_minimal()
```

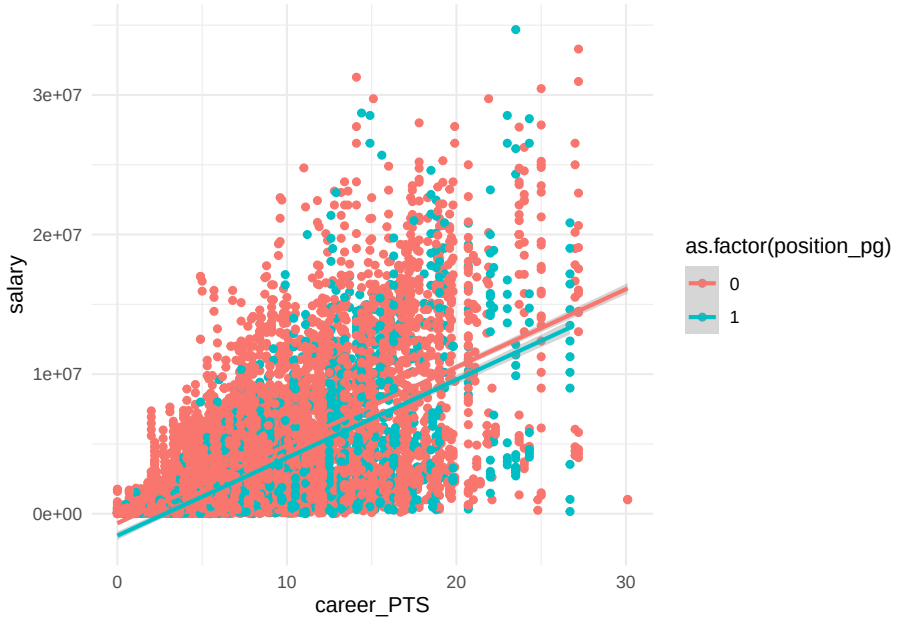



```
# let's look at it by position
scatter_pg <- data_nba %>% filter(position_pg ==1) %>%
  ggplot(aes(y=salary, x=career_PTS)) +
    geom_point() +
     stat_smooth(method = "lm") +
       theme_minimal()

scatter_center <- data_nba %>% filter(position_center ==1) %>%
  ggplot(aes(y=salary, x=career_PTS)) +
    geom_point() +
     stat_smooth(method = "lm") +
       theme_minimal()

scatter_pf <- data_nba %>% filter(position_pf ==1) %>%
  ggplot(aes(y=salary, x=career_PTS)) +
    geom_point() +
     stat_smooth(method = "lm") +
       theme_minimal()

scatter_sf <- data_nba %>% filter(position_sf ==1) %>%
  ggplot(aes(y=salary, x=career_PTS)) +
    geom_point() +
     stat_smooth(method = "lm") +
       theme_minimal()
```

```
library("ggpubr")
ggarrange(scatter_center, scatter_pf,
          scatter_sf, scatter_pg,
          ncol = 2, nrow = 2,
          labels = c("Center",
                     "Power Forward",
                     "Small Forward",
                     "Point Guard"))
```




```
# yet a better way to compare this could be:

data_nba %>%
  ggplot(aes(y=salary, x=career_PTS, color=as.factor(position_pg))) +
    geom_point() +
     stat_smooth(method = "lm",
                 aes(group=as.factor(position_pg))) +
       theme_minimal()
```




```
data_nba %>%
  ggplot(aes(y=salary, x=career_PTS, color=as.factor(position_center))) +
    geom_point() +
     stat_smooth(method = "lm",
                 aes(group=as.factor(position_center))) +
       theme_minimal()
```

Let's reflect a moment what we can learn from all this.

First, there seems to be a somewhat linear relationship between how many points a player scores and how much they are paid. This relationship seems pretty robust across years and teams. It also holds true for point guards just as much as for non-point guards. However, the average salary for point guards is lower in comparison. We also learn that the link between salary and points is stronger for centers. They seems to be paid more, the more they score.

We have set the stage now for linear regression (next week). Linear regression is all about further exploring the relationship between two variables, one outcome (often called "y") and one independent variable (often called "x"). Independent variables have many names. They are sometimes called "covariates"; "predictors"; "exposure", depending on the context.

There are a number of cool things that regression can do for us that simple EDA cannot:

1) It can model the relationship between two variables while considering simultaneously the potential influence of other factors. Image we are interested in the effect of points per game on salary regardless of position, season or team. With regression we can estimate how much more a player would ear every season if he scored 10 more points a game (regardless of the position he plays).

2) It can assess what explains the effect of one variable on another (i.e. mediation). Maybe we find that point guards earn less and we want to know

why. Is it because they score less? Is it because they play less time on average?

3) It can be used to predict salaries for players for whom we don't know the salary or even for hypothetical players. We could also look at the performance trend of players and predict whether they earn more next season or not.

Chapter 3

Exploratory Data Analysis - II

This week we will try to apply our last weeks knowledge into analysis.

3.1 Markdown Introduction

R Markdown is a powerful tool that allows you to create dynamic documents, presentations, and reports using R code. It combines the core syntax of markdown (an easy-to-write plain text format) with embedded R code chunks that are run when the document is rendered[1].

R Markdown documents are fully reproducible, meaning that anyone can re-run the code and generate the same results. This makes it easy to share your work with others and ensure that your results are accurate and reliable.

One of the great things about R Markdown is its flexibility. You can use it to create a wide variety of output formats, including HTML, PDF, and Microsoft Word documents. You can even create interactive documents with Shiny components[1].

To get started with R Markdown, you'll need to install the `rmarkdown` package from CRAN. This can be done by running the command `install.packages("rmarkdown")` in the R console[2]. Once you have the package installed, you can create a new R Markdown document in RStudio by going to `File > New File > R Markdown`.

An R Markdown document is made up of text written in markdown syntax and chunks of R code. When you render the document, the R code is executed and its output (such as plots or tables) is inserted into the final document[1].

When you render this document, the text and code will be combined to create an HTML file that includes both the markdown text and the output of the R code chunks.

You can easily add images to an R Markdown document using the standard markdown syntax for images. The basic syntax for adding an image is `![Alt text](image_url)`, where `Alt text` is the text that will be displayed if the image cannot be loaded, and `image_url` is the URL of the image you want to include.

I hope this helps you understand how to add images to an R Markdown document! Let me know if you have any further questions

R Markdown is a powerful tool that offers many advantages for data analysis and reporting. Some of the key benefits of using R Markdown include:

1. **Reproducibility**: R Markdown documents are fully reproducible, meaning that anyone can re-run the code and generate the same results. This makes it easy to share your work with others and ensure that your results are accurate and reliable.

2. **Flexibility**: R Markdown is incredibly flexible and can be used to create a wide variety of output formats, including HTML, PDF, and Microsoft Word documents. You can even create interactive documents with Shiny components.

3. **Ease of use**: R Markdown is easy to use, even for people with little or no programming experience. The core syntax of markdown is simple and intuitive, and the ability to embed R code directly into the document makes it easy to include dynamic content.

4. **Integration with R**: R Markdown is tightly integrated with R, making it easy to access and use the full power of the R language for data analysis and visualization.

5. **Collaboration**: R Markdown makes it easy to collaborate with others on data analysis projects. You can share your code and results with others, and they can easily reproduce your work and build on it.

Overall, R Markdown is a powerful tool that offers many advantages for data analysis and reporting. It's a great way to create dynamic, reproducible documents that are easy to share and collaborate on

I hope this introduction helps you understand what R Markdown is and how it can be used. If you want to learn more, there are many great resources available online, including the R Markdown website

3.2 Applying EDA(WVS/own data)

3.2.1 Exercise - 1

Boston Housing Dataset

Housing data contains 506 census tracts of Boston from the 1970 census. The dataframe BostonHousing contains the original data by Harrison and Rubinfeld (1979), the dataframe BostonHousing2 the corrected version with additional spatial information.

You can include this data by installing mlbench library:

```
#install.packages("mlbench") ## installing the library
library(mlbench) #adding the library
library(openxlsx)
data(BostonHousing2)
housing <- BostonHousing2
write.xlsx(housing, "../datasets/boston.xlsx")
```

1. **Read the dataset**: Read the Boston Housing Dataset from the Excel file.

```
library(readxl)
BostonHousing <- read_excel("../datasets/boston.xlsx")
```

2. **Inspect the dataset**: Use the proper functions to inspect the structure and contents of the dataset. How many Categorical variables are there? How many numerical variables are there? Is there any null values?

```
str(BostonHousing)
```

```
## tibble [506 x 19] (S3: tbl_df/tbl/data.frame)
##  $ town   : chr [1:506] "Nahant" "Swampscott" "Swampscott" "Marblehead" ...
##  $ tract  : num [1:506] 2011 2021 2022 2031 2032 ...
##  $ lon    : num [1:506] -71 -71 -70.9 -70.9 -70.9 ...
##  $ lat    : num [1:506] 42.3 42.3 42.3 42.3 42.3 ...
##  $ medv   : num [1:506] 24 21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 ...
##  $ cmedv  : num [1:506] 24 21.6 34.7 33.4 36.2 28.7 22.9 22.1 16.5 18.9 ...
##  $ crim   : num [1:506] 0.00632 0.02731 0.02729 0.03237 0.06905 ...
##  $ zn     : num [1:506] 18 0 0 0 0 0 12.5 12.5 12.5 12.5 ...
##  $ indus  : num [1:506] 2.31 7.07 7.07 2.18 2.18 2.18 7.87 7.87 7.87 7.87 ...
##  $ chas   : chr [1:506] "0" "0" "0" "0" ...
##  $ nox    : num [1:506] 0.538 0.469 0.469 0.458 0.458 0.458 0.524 0.524 0.524 0.524 ...
```

```
##  $ rm     : num [1:506] 6.58 6.42 7.18 7 7.15 ...
##  $ age    : num [1:506] 65.2 78.9 61.1 45.8 54.2 58.7 66.6 96.1 100 85.9 ...
##  $ dis    : num [1:506] 4.09 4.97 4.97 6.06 6.06 ...
##  $ rad    : num [1:506] 1 2 2 3 3 3 5 5 5 5 ...
##  $ tax    : num [1:506] 296 242 242 222 222 222 311 311 311 311 ...
##  $ ptratio: num [1:506] 15.3 17.8 17.8 18.7 18.7 18.7 15.2 15.2 15.2 15.2 ...
##  $ b      : num [1:506] 397 397 393 395 397 ...
##  $ lstat  : num [1:506] 4.98 9.14 4.03 2.94 5.33 ...
```

```
glimpse(BostonHousing)
```

```
## Rows: 506
## Columns: 19
## $ town    <chr> "Nahant", "Swampscott", "Swampscott", "Marblehead", "Marblehea~
## $ tract   <dbl> 2011, 2021, 2022, 2031, 2032, 2033, 2041, 2042, 2043, 2044, 20~
## $ lon     <dbl> -70.9550, -70.9500, -70.9360, -70.9280, -70.9220, -70.9165, -7~
## $ lat     <dbl> 42.2550, 42.2875, 42.2830, 42.2930, 42.2980, 42.3040, 42.2970,~
## $ medv    <dbl> 24.0, 21.6, 34.7, 33.4, 36.2, 28.7, 22.9, 27.1, 16.5, 18.9, 15~
## $ cmedv   <dbl> 24.0, 21.6, 34.7, 33.4, 36.2, 28.7, 22.9, 22.1, 16.5, 18.9, 15~
## $ crim    <dbl> 0.00632, 0.02731, 0.02729, 0.03237, 0.06905, 0.02985, 0.08829,~
## $ zn      <dbl> 18.0, 0.0, 0.0, 0.0, 0.0, 0.0, 12.5, 12.5, 12.5, 12.5, 12.5, 1~
## $ indus   <dbl> 2.31, 7.07, 7.07, 2.18, 2.18, 2.18, 7.87, 7.87, 7.87, 7.87, 7.~
## $ chas    <chr> "0", "0", "0", "0", "0", "0", "0", "0", "0", "0", "0", "0", "0~
## $ nox     <dbl> 0.538, 0.469, 0.469, 0.458, 0.458, 0.458, 0.524, 0.524, 0.524,~
## $ rm      <dbl> 6.575, 6.421, 7.185, 6.998, 7.147, 6.430, 6.012, 6.172, 5.631,~
## $ age     <dbl> 65.2, 78.9, 61.1, 45.8, 54.2, 58.7, 66.6, 96.1, 100.0, 85.9, 9~
## $ dis     <dbl> 4.0900, 4.9671, 4.9671, 6.0622, 6.0622, 6.0622, 5.5605, 5.9505~
## $ rad     <dbl> 1, 2, 2, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, 4, 4, 4, 4, 4, 4, 4, 4,~
## $ tax     <dbl> 296, 242, 242, 222, 222, 222, 311, 311, 311, 311, 311, 311, 31~
## $ ptratio <dbl> 15.3, 17.8, 17.8, 18.7, 18.7, 18.7, 15.2, 15.2, 15.2, 15.2, 15~
## $ b       <dbl> 396.90, 396.90, 392.83, 394.63, 396.90, 394.12, 395.60, 396.90~
## $ lstat   <dbl> 4.98, 9.14, 4.03, 2.94, 5.33, 5.21, 12.43, 19.15, 29.93, 17.10~
```

```
# Check for missing values in all columns
colSums(is.na(BostonHousing))
```

```
##    town   tract     lon     lat    medv   cmedv    crim      zn   indus    chas
##       0       0       0       0       0       0       0       0       0       0
##     nox      rm     age     dis     rad     tax ptratio       b   lstat
##       0       0       0       0       0       0       0       0       0
```

3. **Summarize categorical variables**: Create frequency table for categorical variables CHAS (Charles River dummy variable).

```
table(BostonHousing$chas)
```

```
##
##   0   1
## 471  35
```

4. **Summarize numerical variables**: Generate summary statistics for numerical variables CRIM (per capita crime rate by town) and RM (average number of rooms per dwelling).

```
summary(BostonHousing$crim)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
##   0.00632   0.08204   0.25651   3.61352   3.67708 88.97620
```

```
summary(BostonHousing$rm)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   3.561   5.886   6.208   6.285   6.623   8.780
```

5. **Create new variables**: Create new variable indicating whether a house is located near the Charles River or not.

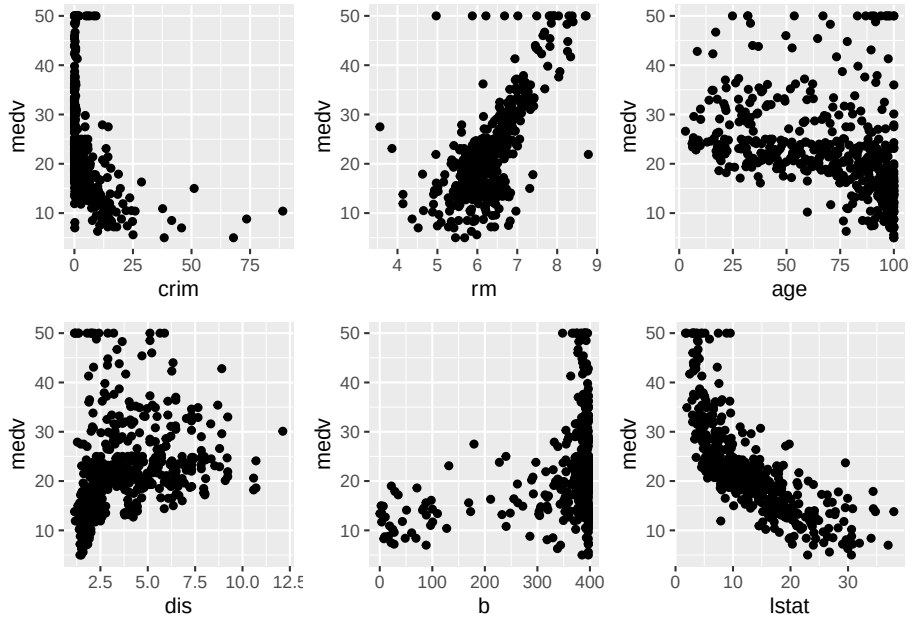
```
library(dplyr)
BostonHousing <- BostonHousing %>%
  mutate(near_river = ifelse(chas == 1, "Yes", "No"))
```

6. **Recoding variables**: Create a new variable that groups houses by their proximity to employment centers.

```
BostonHousing <- BostonHousing %>%
  mutate(distance_group = case_when(
    dis < 2 ~ "Near",
    dis >= 2 & dis < 5 ~ "Medium",
    dis >= 5 ~ "Far"
  ))
```

7. **Visualize relationships**: Create scatter plots to explore the relationship between housing prices and other numeric variables. Interpret the plots. Also show the distribution of ptratio by distance_group.

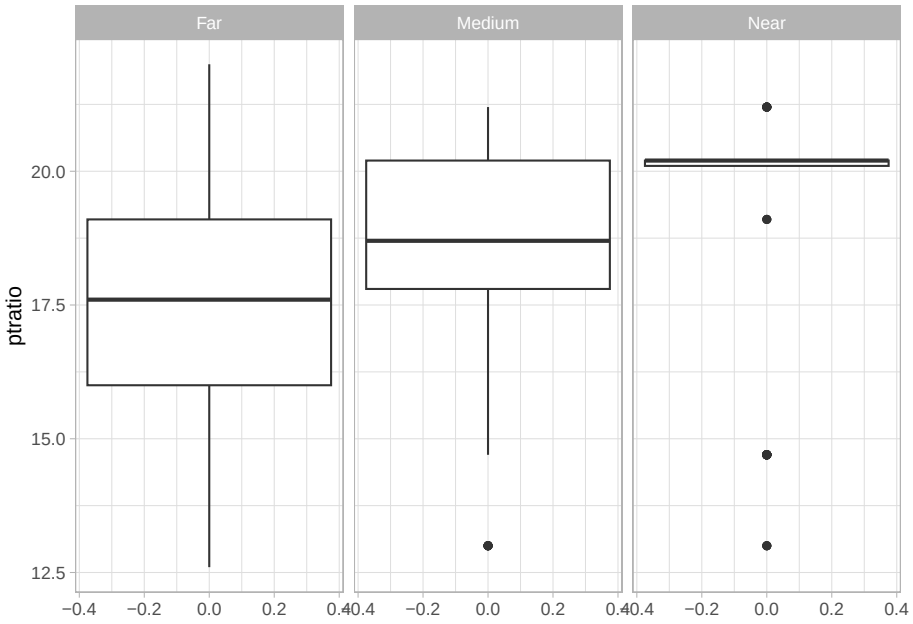
```
# Create a list of numeric variables to plot against MEDV
vars <- c("crim", "rm", "age", "dis", "b", "lstat")

# Create a scatter plot for each variable
plots <- lapply(vars, function(var) {
  ggplot(BostonHousing, aes_string(x = var, y = "medv")) +
    geom_point() +
    labs(x = var, y = "medv")
})
library(gridExtra)
# Combine the plots into a single plot
combined_plot <- do.call(grid.arrange, c(plots, ncol = 3))
```




```
# Display the combined plot
combined_plot
```

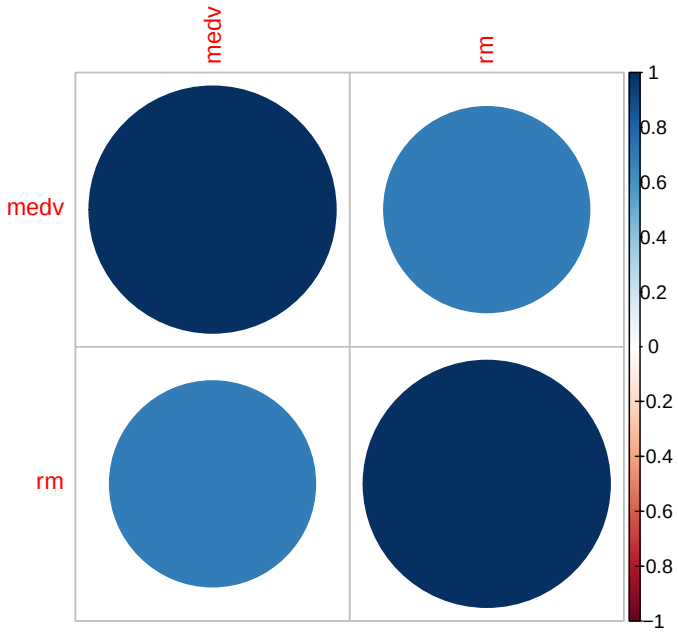
```
## TableGrob (2 x 3) "arrange": 6 grobs
##   z     cells    name           grob
## 1 1 (1-1,1-1) arrange gtable[layout]
## 2 2 (1-1,2-2) arrange gtable[layout]
## 3 3 (1-1,3-3) arrange gtable[layout]
## 4 4 (2-2,1-1) arrange gtable[layout]
## 5 5 (2-2,2-2) arrange gtable[layout]
## 6 6 (2-2,3-3) arrange gtable[layout]
```

```
# Create a boxplot of PTRATIO grouped by distance_group
BostonHousing %>% ggplot() +
geom_boxplot(aes(y=ptratio)) +
  facet_wrap(~distance_group) +
  theme_light()
```

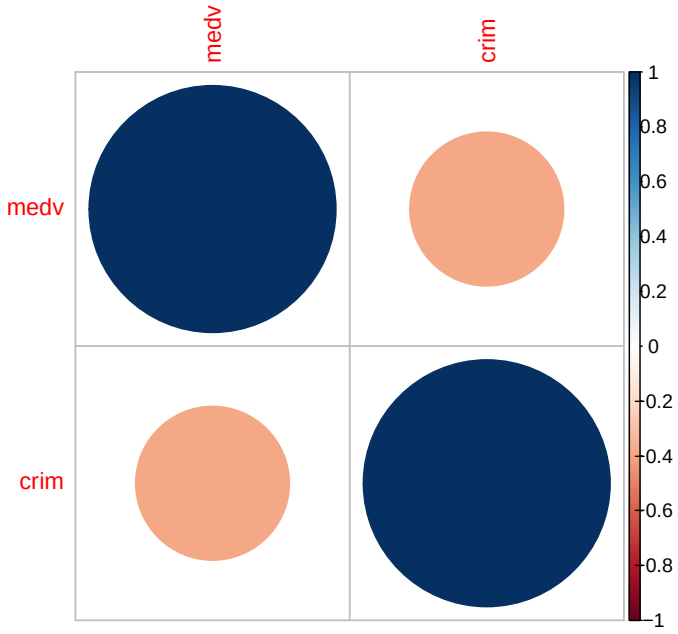


8. **Correlation analysis**: Calculate correlation coefficients between housing prices and average number of rooms per dwelling, also between housing prices and crime rate. Interpret the result.

