running_an_Nbody_code

September 22, 2023

1 How to run a simple N-body code

Here we will generate initial conditions for an N-body code, run a small simulation and analyse the results. This analysis is performed on a 100 star cluster in a 1 pc virial-radius King model. Stellar masses are taken randomly from a Salpeter distribution. Stellar evolution is ignored in this simulation.

For reference you can read chapter 2 of Portegies Zwart & McMillan 2018 (2018araa.book.....P).

With this tutorial you will learn - how to generate initial conditions (mass function, King-model) - to initialize a gravitational N-body code - channels and intra-code data transfer - detecting binaries - plotting results - making cumulative distributions

[1]:
```
%matplotlib inline
import numpy
from matplotlib import pyplot
from amuse.units import units
from amuse.lab import new_powerlaw_mass_distribution
from amuse.units import nbody_system
from amuse.ic.kingmodel import new_king_model
from amuse.community.ph4.interface import ph4
from amuse.ext.LagrangianRadii import LagrangianRadii
from amuse.ext import orbital_elements as oe
from scipy import stats
numpy.random.seed(63)
```

We start by setting-up a simulation by specifying a stellar mass distribution

[4]:
```
"""
n_stars = 100
alpha_IMF = -2.35

m_stars = new_powerlaw_mass_distribution(n_stars, 0.1|units.MSun,
                                         100|units.MSun, alpha_IMF)
"""
```

Now check to see if this indeed gives one the expected mass function by plotting the cumulative distribution against a theoretical distribution.

[2]:
```
"""
def plot_cdf(m, alpha_IMF=-2.35):
    m = sorted(m.value_in(units.MSun))
    fm = numpy.array([0])
    for mi in m:
        fm = numpy.append(fm, fm[-1] + mi)
    fm /= max(fm)
    from amuse.plot import plot
    pyplot.plot(m, fm[:len(m)], lw=4, ls="-")
    x = 10**numpy.arange(-1.0, 1.0, 1./len(m))
    y = x**(alpha_IMF+2)
    fy = numpy.array([0])
    for yi in y:
        fy = numpy.append(fy, fy[-1] + yi)
    fy /= max(fy)
    pyplot.plot(x, fy[:len(x)], lw=2, ls=":")
    pyplot.semilogx()
    pyplot.show()
plot_cdf(m_stars)
"""
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In[2], line 18
     16     pyplot.semilogx()
     17     pyplot.show()
---> 18 plot_cdf(m_stars)

NameError: name 'm_stars' is not defined
```

1.0.1 Question:

Can you explain why the dotted curve does not overplot with the solid curve?

Now we can initialize the cluster size and declare the converter.

1.0.2 Converter

N-body codes operate using special dimensionless N-body units (where the gravitational constant is set to unity). In order to make the code understand SI units, we must pass it a unit converter. This contains two quantities in the relevant unit system, from which AMUSE derives the proper unit conversion. Best practice is to keep those quantities on the order of the system's scale (e.g. the total mass and the radius of the outermost planet in a planetary system) to ensure numerical stability.

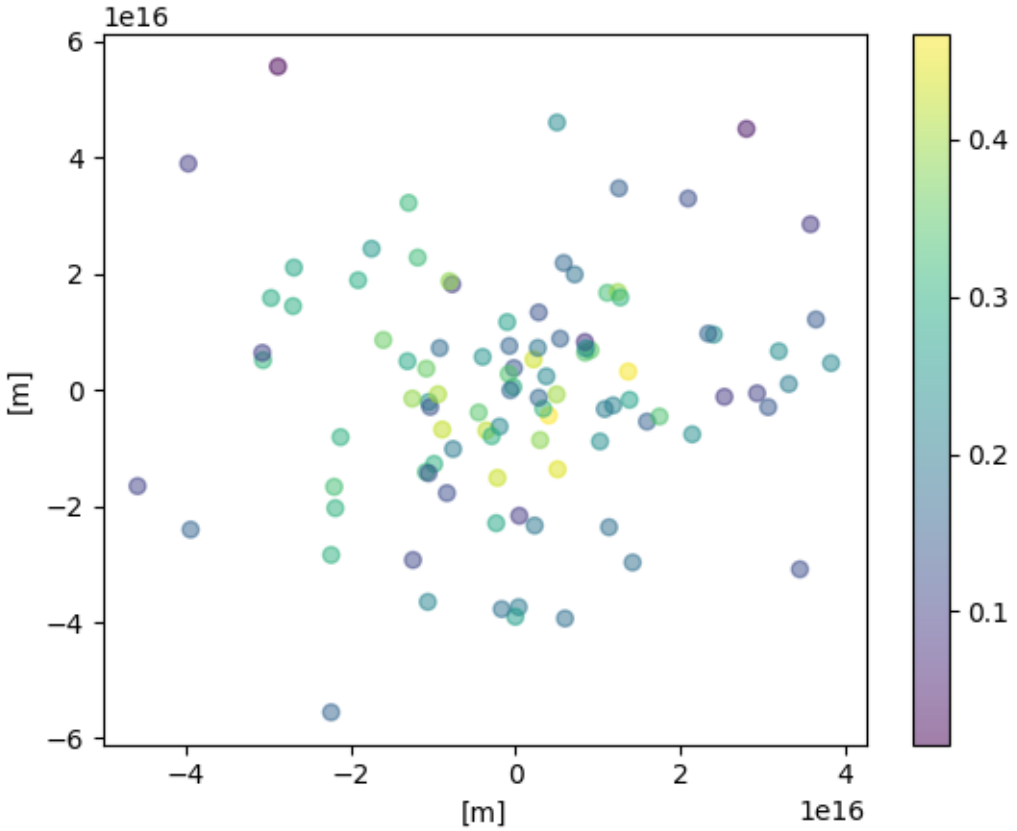
Also see the documentation on the importance of converters in AMUSE.

