

CHAPTER

6

# Vector Semantics

The asphalt that Los Angeles is famous for occurs mainly on its freeways. But in the middle of the city is another patch of asphalt, the La Brea tar pits, and this asphalt preserves millions of fossil bones from the last of the Ice Ages of the Pleistocene Epoch. One of these fossils is the *Smilodon*, or sabre-toothed tiger, instantly recognizable by its long canines. Five million years ago or so, a completely different sabre-tooth tiger called *Thylacosmilus* lived in Argentina and other parts of South America. *Thylacosmilus* was a marsupial whereas *Smilodon* was a placental mammal, but *Thylacosmilus* had the same long upper canines and, like *Smilodon*, had a protective bone flange on the lower jaw. The similarity of these two mammals is one of many examples of parallel or convergent evolution, in which particular contexts or environments lead to the evolution of very similar structures in different species (Gould, 1980).



The role of context is also important in the similarity of a less biological kind of organism: the word. Words that occur in *similar contexts* tend to have *similar meanings*. This link between similarity in how words are distributed and similarity in what they mean is called the **distributional hypothesis**. The hypothesis was first formulated in the 1950s by linguists like Joos (1950), Harris (1954), and Firth (1957), who noticed that words which are synonyms (like *oculist* and *eye-doctor*) tended to occur in the same environment (e.g., near words like *eye* or *examined*) with the amount of meaning difference between two words “corresponding roughly to the amount of difference in their environments” (Harris, 1954, 157).

distributional  
hypothesis

vector  
semantics

representation  
learning

In this chapter we introduce a model known as **vector semantics**, which instantiates this linguistic hypothesis by learning representations of the meaning of words directly from their distributions in texts. These representations are used in every natural language processing application that makes use of meaning. These word representations are also the first example we will see in the book of **representation learning**, automatically learning useful representations of the input text. Finding such **unsupervised** ways to learn representations of the input, instead of creating representations by hand via **feature engineering**, is an important focus of recent NLP research (Bengio et al., 2013).

We’ll begin, however, by introducing some basic principles of word meaning, which will motivate the vector semantic models of this chapter as well as extensions that we’ll return to in Appendix C, Chapter 19, and Chapter 18.

## 6.1 Lexical Semantics

How should we represent the meaning of a word? In the N-gram models we saw in Chapter 3, and in many traditional NLP applications, our only representation of a word is as a string of letters, or perhaps as an index in a vocabulary list. This representation is not that different from a tradition in philosophy, perhaps you’ve seen it in introductory logic classes, in which the meaning of words is often represented by just spelling the word with small capital letters; representing the meaning of “dog” as DOG, and “cat” as CAT).

Representing the meaning of a word by capitalizing it is a pretty unsatisfactory model. You might have seen the old philosophy joke:

Q: What’s the meaning of life?  
A: LIFE

Surely we can do better than this! After all, we’ll want a model of word meaning to do all sorts of things for us. It should tell us that some words have similar meanings (*cat* is similar to *dog*), other words are antonyms (*cold* is the opposite of *hot*). It should know that some words have positive connotations (*happy*) while others have negative connotations (*sad*). It should represent the fact that the meanings of *buy*, *sell*, and *pay* offer differing perspectives on the same underlying purchasing event (If I buy something from you, you’ve probably sold it to me, and I likely paid you).

More generally, a model of word meaning should allow us to draw useful inferences that will help us solve meaning-related tasks like question-answering, summarization, paraphrase or plagiarism detection, and dialogue.

In this section we summarize some of these desiderata, drawing on results in the linguistic study of word meaning, which is called **lexical semantics**.

lexical  
semantics

**Lemmas and Senses** Let’s start by looking at how one word (we’ll choose *mouse*) might be defined in a dictionary: <sup>1</sup>

mouse (N)  
1. any of numerous small rodents...  
2. a hand-operated device that controls a cursor...

lemma  
citation form

Here the form *mouse* is the **lemma**, also called the **citation form**. The form *mouse* would also be the lemma for the word *mice*; dictionaries don’t have separate definitions for inflected forms like *mice*. Similarly *sing* is the lemma for *sing*, *sang*, *sung*. In many languages the infinitive form is used as the lemma for the verb, so Spanish *dormir* “to sleep” is the lemma for *duermes* “you sleep”. The specific forms *sung* or *carpets* or *sing* or *duermes* are called **wordforms**.

wordform

As the example above shows, each lemma can have multiple meanings; the lemma *mouse* can refer to the rodent or the cursor control device. We call each of these aspects of the meaning of *mouse* a **word sense**. The fact that lemmas can be **homonymous** (have multiple senses) can make interpretation difficult (is someone who types “mouse info” to a search engine looking for a pet or a tool?). Appendix C will discuss the problem of homonymy, and introduce *word sense disambiguation*, the task of determining which sense of a word is being used in a particular context.

**Relationships between words or senses** One important component of word meaning is the relationship between word senses. For example when one word has a sense

<sup>1</sup> This example shortened from the online dictionary WordNet, discussed in Appendix C.

whose meaning is identical to a sense of another word, or nearly identical, we say the two senses of those two words are **synonyms**. Synonyms include such pairs as

*couch/sofa vomit/throw up filbert/hazelnut car/automobile*

A more formal definition of synonymy (between words rather than senses) is that two words are synonymous if they are substitutable one for the other in any sentence without changing the *truth conditions* of the sentence, the situations in which the sentence would be true. We often say in this case that the two words have the same **propositional meaning**.

While substitutions between some pairs of words like *car / automobile* or *water / H<sub>2</sub>O* are truth preserving, the words are still not identical in meaning. Indeed, probably no two words are absolutely identical in meaning. One of the fundamental tenets of semantics, called the **principle of contrast** (Bréal 1897, ?, Clark 1987), is the assumption that a difference in linguistic form is always associated with at least some difference in meaning. For example, the word *H<sub>2</sub>O* is used in scientific contexts and would be inappropriate in a hiking guide—*water* would be more appropriate—and this difference in genre is part of the meaning of the word. In practice, the word *synonym* is therefore commonly used to describe a relationship of approximate or rough synonymy.

Where synonyms are words with identical or similar meanings, **Antonyms** are words with an opposite meaning, like:

*long/short big/little fast/slow cold/hot dark/light*  
*rise/fall up/down in/out*

Two senses can be antonyms if they define a binary opposition or are at opposite ends of some scale. This is the case for *long/short*, *fast/slow*, or *big/little*, which are at opposite ends of the *length* or *size* scale. Another group of antonyms, **reversives**, describe change or movement in opposite directions, such as *rise/fall* or *up/down*.

Antonyms thus differ completely with respect to one aspect of their meaning—their position on a scale or their direction—but are otherwise very similar, sharing almost all other aspects of meaning. Thus, automatically distinguishing synonyms from antonyms can be difficult.

**Word Similarity:** While words don't have many synonyms, most words do have lots of *similar* words. *Cat* is not a synonym of *dog*, but *cats* and *dogs* are certainly similar words. In moving from synonymy to similarity, it will be useful to shift from talking about relations between word senses (like synonymy) to relations between words (like similarity). Dealing with words avoids having to commit to a particular representation of word senses, which will turn out to simplify our task.

The notion of word **similarity** is very useful in larger semantic tasks. For example knowing how similar two words are is helpful if we are trying to decide if two phrases or sentences mean similar things. Phrase or sentence similarity is useful in such natural language understanding tasks as question answering, paraphrasing, and summarization.

One way of getting values for word similarity is to ask humans to judge how similar one word is to another. A number of datasets have resulted from such experiments. For example the SimLex-999 dataset (Hill et al., 2015) gives values on a scale from 0 to 10, like the examples below, which range from near-synonyms (*vanish, disappear*) to pairs that scarcely seem to have anything in common (*hole, agreement*):

vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

relatedness  
association

**Word Relatedness:** The meaning of two words can be related in ways others than similarity. One such class of connections is called word **relatedness** (Budanitsky and Hirst, 2006), also traditionally called word **association** in psychology.

Consider the meanings of the words *coffee* and *cup*; Coffee is not similar to cup; they share practically no features (coffee is a plant or a beverage, while a cup is an manufactured object with a particular shape).

But coffee and cup are clearly related; they are associated in the world by commonly co-participating in a shared event (the event of drinking coffee out of a cup). Similarly the nouns *scalpel* and *surgeon* are not similar but are related eventively (a surgeon tends to make use of a scalpel).

semantic field

One common kind of relatedness between words is if they belong to the same **semantic field**. A semantic field is a set of words which cover a particular semantic domain and bear structured relations with each other.

For example, words might be related by being in the semantic field of hospitals (*surgeon, scalpel, nurse, anaesthetic, hospital*), restaurants (*waiter, menu, plate, food, chef*), or houses (*door, roof, kitchen, family, bed*).

topic models

Semantic fields are also related to **topic models**, like **Latent Dirichlet Allocation**, **LDA**, which apply unsupervised learning on large sets of texts to induce sets of associated words from text. Semantic fields and topic models are a very useful tool for discovering topical structure in documents.

semantic frame

**Semantic Frames and Roles:** Closely related to semantic fields is the idea of a **semantic frame**. A semantic frame is a set of words that denote perspectives or participants in a particular type of event. A commercial transaction, for example, is a kind of event in which one entity trades money to another entity in return for some good or service, after which the good changes hands or perhaps the service is performed. This event can be encoded lexically by using verbs like *buy* (the event from the perspective of the buyer) *sell* (from the perspective of the seller), *pay* (focusing on the monetary aspect), or nouns like *buyer*. Frames have semantic roles (like *buyer, seller, goods, money*), and words in a sentence can take on these roles.