```
library(corrplot)
corr_matrix0 <- cor(BostonHousing[, c("medv", "rm")])
corr_matrix1 <- cor(BostonHousing[, c("medv", "crim")])
corrplot(corr_matrix0)
```




```
corrplot(corr_matrix1)
```



This is a draft of questions, dataframe and the variables will be specified later

Chapter 4

DAGs

- Before Linear Regression
- 2 main reasons to model
  - understand causal effects -> DAGs
  - predict -> later

4.1 Causality

- Correlation does not equal Causation
  - Examples
- Identification of (causal) effect(s) of interest

4.2 DAGs

- Indirect causal effect/Overcontrol Bias/Pipe
- Confounders/Fork
- Colliders
- Each with individual examples
  - Maybe one big example

4.3 Total/Direct effect

- Identification

4.4 dagitty.net

Chapter 5

Linear Regression Theory I: Simple Linear Regression

5.1 What is Linear Regression

When we use statistical modelling in social sciences there are two main approaches. The more classical approach is to use modelling for estimating the effect that one or several independent variables have on one dependent variable. Maybe we are interested in knowing if a higher income has an effect on life satisfaction and if yes, what the direction and magnitude of this effect is. Does more money actually make you happier?

The other and more recent approach is to use modelling for making predictions with high accuracy. Based on the relationships between many independent variables and one dependent variable, we try to predict the latter for actual or hypothetical cases and their individual values for the independent variables. This approach lies at the heart of *machine learning* and drives many of the technologies we use on a daily basis from E-Mail spam filters to ChatGPT.

Linear regression is one of the many available modelling techniques and it can serve both approaches lined out above. In this session we will focus on using linear regression for estimating an effect of interest but we will return to prediction at a later point in this course.

But how do we know if we should choose linear regression for a specific task? This is not easy to answer as there are many alternatives and even variations of linear regression which may be better suited for a specific empirical problem. As this is an introduction to modelling and time is of the essence we opted to reduce the options and focus on two kind of models over the next weeks. Linear regression and logistic regression. Both are comparably easy to understand and use. Also, if we understand both of these techniques, we are in a good position

to build upon our knowledge and learn all of those more complex and specific models that we will encounter in textbooks and scientific papers.

With the pool of options trimmed down to two, the question remains unanswered. Should I use linear or logistic regression for my task? But now the answer is relatively straightforward. What is the type of our dependent variable? Is it metric? Then we use linear regression. Is it binary or categorical? Then we use logistic regression. Linear regression will be the focus of this and the next weeks and then we will turn to logistic regression.

Now let us dive in and learn what linear regression is all about.

5.2 Examplary research question & data

Let us imagine that we are interested in a research question that asks what makes a good grade in a seminar paper. In particular we are interested in the effect that the hours a student invests in working on it has on the grade. Based on some theoretical considerations, and maybe some idealistic views, we derive our main hypotheses that putting in more hours will result in a better grade.

Now we also - hypothetically - held a small survey and asked 200 imaginary students some questions on how they approached writing a seminar paper. In particular we asked them how much time they spent working on the paper, if they have attended (almost) all seminar sessions, how closely they worked with their lecturers in preparing the paper and what the mean grade for previous papers was. As these imaginary students have already turned in their papers, we also know the grades they achieved.

Please note, that this is data on **imaginary** students, meaning we have simulated the data making some assumptions on how to achieve a good (or bad) grade in a paper. The assumptions we made do not necessarily reflect the way *you* write a good paper while still being based in our experience on what it takes to achieve a good grade. But remember, no real students were harmed in making up this example data.

Let us have a first look on the data: XXX ALIGN WITH EDA XXX

```
## Warning: package 'skimr' was built under R version 4.2.3
```

```
##
## Attaching package: 'skimr'
```

```
## The following object is masked from 'package:corrr':
##
##     focus
```

```
## Warning: package 'knitr' was built under R version 4.2.3
```

skim_type	skim_variable	n_missing	complete_rate	factor.ordered	factor.n_unique	factor
factor	contact	0	1	FALSE	3	No :
logical	attendance	0	1	NA	NA	NA
numeric	grade	0	1	NA	NA	NA
numeric	hours	0	1	NA	NA	NA
numeric	previous_grades	0	1	NA	NA	NA
numeric	previous_grades_centered	0	1	NA	NA	NA
numeric	hours_centered	0	1	NA	NA	NA

Right now, the observations are ordered by the grade of the seminar paper which run from 1.0 to 5.0 in increments of 0.1. While this is somewhat unrealistic - the german grading system actually only uses theincrements .0, .3 and .7 - simulating the data in this way will make the demonstrations on linear regression easier and more straightforward. The variable `previous_grades` is set up in the same way and represents the mean of the grades the student received up to this point `hours` represents the time a student spent on writing the paper, ranging from $23-57$ hours, with a mean of about 40. Besides these metric variables, the data set also contains two categorical measures. `attendance` is a *dummy variable*, meaning it can only have the values 1 or 0 or `TRUE` and `FALSE` in this case, as it is saved as a logical variable. `TRUE` represents that a student attended almost all seminar sessions before writing the paper - which about 77 did -, `FALSE` states that they did not. `contact` is a factor variable with three categories and shows the answers to the imaginary question on how much contact the student had to the lecturer before starting the writing process. Besides `No contact` the students could have had `E-Mail` contact to state ther research question and get some short written feedback or meet the lecturer `In Person` to achieve a deeper discussion of the question and laid out plan for writing the paper. The two additional variables are versions of `previous_grades` and `hours` that are centered on their respective means. They will come into play at a later point in this session.

Let's have a look at some observations.

```
## # A tibble: 10 x 7
##    grade hours previous_grades attendance contact   previous_grades_centered
##    <dbl> <int>           <dbl> <lgl>      <fct>                       <dbl>
##  1     1    50             1.4 TRUE       E-Mail                     -1.54
##  2     1    46             1   TRUE       E-Mail                     -1.94
##  3     1    42             1   TRUE       In Person                  -1.94
##  4     1    49             1   FALSE      In Person                  -1.94
##  5     1    42             1.2 TRUE       In Person                  -1.74
##  6     1    46             1.8 TRUE       In Person                  -1.14
##  7     1    44             1.4 FALSE      In Person                  -1.54
##  8     1    45             2   TRUE       In Person                  -0.935
```

```
##  9     1    48             1   TRUE       In Person                    -1.94
## 10     1    45             2   TRUE       In Person                    -0.935
## # i 1 more variable: hours_centered <dbl>
```

From this first 10 rows, we can see that the students with the best grades spent more than 40 hours on writing, have already achieved good grades in their papers up to this point and at least had some contact to the lecturers. Most also regularly attended the seminar but two did not and still achieved a 1.0 in their grade.

So what makes a bad grade?

```
## # A tibble: 10 x 7
##    grade hours previous_grades attendance contact    previous_grades_centered
##    <dbl> <int>           <dbl> <lgl>      <fct>                         <dbl>
##  1   4.8    37             4.2 TRUE       No contact                    1.27
##  2   4.8    38             4.3 TRUE       E-Mail                        1.36
##  3   4.8    35             4.4 TRUE       E-Mail                        1.47
##  4   4.9    40             4.2 TRUE       E-Mail                        1.27
##  5   5      35             3.9 FALSE      No contact                    0.965
##  6   5      41             4.9 TRUE       No contact                    1.97
##  7   5      24             4.7 TRUE       E-Mail                        1.76
##  8   5      33             5   TRUE       E-Mail                        2.06
##  9   5      29             4.1 FALSE      E-Mail                        1.16
## 10   5      50             4.6 FALSE      E-Mail                        1.66
## # i 1 more variable: hours_centered <dbl>
```

Here the picture seems less clear. While most students did not put in as many hours, some did and still failed to pass. Half of the students that received a 5.0 regularly attended and most at least had E-Mail contact before writing their paper. What seems to be more consistent though is that the mean of the previous grades is rather low.

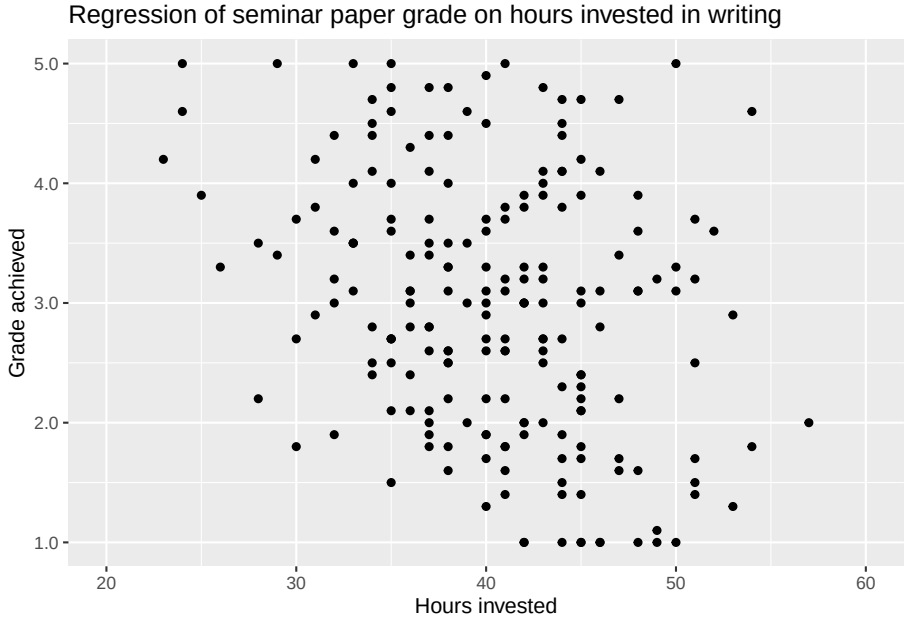
So what do we know now? Does a good or bad track record in grades predict all future grades? This seems not only unrealistic but also a kind of sad take home message. To get a better understanding on which of the potential influential variables had an effect on the final grade and what the magnitude and direction of these effects was, we now turn to linear regression.

5.3 Simple Linear Regression

In a *simple linear regression*, the model is used to describe the relationship between *one* dependent and *one* independant or explanatory variable. The question this model can answer for us is, by how much does the dependent variable increase or decrease, when the explanatory variable increases by 1?

Returning to our exemplary research question on what makes a good grade in a seminar paper an intuitive hypotheses would be, that the grade gets better the more hours a student invests in writing the paper. In this case we assume a linear relationship between the independent variable `hours` and the dependent variable `grade`. As german grades are better the lower their value, we thus would assume a negative effect from `hours` on `grade`.

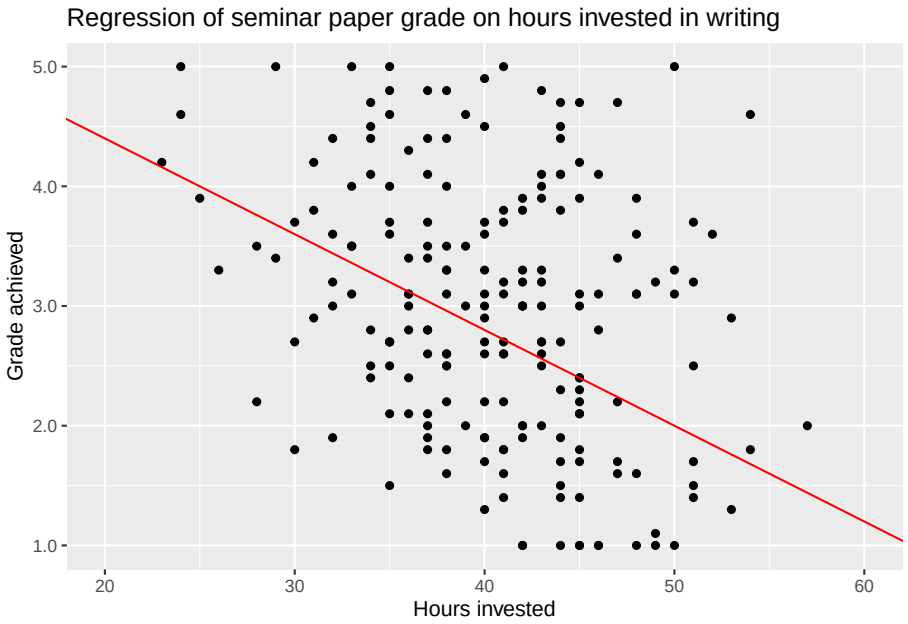
Before turning to the formalities and practical application of a simple linear regression model, let us first have a look on this relationship by plotting the variables against each other.



When we are talking about dependent and independent variables, there is the convention to plot former on the x-axis and the latter on the y-axis. So the *y-variable* is to be explained and the *x-variable* is used to explain it. This convention will also be used in all formulas in this seminar.

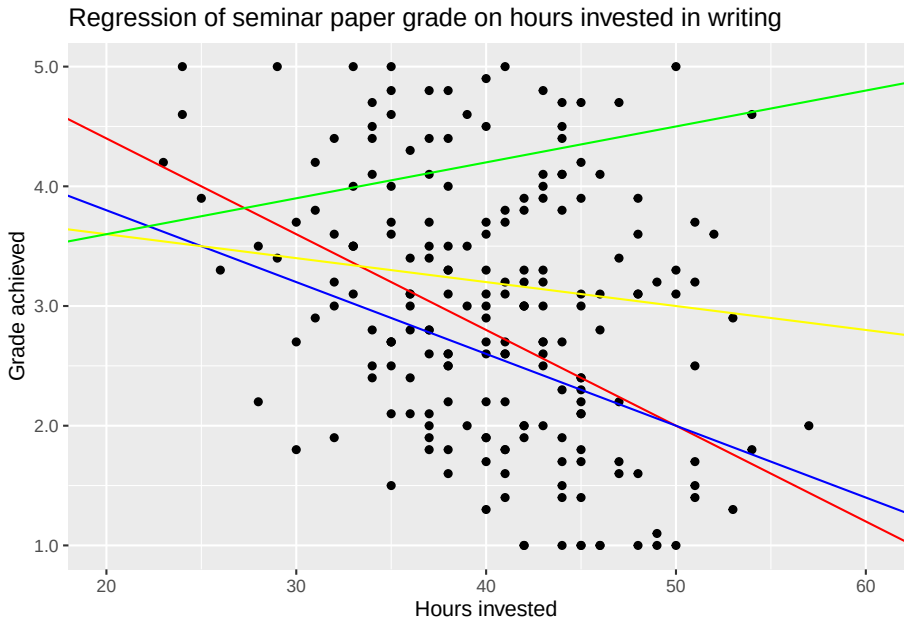
Looking at the plot we first see a cloud of dots, representing all combinations of `hours` and `grade` in all our 200 observations. It may be hard to pick out any pattern, but looking closely we can observe that overall the dots seem to follow a downward slope from the upper left - indicating few hours worked and a worse grade - towards the lower right - indicating more invested hours and a better grade. This would be the relationship stated in our hypotheses. The more hours a student works on a seminar paper the better the final grade will be.

We can try to describe this pattern by adding a line from the upper left to the lower right.



This describes the realtionship between the two variables as linear. Each hour invested decreases the grade by a certain amount, for this proposed line by exactly 0.08 points. Remember that decreasing the value of the grade actually means getting a better grade.

But is this the only possible line or even the *correct* one? Most certainly not as the values used to draw the line were only a wild guess by the authors. We could imagine several other lines that also look more or less reasonable - as well as some that look unreasonable - and add them to the plot.



While we have some intution, that especially the green line misses the mark by a lot, we can't really decide between the others just by looking at the plot. The data points are way to dispersed to see the relationship clearly.

The goal of using a simple linear regression model is to identify the *one* line that describes the relationship the best. The *best* meaning, with as little error as possible.

XXX OVERLAPPING DOTS? XXX

5.3.1 Regression Formula

To understand how these lines in the above plot were conceived and how to find the line with the best *fit*, i.e. the lowest error, we have to understand the formular for linear regression. While formulas may always be kind of daunting, we are in luck as tis particular one is actually quite easy to understand, especially when paired with a graphical representation.

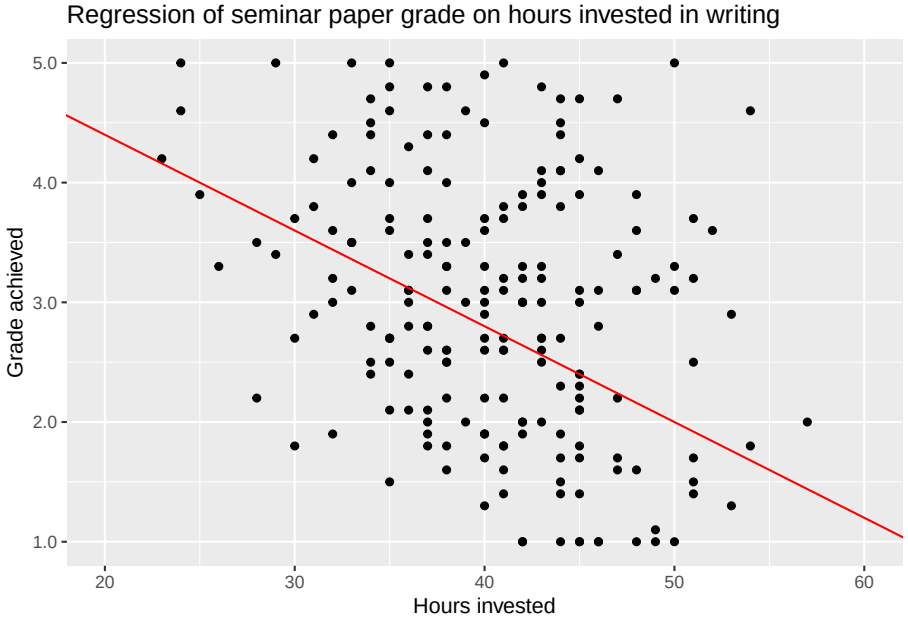
$$y = \beta_0 + \beta_1 * x_1 + \epsilon$$

Let us first look at the parts we already know. y is the dependent variable, in our case the grade achieved. So one thing is for sure, the whole right part of the equation has to be used to calculate the value of y from the data, i.e. the dependent variable x. Here we have three terms. Let us skip the first one for now and focus on the second one $\beta_1 * x_1$.

x_1 is the dependent variable, in our case `hours`. β_1 is the *redression coefficient* for x_1. This value gives us the *slope* of the regression line. Based on this, we can start rewriting the general formula and tailor it to our specific use case.

$$y_{grade} = \beta_0 + \beta_{hours} * x_{hours} + \epsilon$$

Let us return to the first wild guess we made above.

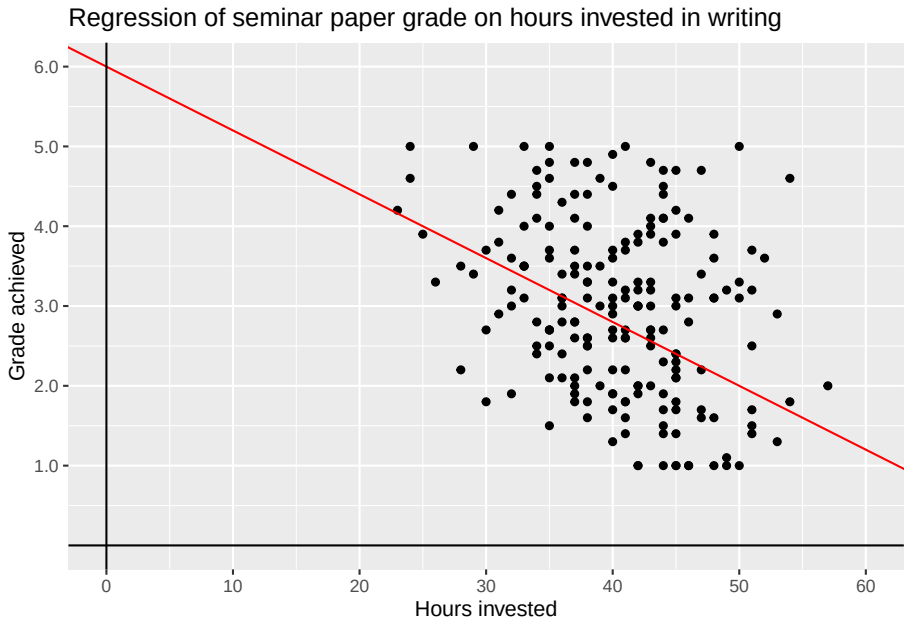


Here we guessed, that an increase in time invested of one hour decreases the value of `grade` by 0.08. This is the slope of the red line and thus also the coefficient in the regression formula that is used in computing said line. So, $\beta_{hours} = -0.08$. We can insert this value into our formula.

$$y_{grade} = \beta_0 - 0.08 * x_{hours} + \epsilon$$

In this way the value of x_{hours} is multiplied by -0.08. Let us assume a student worked 40 hours on their paper. $-0.08 * 40$ being -3.2, we know, that working 40 hours on a paper *on average* - more on that later - leads to a 3.2 lower grade value and thus a better grade. But 3.2 lower than what?

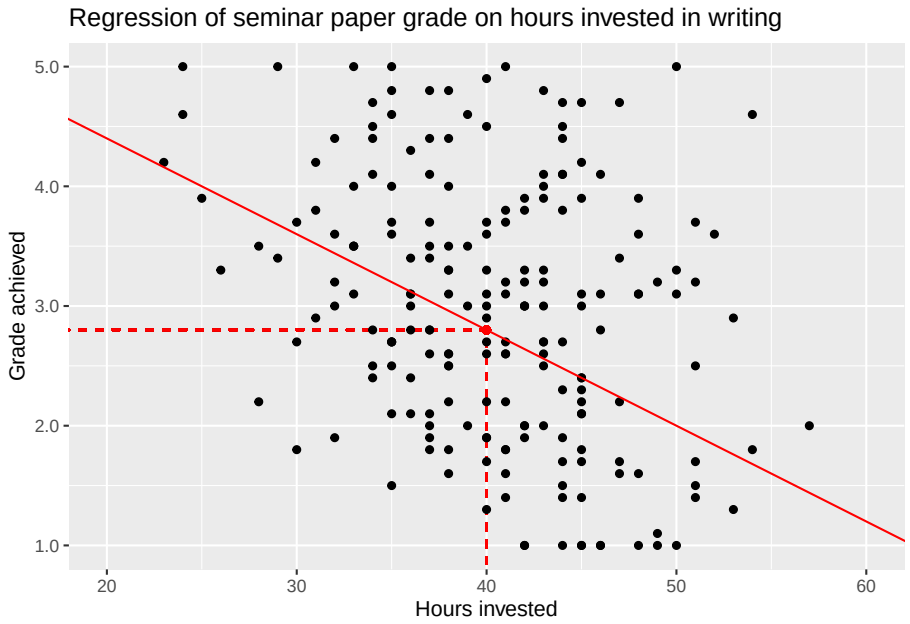
Looking at the formula again, we see that subtract this value from β_0. This is the *intercept*, the value at which the line intersects with the y-axis. Let us zoom out on our plot, to see what happens.



We can now see the point where the red line intersects with the y-axis. This is the intercept of this line, i.e. $\beta_0 = 6$.

$$y_{grade} = 6 - 0.08 * x_{hours} + \epsilon$$

If we now again assume a time investment of 40 hours, we can compute $6 - 0.08 * 40 = 2.8$. So our red regression line - which is still only a wild guess - assumes, that working 40 hours on a seminar paper will result in a grade of 2.8, on avarage. We can mark these values in our plot



The red dot is the intersection of the values `hours = 40` and `grade = 2.8`. As this is the value for y our regression line assumes a student with a time investment of 40 hours achieves, the red dot also lies exactly on the red line.

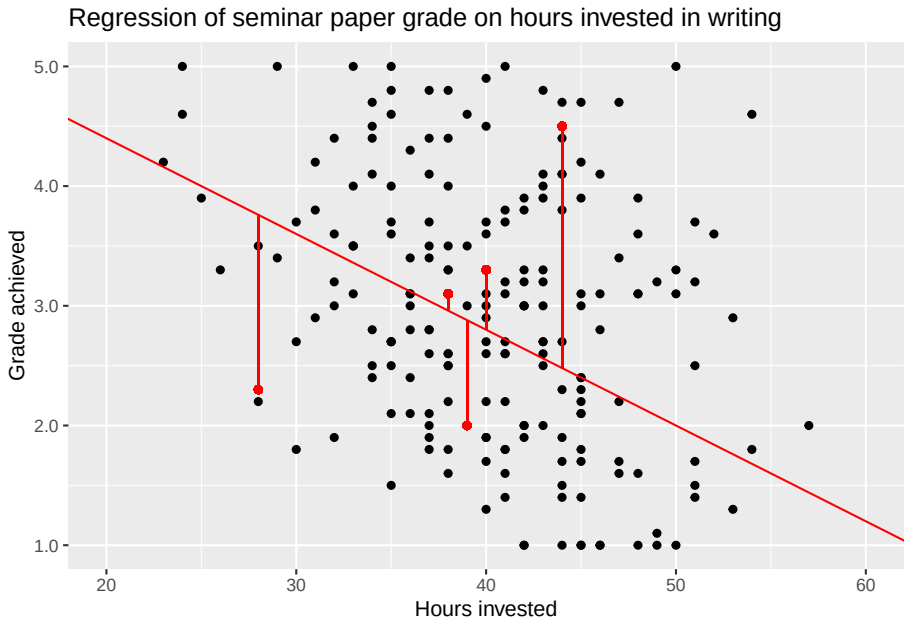
But if we look at the plot once again, we can see that most actual observations for students that invested 40 hours do not actually lie on the regression line but are scattered above and below the line. So some of these students achieve much worse or much better grades than 2.8 investing the same amount of time in their work. This leads us to the last part of the formula, ϵ.

This is the *error term*. Having data that is dispersed like this - and any real world data will always be - our linear line will never be able to pass exactly through every data point. Some points may lie exactly on the line, but many or most will not.

We can visualize this. To keep the plot readable, we only do this for some random observations but in reality the distance of every data point from the regression line is taken into account.

