Topic 9: Models for Matched Pairs

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Questions to be solved

1. Pp 20 interpretation from othose odds ratios
2. Pp 56 notes
3. pp 64 population-averaged
4. pp80 same subject?
5. pp92 kappa k = 0
6. 9.11typo ?????
7. Pi_ii calculation

Models for Matched Pairs

1. Responses in the two samples are statistically dependent.

   eg. sample from two different time points

2. **McNemar's Test: test of marginal homogeneity**

3. π_{ab} the probability of outcome a for the D 2004 observation and b for D 2008

4. n_{ab}: the number of such paris in a sample of n matched pairs with $p_{ab} = n_{ab}/n$ the sample proportion

5. p_{a+} is the proportion in cate a for observagtion 1. and p_{+a} is the corresponding proportion for observation 2, compare these two marginal proportions

6. **Marginal Homogeneity**: $\pi_{1+} = \pi_{+1}$ then $\pi_{2+} = \pi_{+2}$ <=> $\pi_{12} = \pi_{21}$

7. $H_0 : \pi_{12} = \pi_{21}$: **Hypothesis of Symmetry**
   1. Note the diagonal elements are not important since they correspond to the proportions of respondents whose approval/disapproval have stayed the same over time.
8. Total number of observations in the off-diagonal is fixed
   1. $n^* = n_{12} + n_{21}$.
   2. n_{12}, n_{21} ~ Bin(n*, 0.5)
   3. $z = \frac{\frac{n_{12}}{n^*} - 0.5}{\sqrt{0.5(1-0.5)/n^*}} = \frac{n_{12} - n_{21}}{\sqrt{n_{12} + n_{21}}}$
   4. 0.5 n* is the expected count for n12 under H0
   5. estimated proportion n12/n* , and the variance of the proportion under H0 is 0.5(1-0.5)/n* .
   6. Under H0, n*>=10,z is approx N(0,1). p-value for a two-sided alternative is doubled.

2P(Z>=z)
7. z^2 to χ^2_1 This test is valid under geneal multinomial sampling when **n* is not fixed but n is.**

9. **small** sample size example:

   Z^2 = (54-16)^2/ (54+ 16) = 20.63, Pvalue is $P(\chi^2_1 \geq 20.63) = 0.000006$: **Very strong evidence of a shift in the democraft direction**.

10. **Difference between the marginal proportions**:

    1. $d = \pi_{1+} - \pi_{+1} = \pi_{12} - \pi_{21}$
    2. $\hat{d} = \frac{n_{12}}{n} - \frac{n_{21}}{n}$
    3. $V(\hat{d}) = n^{-2}V(n_{12} - n_{21}) = n^{-1}[\pi_{12}(1-\pi_{12}) + \pi_{21}(1-\pi_{21}) + 2\pi_{12}\pi_{21}]$

    $= n^{-1}[\frac{n_{12}}{n}(1-\frac{n_{12}}{n}) + \frac{n_{21}}{n}(1-\frac{n_{21}}{n}) + 2\frac{n_{12}n_{21}}{n^2}]$

    4. 95%CI is $\hat{d} \pm 1.96\sqrt{\hat{V}(\hat{d})}$

11. Male and Female example

    1. Male:95CI for $\pi_{1+} - \pi_{+1}$ = 0.088 $\pm$ 1.96(0.0189) = (0.051, 0.125) We infer that the population percentage of males voting D increased by between 5% and 13%
    2. Female: (0.106, 0.171), shift toward D seems that it may be greater for females than males.
    3. 95CI for a difference of differences: (0.138 - 0.088) $\pm$ 1.96 $\sqrt{(0.0189)^2 + (0.0167)^2}$ = (0.001, 0.1) There is evidence of a greater shift for females than males, as much as 10%.
    4. Sample odds ratio 175*188/(16*54) = 38.1 Two observations have strong association
    5. independent samples has SE of 0.034 nearly twice as large as dependent

12. **Increased precision**

    1. Dependent samples can help improve the precision of statistical inferences for **Within-Subject Effects**
    2. **Substaintial** when samples are highly correlated
    3. **dependent var / independent var difference**: $diffvar(\sqrt{n}d) = -2(\pi_{11}\pi_{22} - \pi_{12}\pi_{21})$
    4. **positive dependence**: $log\theta = log[\pi_{11}\pi_{22}/\pi_{12}\pi_{21}] > 0$ that is $\pi_{11}\pi_{22} > \pi_{12}\pi_{21}$ implies a smaller variance for d for dependent samples.