Knowing that *buy* and *sell* have this relation makes it possible for a system to know that a sentence like *Sam bought the book from Ling* could be paraphrased as *Ling sold the book to Sam*, and that Sam has the role of the *buyer* in the frame and Ling the *seller*. Being able to recognize such paraphrases is important for question answering, and can help in shifting perspective for machine translation.

hyponym

**Taxonomic Relations:** Another way word senses can be related is taxonomically. A word (or sense) is a **hyponym** of another word or sense if the first is more specific, denoting a subclass of the other. For example, *car* is a hyponym of *vehicle*; *dog* is a hyponym of *animal*, and *mango* is a hyponym of *fruit*. Conversely, we say that *vehicle* is a **hypernym** of *car*, and *animal* is a hypernym of *dog*. It is unfortunate that the two words (hypernym and hyponym) are very similar and hence easily confused;

superordinate

for this reason, the word **superordinate** is often used instead of **hypernym**.

<b>Superordinate</b>	vehicle	fruit	furniture	mammal
<b>Subordinate</b>	car	mango	chair	dog

We can define hypernymy more formally by saying that the class denoted by the superordinate extensionally includes the class denoted by the hyponym. Thus, the class of animals includes as members all dogs, and the class of moving actions includes all walking actions. Hyponymy can also be defined in terms of **entailment**. Under this definition, a sense *A* is a hyponym of a sense *B* if everything that is *A* is also *B*, and hence being an *A* entails being a *B*, or  $\forall x A(x) \Rightarrow B(x)$ . Hyponymy/hypernymy is usually a transitive relation; if *A* is a hyponym of *B* and *B* is a hyponym of *C*, then *A* is a hyponym of *C*. Another name for the hypernym/hyponym structure is the **IS-A** hierarchy, in which we say *A* IS-A *B*, or *B* **subsumes** *A*.

Hyponymy is useful for tasks like textual entailment or question answering; knowing that *leukemia* is a type of *cancer*, for example, would certainly be useful in answering questions about leukemia.

**connotations** **Connotation:** Finally, words have *affective meanings* or **connotations**. The word *connotation* has different meanings in different fields, but here we use it to mean the aspects of a word's meaning that are related to a writer or reader's emotions, sentiment, opinions, or evaluations. For example some words have positive connotations (*happy*) while others have negative connotations (*sad*). Some words describe positive evaluation (*great*, *love*) and others negative evaluation (*terrible*, *hate*). Positive or negative evaluation expressed through language is called **sentiment**, as we saw in Chapter 4, and word sentiment plays a role in important tasks like sentiment analysis, stance detection, and many aspects of natural language processing to the language of politics and consumer reviews.

Early work on affective meaning (Osgood et al., 1957) found that words varied along three important dimensions of affective meaning. These are now generally called *valence*, *arousal*, and *dominance*, defined as follows:

**valence:** the pleasantness of the stimulus

**arousal:** the intensity of emotion provoked by the stimulus

**dominance:** the degree of control exerted by the stimulus

Thus words like *happy* or *satisfied* are high on valence, while *unhappy* or *annoyed* are low on valence. *Excited* or *frenzied* are high on arousal, while *relaxed* or *calm* are low on arousal. Important or controlling are high on dominance, while awed or influenced are low on dominance. Each word is thus represented by three numbers, corresponding to its value on each of the three dimensions, like the examples below:

	Valence	Arousal	Dominance
courageous	8.05	5.5	7.38
music	7.67	5.57	6.5
heartbreak	2.45	5.65	3.58
cub	6.71	3.95	4.24
life	6.68	5.59	5.89

Osgood et al. (1957) noticed that in using these 3 numbers to represent the meaning of a word, the model was representing each word as a point in a three-dimensional space, a vector whose three dimensions corresponded to the word's rating on the three scales. This revolutionary idea that word meaning word could be represented as a point in space (e.g., that part of the meaning of *heartbreak* can be represented as the point [2.45, 5.65, 3.58]) was the first expression of the vector semantics models that we introduce next.

## 6.2 Vector Semantics

How can we build a computational model that successfully deals with the different aspects of word meaning we saw in the previous section (word senses, word similarity and relatedness, lexical fields and frames, connotation)?

vector  
semantics

A perfect model that completely deals with each of these aspects of word meaning turns out to be elusive. But the current best model, called **vector semantics**, draws its inspiration from linguistic and philosophical work of the 1950's.

During that period, the philosopher Ludwig Wittgenstein, skeptical of the possibility of building a completely formal theory of meaning definitions for each word, suggested instead that “the meaning of a word is its use in the language” (Wittgenstein, 1953, PI 43). That is, instead of using some logical language to define each word, we should define words by some representation of how the word was used by actual people in speaking and understanding.

Linguists of the period like Joos (1950), Harris (1954), and Firth (1957) (the linguistic distributionalists), came up with a specific idea for realizing Wittgenstein's intuition: define a word by the environment or distribution it occurs in in language use. A word's distribution is the set of contexts in which it occurs, the neighboring words or grammatical environments. The idea is that two words that occur in very similar distributions (that occur together with very similar words) are likely to have the same meaning.

Let's see an example illustrating this distributionalist approach. Suppose you didn't know what the Cantonese word *ongchoi* meant, but you do see it in the following sentences or contexts:

- (6.1) Ongchoi is delicious sauteed with garlic.
- (6.2) Ongchoi is superb over rice.
- (6.3) ...ongchoi leaves with salty sauces...

And furthermore let's suppose that you had seen many of these context words occurring in contexts like:

- (6.4) ...spinach sauteed with garlic over rice...
- (6.5) ...chard stems and leaves are delicious...
- (6.6) ...collard greens and other salty leafy greens

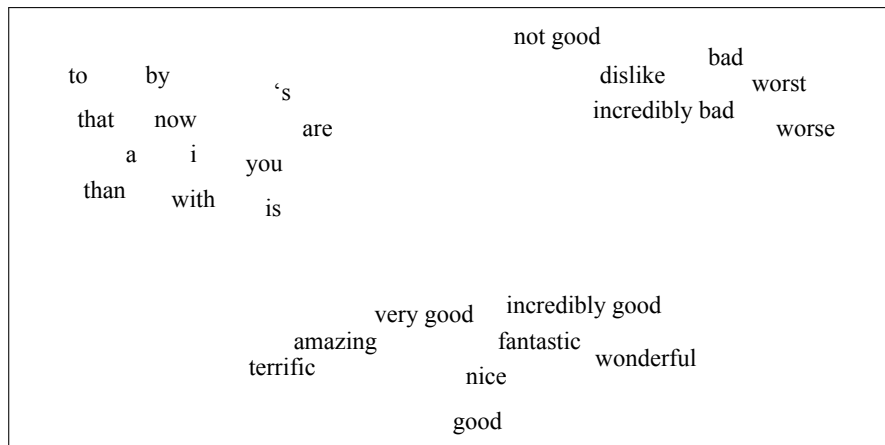
The fact that *ongchoi* occurs with words like *rice* and *garlic* and *delicious* and *salty*, as do words like *spinach*, *chard*, and *collard greens* might suggest to the reader that *ongchoi* is a leafy green similar to these other leafy greens.<sup>2</sup>

We can do the same thing computationally by just counting words in the context of *ongchoi*; we'll tend to see words like *sauteed* and *eaten* and *garlic*. The fact that these words and other similar context words also occur around the word *spinach* or *collard greens* can help us discover the similarity between these words and *ongchoi*.

Vector semantics thus combines two intuitions: the distributionalist intuition (defining a word by counting what other words occur in its environment), and the vector intuition of Osgood et al. (1957) we saw in the last section on connotation: defining the meaning of a word *w* as a vector, a list of numbers, a point in *N*-dimensional space. There are various versions of vector semantics, each defining the numbers in the vector somewhat differently, but in each case the numbers are based in some way on counts of neighboring words.

<sup>2</sup> It's in fact *Ipomoea aquatica*, a relative of morning glory sometimes called *water spinach* in English.





**Figure 6.1** A two-dimensional (t-SNE) projection of embeddings for some words and phrases, showing that words with similar meanings are nearby in space. The original 60-dimensional embeddings were trained for a sentiment analysis task. Simplified from [Li et al. \(2015\)](#).

The idea of vector semantics is thus to represent a word as a point in some multi-dimensional semantic space. Vectors for representing words are generally called **embeddings**, because the word is embedded in a particular vector space. Fig. 6.1 displays a visualization of embeddings that were learned for a sentiment analysis task, showing the location of some selected words projected down from the original 60-dimensional space into a two dimensional space.

Notice that positive and negative words seem to be located in distinct portions of the space (and different also from the neutral function words). This suggests one of the great advantages of vector semantics: it offers a fine-grained model of meaning that lets us also implement word similarity (and phrase similarity). For example, the sentiment analysis classifier we saw in Chapter 4 only works if enough of the important sentimental words that appear in the test set also appeared in the training set. But if words were represented as embeddings, we could assign sentiment as long as words with *similar meanings* as the test set words occurred in the training set. Vector semantic models are also extremely practical because they can be learned automatically from text without any complex labeling or supervision.