```
## # A tibble: 5 x 7
##   grade hours previous_grades attendance contact    previous_grades_centered
##   <dbl> <int>           <dbl> <lgl>      <fct>                         <dbl>
## 1   2.3    45             2.6 TRUE       No contact                   -0.335
## 2   1.9    44             2.8 TRUE       In Person                    -0.135
## 3   3.1    38             3.7 TRUE       In Person                     0.765
## 4   3.3    40             3.4 TRUE       E-Mail                        0.465
## 5   4.6    54             4.5 TRUE       No contact                    1.56
```

```
## # i 1 more variable: hours_centered <dbl>
```



The distance of these or rather all points from the line, the *residuals*, are represented in the error term ϵ. It is a measure for how wrong our line is in describing the data in it's entirety. So why is it wrong? We can not say for sure, but there are two common main reasons.

For one, there may be other variables that also influence the relationship between invested hours and achieved grade, something that we will return to later in this session when we expand the idea of linear regression to multiple independent variables.

But there is also random variation present in every bit of real world data. While our data is simulated we also added random variation on purpose. Because this is what real world data is, it's messy and it's noisy.

Not every seminar paper that had the same time investment, e.g. 40 hours, will have the same quality in results. There may be other influential variables, e.g. the student's general skill level or if they sought assistance by their lecturer in preparing the paper, influencing the final grade. But even if the quality of the paper after working 40 hours would be the same for each student, measurement error, i.e. noise, will be introduced because not every lecturer will grade exactly the same or maybe because papers were submitted at different time points and grading standards may have changed. If we can not measure these variables we have to accept these unobservable sources of noise and hope, where *hope* actually means thorough theoretical and methodical thinking, that we can still measure

our effect of interest. But his also means, that measuring and modelling **always** includes uncertainty. We never know for certain if and to what extent our results are influenced by unobservable variables and random variation. Still, there are ways to assess this uncertainty, which we will regularly return to during the course. This should not stop quantitative social scientists from making strong or even bold arguments based in thorough theoretical thinking and responsible data analysis, but we always have to acknowledge the uncertainty included in every step and make it a part of our interpretations and conclusions.

XXX MAYBE SHORTEN THE SERMON… XXX

The error term ϵ is the final piece of the puzzle in actually computing a linear regression model. Without jumping into the mathematics of it all, the technique that is used to estimate the coefficients β_0 and β_1 is called *OLS* - Ordinary Least Squares. What it basically does, is to take the squares of all residuals, i.e. the distances of the data points from the regression line, sum them up and minimise this value. All this substantialy means is, that OLS searches the regression line with lowest amount of error, i.e. the lowest overall distance from the actual data points.

This computation gives us estimates for the regression coefficients in this formula:

$\hat{y} = b_0 + b_1 * x_1$

We can see two differences to the formula we started with. First, we write $\hat{y}$ - pronounced as "y hat" - instead of y. At the same time, we exclude the error term ϵ. This means that we are no longer computing the actual value of y, as in the point on the regression line for a certain value of x_1 + the error, but the estimate $\hat{y}$, as in the point on the regression line that is predicted for a certain value of x_1. Second, we write b instead of β. This also alludes to the fact that we are now computing an estimate for the coefficients based on the data available and not the real and unknown value of β.

XXX MAYBE ALSO SOME CUTTING REQUIRED XXX

XXX include "on average" XXX

5.3.2 Regressing `grade` on `hours`

Now that we have a firmer understanding on what linear regression actually is and does, we can finally get to the fun part and use the technique for estimating the effect of `hours` on `grade` or in other words, regress `grade` on `hours`.

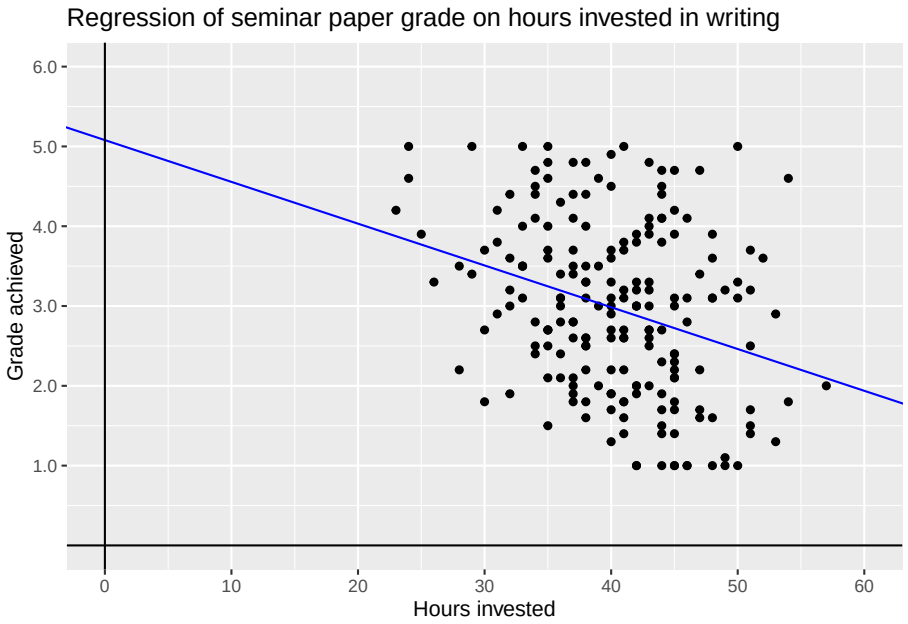
```
##
## Call:
## lm(formula = grade ~ hours, data = grades)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.88006 -0.83961 -0.08006  0.77006  2.53881
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.07912    0.47306  10.737  < 2e-16 ***
## hours       -0.05236    0.01159  -4.517 1.07e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 198 degrees of freedom
## Multiple R-squared:  0.09344,    Adjusted R-squared:  0.08886
## F-statistic: 20.41 on 1 and 198 DF,  p-value: 1.075e-05
```

This is the output from a simple linear regression for `grade` on `hours` from R. How to do this in practice and what the first to lines mean will be the topic of the next sesion. For now we will focus on the coefficient block and introduce the additional elements of the output on by one during this session.

The column `Estimate` gives us the values for β_0 and β_1 discussed above. The estimated coefficient for `hours` tells us that out intuition was right, the more hours a student invests in writing a paper, the better the grade will be. In this case every additional hour spent on working will decrease the value of the grade by -0.05236 points. In keeping with the example of a 40 hour workload this leads to a decrease of $-0.05236 * 40 = -2.0944$ points. Adding the intercept from the same column, the estimated grade after working 40 hours is $5.07912 - 0.05236 * 40 = 2.98472$. So on average a student from our simulated data set will pass after 40 hours of work but will not get a great grade. Remember, this is the expected average value. This does not mean that some students will not get better or worse grades, or even fail to pass, after this amount of time investment, as we hav seen in the plot.

Now that we know the coefficients for the regression line with the best fit, i.e. the lowest error, we can again visualise the result.



Now, what grade can a student expect, on average, if they invest exactly 0 hours, i.e. do nothing and hand in a blank paper. We can look at the graph or to achieve a more precise result, calculate it.

$$5.07912 - 0.05236 * 0 = 5.07912$$

As $x_{hours} = 0$ for this theoretical example, the estimated value $\hat{y}$ or y_{grade} is the same as the intercept β_0. This is what the intercept represents in general, the estimated value $\hat{y}$ when the dependent variable is 0.

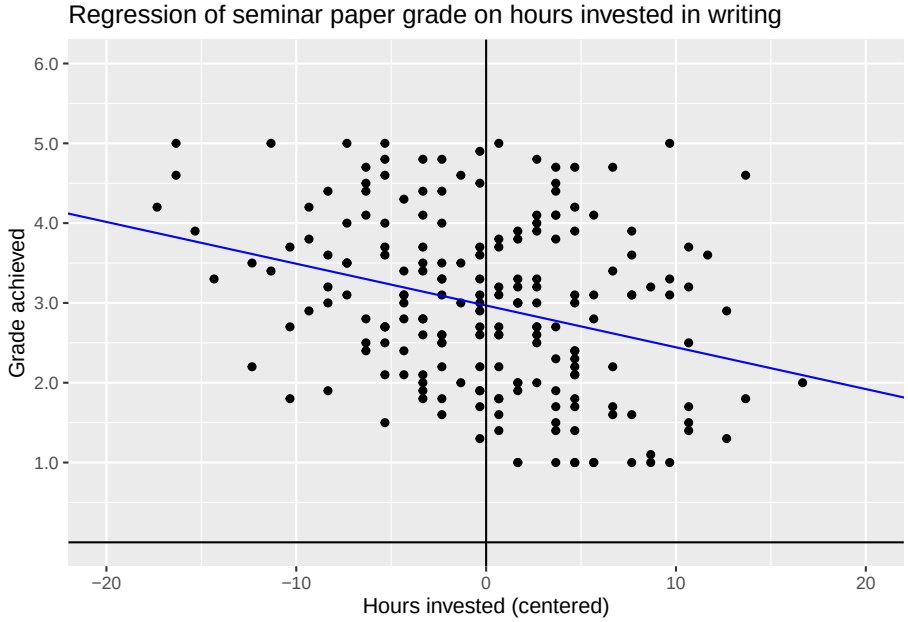
Now, investing zero hours in a seminar paper is not only not advisable, it is also not a value we observed in our data. If the data would include observations with zero hours of time invested, the grade would be a firm 5.0 and the same would be true for low single digits, i.e. turning in a two-pager as a seminar paper. The takeaway is, that the model is highly dependent on the data that it is trained on. If the data would have included such cases we could expect a higher intercept and a steeper slope, i.e. stronger negative coefficient.

Luckily all our simulated students have put in at least some hours. But as we do not have data for zero to 22 hours, we can not really make reliable estimates in this range. Because of this, it does not really make sense to enter `hours` into the regression model as ranging from 0 to 57. One solution that is often used for metric variables is to center them on their mean. This can be achieved by simply subtracting the mean of x from each individual value: $x_i - \bar{x}$.

We can now rerun the regression.

```
##
## Call:
## lm(formula = grade ~ hours_centered, data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.88006 -0.83961 -0.08006  0.77006  2.53881
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)     2.96750    0.07267  40.835  < 2e-16 ***
## hours_centered -0.05236    0.01159  -4.517 1.07e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 198 degrees of freedom
## Multiple R-squared:  0.09344,    Adjusted R-squared:  0.08886
## F-statistic: 20.41 on 1 and 198 DF,  p-value: 1.075e-05
```

Comparing the results to the first model shows us, that the coefficient for $b_{hours\_centered}$ is exactly the same as for b_{hours}. So the effect of working more hours has not changed. What has changed is the value of the intercept. This will make more sense if we again plot the regression line.



By centering the x-Variable on its mean we have changed its interpretation. A value of `hours = 0` now stands for investing as much time as is the mean of

`hours` in the whole data set, which in this case is 40.33 hours. Positive values indicate that a student worked x hours more, negatives indicate $-x$ hours less compared to the mean. In this way, we also moved the y-axis and thus changed the interpretation of the intercept. Its new value of 2.9675 now indicates the estimate for a student who invests the mean value of `hours` in their work, i.e. 40.33. We will use this version of the variable for the rest of the session.

XXX OUTRO XXX

Chapter 6

Linear Regression Theory II: Multiple Linear Regression

XXX INTRO XXX

6.1 Multiple Linear Regression

Maybe explaining the grade a student receives solely based on the hours of invested time, does not paint the whole picture. As we have alluded to, there may be other variables that could also have an effect of the final grade.

A *simple linear regression* only allows for one independent variable. This is why we need *multiple linear regression* if we want to start introducing additional variables into the model. Luckily this is easy to understand as we already know the formula for a simple linear regression.

$$y = \beta_0 + \beta_1 * x_1 + \epsilon$$

To change a simple into a multiple linear regression, we just start adding the additional variables and their coefficients additively to the formula.

$$y = \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + ... + \beta_k * x_k + \epsilon$$

So to add a second variable and its coeffcient we add the term $+\beta_2 * x_2$ and so on until we added all independent varibles of interest k to the model. Everything else works exactly as described above for the simple model.

6.1.1 Adding additional metric variables

We already expected that the mean of the previous grades could be a strong predictor for future grades. We could understand these as a *proxy* variable for the general skill level of a student. The higher the skill level, the higher previous grades will have been.

How we can add additional variables in R code will again be a topic for the next session, but let us look at the results of a regression of `grade` on `hours_centered` and `previous_grades_centered`, the latter being centered on the mean previous grade of 2.935.