[6]:
```
"""
r_cluster = 1.0 | units.parsec
```

```
from amuse.units import nbody_system
converter=nbody_system.nbody_to_si(m_stars.sum(),r_cluster)
from amuse.ic.kingmodel import new_king_model
W0 = 3.0
bodies=new_king_model(n_stars, W0, convert_nbody=converter)
bodies.scale_to_standard(converter)
"""
```

Check the result by plotting the X-Y positions of the bodies.

[7]:
```
"""
from amuse.plot import scatter
def plot_snapshot(bodies):
    v = (bodies.vx**2 + bodies.vy**2 + bodies.vz**2).sqrt()
    scatter(bodies.x, bodies.y, c=v.value_in(units.kms), alpha=0.5)
    pyplot.colorbar()
    pyplot.show()
plot_snapshot(bodies)
"""
```



[8]:
```
"""

from amuse.community.ph4.interface import ph4
from amuse.ext.LagrangianRadii import LagrangianRadii
"""
```

[9]:
```
"""

gravity = ph4(converter)
gravity.particles.add_particles(bodies)
# A channel is a 'permanent' connection to a code's particle
# set. Multiple calls to a code's particle set need to set up
# a new connection every time; with a channel, we can copy
# information back without opening a new connection.
# This does not automatically update bodies! See below
channel = gravity.particles.new_channel_to(bodies)

times = numpy.arange(0, 100, 0.1) | units.Myr
RL25 = [] | units.parsec
Rvir = [] | units.parsec
for time in times:
    gravity.evolve_model(time)
    channel.copy() # Copy from gravity.particles to bodies
    Rvir.append(bodies.virial_radius())
    #L = LagrangianRadii(bodies)
    RL25.append(LagrangianRadii(bodies)[5])

    if not time.value_in(units.Myr)%10.0:
        print("cluster at Time=", time.in_(units.Myr),
              "Mass=", bodies.mass.sum().in_(units.MSun),
              "Rvir=", Rvir[-1].in_(units.parsec))
    b = bodies.get_binaries()
    if(len(b)>0):
        print("Number of binaries found:", len(b))

last_snapshot = bodies

gravity.stop()
pyplot.plot(times.value_in(units.Myr), RL25.value_in(units.parsec),␣
 ↪label="RL25")
pyplot.plot(times.value_in(units.Myr), Rvir.value_in(units.parsec),␣
 ↪label="Rvir")
pyplot.xlabel("t [Myr]")
pyplot.ylabel("r [pc]")
pyplot.legend()
pyplot.show()
```

```
"""
```

```
cluster at Time= 0.0 Myr Mass= 31.9876328986 MSun Rvir= 1.0 parsec
cluster at Time= 10.0 Myr Mass= 31.9876328986 MSun Rvir= 1.01567674079 parsec
cluster at Time= 20.0 Myr Mass= 31.9876328986 MSun Rvir= 0.99430237301 parsec
cluster at Time= 30.0 Myr Mass= 31.9876328986 MSun Rvir= 1.00198032161 parsec
cluster at Time= 40.0 Myr Mass= 31.9876328986 MSun Rvir= 0.963221818863 parsec
cluster at Time= 50.0 Myr Mass= 31.9876328986 MSun Rvir= 1.06564080317 parsec
Number of binaries found: 1
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cluster at Time= 60.0 Myr Mass= 31.9876328986 MSun Rvir= 1.0409964845 parsec
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cluster at Time= 70.0 Myr Mass= 31.9876328986 MSun Rvir= 1.02812695558 parsec
Number of binaries found: 1
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cluster at Time= 80.0 Myr Mass= 31.9876328986 MSun Rvir= 0.88113559406 parsec
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cluster at Time= 90.0 Myr Mass= 31.9876328986 MSun Rvir= 1.01987036861 parsec
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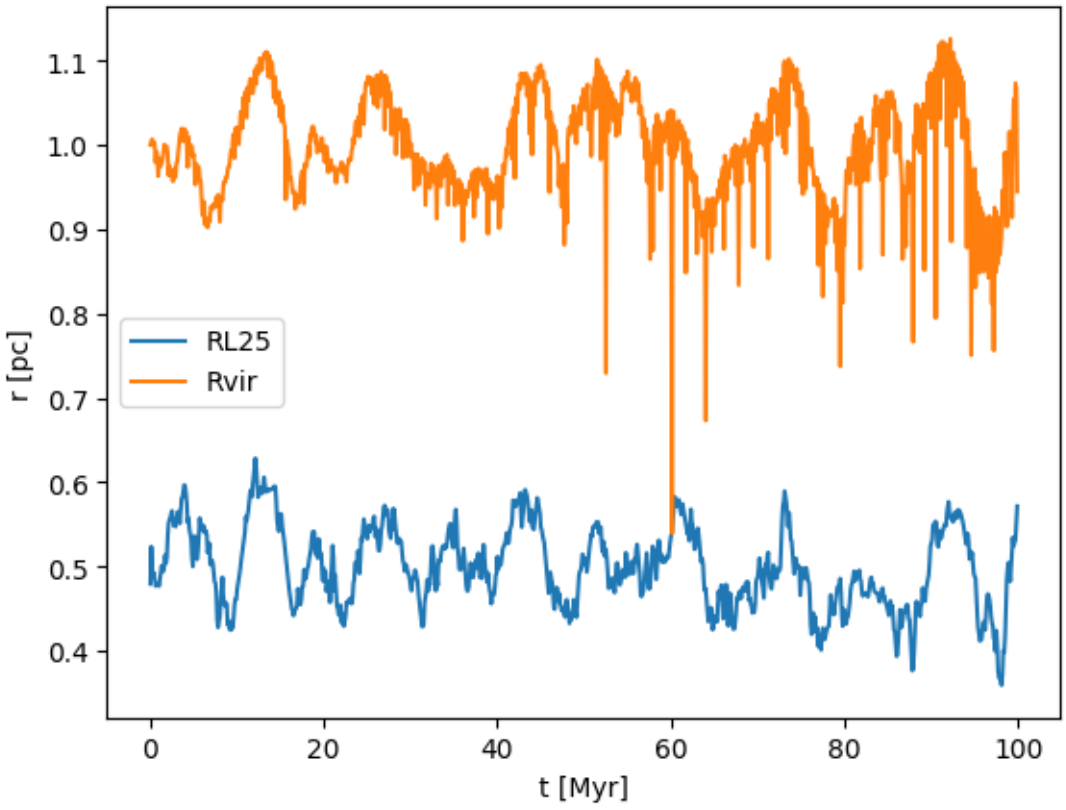

[83]:
```
def sim(r_cluster, n_stars, alpha_IMF, W0, inf=False):
    m_stars = new_powerlaw_mass_distribution(n_stars, 0.1|units.MSun,
                                             100|units.MSun, alpha_IMF)
    converter=nbody_system.nbody_to_si(m_stars.sum(),r_cluster)
    bodies=new_king_model(n_stars, W0, convert_nbody=converter)
    bodies.scale_to_standard(converter)
    gravity = ph4(converter)
    gravity.particles.add_particles(bodies)
    channel = gravity.particles.new_channel_to(bodies)
    RL25 = [] | units.parsec
    Rvir = [] | units.parsec
    if inf==False:
        times = numpy.arange(0, 100, 0.1) | units.Myr