As a result of these advantages, vector models of meaning are now the standard way to represent the meaning of words in NLP. In this chapter we'll introduce the two most commonly used models. First is the **tf-idf** model, often used as a baseline, in which the meaning of a word is defined by a simple function of the counts of nearby words. We will see that this method results in very long vectors that are **sparse**, i.e. contain mostly zeros (since most words simply never occur in the context of others).

Then we'll introduce the **word2vec** model, one of a family of models that are ways of constructing short, **dense** vectors that have useful semantic properties.

We'll also introduce the **cosine**, the standard way to use embeddings (vectors) to compute functions like *semantic similarity*, the similarity between two words, two sentences, or two documents, an important tool in practical applications like question answering, summarization, or automatic essay grading.

## 6.3 Words and Vectors

Vector or distributional models of meaning are generally based on a **co-occurrence matrix**, a way of representing how often words co-occur. This matrix can be constructed in various ways; let's begin by looking at one such co-occurrence matrix, a term-document matrix.

### 6.3.1 Vectors and documents

term-document  
matrix

In a **term-document matrix**, each row represents a word in the vocabulary and each column represents a document from some collection of documents. Fig. 6.2 shows a small selection from a term-document matrix showing the occurrence of four words in four plays by Shakespeare. Each cell in this matrix represents the number of times a particular word (defined by the row) occurs in a particular document (defined by the column). Thus *fool* appeared 58 times in *Twelfth Night*.

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

**Figure 6.2** The term-document matrix for four words in four Shakespeare plays. Each cell contains the number of times the (row) word occurs in the (column) document.

vector space  
model

vector

vector space  
dimension

The term-document matrix of Fig. 6.2 was first defined as part of the **vector space model** of information retrieval (Salton, 1971). In this model, a document is represented as a count vector, a column in Fig. 6.3.

To review some basic linear algebra, a **vector** is, at heart, just a list or array of numbers. So *As You Like It* is represented as the list [1,114,36,20] and *Julius Caesar* is represented as the list [7,62,1,2]. A **vector space** is a collection of vectors, characterized by their **dimension**. In the example in Fig. 6.3, the vectors are of dimension 4, just so they fit on the page; in real term-document matrices, the vectors representing each document would have dimensionality  $|V|$ , the vocabulary size.

The ordering of the numbers in a vector space is not arbitrary; each position indicates a meaningful dimension on which the documents can vary. Thus the first dimension for both these vectors corresponds to the number of times the word *battle* occurs, and we can compare each dimension, noting for example that the vectors for *As You Like It* and *Twelfth Night* have similar values (1 and 0, respectively) for the first dimension.

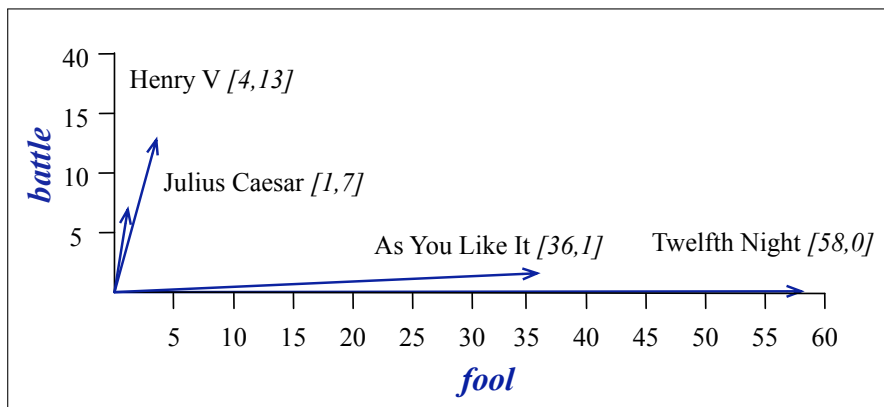
	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

**Figure 6.3** The term-document matrix for four words in four Shakespeare plays. The red boxes show that each document is represented as a column vector of length four.

We can think of the vector for a document as identifying a point in  $|V|$ -dimensional space; thus the documents in Fig. 6.3 are points in 4-dimensional space. Since 4-dimensional spaces are hard to draw in textbooks, Fig. 6.4 shows a visualization in



two dimensions; we've arbitrarily chosen the dimensions corresponding to the words *battle* and *fool*.



**Figure 6.4** A spatial visualization of the document vectors for the four Shakespeare play documents, showing just two of the dimensions, corresponding to the words *battle* and *fool*. The comedies have high values for the *fool* dimension and low values for the *battle* dimension.

Term-document matrices were originally defined as a means of finding similar documents for the task of document **information retrieval**. Two documents that are similar will tend to have similar words, and if two documents have similar words their column vectors will tend to be similar. The vectors for the comedies *As You Like It* [1,114,36,20] and *Twelfth Night* [0,80,58,15] look a lot more like each other (more fools and wit than battles) than they do like *Julius Caesar* [7,62,1,2] or *Henry V* [13,89,4,3]. We can see the intuition with the raw numbers; in the first dimension (battle) the comedies have low numbers and the others have high numbers, and we can see it visually in Fig. 6.4; we'll see very shortly how to quantify this intuition more formally.

A real term-document matrix, of course, wouldn't just have 4 rows and columns, let alone 2. More generally, the term-document matrix  $X$  has  $|V|$  rows (one for each word type in the vocabulary) and  $D$  columns (one for each document in the collection); as we'll see, vocabulary sizes are generally at least in the tens of thousands, and the number of documents can be enormous (think about all the pages on the web).

information  
retrieval

**Information retrieval (IR)** is the task of finding the document  $d$  from the  $D$  documents in some collection that best matches a query  $q$ . For IR we'll therefore also represent a query by a vector, also of length  $|V|$ , and we'll need a way to compare two vectors to find how similar they are. (Doing IR will also require efficient ways to store and manipulate these vectors, which is accomplished by making use of the convenient fact that these vectors are sparse, i.e., mostly zeros).

Later in the chapter we'll introduce some of the components of this vector comparison process: the tf-idf term weighting, and the cosine similarity metric.

### 6.3.2 Words as vectors

We've seen that documents can be represented as vectors in a vector space. But vector semantics can also be used to represent the meaning of *words*, by associating each word with a vector.

row vector

The word vector is now a **row vector** rather than a column vector, and hence the dimensions of the vector are different. The four dimensions of the vector for *fool*,

[36,58,1,4], correspond to the four Shakespeare plays. The same four dimensions are used to form the vectors for the other 3 words: *wit*, [20, 15, 2, 3]; *battle*, [1,0,7,13]; and *good* [114,80,62,89]. Each entry in the vector thus represents the counts of the word's occurrence in the document corresponding to that dimension.

For documents, we saw that similar documents had similar vectors, because similar documents tend to have similar words. This same principle applies to words: similar words have similar vectors because they tend to occur in similar documents. The term-document matrix thus lets us represent the meaning of a word by the documents it tends to occur in.

term-term  
matrix  
word-word  
matrix

However, it is most common to use a different kind of context for the dimensions of a word's vector representation. Rather than the term-document matrix we use the **term-term matrix**, more commonly called the **word-word matrix** or the **term-context matrix**, in which the columns are labeled by words rather than documents. This matrix is thus of dimensionality  $|V| \times |V|$  and each cell records the number of times the row (target) word and the column (context) word co-occur in some context in some training corpus. The context could be the document, in which case the cell represents the number of times the two words appear in the same document. It is most common, however, to use smaller contexts, generally a window around the word, for example of 4 words to the left and 4 words to the right, in which case the cell represents the number of times (in some training corpus) the column word occurs in such a  $\pm 4$  word window around the row word.

For example here are 7-word windows surrounding four sample words from the Brown corpus (just one example of each word):

sugar, a sliced lemon, a tablespoonful of	<b>apricot</b>	jam, a pinch each of,
their enjoyment. Cautiously she sampled her first	<b>pineapple</b>	and another fruit whose taste she likened
well suited to programming on the digital	<b>computer.</b>	In finding the optimal R-stage policy from
for the purpose of gathering data and	<b>information</b>	necessary for the study authorized in the

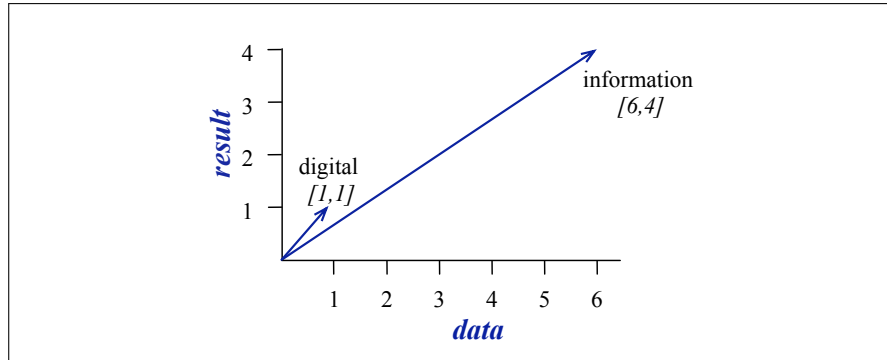
For each word we collect the counts (from the windows around each occurrence) of the occurrences of context words. Fig. 6.5 shows a selection from the word-word co-occurrence matrix computed from the Brown corpus for these four words.

	aardvark	...	computer	data	pinch	result	sugar	...
apricot	0	...	0	0	1	0	1	
pineapple	0	...	0	0	1	0	1	
digital	0	...	2	1	0	1	0	
information	0	...	1	6	0	4	0	

**Figure 6.5** Co-occurrence vectors for four words, computed from the Brown corpus, showing only six of the dimensions (hand-picked for pedagogical purposes). The vector for the word *digital* is outlined in red. Note that a real vector would have vastly more dimensions and thus be much sparser.

