APG4005F
Assignment 1 report
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Title
Combined least squares adjustment on a hemisphere

Introduction
In this assignment I aimed to solve a least squares adjustment with points on a hemisphere using the combined case of least squares. Combined least squares contains the parametric and the condition case, that is, the combined case is the superset of the parametric and condition case of least squares.
The least squares adjustment would be programmed in Python using libraries such as scipy, numpy as well as sympy.
A Chi Squared hypothesis test would also be used to decide whether or not the points lie on a hemisphere or not to some level of significance.

Background
A few hundred points situated on a hemisphere would be provided and a least squares adjustment program would need to be constructed in order to determine whether or not those points formed a hemisphere which was to be based on some constraint (hypothesis test).
The formulation for the method of the combined case of least squares is as follows:
$Ax + Bv + w = 0$
where A is the matrix of coefficients of the unknown parameters
x is the vector of corrections for the unknowns
B is the matrix of coefficients for the observations
v is the vector corrections for the observations
w is the vector or the residuals for each observation

Problem Statement
Do the supplied 3D data points lie on a hemisphere to some degree?
A least squares adjustment program will need to be created in order to determine whether or not the supplied points form a hemisphere or not.

Method
The combined case of least squares is an effective way of solving this problem as it takes into account the unknowns as well as observations.

The least squares adjustment model was constructed as follows:

$$(x - xo)^2 + (y - yo)^2 + (z - zo)^2 - R^2 = 0 = F$$

The A matrix was populated using the partials of F with respect to each xo, yo, zo and ro, these being the unknowns.
The B matrix was populated using the partials of F with respect to each x, y, z, these being the observations.

Using the data from the supplied points, and initial estimates:

xo = 2, yo = 2, zo = -2, ro = 5

The adjustment was iterated and final values were determined for each unknown. The adjustment would be iterated until the values for the estimates began to converge. The number of iterations used was hardcoded into the program based on inspection of the results over multiple iterations.

Results
After performing the adjustment three times, it was observed that the solutions began to converge. After a total of five iterations, the following results were obtained

xo = -0.00021036271491253278
yo = 0.002105401614872917
zo = 0.0321910416183761
ro = 3.1265295315073707

The variance-covariance matrix for the unknowns: Σ_X

	x	**y**	**z**	**r**
x	-2.18144977e-02	2.76851141e-04	-1.59196246e-04	8.62649892e-05
y	2.76851141e-04	-2.05146280e-02	-1.65009269e-03	9.63001890e-04
z	-1.59196246e-04	-1.65009269e-03	-6.62829236e-02	2.64345528e-02
r	8.62649892e-05	9.63001890e-04	2.64345528e-02	-1.56325075e-02

Inspecting the variances (the diagonal entries) of each unknown we can see that they are very small which is a good indicator that our results are sound. Had the variances been too large, we would have to check for errors in the calculation process.

Discussion
The results of the iterated seem to be reasonable. The results converge consistently, even with rather extreme initial estimates. There do not appear to be any outliers or anomalies with the supplied data which may cause issues with the adjustment.

Conclusion
It can be said that with a reasonable level of significance that the supplied points do in fact lie on a hemisphere.