        for time in times:
            gravity.evolve_model(time)
            channel.copy() # Copy from gravity.particles to bodies
            Rvir.append(bodies.virial_radius())
            #L = LagrangianRadii(bodies)
            RL25.append(LagrangianRadii(bodies)[5])
```

```
            if not time.value_in(units.Myr)%10.0:
                print("cluster at Time=", time.in_(units.Myr),
                      "Mass=", bodies.mass.sum().in_(units.MSun),
                      "Rvir=", Rvir[-1].in_(units.parsec))
            b = bodies.get_binaries()
            if(len(b)>0):
                print("Number of binaries found:", len(b))
                oe.get_orbital_elements_from_binary(b)
    else:
        time = 0 | units.Myr
        times = numpy.array([]) | units.Myr
        b = []
        while len(b)==0:
            gravity.evolve_model(time)
            channel.copy() # Copy from gravity.particles to bodies
            Rvir.append(bodies.virial_radius())
            #L = LagrangianRadii(bodies)
            RL25.append(LagrangianRadii(bodies)[5])
            if not round(time.value_in(units.Myr),1)%50.0:
                print("cluster at Time=", round(time.value_in(units.Myr),1),
                      "Mass=", bodies.mass.sum().in_(units.MSun),
                      "Rvir=", Rvir[-1].in_(units.parsec))
            b = bodies.get_binaries()
            if(len(b)>0):
                print("Binary has been found!", len(b))
                oe.get_orbital_elements_from_binary(b[0])
            times = numpy.append(times, time)
            time += 0.1 | units.Myr
        b = b[0]

    last_snapshot = bodies
    gravity.stop()
    return times, RL25, Rvir, b, bodies
```

You have now calculated the dynamical evolution of a small cluster of stars including a realistic mass function.

1.1 Assignments and questions:

1.1.1 Question 1:

Why do we have to *scale_to_standard()* after assigning the masses and positions to all the particles?

1.1.2 Question 2:

Run the notebook and describe the evolution of the virial radius.

1.1.3 Assignment 2:

There is a check for binary detection in the script. If no binary formed in your run, that is okay. Run the script again until a binary forms. Remember the random number seed for this particular run, and reproduce the result.

1.1.4 Question 3:

One of the parameters in the binary is the *hardness*. What is the hardness of the binary from your last run? and explain what is meant with this parameter.

1.1.5 Assignment 3:

Rerun the calculation using the same random-number seed, and store the final snapshot (at t=100Myr). Write a script that reads the last snapshot, finds the binaries in the snapshot and calculate their orbital elements. For calculating the orbital elements of a binary you may want to use the following routine: *get_orbital_elements_from_binary()*

1.1.6 Assignment 4:

Rewrite the script in such a way that the event loops over time continues indefinitely, but that the code stops as soon as the first binary is detected.

Run the script several times until you have enough first binaries formed to make a histogram of the first-binary formation time-scale.

Make a historgram of the eccentricities of the first formed binaries. What can you say about the eccentricity distribution of the first formed binaries in a star cluster?

Now, make a small change to your initial conditions. A variation could include the dimensionless depth of the potential of the initial King model, the slope of the initial-mass function, its lower limit, or the number of stars in the simulation).

Now redo the calculation in which you generate a histogram of moments of first-binary formation and the eccentricity distribution.

Make figures with both distributions: the original distribution for first-binary formation time and the second series of runs (with the altered initial conditions), and do the same for the eccentricity distribtion. Present both distributions as histograms and as cumulative distributions.

1.1.7 Question 4:

Can you understand the difference in first-binary formation-time based on the changes you introduced in the initial conditions? What can you say about the two eccentricity distributions?

Perform a Kolmogorov-Smirnoff test on the two cumulative distributions for the formation times, and the eccentricities.

Did you perform enough runs to make a statistically significant statement about the time of first-binary formation?

1.1.8 Assignment 4:

Make a cumulative distribution of the masses of the two stars for each first binary and compare it to the initial mass-function.

1.1.9 Question 5:

Can you understand the difference between the typical masses of the binaries that formed first and the initial mass-function?

1.1.10 Question 1

Because the N-body code uses G=1, but this is not realistic. scale_to_standard() is used to scale the different codes used by AMUSE to the same standard units.

1.1.11 Question 2

The virial radius bounces around 1, so the cluster is stable

1.1.12 Assignment 2

[ ]:
```
n_stars = 100
alpha_IMF = -2.35
r_cluster = 1.0 | units.parsec
W0 = 3.0

times, RL25, Rvir, detected_binary, last_timestep = sim(r_cluster, n_stars,␣
 ↪alpha_IMF, W0)

pyplot.plot(times.value_in(units.Myr), RL25.value_in(units.parsec),␣
 ↪label="RL25")
pyplot.plot(times.value_in(units.Myr), Rvir.value_in(units.parsec),␣
 ↪label="Rvir")
pyplot.xlabel("t [Myr]")
pyplot.ylabel("r [pc]")
pyplot.legend()
pyplot.show()
```