Note in Fig. 6.5 that the two words *apricot* and *pineapple* are more similar to each other (both *pinch* and *sugar* tend to occur in their window) than they are to other words like *digital*; conversely, *digital* and *information* are more similar to each other than, say, to *apricot*. Fig. 6.6 shows a spatial visualization.

Note that  $|V|$ , the length of the vector, is generally the size of the vocabulary, usually between 10,000 and 50,000 words (using the most frequent words in the training corpus; keeping words after about the most frequent 50,000 or so is generally not helpful). But of course since most of these numbers are zero these are **sparse** vector representations, and there are efficient algorithms for storing and computing with sparse matrices.



**Figure 6.6** A spatial visualization of word vectors for *digital* and *information*, showing just two of the dimensions, corresponding to the words *data* and *result*.

Now that we have some intuitions, let's move on to examine the details of computing word similarity. Afterwards we'll discuss the tf-idf method of weighting cells.

## 6.4 Cosine for measuring similarity

To define similarity between two target words  $v$  and  $w$ , we need a measure for taking two such vectors and giving a measure of vector similarity. By far the most common similarity metric is the **cosine** of the angle between the vectors.

dot product  
inner product

The cosine—like most measures for vector similarity used in NLP—is based on the **dot product** operator from linear algebra, also called the **inner product**:

$$\text{dot-product}(\vec{v}, \vec{w}) = \vec{v} \cdot \vec{w} = \sum_{i=1}^N v_i w_i = v_1 w_1 + v_2 w_2 + \dots + v_N w_N \quad (6.7)$$

As we will see, most metrics for similarity between vectors are based on the dot product. The dot product acts as a similarity metric because it will tend to be high just when the two vectors have large values in the same dimensions. Alternatively, vectors that have zeros in different dimensions—orthogonal vectors—will have a dot product of 0, representing their strong dissimilarity.

vector length

This raw dot-product, however, has a problem as a similarity metric: it favors **long** vectors. The **vector length** is defined as

$$|\vec{v}| = \sqrt{\sum_{i=1}^N v_i^2} \quad (6.8)$$

The dot product is higher if a vector is longer, with higher values in each dimension. More frequent words have longer vectors, since they tend to co-occur with more words and have higher co-occurrence values with each of them. The raw dot product thus will be higher for frequent words. But this is a problem; we'd like a similarity metric that tells us how similar two words are regardless of their frequency.

The simplest way to modify the dot product to normalize for the vector length is to divide the dot product by the lengths of each of the two vectors. This **normalized dot product** turns out to be the same as the cosine of the angle between the two

vectors, following from the definition of the dot product between two vectors  $\vec{a}$  and  $\vec{b}$ :

$$\begin{aligned}\vec{a} \cdot \vec{b} &= |\vec{a}| |\vec{b}| \cos \theta \\ \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} &= \cos \theta\end{aligned}\tag{6.9}$$

**cosine** The **cosine** similarity metric between two vectors  $\vec{v}$  and  $\vec{w}$  thus can be computed as:

$$\text{cosine}(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}\tag{6.10}$$

**unit vector** For some applications we pre-normalize each vector, by dividing it by its length, creating a **unit vector** of length 1. Thus we could compute a unit vector from  $\vec{a}$  by dividing it by  $|\vec{a}|$ . For unit vectors, the dot product is the same as the cosine.

The cosine value ranges from 1 for vectors pointing in the same direction, through 0 for vectors that are orthogonal, to -1 for vectors pointing in opposite directions. But raw frequency values are non-negative, so the cosine for these vectors ranges from 0–1.

Let's see how the cosine computes which of the words *apricot* or *digital* is closer in meaning to *information*, just using raw counts from the following simplified table:

	large	data	computer
apricot	2	0	0
digital	0	1	2
information	1	6	1

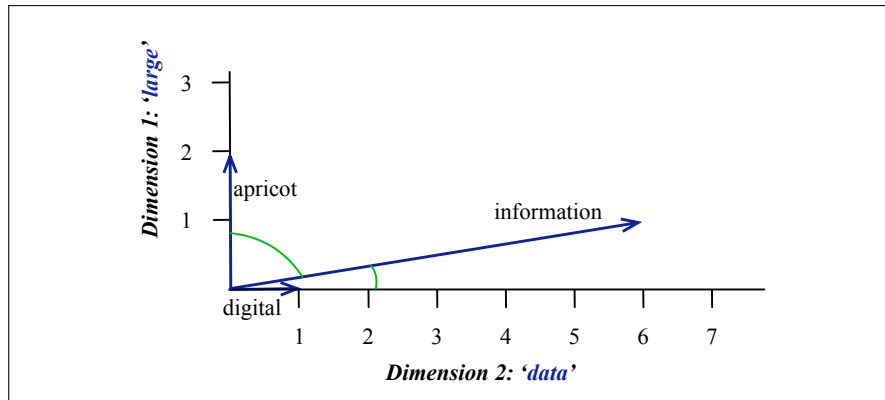
$$\begin{aligned}\cos(\text{apricot}, \text{information}) &= \frac{2 + 0 + 0}{\sqrt{4 + 0 + 0} \sqrt{1 + 36 + 1}} = \frac{2}{2\sqrt{38}} = .16 \\ \cos(\text{digital}, \text{information}) &= \frac{0 + 6 + 2}{\sqrt{0 + 1 + 4} \sqrt{1 + 36 + 1}} = \frac{8}{\sqrt{38} \sqrt{5}} = .58\end{aligned}\tag{6.11}$$

The model decides that *information* is closer to *digital* than it is to *apricot*, a result that seems sensible. Fig. 6.7 shows a visualization.

## 6.5 TF-IDF: Weighing terms in the vector

The co-occurrence matrix in Fig. 6.5 represented each cell by the raw frequency of the co-occurrence of two words.

It turns out, however, that simple frequency isn't the best measure of association between words. One problem is that raw frequency is very skewed and not very discriminative. If we want to know what kinds of contexts are shared by *apricot* and *pineapple* but not by *digital* and *information*, we're not going to get good discrimination from words like *the*, *it*, or *they*, which occur frequently with all sorts of words and aren't informative about any particular word. We saw this also in Fig. 6.3 for the Shakespeare corpus; the dimension for the word *good* is not very discriminative between plays; *good* is simply a frequent word and has roughly equivalent high frequencies in each of the plays.



**Figure 6.7** A graphical demonstration of cosine similarity, showing vectors for three words (*apricot*, *digital*, and *information*) in the two dimensional space defined by counts of the words *data* and *large* in the neighborhood. Note that the angle between *digital* and *information* is smaller than the angle between *apricot* and *information*. When two vectors are more similar, the cosine is larger but the angle is smaller; the cosine has its maximum (1) when the angle between two vectors is smallest ( $0^\circ$ ); the cosine of all other angles is less than 1.

It's a bit of a paradox. Word that occur nearby frequently (maybe *sugar* appears often in our corpus near *apricot*) are more important than words that only appear once or twice. Yet words that are too frequent—ubiquitous, like *the* or *good*— are unimportant. How can we balance these two conflicting constraints?

The **tf-idf algorithm** (the '-' here is a hyphen, not a minus sign) algorithm is the product of two terms, each term capturing one of these two intuitions:

#### term frequency

1. The first is the **term frequency** (Luhn, 1957): the frequency of the word in the document. Normally we want to downweight the raw frequency a bit, since a word appearing 100 times in a document doesn't make that word 100 times more likely to be relevant to the meaning of the document. So we generally use the  $\log_{10}$  of the frequency, resulting in the following definition for the term frequency weight:

$$\text{tf}_{t,d} = \begin{cases} 1 + \log_{10} \text{count}(t,d) & \text{if } \text{count}(t,d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Thus terms which occur 10 times in a document would have a  $\text{tf}=2$ , 100 times in a document  $\text{tf}=3$ , 1000 times  $\text{tf}=4$ , and so on.

#### document frequency

2. The second factor is used to give a higher weight to words that occur only in a few documents. Terms that are limited to a few documents are useful for discriminating those documents from the rest of the collection; terms that occur frequently across the entire collection aren't as helpful. The **document frequency**  $\text{df}_t$  of a term  $t$  is simply the number of documents it occurs in. By contrast, the **collection frequency** of a term is the total number of times the word appears in the whole collection in any document. Consider in the collection Shakespeare's 37 plays the two words *Romeo* and *action*. The words have identical collection frequencies of 113 (they both occur 113 times in all the plays) but very different document frequencies, since *Romeo* only occurs in a single play. If our goal is find documents about the romantic tribulations of *Romeo*, the word *Romeo* should be highly weighted:

	Collection Frequency	Document Frequency
Romeo	113	1
action	113	31

**idf** We assign importance to these more discriminative words like *Romeo* via the **inverse document frequency** or **idf** term weight (Sparck Jones, 1972). The idf is defined using the fraction  $N/df_t$ , where  $N$  is the total number of documents in the collection, and  $df_t$  is the number of documents in which term  $t$  occurs. The fewer documents in which a term occurs, the higher this weight. The lowest weight of 1 is assigned to terms that occur in all the documents. It's usually clear what counts as a document: in Shakespeare we would use a play; when processing a collection of encyclopedia articles like Wikipedia, the document is a Wikipedia page; in processing newspaper articles, the document is a single article. Occasionally your corpus might not have appropriate document divisions and you might need to break up the corpus into documents yourself for the purposes of computing idf.