```
##
## Call:
## lm(formula = grade ~ hours_centered + previous_grades_centered,
##     data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.44462 -0.30556  0.00622  0.32878  1.31002
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                2.967500   0.038316  77.449   <2e-16 ***
## hours_centered            -0.056543   0.006114  -9.248   <2e-16 ***
## previous_grades_centered   0.904079   0.039830  22.699   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5419 on 197 degrees of freedom
## Multiple R-squared:  0.7492, Adjusted R-squared:  0.7467
## F-statistic: 294.3 on 2 and 197 DF,  p-value: < 2.2e-16
```

As we added a new variable, we now see three coefficients. The intercept has not changed. It now indicates the estimated grade for a student who invests the mean amount of hours, 40.33, and whose previous grades are exactly 2.935.

The coefficient for `hours_centered` got mildly more negative, still telling us that the value of `grade` gets lower, the more hours are invested in writing the paper. This coefficient now gives us the effect while *controlling* for the effect of `previous_grades_centered`. This is what multiple linear regression does, giving us the coefficients for our variables of interest while keeping all other independent variables at specific values. As we have centered the variable for previous grades, the coefficient for `hours centered` gives us the effect when the previous grades were exactly at the mean of $2,935$.

In the same way, the coefficient for `previous_grades_centered` gives us the effect of previous grades when the invested hours are controlled for, in this case

when the invested hours were exactly 40.33. The coefficient is rather high and positive. This indicates that a student with a previous grade value that is 1 above the mean, is estimated to receive a new grade that is 0.9 points above the intercept. This means, that the previous grade is a very strong predictor for the new grade.

While plotting in more than two dimensions gets really hard, we can still calculate $\hat{y}$ for certain values of both independent variables. We already know the predicted grade for a student with mean values on both independent variables, as this is the intercept. To make sure that we correct, we can calculate it again.

$$b_0 + b_{hours\_centered} * 0 + b_{previous\_grades\_centered} * 0 = 2.9675$$

For this case we can see, that the previous grade actually is a strong predictor, as the previous and new grades are substantially the same.

What if a student whose previous grades were 1 above the mean, so just below 4.0 but who decides to invest 10 hours more than the mean for the new paper?

$$2.9675 - 0.056543 * 10 + 0.904079 * 1 = 3.306149$$

So the good message is, while previous grades are a strong predictor, putting in more hours still leads to better grades.

What if a really good student decides to rely on their skill and to work less this time?

$$2.9675 - 0.056543 * -10 + 0.904079 * -2 = 1.724772$$

While 1.7 is still a very good grade, working 10 less hours than the mean of students leads to a substantially worse estimate compared to the about 1.0 received in previous grades.

6.1.2 Adding dummy variables

Another variable that could be of interest in explaining the received grade, is if a student attended most of the seminar sessions. `attendance` holds this information in the form of a dummy variable. Dummies can only have two states. "Yes" or "No", "1" or "0" or in this case "`TRUE`" or "`FALSE`".

Let us add the variable to our model.

```
##
## Call:
## lm(formula = grade ~ hours_centered + previous_grades_centered +
```

```
##     attendance, data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.41059 -0.30910  0.01667  0.35607  1.29849
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                3.157411   0.078658  40.141  < 2e-16 ***
## hours_centered            -0.053942   0.006088  -8.860 4.85e-16 ***
## previous_grades_centered   0.911802   0.039282  23.212  < 2e-16 ***
## attendanceTRUE            -0.248250   0.090246  -2.751   0.0065 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5331 on 196 degrees of freedom
## Multiple R-squared:  0.7586, Adjusted R-squared:  0.7549
## F-statistic: 205.3 on 3 and 196 DF,  p-value: < 2.2e-16
```

This gives us a new line in the R Output holding an estimate for `attendanceTRUE`. What is meant by this? In contrast to the metric variables we have uses in our model up to this point, a dummy variable - or binary variable - can only have two states. As we are using a logical variable here, it can only have the value `TRUE` - here indicating regular attendance - or `FALSE`. So what the output shows us, is the effect of attendance being `TRUE` compared to being `FALSE`. If a student did regularly attend the seminar, the estimated grade is -0.248250 lower compared to when they did not.

We can observe what happens in the formula:

$$\hat{y} = b_0 + b_{hours\_centered} * x_{hours\_centered} + b_{previous\_grades\_centeterd} * x_{previous\_grades\_centeterd} + b_{attendance} * x$$

If you calculate with `TRUE` and `FALSE` in R, the values 1 and 0 are used respectively. So $x_{attendance}$ can either have the value 1 for regular attendance or 0 for not so regular attendance.

If a student did regularly attend, the coefficient $b_{attendance} becomes a part of the estimate $\hat{y}$:

$$\hat{y} = b_0 + b_{hours\_centered} * x_{hours\_centered} + b_{previous\_grades\_centeterd} * x_{previous\_grades\_centeterd} + b_{attendance} * 1$$

If stduent did not regularly attended, this happens:

$$\hat{y} = b_0 + b_{hours\_centered} * x_{hours\_centered} + b_{previous\_grades\_centeterd} * x_{previous\_grades\_centeterd} + b_{attendance} * 0$$

$\hat{y} = b_0 + b_{hours\_centered} * x_{hours\_centered} + b_{previous\_grades\_centeterd} * x_{previous\_grades\_centeterd}$

The coeeffcient is no longer a part of the estimate. One can basically say, the coefficient gets switched on or off by the value of the dummy variable.

So while the estimate for a student with mean values for invested hours and previous grades who did not attend is the intercept of 3.157411 for the same student with attendance we can calculate the estimate as:

$$3.157411 - 0.053942 * 0 + 0.911802 * 0 - 0.248250 * 1 = 3.157411 - 0.248250 = 2.909161$$

It seems attending class is an easy way to raise one's grades.

6.1.3 Adding categorical variables

We have one further variable in our simulated data set that could be of interest in explaining, what makes a good grade in a seminar paper.`contact` is a factor variable. It can take three different categories. `No contact` indicates that the student did not contact the lecturer to discuss a research question or the laid out plan for the paper. `E-Mail` means that there was some written contact and at least the basics for the paper were discussed before writing. Lastly, `In Person` stands for an in depth discussion with the lecturer, clearing up problems beforehand and having a more stringent vision for the paper before writing the first word.

Let us add the variable to our model.

```
##
## Call:
## lm(formula = grade ~ hours_centered + previous_grades_centered +
##     attendance + contact, data = grades)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.3835 -0.2525  0.0167  0.2678  0.9347
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                3.617949   0.068077  53.145  < 2e-16 ***
## hours_centered            -0.050830   0.004433 -11.466  < 2e-16 ***
## previous_grades_centered   0.874123   0.028657  30.503  < 2e-16 ***
## attendanceTRUE            -0.324653   0.065781  -4.935 1.72e-06 ***
## contactE-Mail             -0.413808   0.069817  -5.927 1.39e-08 ***
## contactIn Person          -0.853252   0.063964 -13.340  < 2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3869 on 194 degrees of freedom
## Multiple R-squared:  0.8741, Adjusted R-squared:  0.8709
## F-statistic: 269.4 on 5 and 194 DF,  p-value: < 2.2e-16
```

Wait, we entered three categories into the model and got estimates for two of them. What happened? What R does is to create two dummy variables on the fly. The first discerns between having E-Mail contact or no contact at all. The second one between having contact in person to no contact at all. So for categorical variables in regression models we always compare being in one category to being in the *base category*. In this case the base category is `No contact` but we could also change the base category. It depends on what we are interested in comparing to. For our example comparing the effects of having more in depth contact to having none makes sense.

Let us look at our formula again:

$$\hat{y} = b_0 + b_{hours\_centered} * x_{hours\_centered} + b_{previous\_grades\_centeterd} * x_{previous\_grades\_centeterd} + b_{attendance} * a$$

Now there are three possibilities. A student can have no contact at all. In this case both dummy variables equal 0. To make our formula easier to read, we have abbreviated the middle part for now:

$$\hat{y} = b_0 + ... + b_{E-Mail} * 0 + b_{InPerson} * 0$$

So in this case, controlling all other independent variables at their default values, the mean for the metric variables, `FALSE` for `attendance`, the intercept gives us the estimate for the grade as both dummy variables that were created for `contact`are "switched off".

THe two other possibilities are that a student either had E-Mail contact or an in person discussion:

$$\hat{y} = b_0 + ... + b_{E-Mail} * 1 + b_{InPerson} * 0$$

$$\hat{y} = b_0 + ... + b_{E-Mail} * 0 + b_{InPerson} * 1$$

In both cases the relevant dummy variable is "switched on" while the other does not factor into the equation.

Looking at the estimates, we can see that having contact to the lecturer before writing has strong negative effects, resulting in better grades. Having E-Mail

contact reduces the value of `grade` by -0.413808 points, having an in person discussion by -0.853252.

So what grade can a student whose previous grades were at the mean of 2.935, but who decided to put in 20 hours more compared to their peers, regularly attend the seminar and have an in-depth personal discussion before writing their paper expect on average as their new grade?

$$3.617949 - 0.050830 * 20 + 0.874123 * 0 - 0.324653 * 1 - 0.413808 * 0 - 0.853252 * 1 = 1.423444$$

Putting in the hours, attending and working with your lecturer seems to pay off, at least in our simulated data set.

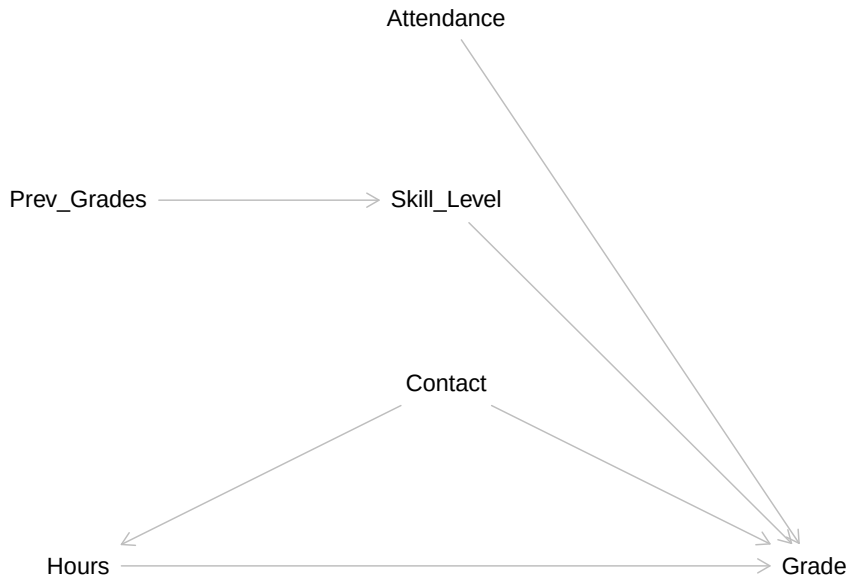
6.2 Returning to our research question

Our exemplary research question concerned itself with what makes a good grade in a seminar paper. In particular we were interested in the effect of the invested hours, as our main hypothesis was that more hours lead to better grades. What do we know now?

All analysis point towards a clear effect from `hours` on `grade`. This effect was consistently visible in all of our models. But did we correctly identify and estimate the effect of interest? Maybe. The problem is, we actually did not approach the analysis correctly. In a real analysis we should **absolutely** refrain from adding variables to our model that *could be* relevant until we are satisfied or all available variables are bunched into one huge model. It was fine to do this in this introduction to linear regression and for learning how different types of variables can be used in a regression model. But in a real project, we have to invest time to think about which variables to add because we assume that they have a relevant effect based on theoretical assumptions about the processes we are interested in.

So let us do this now and vow do make this our first step in all future endeavors. While we do not have a clear theoretical basis, we can make clear assumptions on the data generating process and draw these in a DAG.

```
## Warning: package 'dagitty' was built under R version 4.2.3
```



XXX MAYBE NICEN IT UP XXX

Our central assumption, and the effect we want to identify and estimate, is the direct effect from `hours` on `grade` in the bottom line. The more hours a student invests, the better the grade should be. This is our variable of interest and it has to be included in the model.

The assumed effect of `contact` is more complex. For one we assume that a more in-depth contact with the lecturer will increase the grade directly. The research question will be more focused, the student will know what is important to a certain lecturer, common mistakes can be avoided if they are cleared up beforehand and so on. But we will also assume that `contact` will have an effect on `hours` in the sense that the hours invested can be used more efficiently if an in-depth discussion has taken place. Instead of wasting time running into problems that could have been avoided most of the invested time can actually go into constructive work. This makes `contact` a confounder for `grade` and `hours`. Tapping into the knowledge from the last session, now we know that we also have to control for `contact` to measure the effect of `hours` correctly.

A student's skill level will also have a direct effect on `grade`. As we do not have a direct measure of skill in our data, we use `previous_grades` as a proxy for skill level. `attendance` also has a direct effect on `grade` as students who were present in the seminar will not only have learned the seminar's contents, but will also have a better understanding of what is expected in their seminar papers. As neither skill level - or `previous_grades` - nor `attendance` has any connection to `hours` we also should not control for them in our model.

That leaves us with `hours` and `contact` to be included in our linear regression, if our goal is to accuarately identify and estimate the effect of invested time on the final grade. So let us do this:

```
##
## Call:
## lm(formula = grade ~ hours_centered + contact, data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85595 -0.74624 -0.02106  0.66648  2.50161
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        3.44352    0.10404  33.098  < 2e-16 ***
## hours_centered    -0.04967    0.01052  -4.723 4.43e-06 ***
## contactE-Mail     -0.46482    0.16785  -2.769  0.00616 **
## contactIn Person -1.02804    0.15240  -6.746 1.67e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9305 on 196 degrees of freedom
## Multiple R-squared:  0.2643, Adjusted R-squared:  0.253
## F-statistic: 23.47 on 3 and 196 DF,  p-value: 5.072e-13
```

This is our estimate. Each hour invested beyond the mean of 40.33 hours changes the grade by about -0.05 points. This supports our hypotheses and we can conclude, that investing more hours into writing a seminar paper actually is a worthwhile investment.

But remember: This is correct as long as our DAG is drawn correctly. This is always debatable. Maybe we should assume an effect from skill level on `hours`. The higher the skill level the more efficiently the available time can be used. For this example we know the DAG is correct, because we have simulated the data exactly in this way. For real world applications we never know if our DAG is correct. All we can - and have to - do is base it on thorough thought, theoretical work and sound arguments.

This all is true *if* our goal is to estimate an effect of interest as precisely as possible. But as we have alluded to in the introduction to this session we could also use modelling with a different goal, i.e. predicting a grade as accurately as possible. For this task, the model which only includes `hours` and `contact` will not do the best job. From our DAG we know that `attendance` and `previous_grade` should have an effect on `grade`, as we have also seen in our models. For this task the full model including all these variables will produce better estimates. We will return to this in a later session, but for now we should remember that

we have to know our task because the task dictates which is the best model to use.

6.3 Interpretation

XXX WHERE TO PUT IT? XXX

Looking at the coefficient block from either of the two outputs, we see more than just our estimate. The `Std. Error` or `std.error` is a measure for the uncertainty of our estimates. It basically tells us, how far away the actual values of the observations used to compute the model are from our estimate *on average*. The smaller the *standard error*, the more accurate is our estimate. The standard error is presented in the units of the estimate and we can thus compare them. A large standard error for a large estimate is far less problematic compared to a large standard error for a small estimate.

The estimate and it's standard error are the basis for *hypothesis testing*. What we are testing is the *alternative hypotheses* H_a that there actually is an effect of our independent variable on the dependent variable against the *null hypothesis* H_0 that there is no effect. To reject the null hypothesis and be confident that we are observing an actual effect, versus an effect that is joust based on random variation in our sample, the estimate has to be far away enough from 0 and be accurate enough, i.e. have a small standard error. This relationship is computed in the *t-statistic*, `t value` and `statistic` in our outputs. From this the *p-value* can be computed, `Pr(>|t|)` and `p.value` in the outputs. The *p-value* tells us the probability to observe a association between the independent and the dependent variable as large or larger than our estimate suggests, if the true association would actually be 0. If the p-value is small enough, we can reject H_0 and conclude that we observed an actual effect. There are certain agreed upon cutoffs in statistics where values that meet this cutoff are considered *statistically significant*. The most common cutoff in social sciences is 0.05 indicated by one `*` in the output from `summary()` There are other common cutoffs indicated by more asterisks.

Interpreting p-values correctly and not falling into the common pitfalls is a topic on its own. We do not have the time to dive into this here, so for now we can agree that p-values below 0.05 indicate that we can reject H_0 and thus conclude that we have actually observed an effect. Still, our interpretation of regression results should not focus solely on p-values or lead us to disregard any effects that did not meet the cutoff. For example, we can have very small p-values for effects that are so small that they are substantially irrelevant. One way to address this is to inspect the actual magnitudes of the effects, something we will return to in our session on prediction. On the other hand, we can have p-values larger than 0.05 for effects that are still relevant. Maybe the problem is not that there is no effect but that we were not able to measure the variable in question precisely enough or that we just did not have enough observations.

We can not go any deeper than this here, but we should remember that the practice of declaring every effect with stars a win and disregarding everything without them may still be common but is not the way to go forward.

XXX CORRECT? MAYBE SHORTEN XXX

XXX OUTRO XXX

Chapter 7

Linear Regression Theory III: Diagnostics

7.1 Model fit

Over the last two sessions we learned the theory and mathematics behind linear regression, how to interpret coefficients for different types of explanatory variables and how we can use p-values to assess our hypothesis. But if you looked closely, there are still parts of the output we have not talked about yet.

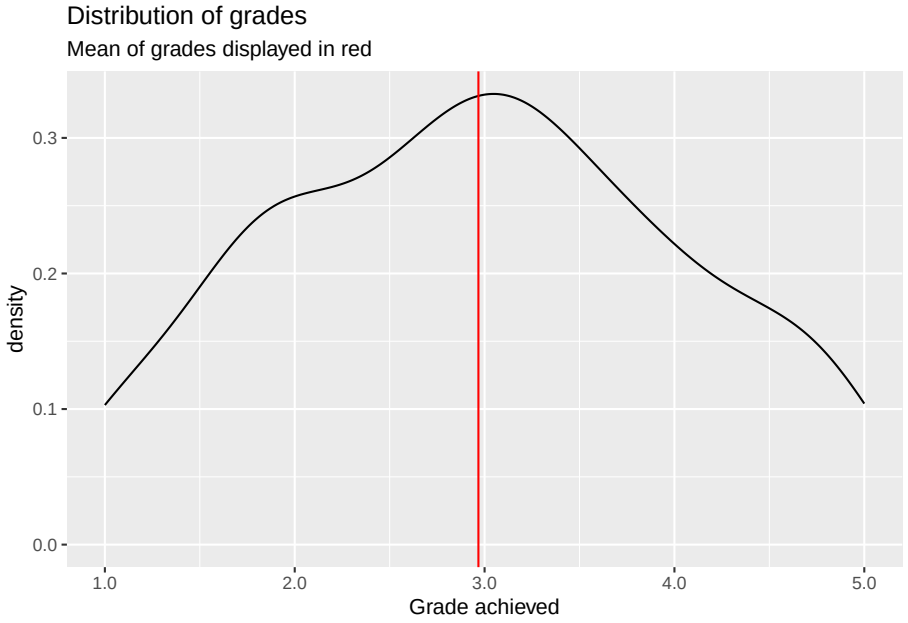
Let us again inspect the output from the simplest model we computed, regressing the grade solely on the invested hours:

```
##
## Call:
## lm(formula = grade ~ hours_centered, data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.88006 -0.83961 -0.08006  0.77006  2.53881
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)     2.96750    0.07267  40.835  < 2e-16 ***
## hours_centered -0.05236    0.01159  -4.517 1.07e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 198 degrees of freedom
## Multiple R-squared:  0.09344,    Adjusted R-squared:  0.08886
```

```
## F-statistic: 20.41 on 1 and 198 DF,  p-value: 1.075e-05
```

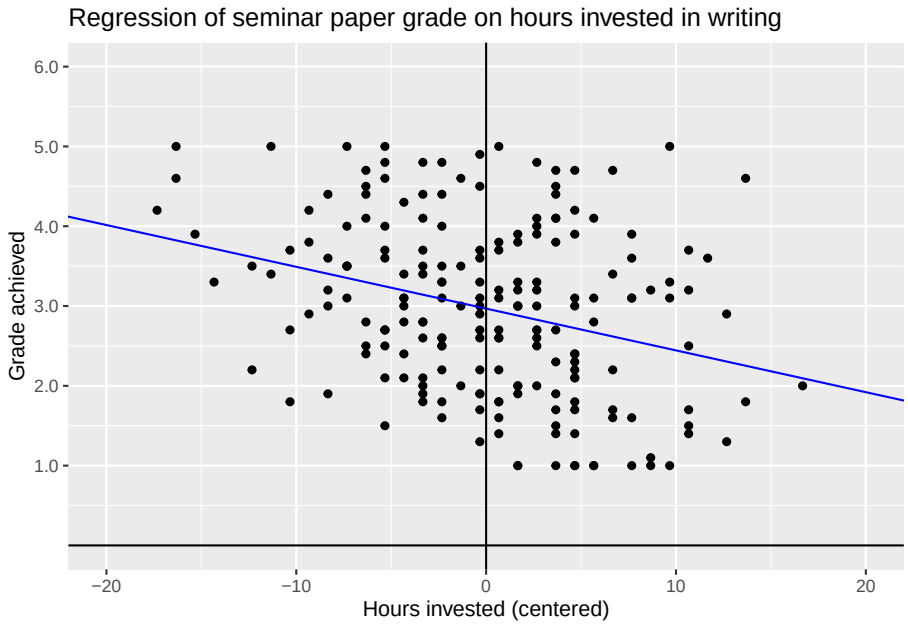
Up to now, we exclusively talked about the coefficient block. We will return to the "Call" next session and to the "Residuals" later in this session. For now let us focus on the bottom block in the output.

R^2 or *R-squared* is a measure for the amount of variance in the data that is "explained" by the model. Real world data will always have variance. Not every value will neatly fall onto the mean value of a variable. Rather the data is dispersed around it. The same is true for our dependent variable:



This is a density plot and shows the distribution of a metric variable as a smoothed line. We do not see every individual actual value but the general shape of the data. The red line represent the mean of `grade`, which is about 2.97. Most actual values are not exactly at the mean but are rather dispersed around it, ranging between 1.0 and 5.0. This is the variance in our outcome variable.

Let us now plot `grade` against `hours_centered` and add the regression line from our model above:



Without our regression line, all we would have is a cloud of points without much order to it. What linear regression does, is trying to bring order into this by fitting a line that best explains the variance of the dependent variable, `grade` in our case by its relationship to one or multiple dependent variables, here `hours_centered`. But this linear line can never explain the variance completely. For this it had to pass through every data point. Our line does not. Actually most data points do not lie on the regression line but at some distance to it. You will remember that OLS computes *the* regression line for which the squared distances are smallest. This is the line that explains most of the variance of y by its relationship to x, but not all variance is explained. An unexplained part remains. These are the residuals, the distance that points fall from the regression line. R^2 tells us the relative amount of how much we reduced the initial variance by fitting the line and thus explaining a part of said variance.

A R^2 of 0 would mean that no variance is explained, a value of 1 that all variance is explained. Two highly unlikely outcomes. We will almost always explain something and never explain everything.

In our model, $R^2 = 0.09344$. This means we explained about 9.3% of the variance of `grade` through its relationship with `hours_centered`. That's nice, but this also means that over 90% are still unexplained. We will not explain all of the variance, i.e. $R^2 = 1$, but in general a higher R^2 is desirable.

So what can we do? We can try to add additional variables to the model that help in explaining the variance in the outcome variable. Last session we concluded that the "correct" model to measure the effect of invested hours on the achieved grade would also have to include `contact`:

```
##
## Call:
## lm(formula = grade ~ hours_centered + contact, data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85595 -0.74624 -0.02106  0.66648  2.50161
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        3.44352    0.10404  33.098  < 2e-16 ***
## hours_centered    -0.04967    0.01052  -4.723 4.43e-06 ***
## contactE-Mail     -0.46482    0.16785  -2.769  0.00616 **
## contactIn Person -1.02804    0.15240  -6.746 1.67e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9305 on 196 degrees of freedom
## Multiple R-squared:  0.2643, Adjusted R-squared:  0.253
## F-statistic: 23.47 on 3 and 196 DF,  p-value: 5.072e-13
```

We can use R^2 to compare the *model fit* of multiple models. Here the larger model achieved a considerably higher value of $R^2 = 0.2643$. The model fit improved as we can now explain a higher ratio of the variance in `grade`.

After R^2 we see another value, *Adjusted R-squared*. This becomes relevant if we add additional variables to our model. R^2 almost always increases, and never decreases, when adding additional variables to the model, especially if we have few observations. Because of this R^2 can get less reliable when we have many variables and few observations. *Adjusted R-squared* corrects for this by including both factors in the calculation. When we have many observations the differences are negligible. This is true for our case. We have relatively many observations and few variables in our model, so the values of both measures are rather close. But in cases where this relationship is not as favorable adjusted R-squared should be used in place of the regular R^2.

The block in our output also gives us the *Residual standard error*. As we have seen above, most actual data point do not lie on the regression line but some distance away from it. These are residuals. Thus their standard error basically tells us how much we miss the spot on average. As it is given in units of the dependent variable, we can say that the estimated for `grade` based on our second model are on average 0.93 off. A considerable amount, as this is almost one whole grading step. This is still an improvement from the 1.028 in the first model but nevertheless a substantial error.

The last line in the output gives us two connected measures. The *F-statistic* is the test statistic for R^2 and is used the compute the corresponding *p-value*.

In this case we are testing if the R^2 our model returned based on our sample is possible, when the actual population value of R^2 if 0. In other words, could we have achieved this R^2 by chance if the independent variables in our model actually do not explain part of the variance in the population? Both of our models have very small p-values, so it is highly unlikely that we have just explained some variance by chance. This gives further credibility to our model specification.

We can conclude that the second model was an improvement over the first. But can we do more? Sure! We can add additional explanatory variables:

```
##
## Call:
## lm(formula = grade ~ hours_centered + previous_grades_centered +
##     attendance + contact, data = grades)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.3835 -0.2525  0.0167  0.2678  0.9347
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                3.617949   0.068077  53.145  < 2e-16 ***
## hours_centered            -0.050830   0.004433 -11.466  < 2e-16 ***
## previous_grades_centered   0.874123   0.028657  30.503  < 2e-16 ***
## attendanceTRUE            -0.324653   0.065781  -4.935 1.72e-06 ***
## contactE-Mail             -0.413808   0.069817  -5.927 1.39e-08 ***
## contactIn Person          -0.853252   0.063964 -13.340  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3869 on 194 degrees of freedom
## Multiple R-squared:  0.8741, Adjusted R-squared:  0.8709
## F-statistic: 269.4 on 5 and 194 DF,  p-value: < 2.2e-16
```

The p-value is even lowe, and the F-statistic even higher, compared to our second model, but this was never an issue. What is more interesting is that we have substantially increased R^2 and decreased the residual standard error. As we have concluded last week, this larger model is better at predicting the actual values of `grade`. Thus the explained variance has to increase and the average error in estimating y has to decrease. But is this the better model? The values on the model fit would suggest so. And this also is true, if our aim is predicting `grade` to the best of our abilities. But if our aim is still measuring the effect of `hours` on `grade` we know from our DAG that we do not have to or even should not control for the additional variables to get an unbiased estimator for the effect of interest.

What can we take away from this? While the model fit measures are an important tool for comparing multiple possible models and better values are desirable in general, it should not be our goal to just max out all measures and declare this model the "winner". It is never that easy in statistics. One thing we can never replace is thorough theoretical work before even computing our first model. Based on our DAG, if it is correct, we know do not have to control for previous grades and attendance. Including them may give us a larger R^2, but is still not the correct way to build our model.

Based on this our best model is still the second one:

```
summary(m2)
```

```
##
## Call:
## lm(formula = grade ~ hours_centered + contact, data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85595 -0.74624 -0.02106  0.66648  2.50161
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        3.44352    0.10404  33.098  < 2e-16 ***
## hours_centered    -0.04967    0.01052  -4.723 4.43e-06 ***
## contactE-Mail     -0.46482    0.16785  -2.769  0.00616 **
## contactIn Person -1.02804    0.15240  -6.746 1.67e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9305 on 196 degrees of freedom
## Multiple R-squared:  0.2643, Adjusted R-squared:  0.253
## F-statistic: 23.47 on 3 and 196 DF,  p-value: 5.072e-13
```

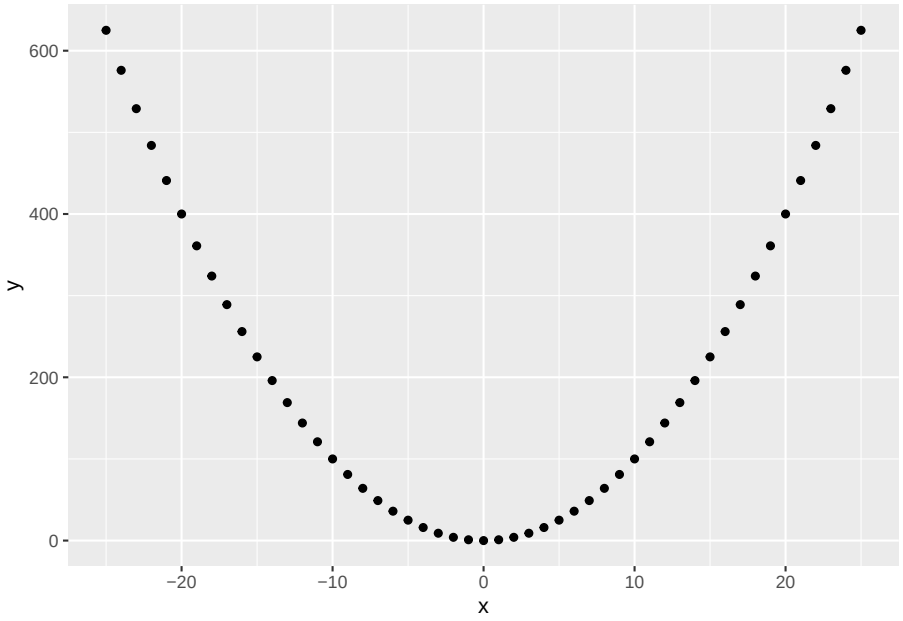
So can we finally call it a day and put the issue of the effect from invested hours on received grade to rest for good? Maybe, but we still do not know for certain befre we apply a last step in constructing a valid linear regression model, *regression diagnostics*.

7.2 Regression diagnostics

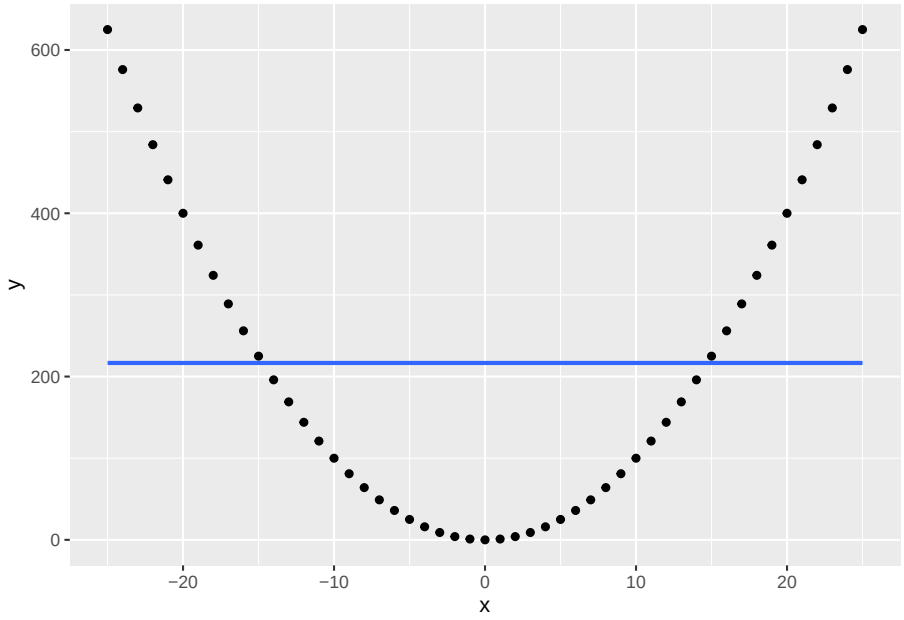
As linear regression is a statistical technique, there are certain statistical assumptions we have to meet. If we violate those, the best laid plans may falter and our results may be not as robust as we hoped. Let us go through these assumptions and the tests to check for them one by one.

7.2.1 Linearity

The name already gives it away, a *linear* regression is used to estimate **linear** relationships between variables. For this to work, the relationships actually have to be linear. But a relationship between two variables can have other functional forms. Consider this for example:

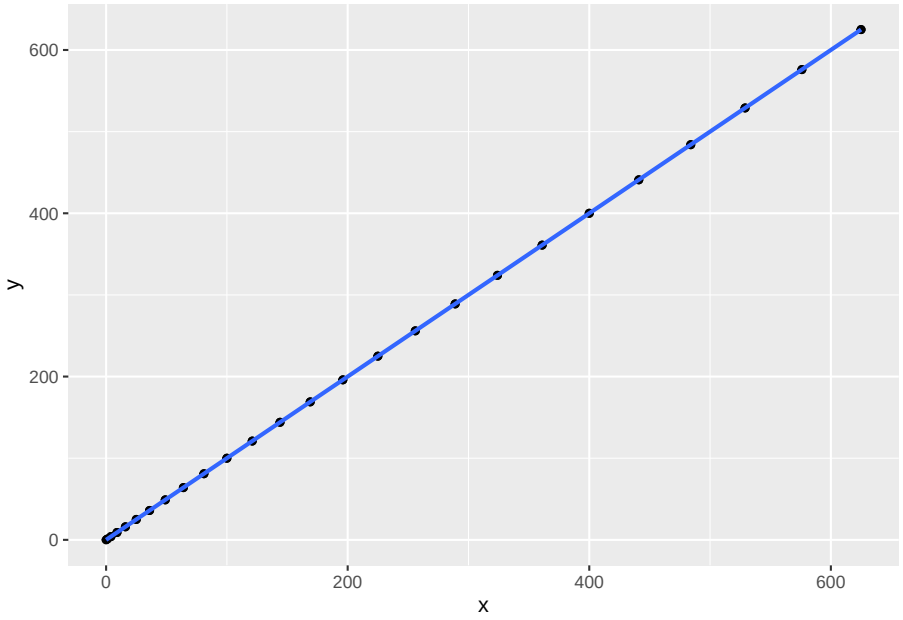


The relationship is clearly not linear. But we can still fit a regression and get a result:



The regression line shows us that `x` and `y` are completely uncorrelated. This is clearly not true, but as our linear regression assumes linearity, it tries to model the relationship in linear terms.

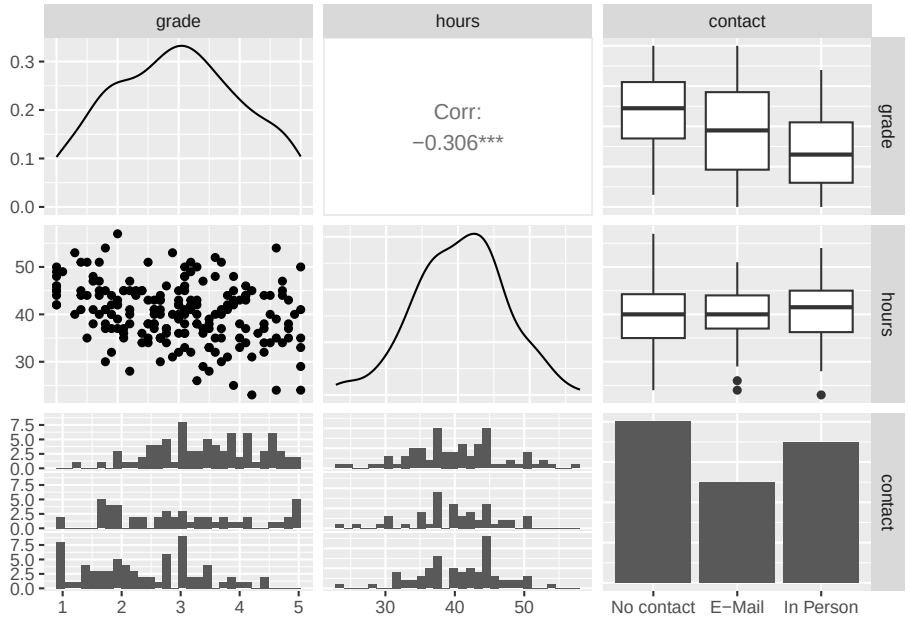
What we can do in such cases is to transform the variable in question in a non-linear way. Here the quadratic relationship is easy to spot, so if we transform x to x^2, this happens:



The non-linear relationship between `x` and `y` has thus been transformed into a linear one.

For real world data, the non-linearity most often is not as straightforward to spot as in this example. A first step to approach this, is inspecting a scatterplot matrix. This is usually done before starting to model to identify relationships between the variables used.