```
0.0 Myr
cluster at Time= 0.0 Myr Mass= 31.9876328986 MSun Rvir= 1.0 parsec
0.1 Myr
0.2 Myr
0.3 Myr
0.4 Myr
0.5 Myr
0.6 Myr
0.7 Myr
0.8 Myr
0.9 Myr
```

1.0 Myr
1.1 Myr
1.2 Myr
1.3 Myr
1.4 Myr
1.5 Myr
1.6 Myr
1.7 Myr
1.8 Myr
1.9 Myr
2.0 Myr
2.1 Myr
2.2 Myr
2.3 Myr
2.4 Myr
2.5 Myr
2.6 Myr
2.7 Myr
2.8 Myr
2.9 Myr
3.0 Myr
3.1 Myr
3.2 Myr
3.3 Myr
3.4 Myr
3.5 Myr
3.6 Myr
3.7 Myr
3.8 Myr
3.9 Myr
4.0 Myr
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4.4 Myr
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4.6 Myr
4.7 Myr
4.8 Myr
4.9 Myr
5.0 Myr
5.1 Myr
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5.6 Myr
5.7 Myr

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5.8 Myr
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6.0 Myr
6.1 Myr
6.2 Myr
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6.7 Myr
6.8 Myr
6.9 Myr
7.0 Myr
7.1 Myr
7.2 Myr
7.3 Myr
7.4 Myr
7.5 Myr
7.6 Myr
7.7 Myr
7.8 Myr
7.9 Myr
8.0 Myr
8.1 Myr
8.2 Myr
8.3 Myr
8.4 Myr
8.5 Myr
8.6 Myr
8.7 Myr
8.8 Myr
8.9 Myr
9.0 Myr
9.1 Myr
9.2 Myr
9.3 Myr
9.4 Myr
9.5 Myr
9.6 Myr
9.7 Myr
9.8 Myr
9.9 Myr
10.0 Myr
cluster at Time= 10.0 Myr Mass= 31.9876328986 MSun Rvir= 1.01567674079 parsec
10.1 Myr
10.2 Myr
10.3 Myr
10.4 Myr
```

10.5 Myr
10.6 Myr
10.7 Myr
10.8 Myr
10.9 Myr
11.0 Myr
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11.2 Myr
11.3 Myr
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11.5 Myr
11.6 Myr
11.7 Myr
11.8 Myr
11.9 Myr
12.0 Myr
12.1 Myr
12.2 Myr
12.3 Myr
12.4 Myr
12.5 Myr
12.6 Myr
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12.9 Myr
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13.3 Myr
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13.6 Myr
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13.8 Myr
13.9 Myr
14.0 Myr
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14.8 Myr
14.9 Myr
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15.2 Myr

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15.6 Myr
15.7 Myr
15.8 Myr
15.9 Myr
16.0 Myr
16.1 Myr
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16.5 Myr
16.6 Myr
16.7 Myr
16.8 Myr
16.9 Myr
17.0 Myr
17.1 Myr
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17.3 Myr
17.4 Myr
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17.6 Myr
17.7 Myr
17.8 Myr
17.9 Myr
18.0 Myr
18.1 Myr
18.2 Myr
18.3 Myr
18.4 Myr
18.5 Myr
18.6 Myr
18.7 Myr
18.8 Myr
18.9 Myr
19.0 Myr
19.1 Myr
19.2 Myr
19.3 Myr
19.4 Myr
19.5 Myr
19.6 Myr
19.7 Myr
19.8 Myr
19.9 Myr
20.0 Myr

```
cluster at Time= 20.0 Myr Mass= 31.9876328986 MSun Rvir= 0.99430237301 parsec
20.1 Myr
20.2 Myr
20.3 Myr
20.4 Myr
20.5 Myr
20.6 Myr
20.7 Myr
20.8 Myr
20.9 Myr
21.0 Myr
21.1 Myr
21.2 Myr
21.3 Myr
21.4 Myr
21.5 Myr
21.6 Myr
21.7 Myr
21.8 Myr
21.9 Myr
22.0 Myr
22.1 Myr
22.2 Myr
22.3 Myr
22.4 Myr
22.5 Myr
22.6 Myr
22.7 Myr
22.8 Myr
22.9 Myr
```

1.1.13 Question 3

[4]:
```
print("The hardness of the binaries is:", detected_binary[0].hardness, "and",␣
↪detected_binary[1].hardness)
```

```
The hardness of the binaries is: [ 11.96512436  11.96512436] and [ 12.27563788
12.27563788]
```

They are the same, as both stars in the binary are binary. Binaries are selected according to a hardness criterion, namely it has to be higher than 10 for a dectection.

1.1.14 Assignment 3

[6]:
```
def orbit_el(time):
    binaries = time.get_binaries()
    print(oe.get_orbital_elements_from_binary(binaries))
```

[7]:
```
orbit_el(last_timestep)
```

```
---------------------------------------------------------------------------
AttributeError                            Traceback (most recent call last)
Cell In[7], line 1
----> 1 orbit_el(last_timestep)

Cell In[6], line 3, in orbit_el(time)
      1 def orbit_el(time):
      2     binaries = time.get_binaries()
----> 3     print(oe.get_orbital_elements_from_binary(binaries))

File ~/Amuse-env/lib/python3.10/site-packages/amuse/ext/orbital_elements.py:403
 ↪in get_orbital_elements_from_binary(binary, G)
    401     secondaries.add_particle(Particle())
    402     secondaries[0].mass = 0 * primaries[0].mass
--> 403     secondaries[0].position = binary.position
    404     secondaries[0].velocity = binary.velocity
    406 (
    407         mass1, mass2, semimajor_axis, eccentricity, true_anomaly,
    408         inclination, long_asc_node, arg_per
    409         ) = get_orbital_elements_from_binaries(primaries, secondaries,␣
 ↪G=G)

AttributeError: 'list' object has no attribute 'position'
```