Because of the large number of documents in many collections, this measure is usually squashed with a log function. The resulting definition for inverse document frequency (idf) is thus

$$\text{idf}_t = \log_{10} \left( \frac{N}{df_t} \right) \quad (6.12)$$

Here are some idf values for some words in the Shakespeare corpus, ranging from extremely informative words which occur in only one play like *Romeo*, to those that occur in a few like *salad* or *Falstaff*, to those which are very common like *fool* or so common as to be completely non-discriminative since they occur in all 37 plays like *good* or *sweet*.<sup>3</sup>

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.074
fool	36	0.012
good	37	0
sweet	37	0

**tf-idf** The **tf-idf** weighting of the value for word  $t$  in document  $d$ ,  $w_{t,d}$  thus combines term frequency with idf:

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t \quad (6.13)$$

Fig. 6.8 applies tf-idf weighting to the Shakespeare term-document matrix in Fig. 6.2. Note that the tf-idf values for the dimension corresponding to the word *good* have now all become 0; since this word appears in every document, the tf-idf algorithm leads it to be ignored in any comparison of the plays. Similarly, the word *fool*, which appears in 36 out of the 37 plays, has a much lower weight.

The tf-idf weighting is by far the dominant way of weighting co-occurrence matrices in information retrieval, but also plays a role in many other aspects of natural

<sup>3</sup> *Sweet* was one of Shakespeare's favorite adjectives, a fact probably related to the increased use of sugar in European recipes around the turn of the 16th century (Jurafsky, 2014, p. 175).

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	0.074	0	0.22	0.28
<b>good</b>	0	0	0	0
<b>fool</b>	0.019	0.021	0.0036	0.0083
<b>wit</b>	0.049	0.044	0.018	0.022

**Figure 6.8** A tf-idf weighted term-document matrix for four words in four Shakespeare plays, using the counts in Fig. 6.2. Note that the idf weighting has eliminated the importance of the ubiquitous word *good* and vastly reduced the impact of the almost-ubiquitous word *fool*.

language processing. It's also a great baseline, the simple thing to try first. We'll look at other weightings like PPMI (Positive Pointwise Mutual Information) in Section 6.7.

## 6.6 Applications of the tf-idf vector model

In summary, the vector semantics model we've described so far represents a target word as a vector with dimensions corresponding to all the words in the vocabulary (length  $|V|$ , with vocabularies of 20,000 to 50,000), which is also sparse (most values are zero). The values in each dimension are the frequency with which the target word co-occurs with each neighboring context word, weighted by tf-idf. The model computes the similarity between two words  $x$  and  $y$  by taking the cosine of their tf-idf vectors; high cosine, high similarity. This entire model is sometimes referred to for short as the **tf-idf** model, after the weighting function.

One common use for a tf-idf model is to compute word similarity, a useful tool for tasks like finding word paraphrases, tracking changes in word meaning, or automatically discovering meanings of words in different corpora. For example, we can find the 10 most similar words to any target word  $w$  by computing the cosines between  $w$  and each of the  $V - 1$  other words, sorting, and looking at the top 10.

The tf-idf vector model can also be used to decide if two documents are similar. We represent a document by taking the vectors of all the words in the document, and computing the **centroid** of all those vectors. The centroid is the multidimensional version of the mean; the centroid of a set of vectors is a single vector that has the minimum sum of squared distances to each of the vectors in the set. Given  $k$  word vectors  $w_1, w_2, \dots, w_k$ , the centroid **document vector**  $d$  is:

$$d = \frac{w_1 + w_2 + \dots + w_k}{k} \quad (6.14)$$

Given two documents, we can then compute their document vectors  $d_1$  and  $d_2$ , and estimate the similarity between the two documents by  $\cos(d_1, d_2)$ .

Document similarity is also useful for all sorts of applications; information retrieval, plagiarism detection, news recommender systems, and even for digital humanities tasks like comparing different versions of a text to see which are similar to each other.



## 6.7 Optional: Pointwise Mutual Information (PMI)

pointwise  
mutual  
information

An alternative weighting function to tf-idf is called PPMI (positive pointwise mutual information). PPMI draws on the intuition that best way to weigh the association between two words is to ask how much **more** the two words co-occur in our corpus than we would have a priori expected them to appear by chance.

**Pointwise mutual information** (Fano, 1961)<sup>4</sup> is one of the most important concepts in NLP. It is a measure of how often two events  $x$  and  $y$  occur, compared with what we would expect if they were independent:

$$I(x, y) = \log_2 \frac{P(x, y)}{P(x)P(y)} \quad (6.16)$$

The pointwise mutual information between a target word  $w$  and a context word  $c$  (Church and Hanks 1989, Church and Hanks 1990) is then defined as:

$$\text{PMI}(w, c) = \log_2 \frac{P(w, c)}{P(w)P(c)} \quad (6.17)$$

The numerator tells us how often we observed the two words together (assuming we compute probability by using the MLE). The denominator tells us how often we would **expect** the two words to co-occur assuming they each occurred independently; recall that the probability of two independent events both occurring is just the product of the probabilities of the two events. Thus, the ratio gives us an estimate of how much more the two words co-occur than we expect by chance. PMI is a useful tool whenever we need to find words that are strongly associated.

PMI values range from negative to positive infinity. But negative PMI values (which imply things are co-occurring *less often* than we would expect by chance) tend to be unreliable unless our corpora are enormous. To distinguish whether two words whose individual probability is each  $10^{-6}$  occur together more often than chance, we would need to be certain that the probability of the two occurring together is significantly different than  $10^{-12}$ , and this kind of granularity would require an enormous corpus. Furthermore it's not clear whether it's even possible to evaluate such scores of 'unrelatedness' with human judgments. For this reason it is more common to use Positive PMI (called **PPMI**) which replaces all negative PMI values with zero (Church and Hanks 1989, Dagan et al. 1993, Niwa and Nitta 1994)<sup>5</sup>:

$$\text{PPMI}(w, c) = \max(\log_2 \frac{P(w, c)}{P(w)P(c)}, 0) \quad (6.18)$$

More formally, let's assume we have a co-occurrence matrix  $F$  with  $W$  rows (words) and  $C$  columns (contexts), where  $f_{ij}$  gives the number of times word  $w_i$  occurs in

<sup>4</sup> Pointwise mutual information is based on the **mutual information** between two random variables  $X$  and  $Y$ , which is defined as:

$$I(X, Y) = \sum_x \sum_y P(x, y) \log_2 \frac{P(x, y)}{P(x)P(y)} \quad (6.15)$$

In a confusion of terminology, Fano used the phrase *mutual information* to refer to what we now call *pointwise mutual information* and the phrase *expectation of the mutual information* for what we now call *mutual information*.

<sup>5</sup> Positive PMI also cleanly solves the problem of what to do with zero counts, using 0 to replace the  $-\infty$  from  $\log(0)$ .

context  $c_j$ . This can be turned into a PPMI matrix where  $ppmi_{ij}$  gives the PPMI value of word  $w_i$  with context  $c_j$  as follows:

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}} \quad p_{i*} = \frac{\sum_{j=1}^C f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}} \quad p_{*j} = \frac{\sum_{i=1}^W f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}} \quad (6.19)$$

$$PPMI_{ij} = \max(\log_2 \frac{p_{ij}}{p_{i*} p_{*j}}, 0) \quad (6.20)$$

Thus for example we could compute  $PPMI(w=\text{information}, c=\text{data})$ , assuming we pretended that Fig. 6.5 encompassed all the relevant word contexts/dimensions, as follows:

$$\begin{aligned} P(w=\text{information}, c=\text{data}) &= \frac{6}{19} = .316 \\ P(w=\text{information}) &= \frac{11}{19} = .579 \\ P(c=\text{data}) &= \frac{7}{19} = .368 \\ ppmi(\text{information}, \text{data}) &= \log 2(.316 / (.368 * .579)) = .568 \end{aligned}$$

Fig. 6.9 shows the joint probabilities computed from the counts in Fig. 6.5, and Fig. 6.10 shows the PPMI values.

	p(w,context)					p(w)
	computer	data	pinch	result	sugar	p(w)
apricot	0	0	0.05	0	0.05	0.11
pineapple	0	0	0.05	0	0.05	0.11
digital	0.11	0.05	0	0.05	0	0.21
information	0.05	.32	0	0.21	0	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	

**Figure 6.9** Replacing the counts in Fig. 6.5 with joint probabilities, showing the marginals around the outside.

	computer	data	pinch	result	sugar
apricot	0	0	2.25	0	2.25
pineapple	0	0	2.25	0	2.25
digital	1.66	0	0	0	0
information	0	0.57	0	0.47	0

**Figure 6.10** The PPMI matrix showing the association between words and context words, computed from the counts in Fig. 6.5 again showing five dimensions. Note that the 0 ppmi values are ones that had a negative pmi; for example  $pmi(\text{information}, \text{computer}) = \log 2(.05 / (.16 * .58)) = -0.618$ , meaning that *information* and *computer* co-occur in this mini-corpus slightly less often than we would expect by chance, and with ppmi we replace negative values by zero. Many of the zero ppmi values had a pmi of  $-\infty$ , like  $pmi(\text{apricot}, \text{computer}) = \log 2(0 / (0.16 * 0.11)) = \log 2(0) = -\infty$ .