```
## Warning: package 'GGally' was built under R version 4.2.3
```

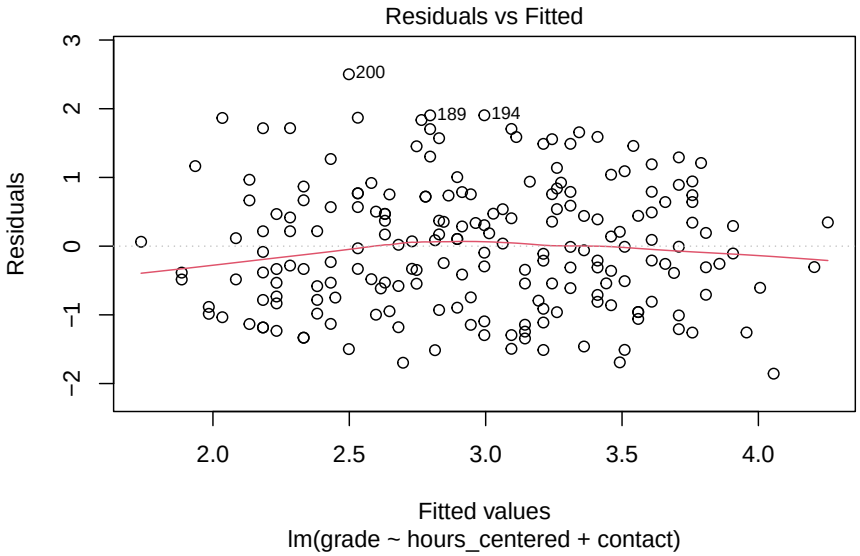


The diagonal displays the distribution of all variables included. Here metric variables are displayed as density plots and categorical variables as bar plots. Below and above the diagonal the relationship between two variables is shown. The scatterplot on the left of the second row is the one between `hours` and `grade` we have already seen several times. There is no indication of non-linearity here. What we have not inspected yet, is the relationship between `contact` and the two other variables. The bottom row contains histograms of the two metric variables by the category of contact, the right column boxplots for the same combination. Without going into too much detail on both types of plots, both show us how the distribution for both metric variables changes by category. The more personal the contact with the lecturer, the lower the distribution of final grades is. This makes sense, as we have already seen this correlation in the results of our model. Between `hours` and `contact` there seems to be no correlation. The amount of hours a student invests in writing the paper, does not lower the hours invested in a systematic way.

XXX MAYBE LINK EXPLANATIONS FOR HISTOGRAMS AND BOXPLOTS XXX

But this does not clear the model of suspicions of non-linearity just yet. Even when all pairwise relationships are linear, controlling for multiple variables at the same time can introduce non-linearity for this specific combination. One way to approach this is to inspect the *Residuals vs. Fitted* plot. As the name suggests, this plots the fitted values, i.e. the estimates for our dependent variables based on the model, against the residuals of the dependent variable. When the relationship is linear, we should see a more or less straight line along the

x-axis, where $y = 0$.



For many use cases, the line is straight enough, indicating no clear and strong patterns on non-linearity. Still the residuals seem to be slightly off for very good and very bad estimated grades.

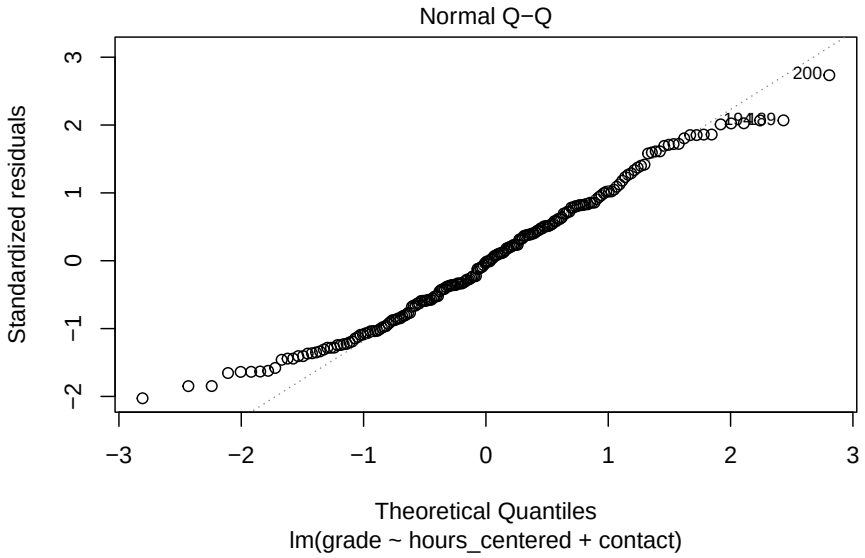
Besides violating the assumption of linearity, patterns in the residuals vs. fitted plot can also indicate, that there is some important explanatory variable missing from the model. We will return to this this, after we have applied the further tests and discussed the remaining assumptions.

7.2.2 Normally distributed residuals

Another assumptions in linear regression is that the residuals are normally distributed. This is especially relevant for sample with a small n as the test statistics tend to get unreliable in these cases if the residuals are not normally distributed. For larger samples, as in our case, this is not as problematic. Still, systematic deviations from normality can indicate that the model is not *parsimonious*. This means that either not all relevant variables are included or that variables are included that are not necessary for the model.

We get a first idea of the distribution from the model summary. This shows us the median as well as the 25- 75-percentiles and the minimum and maximum values. While strong and clear violations against the normality assumption could already be visible here, these measures are not enough to actually test for normality. A more informative and accurate approach is using a *Q-Q plot*.

This plots the standardizes residuals, the residuals divided by their standard error, against a theoretical normal distribution. If the residuals are perfectly normally distributed, each data point lies on the diagonal, if they are not they move away from the line. Small deviations, especially in the tails, should not be over emphasized. What we are looking for are clear and systematic deviations.



In the case of our model, most data points lie on the diagonal, while there are some small deviations in the tails. As our n is large enough, this should not be problematic. It may indicate that the model is not parsimonious but there is no clear cause for concern here.

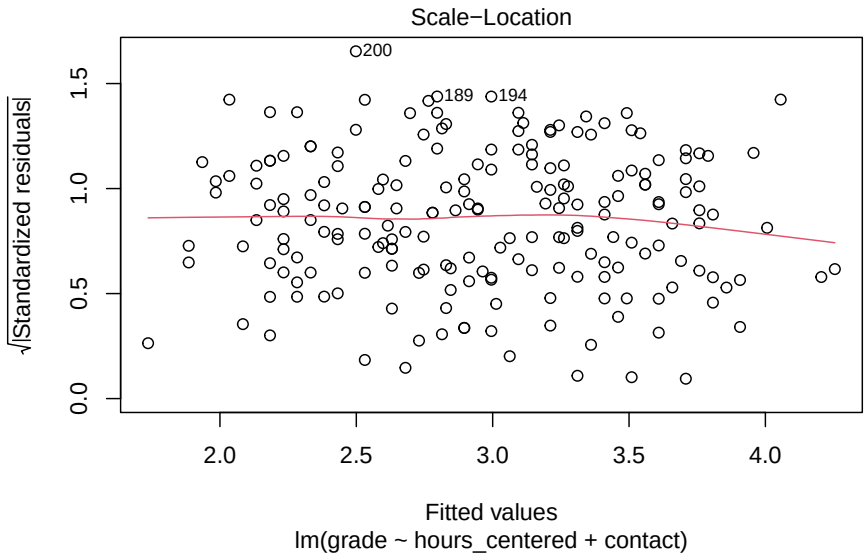
7.2.3 Homoscedasticity

The *homoscedasticity* assumption states that the residuals are expected to have a constant variance over the whole range of the dependent variable. Let us assume that the variance of our residuals would be lower for very good grades and higher for very bad ones. This would indicate that we can make more accurate estimates for better grades than for worse ones as a small variance would indicate smaller residuals and thus a smaller error. For the assumption to hold we must be able to make about the same quality, be it high or low, for all values of `grade`.

The problem is that the computation of the standard errors, test statistics and p-values depends on this assumption. If the assumption is violated, if we have *heteroscedasticity*, these measures are not reliable anymore.

The problem often occurs, if the dependent variable is not symmetric. In the scatterplot matrix above, we already saw that `grade` is fairly symmetrically distributed, so we would not expect problems here. If our dependent variable was unsymmetrical, transforming it to be more symmetrical, e.g. by using the logarithm or a square root, could help.

To check for problems with heteroscedasticity, we can use the *Scale-Location* plot. This plots the fitted values against the square root of the standardized residuals. For homoscedasticity to hold, we should see our data points as a horizontal band with more or less constant width running from the left to the right. The same goes for the plotted line.



In our case the homoscedasticity assumption holds. Slight variations are not problematic and overall the variance seems to be constant.

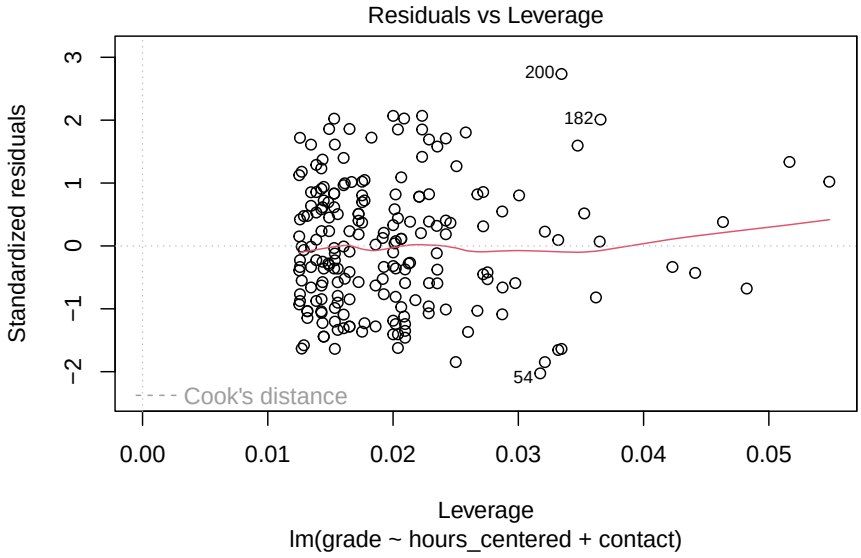
7.2.4 No overly influential data points

Observations can get highly influential if they have unusual values. Sometimes these are extremely low or high values on some variable. But even "normal" values on two or more variables can get unusual in their combination. Imagine a student with 60 invested hours. A high value but not overly extreme. Now the same student had in person contact with their lecturer but still received a 5.0. This could potentially be an outlier as this combination is unusual in terms of what the model expects. In case of our model this observation would most probably not be overly influential. But imagine the same observation with 300

invested hours. Such extreme cases can influence the fit by figuratively "puling" the regression line in their direction.

We can divide influential data points into unusual values on the dependent variable, *outliers*, and unusual values on independent variables, *high leverage points*. The latter have high leverage because the "pull" on the regression lines and thus change the slope. As a rule of thumb, we can consider values with standardized residuals over 3 or under -3 outliers. Concerning the dependent variables we can compute the *leverage statistic*. Here values that exceed $2 * (p + 1)/n$, where p is the number of predictors, are considered as having high leverage. We can inspect both at the same time in the *Residuals vs. Leverage* plot.

XXX LINK TO LEVERAGE STATISTIC FORMULA XXX



We can see that there are no clear outliers. To assess points with high leverage we first have to compute the threshold as: $2 * (3 + 1)/200 = 0.04$. Note that while we have two independent variables in our model, we actually have three predictors due to our categorical variable. Thus we have to compute with $p = 3$ instead of $p = 2$. We can see that there are a number of points that exceed this value. The question is, why do these values exist? Sometimes these are measurement errors, extreme values or unusual combinations that come down to the researcher recording the wrong values into the data set. In these cases we can try to fix the errors or remove the observations from the data. As we have several values with high leverage, this seems highly unlikely. But if we had not simulated the data ourselves and knew that there is no error, we should at least check. What seems more probable though, and is often the actual root of high

leverage, is that there are variables missing from the model that could explain the high leverage. In this case the values should be lower after we include the missing variables into the model. We will return to this later.

XXX IS LEVERAGE CORRECTLY COMPUTED? HOW TO INCLUDE A CAT. VAR? XXX XXX ALSO INCLUDE COOKS D? XXXX

7.2.5 No (multi)collinearity

The final assumption we will discuss here, is the absence of (high) collinearity between the predictor variables. Collinearity is present, if two independent variables are highly correlated with each other. This can become a problem as it gets harder to individually estimate the effects for both variables on the outcome as the collinear variables vary together at the same time.

Often collinearity can already be spotted in the correlation matrix. Considering our matrix above we saw no clear indication that `hours` and `contact` are correlated. But the problem can get more complicated if we include three or more independent variables in our model. While none of the pairs of variables may be highly correlated, correlation may exist for a set of three or more of those variables. In these cases we speak of *multicollinearity*. We can not spot this in a correlation matrix, but there is an easy to use measure available.

The *variance inflation factor* (VIF) can be used to inspect (multi)collinearity between two or more independent variables accurately. A VIF of 1 would indicate no collinearity. For real world data this is almost never true as some amount of collinearity always exists. But in general we can say that the VIF should be near 1 and should not exceed a value of 5.

Let us compute the measure for our model with `hours_centered` and `contact`:

```
## Warning: package 'car' was built under R version 4.2.3
```

```
## hours_centered        contact
##       1.004352       1.004352
```

Both values are very close to 1 so we can conclude that we did not violate the assumption. But what could we do, if we did? One approach is to just delete one of the highly correlated independent variables from the model. As they vary together, it may be save to exclude one of them witout losing to much information. Another approach would be, to combine both variables into a new measure. Let us imagine that besides `contact` we would have another variable in our model, measuring how well a student feels supported by their lecturer in writing the paper. We could also imagine both variables being strongly correlated as they measure comparable concepts. We could then either drop one of the variables, maybe losing some information in the process, or we could

combine both into a new variable which measures the form and the feeling of support at the same time, maybe leading to a more accurate estimate while at the same time eliminating the problem of collinearity. Which one is the right solution depends on the specific case.

7.3 Returning to our research question

When we tested for linearity above, we saw a mild pattern in the data which is not explainable by a violation of the assumption of linearity and thus could be an indication of a missing relevant explanatory variable in our model. Some of the other tests also supported this notion. The Q-Q plot showed us that the residuals have some slight deviations from normality in the tails. While these deviations are small enough to not cause concern on their own, taken together with the residuals vs. fitted plot this gives more weight to the suspicion that some important variable is missing. We also identified some of observations with high leverage. While we can rule out errors in our data, the high leverage could also be explainable by a missing variable.

But which variable could be missing from the model? If our DAG is correct, we can rule out `attendance` and `previous_grades`. We did assume that `contact` is a confounder for `hours` and `grade` and thus included it in our model. Did we go wrong there?

We did assume, that the more personal the contact, the more efficiently the time working on the paper can be used. And here may lie the problem. The way we included `contact` in the model is not the way we reasoned in our DAG. It would be correct if we assumed that the more personal the contact, the less time has to be invested. But we already saw in the scatterplot matrix that there is no such relationship between the variables. To specify the effect of `contact` in the model correctly, reflecting the idea of a more efficient use of time the closer the contact was, we have to include it as an interaction with `hours`.

7.3.1 Interactions

In an *interaction*, we assume that the effect of one variable differs based on the value of another variable. Let us return to the formula for a multiple regression with two variables:

$$y = \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + \epsilon$$

Here we assume that the value of y varies with the value of x_1 and x_2 as indicated by the coefficients β_1 and β_2.

But we could also follow the notion that the value of x_1 influences y differently based on the value of x_2. For example the effect of x_1 on y could be higher when

x_2 also has a high value. This is an interaction and is reflected in the formula by adding an additional multiplicative term between the two dependent variables with an additional associated coefficient:

$$y = \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + \beta_3 * x_1 * x_2 + \epsilon$$

To get a better understanding of this, let us return to our model and add an interaction between `hours_centered` and `contact`.

```
##
## Call:
## lm(formula = grade ~ hours_centered + contact + hours_centered *
##     contact, data = grades)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.77816 -0.72882 -0.08719  0.56140  2.53271
##
## Coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                      3.44466    0.10345  33.298  < 2e-16 ***
## hours_centered                  -0.02893    0.01535  -1.885  0.06098 .
## contactE-Mail                   -0.46775    0.16729  -2.796  0.00569 **
## contactIn Person                -1.01493    0.15171  -6.690 2.33e-10 ***
## hours_centered:contactE-Mail    -0.02377    0.02657  -0.895  0.37204
## hours_centered:contactIn Person -0.05017    0.02442  -2.055  0.04125 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9253 on 194 degrees of freedom
## Multiple R-squared:   0.28,  Adjusted R-squared:  0.2615
## F-statistic: 15.09 on 5 and 194 DF,  p-value: 1.63e-12
```

How can we interpret these results? While the estimates for the intercept and for having e-mail or personal contact in comparison to having no contact at all have barely changed, the coefficient for the amount of hours invested substantially shrunk to almost half its former value. Up till now, we assumed that the effect of `hours` would be the same for each student. Not that we have included an interaction we assume that the effect of `hours` differs, based on the form of contact a student had.

Let us rewrite our formula for $\hat{y}$ including the interaction. As we are interacting with a categorical variable with three categories, we have to add two interaction terms. The first for the effect of invested hours when e-mail contact was made and the second for the effect of hours when contact was made in person, in both cases compared to having had no contact.

$$\hat{y} = b_0 + b_{hours\_centered} * x_{hours\_centered} + b_{E-Mail} * x_{E-Mail} + b_{InPerson} * x_{InPerson} + b_{hours\_E-Mail} * x_{hours\_c}$$

Let us now also add the coefficients from the model:

$$\hat{y} = 3.44466 - 0.02893 * x_{hours\_centered} - 0.46775 * x_{E-Mail} - 1.01493 * x_{InPerson} - 0.02377 * x_{hours\_centered} * x_I$$

We can now consider the three possible forms of contact one by one.

What happens, when a student had no contact? To explore this, we return to the regression formula and equal x_{E-Mail} and $x_{InPerson}$ to 0, which means that no contact was made beforehand. Note that for now we do not care about the actual value of `hours_centered`.

$$\hat{y} = 3.44466 - 0.02893 * x_{hours\_centered} - 0.46775 * 0 - 1.01493 * 0 - 0.02377 * x_{hours\_centered} * 0 - 0.05017 * x_{hours}$$

This shortens to:

$$\hat{y} = 3.44466 - 0.02893 * x_{hours\_centered}$$

For a student who did not make contact, we would estimate the final grade as the intercept minus 0.02893 per hour invested more than the mean of `hours_centered`. As having equaled x_{E-Mail} and $x_{InPerson}$ to 0 not only "switched off" the effects of `contact` but also removed the interaction effects from the equation, the estimated effect for `hours_centered` is only its coefficient of −0.02893.

What happens, when a student had e-mail contact?

$$\hat{y} = 3.44466 - 0.02893 * x_{hours\_centered} - 0.46775 * 1 - 1.01493 * 0 - 0.02377 * x_{hours\_centered} * 1 - 0.05017 * x_{hours}$$

This shortens to:

$$\hat{y} = 3.44466 - 0.02893 * x_{hours\_centered} - 0.46775 - 0.02377 * x_{hours\_centered}$$

and further to:

$$\hat{y} = 2.97691 - 0.0527 * x_{hours\_centered}$$

The intercept is reduced by the coefficient of having e-mail contact, but what is actually of interest here is the effect that `hours_centerd` has. For a student who had e-mail contact, each hour invested above the mean reduces the estimated grade by -0.0527.

We can compute the same for a student with personal contact:

$$\hat{y} = 3.44466 - 0.02893 * x_{hours\_centered} - 0.46775 * 0 - 1.01493 * 1 - 0.02377 * x_{hours\_centered} * 0 - 0.05017 * x_{hours\_centered} * 1$$

$$\hat{y} = 3.44466 - 0.02893 * x_{hours\_centered} - 1.01493 - 0.05017 * x_{hours\_centered}$$

$$\hat{y} = 2.42973 - 0.0791 * x_{hours\_centered}$$

For a student who had in person contact, each hour invested above the mean reduces the estimated grade by -0.0791.

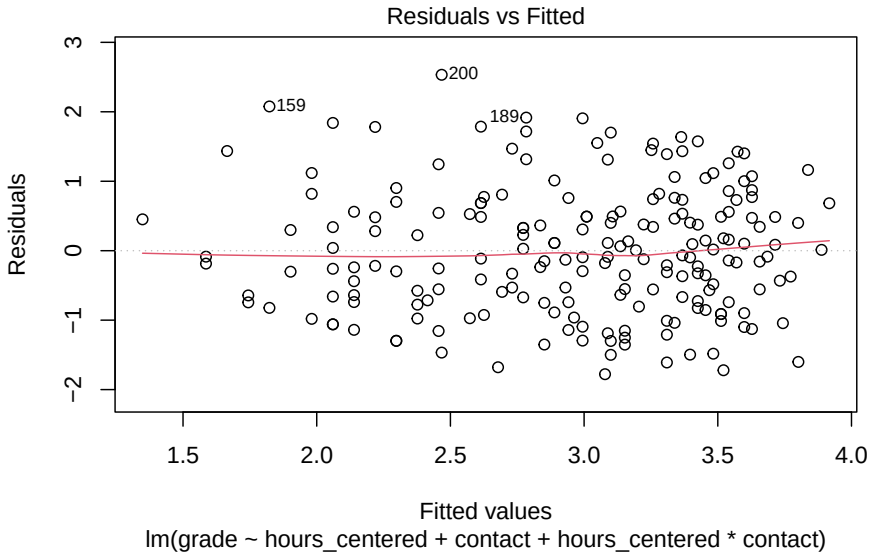
In practice we would have reached these conclusions more quickly by inspecting the output from our model and just subtracting the corresponding interaction effect from the effect for `hours_centered`.

In the model without the added interaction we concluded, that on average each hour invested above the mean decreases the final grade by about -0.5. Now we see that the effect of hours depends on the form of contact had. This reflects the theoretical assumption from our DAG that time can be used more efficiently the more personal the form of contact was.

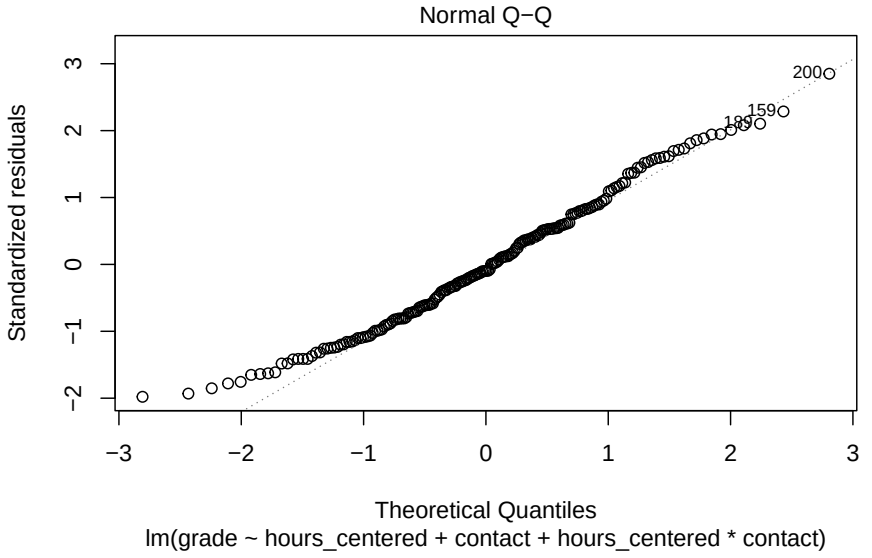
The same DAG that informed our best model from last week now lead us to including the interaction. This underlines the importance of thorough theoretical thinking before starting to model. If we had invested more time, we could have concluded the correct model directly. In our case we first needed the regression diagnostics to tell us that something might be off before we figured out our error.

7.3.2 Regression diagnostics (revisited)

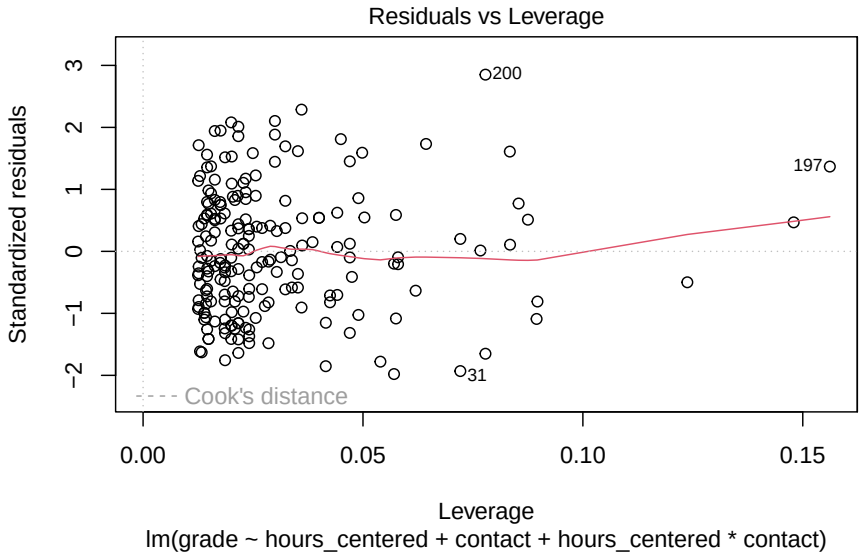
We now have a theoretically sound model, but did we also solve the problems indicated in the regression diagnostics?



The residuals vs. fitted plot now shows an almost straight horizontal line with no clear visible patterns. This indicates that the problem we saw with our former model actually came down to a missing variable, or to be more precise a missing term in our case.



The Q-Q plot now also shows more normally distributed residuals. While there are still some small deviations at the lower tail these do not indicate a remaining severe problem. THe deviations are smaller than before and also not drastic in absolute terms. Also, as stated above, violating the normality assumption is less problematic with a high n and few variables in the model, which is true for our case.



Adding the interaction term actually increased the leverage of the more influential observations.

XXX REVIST LEVERAGE, I THNIK I SHOULD INCLUDE COOK'S D BECAUSE LEVERAGE MAY NOT BE PROBLEMATIC IN ITSELF XXX

XXX MAYBE PUT DIAGNOSTICS FOR M2 AND M4 NEXT TO EACH OTHER XXX

XXX OUTRO: here we know, never that easy in reality XXX

Chapter 8

Linear Regression - Application

8.1 Objectives

8.2 R functions covered this week

- `lm()`

8.3 Temporary from EDA I

This is interesting for a first look. For example, it seems that the weight is strongly correlated with whatever position you play. Centers are heavy, point guards are light weights. We also see that most performance metrics ("career_…") are correlated with each other and also with salary. Good players seem to be good in many things, and good players seem to be paid more.

Now, that we have a feeling for the whole dataset, we want to explore individual variables. To keep it focused, we want to further explore the question of whether players that score more on average are also paid more.

Maybe roles are clearly divided on the team. Maybe really good passers are highly paid because they give great passes to people who then score. Or maybe teams don't care about passers and just pay more to people who score more.

So, now, let's look at salary and average number of points scored by game. Also, we want to know whether point guards (short people who pass a lot) are paid less then other positions (who score more).

So, it seems that point guards are paid a little less even though they make a few more points on average. Interesting puzzle to explore.

Let's reflect a moment what we can learn from all this.

First, there seems to be a somewhat linear relationship between how many points a player scores and how much they are paid. This relationship seems pretty robust across years and teams. It also holds true for point guards just as much as for non-point guards. However, the average salary for point guards is lower in comparison. We also learn that the link between salary and points is stronger for centers. They seems to be paid more, the more they score.

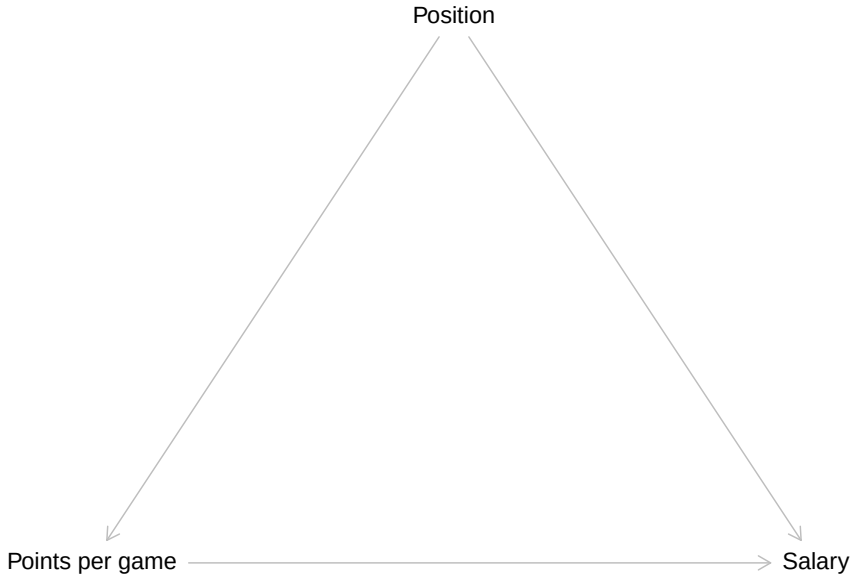
1) It can model the relationship between two variables while considering simultaneously the potential influence of other factors. Image we are interested in the effect of points per game on salary regardless of position, season or team. With regression we can estimate how much more a player would ear every season if he scored 10 more points a game (regardless of the position he plays).

8.4 Research question

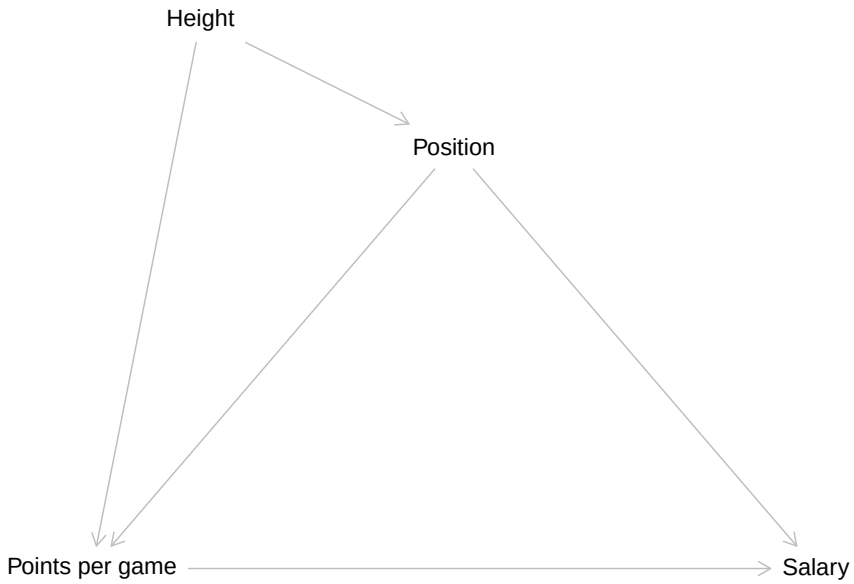
We will now pick up where we left several sessions ago and return to the NBA data. We already saw, that there was an interesting relationship between the points a player makes per game and the salary he receives. The more points, the higher the salary. This makes intuitive sense as high scoring player is more valuable to the team and thus receives a higher monetary compensation. Our goal for this session is to estimate the effect of points on salary and to assess if it really exists and what its magnitude is.

Let us start building a DAG with the information we already have.

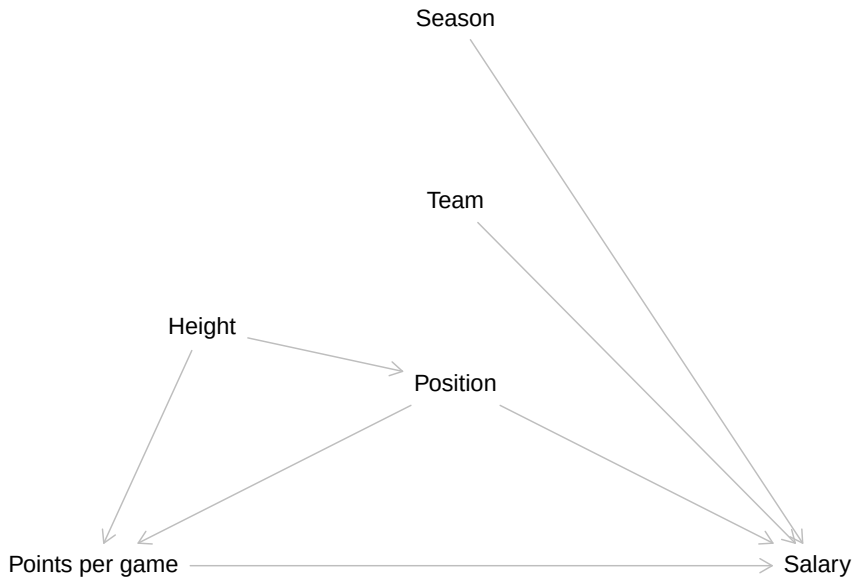
Now we have to think about other factors that could influence the relationship between points and salary. One variable we already identified as having an effect on both was the position a player occupies. The position influences how many points per game a player can score and we also already saw that centers make more money compared to point guards. Right now we have no reason to believe that other positions do not also have an effect on the received salary. Following this reasoning, position is a confounder for points and salary.



It is also reasonable that the height influences which position a player can occupy. Centers have to be big while smaller players tend to play on other positions. At the same time, height is an advantage if you want to score in a basketball game. Thus height is a confounder for points and position.



Another factor that will have an effect on the salary is the team a player play for. More successful teams will be able to pay higher salaries. The season an observation was recorded in, should also influence the paid salary as we can expect a general inflation of salaries over time.



This is our final DAG. Now what does this imply for our model? Our goal is to measure the effect of the points per game on the received salary. Position is a confounder for both, so we have to control for it to get an unbiased estimate for points. Height is also a confounder, but for points and position. So we have to control for this variable too? As we already control for position, the path from height -> position -> points is already closed. This means that we do **not** have to onclude height in our model to get a unbiased estimate. The two remaining variables, team and season, have direct effects on salary but no other connections. This implies, that we also do **not** have to control for them if our goal is estimating the effect of points. Remember, if our goal was predicting the salary as accurately as possible, we should maybe include both variables, but this is not our goal today.

Now let us get to it and load the NBA data we prepared in week 2.