1.1.15 Assignment 4

[107]:
```
n_sim = 25

#First simulation

n_stars = 100
alpha_IMF = -2.35
r_cluster = 1.0 | units.parsec
W0 = 3.0

formation_time = [] | units.Myr
e_first_binary = []
mass_first_binary = [] | units.MSun

for i in range(n_sim):
    print("simulation numer:", i)
    times, RL25, Rvir, detected_binary, last_timestep = sim(r_cluster, n_stars,␣
 ↪alpha_IMF, W0, inf=True)
    formation_time.append(times[-1])
```

```
    e_first_binary.append(oe.
 ↪get_orbital_elements_from_binary(detected_binary)[3])
    mass_first_binary.append(oe.
 ↪get_orbital_elements_from_binary(detected_binary)[0])


#Second simulation

n_stars = 100
alpha_IMF = -2
r_cluster = 1.0 | units.parsec
W0 = 3.0

formation_time_second = [] | units.Myr
e_first_binary_second = []
mass_first_binary_second = [] | units.MSun

for i in range(n_sim):
    print("simulation numer:", i)
    times, RL25, Rvir, detected_binary, last_timestep = sim(r_cluster, n_stars,␣
 ↪alpha_IMF, W0, inf=True)
    formation_time_second.append(times[-1])
    e_first_binary_second.append(oe.
 ↪get_orbital_elements_from_binary(detected_binary)[3])
    mass_first_binary_second.append(oe.
 ↪get_orbital_elements_from_binary(detected_binary)[0])
```

```
simulation numer: 0
cluster at Time= 0 Mass= 46.5128509562 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 46.5128509562 MSun Rvir= 1.00075201353 parsec
cluster at Time= 100.0 Mass= 46.5128509562 MSun Rvir= 1.09333877111 parsec
Binary has been found! 1
simulation numer: 1
cluster at Time= 0 Mass= 32.7728917788 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 32.7728917788 MSun Rvir= 1.08634555598 parsec
cluster at Time= 100.0 Mass= 32.7728917788 MSun Rvir= 0.948758172935 parsec
cluster at Time= 150.0 Mass= 32.7728917788 MSun Rvir= 1.01113979025 parsec
Binary has been found! 1
simulation numer: 2
cluster at Time= 0 Mass= 52.8871175971 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 52.8871175971 MSun Rvir= 1.00763455747 parsec
cluster at Time= 100.0 Mass= 52.8871175971 MSun Rvir= 1.04493536344 parsec
Binary has been found! 1
simulation numer: 3
cluster at Time= 0 Mass= 23.7602217249 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 23.7602217249 MSun Rvir= 1.01331316669 parsec
cluster at Time= 100.0 Mass= 23.7602217249 MSun Rvir= 1.04007168249 parsec
```

```
Binary has been found! 1
simulation numer: 4
cluster at Time= 0 Mass= 26.8667679367 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 26.8667679367 MSun Rvir= 1.03254010174 parsec
cluster at Time= 100.0 Mass= 26.8667679367 MSun Rvir= 0.931491855947 parsec
Binary has been found! 1
simulation numer: 5
cluster at Time= 0 Mass= 38.6365813011 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 38.6365813011 MSun Rvir= 1.00955242197 parsec
cluster at Time= 100.0 Mass= 38.6365813011 MSun Rvir= 1.01014000438 parsec
cluster at Time= 150.0 Mass= 38.6365813011 MSun Rvir= 0.984468080948 parsec
Binary has been found! 1
simulation numer: 6
cluster at Time= 0 Mass= 37.7493607532 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 37.7493607532 MSun Rvir= 0.978427897386 parsec
cluster at Time= 100.0 Mass= 37.7493607532 MSun Rvir= 1.03959739875 parsec
cluster at Time= 150.0 Mass= 37.7493607532 MSun Rvir= 0.847430797607 parsec
Binary has been found! 2
simulation numer: 7
cluster at Time= 0 Mass= 28.9628445707 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 28.9628445707 MSun Rvir= 0.967078543142 parsec
cluster at Time= 100.0 Mass= 28.9628445707 MSun Rvir= 1.04530978271 parsec
Binary has been found! 1
simulation numer: 8
cluster at Time= 0 Mass= 37.8372119238 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 37.8372119238 MSun Rvir= 0.959037749282 parsec
Binary has been found! 1
simulation numer: 9
cluster at Time= 0 Mass= 26.8811417736 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 26.8811417736 MSun Rvir= 1.06287596465 parsec
cluster at Time= 100.0 Mass= 26.8811417736 MSun Rvir= 1.0367261286 parsec
cluster at Time= 150.0 Mass= 26.8811417736 MSun Rvir= 0.998498361836 parsec
Binary has been found! 1
simulation numer: 10
cluster at Time= 0 Mass= 33.5263907964 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 33.5263907964 MSun Rvir= 0.958058474053 parsec
cluster at Time= 100.0 Mass= 33.5263907964 MSun Rvir= 0.975550902584 parsec
cluster at Time= 150.0 Mass= 33.5263907964 MSun Rvir= 1.0070607783 parsec
Binary has been found! 1
simulation numer: 11
cluster at Time= 0 Mass= 33.2801486542 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 33.2801486542 MSun Rvir= 0.984808744605 parsec
cluster at Time= 100.0 Mass= 33.2801486542 MSun Rvir= 1.03034201832 parsec
cluster at Time= 150.0 Mass= 33.2801486542 MSun Rvir= 1.04281627123 parsec
cluster at Time= 200.0 Mass= 33.2801486542 MSun Rvir= 0.919691817392 parsec
Binary has been found! 1
simulation numer: 12
cluster at Time= 0 Mass= 43.5399976444 MSun Rvir= 1.0 parsec
```

```
cluster at Time= 50.0 Mass= 43.5399976444 MSun Rvir= 0.962893298641 parsec
cluster at Time= 100.0 Mass= 43.5399976444 MSun Rvir= 0.892827550601 parsec
Binary has been found! 1
simulation numer: 13
cluster at Time= 0 Mass= 32.758888114 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 32.758888114 MSun Rvir= 1.08693109322 parsec
cluster at Time= 100.0 Mass= 32.758888114 MSun Rvir= 0.905191485386 parsec
Binary has been found! 1
simulation numer: 14
cluster at Time= 0 Mass= 42.4341721721 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 42.4341721721 MSun Rvir= 1.00092876264 parsec
cluster at Time= 100.0 Mass= 42.4341721721 MSun Rvir= 1.0433669614 parsec
Binary has been found! 1
simulation numer: 15
cluster at Time= 0 Mass= 25.2302946397 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 25.2302946397 MSun Rvir= 1.03262510551 parsec
cluster at Time= 100.0 Mass= 25.2302946397 MSun Rvir= 0.889641971441 parsec
Binary has been found! 1
simulation numer: 16
cluster at Time= 0 Mass= 31.5218713014 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 31.5218713014 MSun Rvir= 1.03629548942 parsec
cluster at Time= 100.0 Mass= 31.5218713014 MSun Rvir= 1.11324971324 parsec
cluster at Time= 150.0 Mass= 31.5218713014 MSun Rvir= 1.13663675058 parsec
cluster at Time= 200.0 Mass= 31.5218713014 MSun Rvir= 0.959729446014 parsec
Binary has been found! 1
simulation numer: 17
cluster at Time= 0 Mass= 105.662941791 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 105.662941791 MSun Rvir= 0.975878916752 parsec
Binary has been found! 1
simulation numer: 18
cluster at Time= 0 Mass= 33.9345667979 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 33.9345667979 MSun Rvir= 1.00835560266 parsec
cluster at Time= 100.0 Mass= 33.9345667979 MSun Rvir= 0.993324242423 parsec
cluster at Time= 150.0 Mass= 33.9345667979 MSun Rvir= 0.999650436691 parsec
Binary has been found! 1
simulation numer: 19
cluster at Time= 0 Mass= 23.6059693814 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 23.6059693814 MSun Rvir= 1.06776754153 parsec
cluster at Time= 100.0 Mass= 23.6059693814 MSun Rvir= 1.02655756613 parsec
Binary has been found! 1
simulation numer: 20
cluster at Time= 0 Mass= 30.4321733665 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 30.4321733665 MSun Rvir= 0.94097717806 parsec
cluster at Time= 100.0 Mass= 30.4321733665 MSun Rvir= 0.989790708262 parsec
Binary has been found! 1
simulation numer: 21
cluster at Time= 0 Mass= 43.9789239896 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 43.9789239896 MSun Rvir= 1.01284580646 parsec
```