PMI has the problem of being biased toward infrequent events; very rare words tend to have very high PMI values. One way to reduce this bias toward low frequency events is to slightly change the computation for  $P(c)$ , using a different function  $P_\alpha(c)$  that raises contexts to the power of  $\alpha$ :

$$PPMI_\alpha(w, c) = \max(\log_2 \frac{P(w, c)}{P(w)P_\alpha(c)}, 0) \quad (6.21)$$

$$P_{\alpha}(c) = \frac{\text{count}(c)^{\alpha}}{\sum_c \text{count}(c)^{\alpha}} \quad (6.22)$$

Levy et al. (2015) found that a setting of  $\alpha = 0.75$  improved performance of embeddings on a wide range of tasks (drawing on a similar weighting used for skip-grams described below in Eq. 6.31). This works because raising the probability to  $\alpha = 0.75$  increases the probability assigned to rare contexts, and hence lowers their PMI ( $P_{\alpha}(c) > P(c)$  when  $c$  is rare).

Another possible solution is Laplace smoothing: Before computing PMI, a small constant  $k$  (values of 0.1-3 are common) is added to each of the counts, shrinking (discounting) all the non-zero values. The larger the  $k$ , the more the non-zero counts are discounted.

	computer	data	pinch	result	sugar
apricot	2	2	3	2	3
pineapple	2	2	3	2	3
digital	4	3	2	3	2
information	3	8	2	6	2

**Figure 6.11** Laplace (add-2) smoothing of the counts in Fig. 6.5.

	computer	data	pinch	result	sugar
apricot	0	0	0.56	0	0.56
pineapple	0	0	0.56	0	0.56
digital	0.62	0	0	0	0
information	0	0.58	0	0.37	0

**Figure 6.12** The Add-2 Laplace smoothed PPMI matrix from the add-2 smoothing counts in Fig. 6.11.

## 6.8 Word2vec

In the previous sections we saw how to represent a word as a sparse, long vector with dimensions corresponding to the words in the vocabulary, and whose values were tf-idf or PPMI functions of the count of the word co-occurring with each neighboring word. In this section we turn to an alternative method for representing a word: the use of vectors that are **short** (of length perhaps 50-500) and **dense** (most values are non-zero).

It turns out that dense vectors work better in every NLP task than sparse vectors. While we don't completely understand all the reasons for this, we have some intuitions. First, dense vectors may be more successfully included as features in machine learning systems; for example if we use 100-dimensional word embeddings as features, a classifier can just learn 100 weights to represent a function of word meaning; if we instead put in a 50,000 dimensional vector, a classifier would have to learn tens of thousands of weights for each of the sparse dimensions. Second, because they contain fewer parameters than sparse vectors of explicit counts, dense vectors may generalize better and help avoid overfitting. Finally, dense vectors may do a better job of capturing synonymy than sparse vectors. For example, *car* and *automobile* are synonyms; but in a typical sparse vector representation, the *car* dimension and the *automobile* dimension are distinct dimensions. Because the

relationship between these two dimensions is not modeled, sparse vectors may fail to capture the similarity between a word with *car* as a neighbor and a word with *automobile* as a neighbor.

In this section we introduce one method for very dense, short vectors, **skip-gram with negative sampling**, sometimes called **SGNS**. The skip-gram algorithm is one of two algorithms in a software package called **word2vec**, and so sometimes the algorithm is loosely referred to as word2vec (Mikolov et al. 2013, Mikolov et al. 2013a). The word2vec methods are fast, efficient to train, and easily available online with code and pretrained embeddings. We point to other embedding methods, like the equally popular *GloVe* (Pennington et al., 2014), at the end of the chapter.

The intuition of word2vec is that instead of counting how often each word  $w$  occurs near, say, *apricot*, we'll instead train a classifier on a binary prediction task: "Is word  $w$  likely to show up near *apricot*?" We don't actually care about this prediction task; instead we'll take the learned classifier *weights* as the word embeddings.

The revolutionary intuition here is that we can just use running text as implicitly supervised training data for such a classifier; a word  $s$  that occurs near the target word *apricot* acts as gold 'correct answer' to the question "Is word  $w$  likely to show up near *apricot*?" This avoids the need for any sort of hand-labeled supervision signal. This idea was first proposed in the task of neural language modeling, when Bengio et al. (2003) and Collobert et al. (2011) showed that a neural language model (a neural network that learned to predict the next word from prior words) could just use the next word in running text as its supervision signal, and could be used to learn an embedding representation for each word as part of doing this prediction task.

We'll see how to do neural networks in the next chapter, but word2vec is a much simpler model than the neural network language model, in two ways. First, word2vec simplifies the task (making it binary classification instead of word prediction). Second, word2vec simplifies the architecture (training a logistic regression classifier instead of a multi-layer neural network with hidden layers that demand more sophisticated training algorithms). The intuition of skip-gram is:

1. Treat the target word and a neighboring context word as positive examples.
2. Randomly sample other words in the lexicon to get negative samples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the regression weights as the embeddings

### 6.8.1 The classifier

Let's start by thinking about the classification task, and then turn to how to train. Imagine a sentence like the following, with a target word *apricot* and assume we're using a window of  $\pm 2$  context words:

```
... lemon,  a [tablespoon of apricot jam,      a] pinch ...
              c1      c2    t    c3          c4
```

Our goal is to train a classifier such that, given a tuple  $(t, c)$  of a target word  $t$  paired with a candidate context word  $c$  (for example (*apricot*, *jam*), or perhaps (*apricot*, *aardvark*) it will return the probability that  $c$  is a real context word (true for *jam*, false for *aardvark*):

$$P(+|t, c) \tag{6.23}$$

The probability that word  $c$  is not a real context word for  $t$  is just 1 minus Eq. 6.23:

$$P(-|t, c) = 1 - P(+|t, c) \quad (6.24)$$

How does the classifier compute the probability  $P$ ? The intuition of the skip-gram model is to base this probability on similarity: a word is likely to occur near the target if its embedding is similar to the target embedding. How can we compute similarity between embeddings? Recall that two vectors are similar if they have a high dot product (cosine, the most popular similarity metric, is just a normalized dot product). In other words:

$$\text{Similarity}(t, c) \approx t \cdot c \quad (6.25)$$

Of course, the dot product  $t \cdot c$  is not a probability, it's just a number ranging from 0 to  $\infty$ . (Recall, for that matter, that cosine isn't a probability either). To turn the dot product into a probability, we'll use the **logistic** or **sigmoid** function  $\sigma(x)$ , the fundamental core of logistic regression:

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (6.26)$$

The probability that word  $c$  is a real context word for target word  $t$  is thus computed as:

$$P(+|t, c) = \frac{1}{1 + e^{-t \cdot c}} \quad (6.27)$$

The sigmoid function just returns a number between 0 and 1, so to make it a probability we'll need to make sure that the total probability of the two possible events ( $c$  being a context word, and  $c$  not being a context word) sum to 1.

The probability that word  $c$  is not a real context word for  $t$  is thus:

$$\begin{aligned} P(-|t, c) &= 1 - P(+|t, c) \\ &= \frac{e^{-t \cdot c}}{1 + e^{-t \cdot c}} \end{aligned} \quad (6.28)$$

Equation 6.27 give us the probability for one word, but we need to take account of the multiple context words in the window. Skip-gram makes the strong but very useful simplifying assumption that all context words are independent, allowing us to just multiply their probabilities:

$$P(+|t, c_{1:k}) = \prod_{i=1}^k \frac{1}{1 + e^{-t \cdot c_i}} \quad (6.29)$$

$$\log P(+|t, c_{1:k}) = \sum_{i=1}^k \log \frac{1}{1 + e^{-t \cdot c_i}} \quad (6.30)$$

In summary, skip-gram trains a probabilistic classifier that, given a test target word  $t$  and its context window of  $k$  words  $c_{1:k}$ , assigns a probability based on how similar this context window is to the target word. The probability is based on applying the logistic (sigmoid) function to the dot product of the embeddings of the target word with each context word. We could thus compute this probability if only we had embeddings for each word target and context word in the vocabulary. Let's now turn to learning these embeddings (which is the real goal of training this classifier in the first place).

### 6.8.2 Learning skip-gram embeddings

Word2vec learns embeddings by starting with an initial set of embedding vectors and then iteratively shifting the embedding of each word  $w$  to be more like the embeddings of words that occur nearby in texts, and less like the embeddings of words that don't occur nearby.

Let's start by considering a single piece of the training data, from the sentence above:

... lemon, a [tablespoon of apricot jam, a] pinch ...  
                                   c1          c2    t      c3          c4

This example has a target word  $t$  (apricot), and 4 context words in the  $L = \pm 2$  window, resulting in 4 positive training instances (on the left below):

positive examples +		negative examples -			
t	c	t	c	t	c
apricot	tablespoon	apricot	aardvark	apricot	twelve
apricot	of	apricot	puddle	apricot	hello
apricot	preserves	apricot	where	apricot	dear
apricot	or	apricot	coaxial	apricot	forever

For training a binary classifier we also need negative examples, and in fact skip-gram uses more negative examples than positive examples, the ratio set by a parameter  $k$ . So for each of these  $(t, c)$  training instances we'll create  $k$  negative samples, each consisting of the target  $t$  plus a 'noise word'. A noise word is a random word from the lexicon, constrained not to be the target word  $t$ . The right above shows the setting where  $k = 2$ , so we'll have 2 negative examples in the negative training set — for each positive example  $t, c$ .