```
library(tidyverse)
load("../datasets/nba/data_nba.RData")
```

8.5 Simple linear regression in R

To conduct a multiple linear regression in R, we can use the built-in *base R* function `lm()`. The function is straightforward to use. As the first argument we write the regression formula in R's *formula syntax*.

In the formula syntac we start by writing the name of our `dependent_variable` followed by a *tilde* `~`. You can read this as an = or as "regress the dependent variable on". After the tilde we add our first `indepedent variable` by again writing out its name. If we have multiple independent variables in our model - so we are running a *multiple linear regression* - we can add those by writing a `+` followed by the name of the variable to be added.

As a second argument, the function needs the name of the object that holds our data.

The goal of our research question is to estimate the effect of the points per game on the received salary. So to regress `salary` on `career_PTS`, we just write:

```
lm(salary ~ career_PTS, data = data_nba)
```

```
##
## Call:
## lm(formula = salary ~ career_PTS, data = data_nba)
##
## Coefficients:
## (Intercept)   career_PTS
##     -851914       552843
```

This gives us a short output. The first line just echoes our code used to run the regression. We have seen this in the last session already, but now we know what the meaning was. After this we have a short block with the estimated coefficients. As we have run a simple linear regression, we only get the intercept and the coefficient for the sole independent variable used in the model. If we would have run a multiple linear regression, the result would basically look the same, only with more coefficients to display.

Before we dive into the results, we should talk about how to receive a more verbose output that does not hide all the other vital information that is associated with the model.

The easiest way is to use the base R function `summary()`. This is a generic R function that returns different summaries, depending on the object it is used on. We can for example use it on a data frame or tibble to get some descriptive statistics for the included variables. For example, we can information on the distribution of points per game by writing:

```
summary(data_nba$career_PTS)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   5.100   8.000   8.908  12.000  30.100
```

When we use `summary()` on a model object, like the one created by `lm()`, we get a different output. Before we apply this we should save our model in an object. This is good practice in most cases as we can now apply all additional analysis of the model on ths object and we do not have to rerun the model every time.

```
m1 <- lm(salary ~ career_PTS, data = data_nba)
```

We can now apply `summary()` on the object `m1`, short for "model 1":

```
summary(m1)
```

```
##
## Call:
## lm(formula = salary ~ career_PTS, data = data_nba)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -14788659  -2023969   -434599   1311807  24326060
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -851915      76417  -11.15   <2e-16 ***
## career_PTS    552843       7453   74.17   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3732000 on 9726 degrees of freedom
## Multiple R-squared:  0.3613, Adjusted R-squared:  0.3612
## F-statistic:  5502 on 1 and 9726 DF,  p-value: < 2.2e-16
```

This is the output we saw over the last weeks and it includes extended and better readable coefficient block as well as the information on the residuals and the model fit.

An alternative method of displaying the coefficients in a regular tibble format, is to use `tidy()` from the `broom` package.

```
library(broom)
```

```
## Warning: package 'broom' was built under R version 4.2.3
```

```
tidy(m1)
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) -851914.    76417.     -11.1 1.09e-28
## 2 career_PTS   552843.     7453.      74.2 0
```

8.5.1 Interpretation

While we know our model is not complete yet, let us still inspect the results. For each point a player scores per game, his salary rises by about $552,000$\$. We see a clear positive and substantial effect. Let us also inspect the intercept. THis tells us that a player who makes no points per game has to pay the team about $850,000$. Wait, this does not make sense... To make the intercept more readily interpretable we should again center our metric dependent variable `career_PTS` on its mean.

```
mean(data_nba$career_PTS)
```

```
## [1] 8.907679
```

```
data_nba <- data_nba %>%
  mutate(PTS_centered = career_PTS - mean(career_PTS))
```

As we have now centered the independent variable of interest on its mean of 8.9 we can rerun the model.

```
m1 <- lm(salary ~ PTS_centered, data = data_nba)

summary(m1)
```

```
##
## Call:
## lm(formula = salary ~ PTS_centered, data = data_nba)
##
## Residuals:
```

```
##       Min        1Q    Median        3Q       Max
## -14788659  -2023969   -434599   1311807  24326060
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4072633      37840  107.63   <2e-16 ***
## PTS_centered   552843       7453   74.17   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3732000 on 9726 degrees of freedom
## Multiple R-squared:  0.3613, Adjusted R-squared:  0.3612
## F-statistic:  5502 on 1 and 9726 DF,  p-value: < 2.2e-16
```

The coefficient for points per game has not changed but its interpretation has. For each point per game over the mean of 8.9 points per game, the salary is estimated to increase by about $552,000$\$. At the same time, for each point below the mean the salary is estimated to decrease by the same amount. The intercept now shows us the estimated salary of a player who scores 8.9 points per game which is slightly upwards of $4,000,000$\$ This makes way more sense.

This model model already achieved a considerable R^2 of 0.36. About 36% of the variance in salaries is explained by the points per game a player scores.

8.6 Multiple linear regression in R

The DAG we have constructed above based on our research question indicated that we also have to include the position a player occupies in our model. We can add additional independent variables to the formula used in `lm()` with a `+` and the name of the additional variable(s). This works the same way for all types of variables, i.e. metric, dummies or categorical variables. So let us do this now by adding the 5 dummies we constructed for the positions:

```
m2 <- lm(salary ~ PTS_centered + position_center + position_sf +  position_pf + position_sg + pos

summary(m2)
```

```
##
## Call:
## lm(formula = salary ~ PTS_centered + position_center + position_sf +
##     position_pf + position_sg + position_pg, data = data_nba)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
```

```
## -14511723  -1950255   -372906   1358768  24433660
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      3679728     114300  32.193  < 2e-16 ***
## PTS_centered      568019       7474  75.994  < 2e-16 ***
## position_center  1380246     114539  12.050  < 2e-16 ***
## position_sf       125384     102174   1.227 0.219790
## position_pf       206505      94882   2.176 0.029547 *
## position_sg      -331033      96841  -3.418 0.000633 ***
## position_pg      -114552     117486  -0.975 0.329572
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3652000 on 9721 degrees of freedom
## Multiple R-squared:  0.3888, Adjusted R-squared:  0.3884
## F-statistic:  1031 on 6 and 9721 DF,  p-value: < 2.2e-16
```

8.6.1 Interpretation

We still see a clear positive effect of points per game on the received salary after controlling for the position a player occupies. Among those centers are by far the top earners, making about $1,400,00$ more than players on other positions. Most other positions show relatively small effects on the earnings. Power and small forwards earn somewhat more than other positions on average while point and especially shooting guards earn less.

We can now compare two fictive cases of a center and a point guard who each make about 20 points per game. What is the estimated salary for them?

As we have extensively worked with the formulas over the last sessions, we can now keep it short and calculate the estimate directly. Remember that we centered the points per game on the mean of about 8.9, so making 20 per game would mean making about 11.1 than the average player. We will keep it simple here and calculate with 11.

$$\hat{y_{center\_20}} = 3679728 + 568019 * 11 + 1380246 = 11,308,183$$

$$\hat{y_{pg\_20}} = 3679728 + 568019 * 11 - 114552 = 9,813,385$$

Despite making the same amount of points per game for their team, the model estimates that a point guard earns about $1,500,000$ less compared to a center.

8.6.2 Sidenote: Adding interactions

We will not use interactions in this session but we briefly want to state how we could add them in the formula syntax.

Remember that interactions are multiplicative terms in our regression formula. Adding them to the R formula syntax works the same way. We add the new term with a `+` and use a `*` between the two variables that we want to interact.

Here is a non running toy example where we interact two x-variables:

```
lm(y ~ x1 + x2 + x1 * x2, data = some_data)
```

8.7 Regression Diagnostics

So how does our model perform? Did we meet all the regression assumptions that were introduced last week?

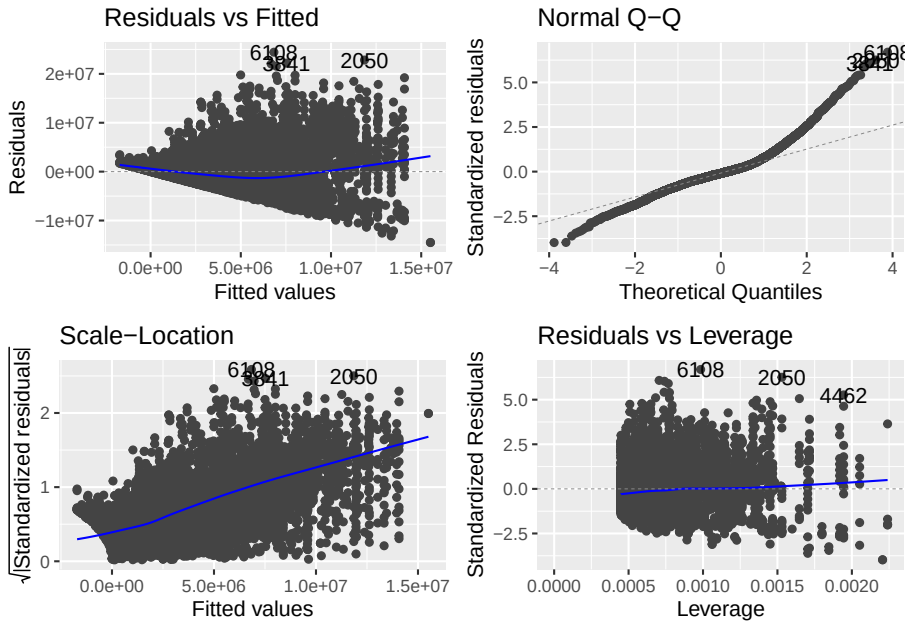
To access the visual tests we used last session, we can just use the base R function plot, applied to the model object. If we just write `plot(m2)`, the console asks us to press `ENTER` to go through each plot ony by one. We can also add a number as a second argument, specifying which plot we want to see. For example, `plot(m2, 1)` gives us the residuals vs. fitted plot.

But there is an easier way to see all four plots at once. The package `ggfortify` expands the functionalities of `ggplot2` so that we can use it's `autoplot()` function to automatically plot all four visual tests of interest. An added benefit, depending on your taste, is that the plots are rendered in the style of `ggplot2`.

```
library(ggfortify)
```

```
## Warning: package 'ggfortify' was built under R version 4.2.3
```

```
autoplot(m2)
```



The residuals vs. fitted does not show us a more or less straight line but starts mildly positive, then dips below 0 and rises again for higher estimated salaries. This could indicate at least wo thing. Either we have missed an important indepedent variable, like in the last session, or we are actually dealing with some amount on non-linearity.

The Q-Q plot this time shows, that the actual residuals are far from being distributed normally. While we can never expect a perfectly normal distribution, here the deviations are striking, especially for high residuals.

The scale-location plot is used to adress the assumption of homoscedasticity. What we want to see, is a straight line with data points equally distributed around it. This clearly is not the case here. As it is, the plot indicates that we may be able to estimate small salaries reasonably well but that the higher the estimate, the more unreliable our model gets.

The residuals va. leverage plot also indicates some problems. There are some observations that have larger or smaller standardized residuals compared to the thresholds of 3 and -3. The threshold for leverage is computed as $2 * (6 + 1)/9728 = 0.001439145$ We also see some observations with higher values. While both are rules of thumb and may not necessarily point to severe problems by themselves, things can get problematic when there are observation that do not meet the thresholds for both measures at the same time. This is indicated by clusters in the lower or upper right corners. We can observe this in the lower right.

We should also test for multicollinearity. We can compute the VIF measures

using a function from the package `car`.