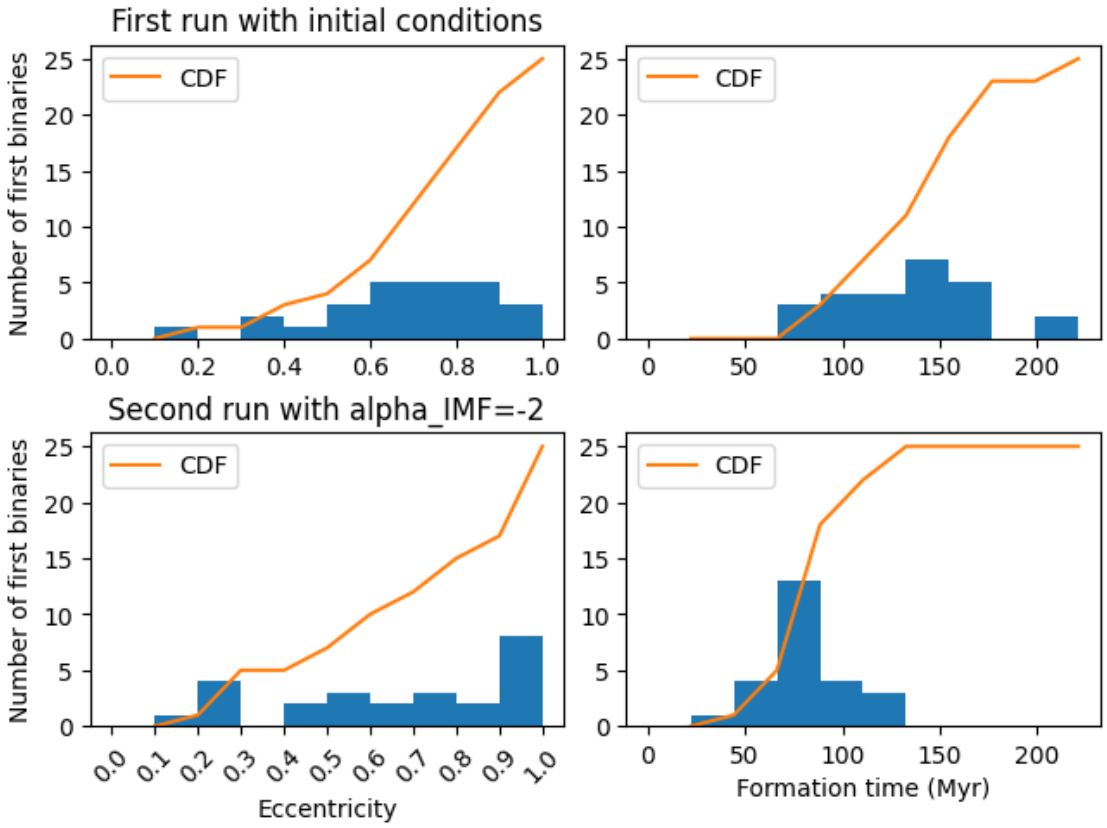
```
Binary has been found! 1
simulation numer: 22
cluster at Time= 0 Mass= 33.6499803717 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 33.6499803717 MSun Rvir= 0.925854892656 parsec
Binary has been found! 1
simulation numer: 23
cluster at Time= 0 Mass= 27.0759316869 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 27.0759316869 MSun Rvir= 0.97605194055 parsec
Binary has been found! 1
simulation numer: 24
cluster at Time= 0 Mass= 34.4341913226 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 34.4341913226 MSun Rvir= 0.981144607964 parsec
Binary has been found! 1
simulation numer: 0
cluster at Time= 0 Mass= 48.2462110329 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 48.2462110329 MSun Rvir= 1.00423004889 parsec
cluster at Time= 100.0 Mass= 48.2462110329 MSun Rvir= 0.950661243815 parsec
Binary has been found! 1
simulation numer: 1
cluster at Time= 0 Mass= 94.6877261512 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 94.6877261512 MSun Rvir= 1.07999968901 parsec
Binary has been found! 1
simulation numer: 2
cluster at Time= 0 Mass= 52.322224789 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 52.322224789 MSun Rvir= 0.957923540676 parsec
cluster at Time= 100.0 Mass= 52.322224789 MSun Rvir= 1.02975211897 parsec
Binary has been found! 1
simulation numer: 3
cluster at Time= 0 Mass= 76.5533347801 MSun Rvir= 1.0 parsec
Binary has been found! 1
simulation numer: 4
cluster at Time= 0 Mass= 77.644611263 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 77.644611263 MSun Rvir= 0.982793486344 parsec
Binary has been found! 1
simulation numer: 5
cluster at Time= 0 Mass= 70.6812576373 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 70.6812576373 MSun Rvir= 1.03504192701 parsec
Binary has been found! 1
simulation numer: 6
cluster at Time= 0 Mass= 85.8157985185 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 85.8157985185 MSun Rvir= 0.911474183385 parsec
Binary has been found! 1
simulation numer: 7
cluster at Time= 0 Mass= 123.475161161 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 123.475161161 MSun Rvir= 1.00267325332 parsec
Binary has been found! 1
simulation numer: 8
cluster at Time= 0 Mass= 142.739220438 MSun Rvir= 1.0 parsec
```

```
cluster at Time= 50.0 Mass= 142.739220438 MSun Rvir= 1.05575468504 parsec
Binary has been found! 1
simulation numer: 9
cluster at Time= 0 Mass= 132.84344222 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 132.84344222 MSun Rvir= 0.935486629861 parsec
Binary has been found! 1
simulation numer: 10
cluster at Time= 0 Mass= 89.7048651966 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 89.7048651966 MSun Rvir= 1.06493944912 parsec
Binary has been found! 1
simulation numer: 11
cluster at Time= 0 Mass= 72.2242997602 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 72.2242997602 MSun Rvir= 1.05798624557 parsec
Binary has been found! 1
simulation numer: 12
cluster at Time= 0 Mass= 82.8024823675 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 82.8024823675 MSun Rvir= 1.03149929569 parsec
Binary has been found! 1
simulation numer: 13
cluster at Time= 0 Mass= 90.3711145095 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 90.3711145095 MSun Rvir= 1.00719868919 parsec
Binary has been found! 1
simulation numer: 14
cluster at Time= 0 Mass= 61.162480918 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 61.162480918 MSun Rvir= 0.971601682774 parsec
Binary has been found! 1
simulation numer: 15
cluster at Time= 0 Mass= 64.3499160393 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 64.3499160393 MSun Rvir= 1.06771017619 parsec
Binary has been found! 1
simulation numer: 16
cluster at Time= 0 Mass= 106.040759943 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 106.040759943 MSun Rvir= 0.870194914145 parsec
Binary has been found! 1
simulation numer: 17
cluster at Time= 0 Mass= 54.7482461624 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 54.7482461624 MSun Rvir= 0.996715023519 parsec
cluster at Time= 100.0 Mass= 54.7482461624 MSun Rvir= 1.11003172814 parsec
Binary has been found! 1
simulation numer: 18
cluster at Time= 0 Mass= 32.1700701089 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 32.1700701089 MSun Rvir= 0.985928952049 parsec
cluster at Time= 100.0 Mass= 32.1700701089 MSun Rvir= 0.986786450211 parsec
Binary has been found! 1
simulation numer: 19
cluster at Time= 0 Mass= 113.527496332 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 113.527496332 MSun Rvir= 0.983440656089 parsec
Binary has been found! 1
```

```
simulation numer: 20
cluster at Time= 0 Mass= 70.1739654117 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 70.1739654117 MSun Rvir= 1.12863586731 parsec
Binary has been found! 1
simulation numer: 21
cluster at Time= 0 Mass= 48.2533181115 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 48.2533181115 MSun Rvir= 1.025044764 parsec
Binary has been found! 1
simulation numer: 22
cluster at Time= 0 Mass= 89.9976560552 MSun Rvir= 1.0 parsec
Binary has been found! 1
simulation numer: 23
cluster at Time= 0 Mass= 100.108578813 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 100.108578813 MSun Rvir= 0.947010635664 parsec
Binary has been found! 1
simulation numer: 24
cluster at Time= 0 Mass= 58.5699972355 MSun Rvir= 1.0 parsec
cluster at Time= 50.0 Mass= 58.5699972355 MSun Rvir= 0.9424422899 parsec
Binary has been found! 1
```