The noise words are chosen according to their weighted unigram frequency  $p_\alpha(w)$ , where  $\alpha$  is a weight. If we were sampling according to unweighted frequency  $p(w)$ , it would mean that with unigram probability  $p(\text{"the"})$  we would choose the word *the* as a noise word, with unigram probability  $p(\text{"aardvark"})$  we would choose *aardvark*, and so on. But in practice it is common to set  $\alpha = .75$ , i.e. use the weighting  $p^{\frac{3}{4}}(w)$ :

$$P_\alpha(w) = \frac{\text{count}(w)^\alpha}{\sum_{w'} \text{count}(w')^\alpha} \quad (6.31)$$

Setting  $\alpha = .75$  gives better performance because it gives rare noise words slightly higher probability: for rare words,  $P_\alpha(w) > P(w)$ . To visualize this intuition, it might help to work out the probabilities for an example with two events,  $P(a) = .99$  and  $P(b) = .01$ :

$$\begin{aligned} P_\alpha(a) &= \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97 \\ P_\alpha(b) &= \frac{.01^{.75}}{.99^{.75} + .01^{.75}} = .03 \end{aligned} \quad (6.32)$$

Given the set of positive and negative training instances, and an initial set of embeddings, the goal of the learning algorithm is to adjust those embeddings such that we

- Maximize the similarity of the target word, context word pairs  $(t, c)$  drawn from the positive examples

- Minimize the similarity of the  $(t, c)$  pairs drawn from the negative examples.

We can express this formally over the whole training set as:

$$L(\theta) = \sum_{(t,c) \in +} \log P(+|t, c) + \sum_{(t,c) \in -} \log P(-|t, c) \quad (6.33)$$

Or, focusing in on one word/context pair  $(t, c)$  with its  $k$  noise words  $n_1 \dots n_k$ , the learning objective  $L$  is:

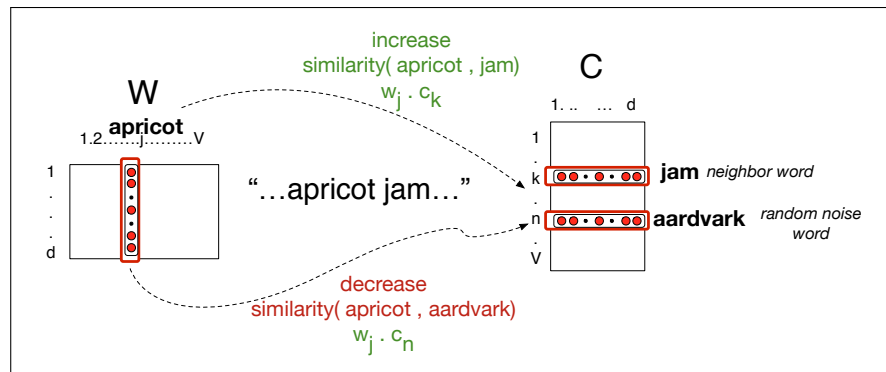
$$\begin{aligned} L(\theta) &= \log P(+|t, c) + \sum_{i=1}^k \log P(-|t, n_i) \\ &= \log \sigma(c \cdot t) + \sum_{i=1}^k \log \sigma(-n_i \cdot t) \\ &= \log \frac{1}{1 + e^{-c \cdot t}} + \sum_{i=1}^k \log \frac{1}{1 + e^{n_i \cdot t}} \end{aligned} \quad (6.34)$$

That is, we want to maximize the dot product of the word with the actual context words, and minimize the dot products of the word with the  $k$  negative sampled non-neighbor words.

We can then use stochastic gradient descent to train to this objective, iteratively modifying the parameters (the embeddings for each target word  $t$  and each context word or noise word  $c$  in the vocabulary) to maximize the objective.

target  
embedding  
context  
embedding

Note that the skip-gram model thus actually learns **two** separate embeddings for each word  $w$ : the **target embedding**  $t$  and the **context embedding**  $c$ . These embeddings are stored in two matrices, the **target matrix**  $T$  and the **context matrix**  $C$ . So each row  $i$  of the target matrix  $T$  is the  $1 \times d$  vector embedding  $t_i$  for word  $i$  in the vocabulary  $V$ , and each column  $i$  of the context matrix  $C$  is a  $d \times 1$  vector embedding  $c_i$  for word  $i$  in  $V$ . Fig. 6.13 shows an intuition of the learning task for the embeddings encoded in these two matrices.



**Figure 6.13** The skip-gram model tries to shift embeddings so the target embedding (here for *apricot*) are closer to (have a higher dot product with) context embeddings for nearby words (here *jam*) and further from (have a lower dot product with) context embeddings for words that don't occur nearby (here *aardvark*).

Just as in logistic regression, then, the learning algorithm starts with randomly initialized  $W$  and  $C$  matrices, and then walks through the training corpus using gradient descent to move  $W$  and  $C$  so as to maximize the objective in Eq. 6.34. Thus the matrices  $W$  and  $C$  function as the parameters  $\theta$  that logistic regression is tuning.



Once the embeddings are learned, we'll have two embeddings for each word  $w_i$ :  $t_i$  and  $c_i$ . We can choose to throw away the  $C$  matrix and just keep  $W$ , in which case each word  $i$  will be represented by the vector  $t_i$ .

Alternatively we can add the two embeddings together, using the summed embedding  $t_i + c_i$  as the new  $d$ -dimensional embedding, or we can concatenate them into an embedding of dimensionality  $2d$ .

As with the simple count-based methods like tf-idf, the context window size  $L$  effects the performance of skip-gram embeddings, and experiments often tune the parameter  $L$  on a dev set. One difference from the count-based methods is that for skip-grams, the larger the window size the more computation the algorithm requires for training (more neighboring words must be predicted).

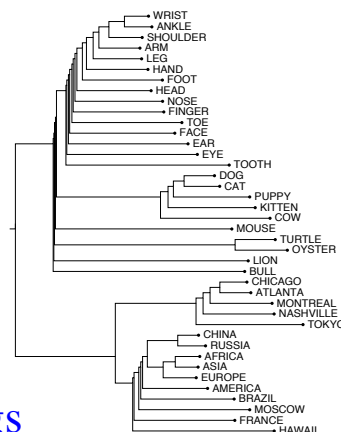
## 6.9 Visualizing Embeddings

Visualizing embeddings is an important goal in helping understand, apply, and improve these models of word meaning. But how can we visualize a (for example) 100-dimensional vector?

The simplest way to visualize the meaning of a word  $w$  embedded in a space is to list the most similar words to  $w$  sorting all words in the vocabulary by their cosines. For example the 7 closest words to *frog* using the GloVe embeddings are: *frogs*, *toad*, *litoria*, *leptodactylidae*, *rana*, *lizard*, and *eleutherodactylus* (Pennington et al., 2014)

Yet another visualization method is to use a clustering algorithm to show a hierarchical representation of which words are similar to others in the embedding space. The example on the right uses hierarchical clustering of some embedding vectors for nouns as a visualization method (Rohde et al., 2006).

Probably the most common visualization method, however, is to project the 100 dimensions of a word down into 2 dimensions. Fig. 6.1 showed one such visualization, using a projection method called t-SNE (van der Maaten and Hinton, 2008).



## 6.10 Semantic properties of embeddings

Vector semantic models have a number of parameters. One parameter that is relevant to both sparse tf-idf vectors and dense word2vec vectors is the size of the context window used to collect counts. This is generally between 1 and 10 words on each side of the target word (for a total context of 3-20 words).

The choice depends on the goals of the representation. Shorter context windows tend to lead to representations that are a bit more syntactic, since the information is coming from immediately nearby words. When the vectors are computed from short context windows, the most similar words to a target word  $w$  tend to be semantically similar words with the same parts of speech. When vectors are computed from long context windows, the highest cosine words to a target word  $w$  tend to be words that are topically related but not similar.