```
library(car)

vif(m2)
```

```
##    PTS_centered position_center     position_sf     position_pf     position_sg
##        1.050400        2.035722        1.686736        1.512765        1.565004
##     position_pg
##        1.916933
```

The values for any variable should not exceed 5 and should be closer to 1. Our value for points shows no signs of multicollinearity. The values for the position dummies have somewhat higher values, which makes sense. While there are some players that play multiple positions, for most a value of 1 on one position predicts the other positions as having a value of 0. But as we are still far away from the threshold of 5, there is no need for concern here.

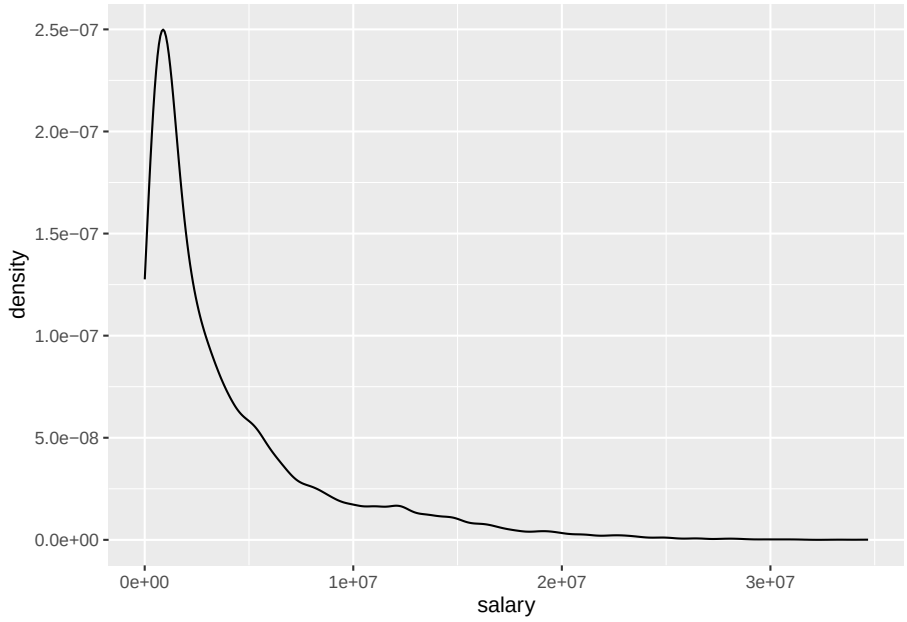
Overall, we have problems! While we do not see signs of problematic multicollinearity, all other tests indicated clear and in parts severe problems. We have to put in some more work before we can be confident that our model accurately predicts the effect of points per game on the received salary.

Before we start addressing the problems, we should not that the four plots are highly interactive. It is entirely possible that solving one of the problems also solves the others or, for added fun, even generates new ones. This means that we should refrain from turning too many dials at once and rather change the model one step at a time, see if it improves things and then adress remaining problems in the same way.

8.7.1 Skewed outcome variable

The deviation from normality and the clearly present heteroscedasticity could both point to the same problem, namely a skewed dependent variable. Let us examine its distribution first.

```
data_nba %>%
  ggplot(aes(x = salary)) +
  geom_density()
```

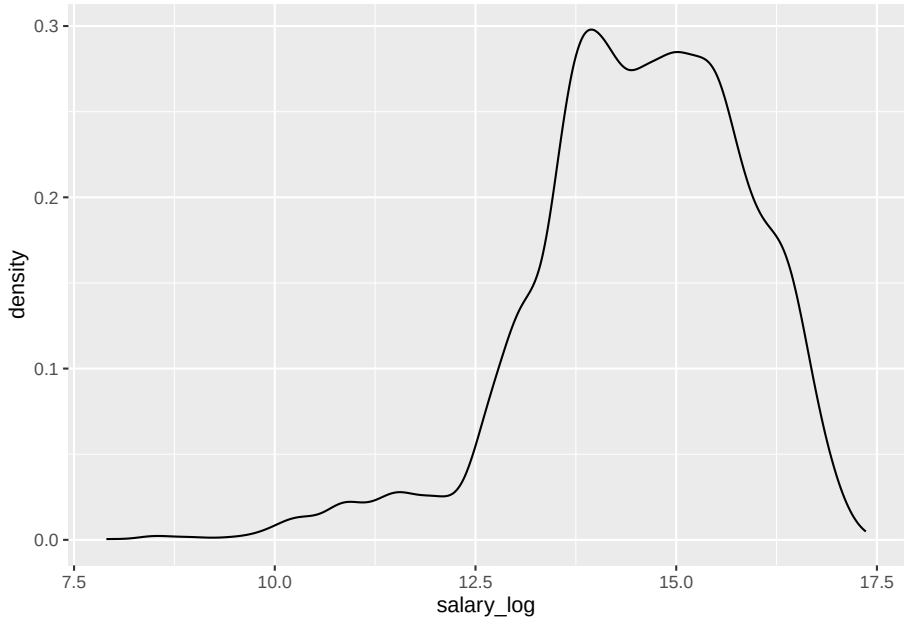


Our outcome variable is not only skewed, it is **highly skewed**. While there are many salaries in the "lower" six to seven digits regions, we also see some extremely high wages up to about $35,000,000$\$. The higher the salary the fewer observations we have. That is why we see such a long flat tail to the right.

This distribution actually is relatively common for income data. In most surveys of the general population we have many people receiving relatively low incomes while fewer individuals receive higher or extremely high incomes. It is still interesting that this also holds true for a population of high earners such as NBA players. That is why inequality is relative. Compared to the general population almost all our players would be somewhere in the long tail to the right. Compared to their own population we still see highly substantial differences in outcomes.

We can transform the dependent variable to a different scale to get a less skewed distribution. A common transformation for income data is to take the *logarithmus naturalis* of the actual value and then use this as our dependent variable. To achive the transformation we can simply use the base R function `log()` which as its default computes the ln.

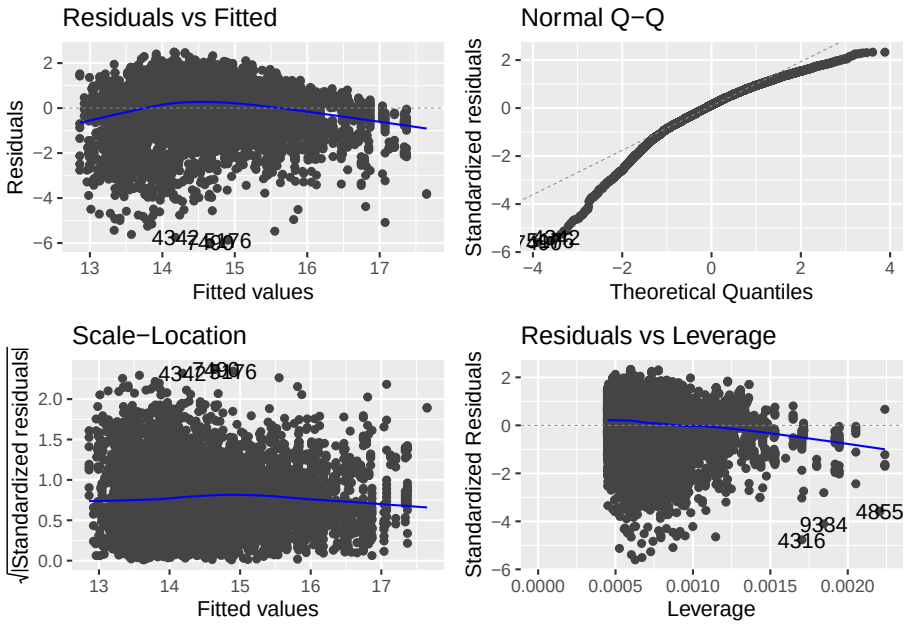
```
data_nba <- data_nba %>%
  mutate(salary_log = log(salary))

data_nba %>%
  ggplot(aes(x = salary_log)) +
  geom_density()
```



While the distribution of the transformed variable also is somewhat skewed, now to the left, overall it is much more evenly distributed.

We should use the new variable as our outcome an check the tests again.

```
m3 <- lm(salary_log ~ PTS_centered + position_center + position_sf +  position_pf + position_sg +

autoplot(m3)
```



Looking at the scale-location plot first, we can now see a straight line with our residuals fairly evenly distributed around it. Thus we now longer see any signs of heteroscedasticity. The Q-Q plot now also indicates a somewhat more normal distribution of our residuals but there are substantial deviations still. While high residuals now appear to more or less follow the normal distribution, small residuals now deviate stronger than they have before. This reflects the transformation and its distribution, which now has long tail on the left and not on the right anymore. Turning to the residuals vs. leverage plot we still see some observations that do not meet the respective thresholds. At the same time, there appear to be less that simultaneously have high absolute standardized residuals and high leverage. The residuals vs. fitted plot now also shows a more even distribution while the signs on non-linearity remain. We do not have to recompute the VIF measure as we did not change any independent variables in the model.

8.7.2 Non-linearity

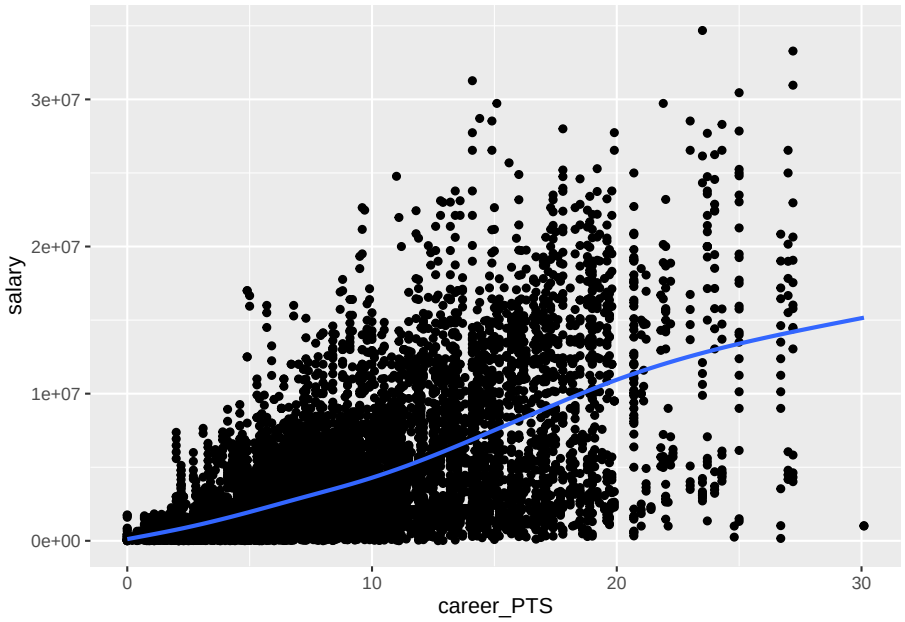
Let us now address the non-linearity that is still indicated in the first plot. We can approach non-linear relationships in our inherently linear model by adding non-linear transformations of a dependent variable to the model. But before we start squaring random variables, we should think about what could be non-linear in our case. We can rule out our dummy variables for position. This leaves the points scored. The model already tells us that our suspicion that salary rises with the points scored could be true. But maybe this relationship is

not linear over its whole range. If you already are among high scorers, scoring one or two points more than your peers may not be such a substantial difference and thus may not have the same strong effect on salary.

We should first inspect the relationship between both variables again. This time we add a LOWESS curve to the plot. This is often helpful in detecting non-linearity as the curve can change its slope over the range of the dependent variable. This is also the default for `geom_smooth()`.

```
data_nba %>%
  ggplot(aes(x = career_PTS, y = salary)) +
  geom_point() +
  geom_smooth(se = FALSE)
```

```
## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```

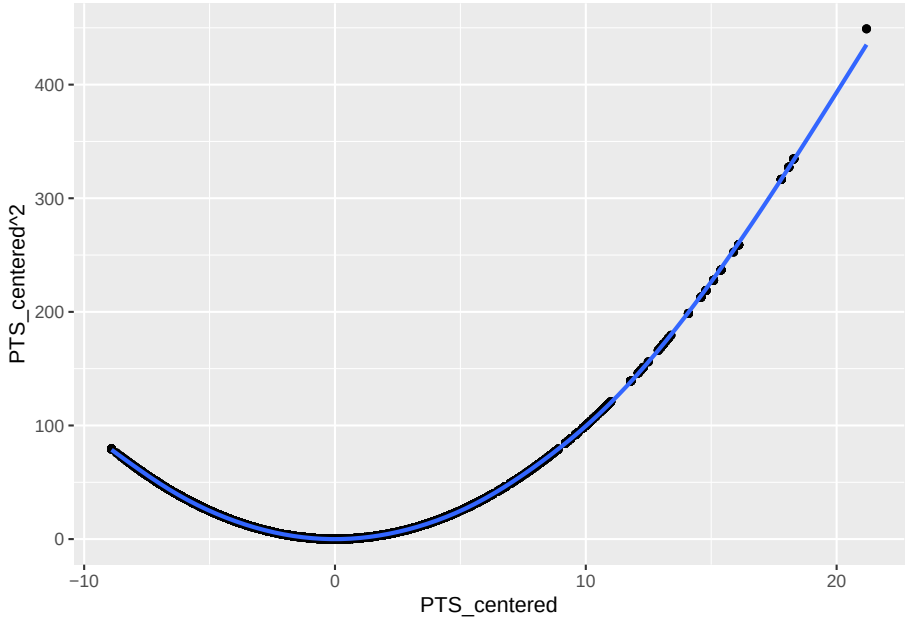


While this is not as clear as we hoped, the line may still indicate some mild non-linearity as it flattens somewhat for really high point values. Also we have to keep in mind that the non-linearity may be stronger when we control for additional variables, as our position dummies.

One common way to address the non-linearity is taking the square of the dependent variable in question. We should not square our centered points variable though. Let us inspect what would happen if we squared it.

```
data_nba %>%
  ggplot(aes(x = PTS_centered, y = PTS_centered ^ 2)) +
  geom_point() +
  geom_smooth(se = FALSE)
```

```
## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```

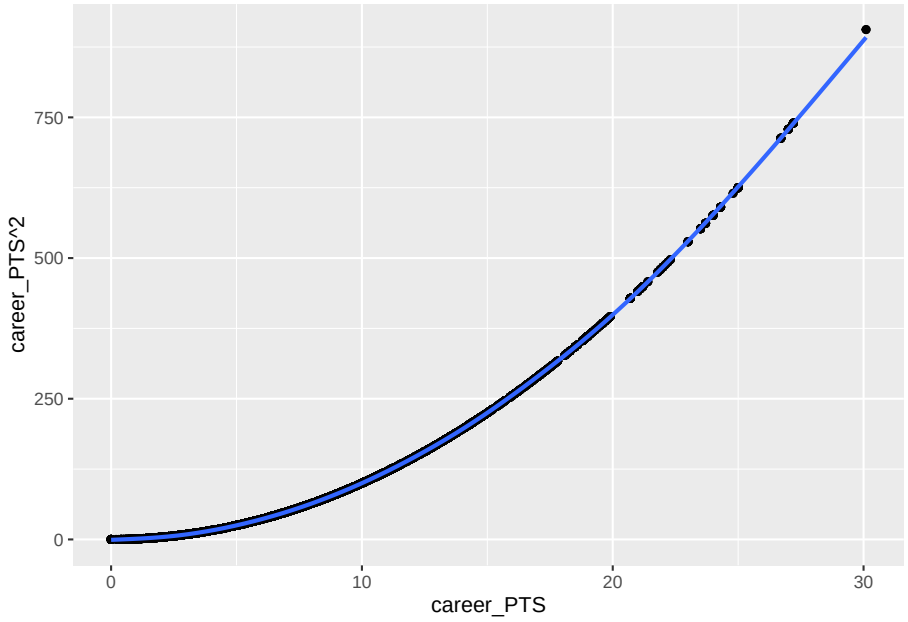


As the square of negative values is positive we would basically introduce the assumption into the model that there is non-linearity for low and high scorers and that the effect will be in the same direction. While our assumption is that there are diminishing returns between being a high scorer and a **really** high scorer, we do not assume that making more points if you are among the lower scorers should have the same effect. If at all, in these regions additional points could have an even larger effect.

Because of this we should return the uncentered version of our variable. What happens if we square this?

```
data_nba %>%
  ggplot(aes(x = career_PTS, y = career_PTS ^ 2)) +
  geom_point() +
  geom_smooth(se = FALSE)
```

```
## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```



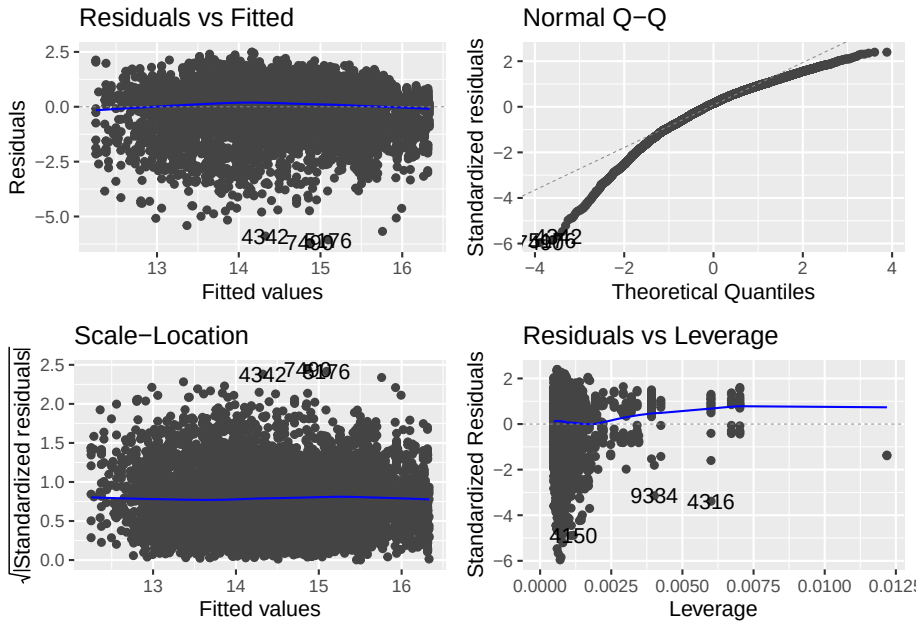
This is what we wanted, a transformation that expects a stronger difference the higher the score value is. For the final model we will thus work with the uncentered variable. We included the centered version because it is more straightforward to interpret. This is not really a concern anymore because dreams of easy interpretability are long gone after transforming two variables.

We could again transform the variable in our data and thus add a second version, but we can also do so directly in the formuly syntax. When we use the function `I()` we tell R to interpret anything within the parantheses as a mathematacial expression. This is waht we will do below. Note that we add the point variable as its untransformed and transformed versions. The first represents the linear parts and the second the non-linear parts of the effect.

```
m4 <- lm(salary_log ~ career_PTS + I(career_PTS^2) + position_center + position_sf + position_pf
```

We can now reasses the tests for the last model.

```
autoplot(m4)
```



The line in the residuals vs. fitted plot got more straight. It seems that we actually captured the mild non-linearity that was present before by adding the squared points value to our model. The scale-location plot also still indicates no more problems of heteroscedasticity. Contrary, the data points now are even more evenly distributed compared to `m3`. The Q-Q plot also has not changed, still showing non-normally distributed residuals. We can not really fix this now, but we also learned, that this test is less consequantial if we have a large N. Turning to the residuals vs. leverage plot, we still se several points that do not meet the thresholds but at the same time we do not see any points with high values for both. Overall there seem to be no overly influential points which we had to address. Let us also reexamine the VIF.

```
vif(m4)
```

```
##      career_PTS I(career_PTS^2) position_center     position_sf     position_pf
##       12.135096       11.883255        2.040563        1.709312        1.520284
##     position_sg     position_pg
##        1.569463        1.949078
```

We now see high VIF values for both versions of our point variable. The only substantial change is that we now see high values for these. Did we introduce a new problem? If we take the measure at face value, yes. But if we think about it, no. All this means is that both versions of our variable are highly correlated. Of course they are. One is computed from the other. We can perfectly predict the value of `career_PTS^2` from `career_PTS`. There is collinearity by design.

If we want to assess multicollinearity we should apply the function to `m3`. If would have used interactions, the situation would be similar. This is just a small reminder that all our tests do not work without thinking about what we are actually doing.

8.8 Returning to our research question

As we now settled on `m4` as our best model, it is time to discuss what we actually found out about the effect of scored points on the received salary.

```
summary(m4)
```

```
##
## Call:
## lm(formula = salary_log ~ career_PTS + I(career_PTS^2) + position_center +
##     position_sf + position_pf + position_sg + position_pg, data = data_nba)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.1886 -0.5744  0.1583  0.7318  2.4876
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     12.2544680  0.0428953 285.684  < 2e-16 ***
## career_PTS       0.3124511  0.0072287  43.224  < 2e-16 ***
## I(career_PTS^2) -0.0071879  0.0003113 -23.092  < 2e-16 ***
## position_center  0.5482072  0.0326289  16.801  < 2e-16 ***
## position_sf      0.1067444  0.0292656   3.647 0.000266 ***
## position_pf      0.1214253  0.0270641   4.487 7.32e-06 ***
## position_sg     -0.0025577  0.0275935  -0.093 0.926150
## position_pg      0.0312286  0.0337077   0.926 0.354234
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.039 on 9720 degrees of freedom
## Multiple R-squared:  0.3978, Adjusted R-squared:  0.3973
## F-statistic: 917.1 on 7 and 9720 DF,  p-value: < 2.2e-16
```

The more points a player scores, the higher the salary is estimated. At the same time we have identified a non-linear aspect to this relationship. The non-linear effect is small but negative. This indicates diminishing returns for high scorers. The higher the score, the less positive the effect of additional points is.

The problem after two transformations of involved variables is, that interpretation has lost all its intuitiveness. The effects now describe the change in