[108]:
```
fig, axs = pyplot.subplots(2, 2, layout='constrained')#, sharey='row',␣
 ↪sharex="col")

axs[0,0].set_title("First run with initial conditions")
count = axs[0,0].hist(e_first_binary, numpy.arange(0,1.1,0.1))
axs[0,0].set_ylabel("Number of first binaries")
cdf_e = numpy.cumsum(count[0])
axs[0,0].plot(count[1][1:], cdf_e, label="CDF")
axs[0,0].legend()

count = axs[0,1].hist(formation_time.number, numpy.linspace(0,␣
 ↪max(max(formation_time.number), max(formation_time_second.number)),11))
cdf_t = numpy.cumsum(count[0])
axs[0,1].plot(count[1][1:], cdf_t, label="CDF")
axs[0,1].legend()

axs[1,0].set_title("Second run with alpha_IMF=-2")
count = axs[1,0].hist(e_first_binary_second, numpy.arange(0,1.1,0.1))
axs[1,0].set_ylabel("Number of first binaries")
axs[1,0].set_xticks(numpy.arange(0,1.1,0.1))
axs[1,0].set_xticklabels(axs[1,0].get_xticks(), rotation=45)
axs[1,0].set_xlabel("Eccentricity")
cdf_e_second = numpy.cumsum(count[0])
axs[1,0].plot(count[1][1:], cdf_e_second, label="CDF")
axs[1,0].legend()
```

```
count = axs[1,1].hist(formation_time_second.number, numpy.linspace(0,␣
 ↪max(max(formation_time.number), max(formation_time_second.number)),11))
axs[1,1].set_xlabel("Formation time (Myr)")
cdf_t_second = numpy.cumsum(count[0])
axs[1,1].plot(count[1][1:], cdf_t_second, label="CDF")
axs[1,1].legend()

pyplot.show()
```



1.1.16 Question 4

For the second simulation I took alpha_IMF=-2. For this saltpeter mass function, which scales with m^alpha_IMF, this means that the masses of the stars are higher, so they have a larger gravitional field and pull on each other and binaries will be formed sooner, which can be seen in a smaller formation time in de second row of plots! The eccentricity does not change much.