13. Inference notes

    1. McNemar statistics depends only on cases classified in **different** categories(12 21) for the two observations.
    2. all cases contribute to inference about how much π_{1+} and π_{+1} differ.
    3. n11 and n22 may not contribute to whether there is marginal heterogeneity, but they do suggest whatever heterogeneity exits is small.

Logistic Regression for Matched Pairs

1. **Marginal Models:** describe the marginal distributions of responses for the two observations.

   1. $P(Y_1 = 1) = \alpha + \delta, P(Y_2 = 1) = \alpha$

$\delta = P(Y_1 = 1) - P(Y_2 = 1)$

Hypothesis of equal marginal probability for McNemar's test: $H_0 : \delta = 0$

2. alternative model applies the logit link:

   $logit[P(Y_1 = 1)] = \alpha + \beta, logit[P(Y_2 = 1)] = \alpha$

   OR $logit[P(Y_t = 1)] = \alpha + \beta x_t$

   where x_t: indicator var that equals 1 when t = 1 and 0 when t = 2

   ML estimate of β is the **log odds ratio of marginal proportions**, $\hat{\beta} = log[(p_{+1}/p_{+2})/(p_{1+}/p_{2+})]$

2. **Subject-specific model**: $link[P(Y_{it} = 1)] = \alpha_i + \beta x_t$.
   1. The effect of beta is defined conditional on the subject.
   2. A model of these tables can allow probabilities to vary by subject. They have subject-specific intercepts.
   3. its estimate decribes conditional association for the 3-way table stratified by subject
   4. The effect in marginal models are **population-averaged**, since they refer to averaging over the entire population.
3. **Subject-specific Tables**: 2x2xn talbe with separate partial table for each of n matched pairs. Models refer to it are **Conditional Models**.

   population-averaged table : 2x2 table that cross-classifies in a single table the two responses for all subjects.

4. $logit[P(Y_{i1} = 1)] = \alpha_i + \beta, logit[P(Y_{i2} = 1)] = \alpha_i$

   $P(Y_{i1} = 1) = \frac{exp(\alpha_i+\beta)}{(1+exp(\alpha_i+\beta))}, P(Y_{i1} = 1) = \frac{exp(\alpha_i)}{(1+exp(\alpha_i))}$

   α_i permits the probability vary among subject

   Subject with relatively large positive α_i: next class use formula

   Subject with relatively large negative α_i:

5. The odds of success for obseration 1 are $exp(\beta)$ times the odds of success for observation2.
6. **Subject-specific effect** Conditional association refers to a single subject.
   1. β = 0 => **Marginal homogeneity**: each subject, the probability of success is the same for both observations
   2. $\hat{\beta} = log(n_{21}/n_{12}), SE = \sqrt{1/n_{21} + 1/n_{12}}, exp(\hat{\beta})$**: Conditional Odds Ratio**
7. **Conditional odds ratio can differ from marginal odds ratio**
8. Alternative way of fitting: treat α_i as random effect
9. **Movie Review**:
   1. π_{ij} Denotes the probability that sickel classifies the movie in category i and Ebert classifies the movie in category i.
   2. π_{ii} is the probability that they both placed the movie in to the same category.
   3. $\sum \pi_{ii}$Total probability of agreement

10. **Cohen's Kappa** Strength of inter-rater aggreement
    1. $K = \frac{\sum \pi_{ii} - \sum \pi_{i+}\pi_{+i}}{1 - \sum \pi_{i+}\pi_{+i}}$: **LARGER value implies stronger agreement.**
    2. H0: $\pi_{ii} = \pi_{i+} \times \pi_{+i}$
    3. $\sum \pi_{ii}$ = 1: perfect agreement $\sum \pi_{ii} = 0$: No agreement
    4. K = 1 perfect aggreement
    5. K = 0 does not mean perfect diagreement , it only means agreement by chance as that would indicate that the diagonal ceel probabilities are simply product of the corresponding marginals
    6. Agreement is greater/less than agreement by chance, k>= / <= 0
    7. Minimum possible value of K = -1
    8. K > 0.75 indicate excellent agreement while lower than 0.4 indicate poor agreement.
11. **Agreement VS Association**: strong agreement implies strong association, strong association **may not imply** strong agreement.
12. weighted kappa: put different weight on different levels,