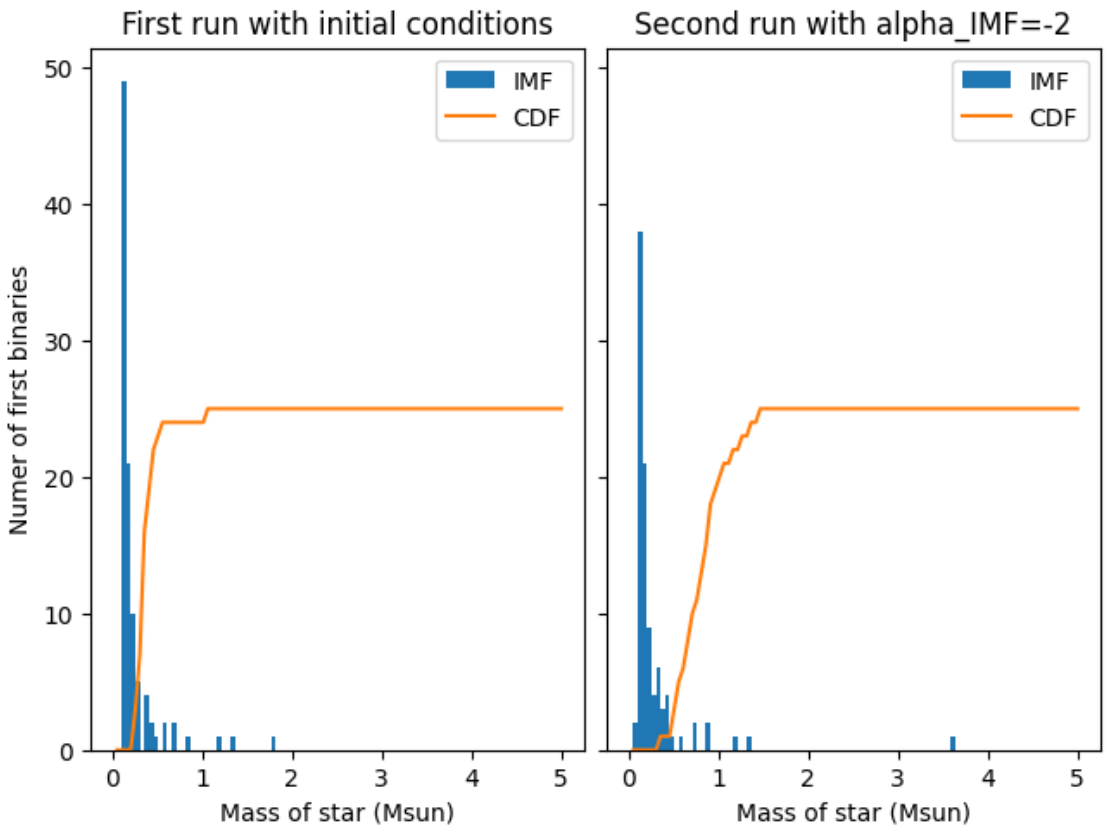
[109]:
```
print("The KS test for the eccentricities is:",stats.kstest(cdf_e,cdf_e_second))
print("The KS test for the formation times is:",stats.
 ↪kstest(cdf_t,cdf_t_second))
```

```
The KS test for the eccentricities is: KstestResult(statistic=0.3,
pvalue=0.78692978847777606, statistic_location=4.0, statistic_sign=1)
The KS test for the formation times is: KstestResult(statistic=0.4,
pvalue=0.41752365281777043, statistic_location=23.0, statistic_sign=1)
```

No I did not, as p>0.05.

1.1.17 Assigment 4

[135]:
```
fig, axs = pyplot.subplots(1, 2, layout='constrained', sharey=True)

axs[0].hist(new_powerlaw_mass_distribution(100, 0.1|units.MSun, 100|units.MSun,␣
 ↪-2.35).number, numpy.linspace(0,5,100), label="IMF")
count = numpy.histogram(mass_first_binary.number, numpy.linspace(0,5,100))
cdf_mass = numpy.cumsum(count[0])
axs[0].plot(count[1][1:], cdf_mass, label="CDF")

axs[1].hist(new_powerlaw_mass_distribution(100, 0.1|units.MSun, 100|units.MSun,␣
 ↪-2).number, numpy.linspace(0,5,100), label="IMF")
count = numpy.histogram(mass_first_binary_second.number, numpy.
 ↪linspace(0,5,100))
cdf_mass = numpy.cumsum(count[0])
axs[1].plot(count[1][1:], cdf_mass, label="CDF")

axs[0].set_title("First run with initial conditions")
axs[1].set_title("Second run with alpha_IMF=-2")
axs[0].set_ylabel("Numer of first binaries")
axs[0].set_xlabel("Mass of star (Msun)")
axs[1].set_xlabel("Mass of star (Msun)")
axs[0].legend()
axs[1].legend()

pyplot.show()
```



As said in Question 4, for alpha_IMF=-2 the masses are higher, as is seen, as the curve is less steep, so more higher masses!

1.1.18 Question 5

The typical masses of the first binaries are very higher, the IMF predicts more lower mass stars. As said, binaries are formed due to gravititional attraction, which is dependent on mass. So the first binaries will be heavier than predicted by the IMF!