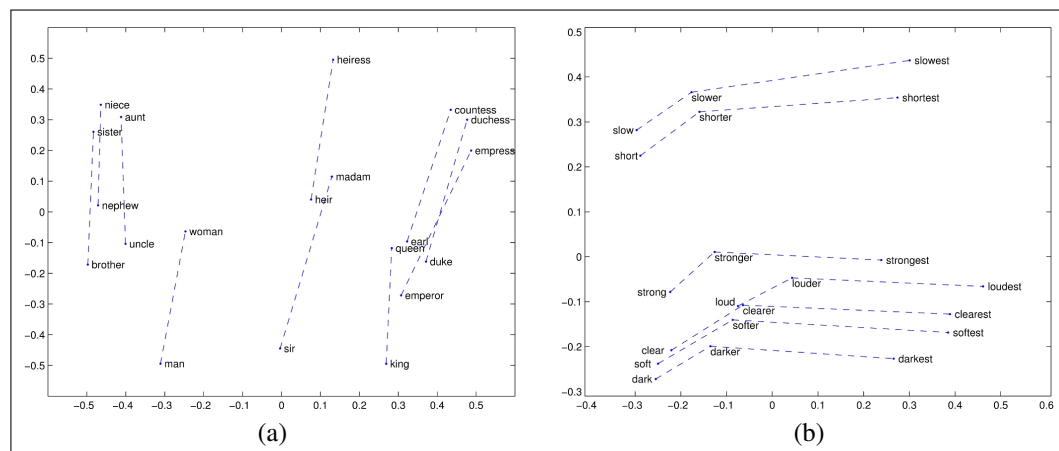
For example [Levy and Goldberg \(2014a\)](#) showed that using skip-gram with a window of  $\pm 2$ , the most similar words to the word *Hogwarts* (from the Harry Potter series) were names of other fictional schools: *Sunnydale* (from *Buffy the Vampire Slayer*) or *Evernight* (from a vampire series). With a window of  $\pm 5$ , the most similar words to *Hogwarts* were other words topically related to the Harry Potter series: *Dumbledore*, *Malfoy*, and *half-blood*.

first-order  
co-occurrence

second-order  
co-occurrence

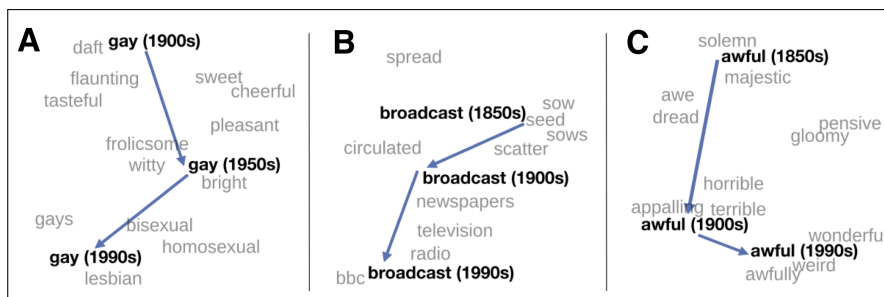
It's also often useful to distinguish two kinds of similarity or association between words ([Schütze and Pedersen, 1993](#)). Two words have **first-order co-occurrence** (sometimes called **syntagmatic association**) if they are typically nearby each other. Thus *wrote* is a first-order associate of *book* or *poem*. Two words have **second-order co-occurrence** (sometimes called **paradigmatic association**) if they have similar neighbors. Thus *wrote* is a second-order associate of words like *said* or *remarked*.

**Analogy** Another semantic property of embeddings is their ability to capture relational meanings. [Mikolov et al. \(2013b\)](#) and [Levy and Goldberg \(2014b\)](#) show that the *offsets* between vector embeddings can capture some analogical relations between words. For example, the result of the expression vector('king') - vector('man') + vector('woman') is a vector close to vector('queen'); the left panel in Fig. 6.14 visualizes this, again projected down into 2 dimensions. Similarly, they found that the expression vector('Paris') - vector('France') + vector('Italy') results in a vector that is very close to vector('Rome').



**Figure 6.14** Relational properties of the vector space, shown by projecting vectors onto two dimensions. (a) 'king' - 'man' + 'woman' is close to 'queen' (b) offsets seem to capture comparative and superlative morphology ([Pennington et al., 2014](#)).

**Embeddings and Historical Semantics:** Embeddings can also be a useful tool for studying how meaning changes over time, by computing multiple embedding spaces, each from texts written in a particular time period. For example Fig. 6.15 shows a visualization of changes in meaning in English words over the last two centuries, computed by building separate embedding spaces for each decade from historical corpora like Google N-grams ([Lin et al., 2012](#)) and the Corpus of Historical American English ([Davies, 2012](#)).



**Figure 6.15** A t-SNE visualization of the semantic change of 3 words in English using word2vec vectors. The modern sense of each word, and the grey context words, are computed from the most recent (modern) time-point embedding space. Earlier points are computed from earlier historical embedding spaces. The visualizations show the changes in the word *gay* from meanings related to “cheerful” or “frolicsome” to referring to homosexuality, the development of the modern “transmission” sense of *broadcast* from its original sense of sowing seeds, and the pejoration of the word *awful* as it shifted from meaning “full of awe” to meaning “terrible or appalling” (Hamilton et al., 2016).

## 6.11 Bias and Embeddings

In addition to their ability to learn word meaning from text, embeddings, alas, also reproduce the implicit biases and stereotypes that were latent in the text. Recall that embeddings model analogical relations; ‘queen’ as the closest word to ‘king’ - ‘man’ + ‘woman’ implies the analogy *man:woman::king:queen*. But embedding analogies also exhibit gender stereotypes. For example Bolukbasi et al. (2016) find that the closest occupation to ‘man’ - ‘computer programmer’ + ‘woman’ in word2vec embeddings trained on news text is ‘homemaker’, and that the embeddings similarly suggest the analogy ‘father’ is to ‘doctor’ as ‘mother’ is to ‘nurse’. Algorithms that used embeddings as part of an algorithm to search for potential programmers or doctors might thus incorrectly downweight documents with women’s names.

Embeddings also encode the implicit associations that are a property of human reasoning. The Implicit Association Test (Greenwald et al., 1998) measures people’s associations between concepts (like ‘flowers’ or ‘insects’) and attributes (like ‘pleasantness’ and ‘unpleasantness’) by measuring differences in the latency with which they label words in the various categories.<sup>6</sup> Using such methods, people in the United States have been shown to associate African-American names with unpleasant words (more than European-American names), male names more with mathematics and female names with the arts, and old people’s names with unpleasant words (Greenwald et al. 1998, Nosek et al. 2002a, Nosek et al. 2002b). Caliskan et al. (2017) replicated all these findings of implicit associations using GloVe vectors and cosine similarity instead of human latencies. For example African American names like ‘Leroy’ and ‘Shaniqua’ had a higher GloVe cosine with unpleasant words while European American names (‘Brad’, ‘Greg’, ‘Courtney’) had a higher cosine with pleasant words. Any embedding-aware algorithm that made use of word sentiment could thus lead to bias against African Americans.

<sup>6</sup> Roughly speaking, if humans associate ‘flowers’ with ‘pleasantness’ and ‘insects’ with ‘unpleasantness’, when they are instructed to push a red button for ‘flowers’ (daisy, iris, lilac) and ‘pleasant words’ (love, laughter, pleasure) and a green button for ‘insects’ (flea, spider, mosquito) and ‘unpleasant words’ (abuse, hatred, ugly) they are faster than in an incongruous condition where they push a red button for ‘flowers’ and ‘unpleasant words’ and a green button for ‘insects’ and ‘pleasant words’.

Recent research focuses on ways to try to remove the kinds of biases, for example by developing a transformation of the embedding space that removes gender stereotypes but preserves definitional gender (Bolukbasi et al. 2016, Zhao et al. 2017).

Historical embeddings are also being used to measure biases in the past. Garg et al. (2018) used embeddings from historical texts to measure the association between embeddings for occupations and embeddings for names of various ethnicities or genders (for example the relative cosine similarity of women’s names versus men’s to occupation words like ‘librarian’ or ‘carpenter’) across the 20th century. They found that the cosines correlate with the empirical historical percentages of women or ethnic groups in those occupation. Historical embeddings also replicated old surveys of ethnic stereotypes; the tendency of experimental participants in 1933 to associate adjectives like ‘industrious’ or ‘superstitious’ with, e.g., Chinese ethnicity, correlates with the cosine between Chinese last names and those adjectives using embeddings trained on 1930s text. They also were able to document historical gender biases, such as the fact that embeddings for adjectives related to competence (‘smart’, ‘wise’, ‘thoughtful’, ‘resourceful’) had a higher cosine with male than female words, and showed that this bias has been slowly decreasing since 1960.

We will return in later chapters to this question about the role of bias in natural language processing and machine learning in general.

## 6.12 Evaluating Vector Models

The most important evaluation metric for vector models is extrinsic evaluation on tasks; adding them as features into any NLP task and seeing whether this improves performance over some other model.

Nonetheless it is useful to have intrinsic evaluations. The most common metric is to test their performance on **similarity**, computing the correlation between an algorithm’s word similarity scores and word similarity ratings assigned by humans. **WordSim-353** (Finkelstein et al., 2002) is a commonly used set of ratings from 0 to 10 for 353 noun pairs; for example (*plane*, *car*) had an average score of 5.77. **SimLex-999** (Hill et al., 2015) is a more difficult dataset that quantifies similarity (*cup*, *mug*) rather than relatedness (*cup*, *coffee*), and including both concrete and abstract adjective, noun and verb pairs. The **TOEFL dataset** is a set of 80 questions, each consisting of a target word with 4 additional word choices; the task is to choose which is the correct synonym, as in the example: *Levied is closest in meaning to: imposed, believed, requested, correlated* (Landauer and Dumais, 1997). All of these datasets present words without context.

Slightly more realistic are intrinsic similarity tasks that include context. The Stanford Contextual Word Similarity (SCWS) dataset (Huang et al., 2012) offers a richer evaluation scenario, giving human judgments on 2,003 pairs of words in their sentential context, including nouns, verbs, and adjectives. This dataset enables the evaluation of word similarity algorithms that can make use of context words. The *semantic textual similarity* task (Agirre et al. 2012, Agirre et al. 2015) evaluates the performance of sentence-level similarity algorithms, consisting of a set of pairs of sentences, each pair with human-labeled similarity scores.

Another task used for evaluate is an analogy task, where the system has to solve problems of the form *a is to b as c is to d*, given *a*, *b*, and *c* and having to find *d*. Thus given *Athens is to Greece as Oslo is to \_\_\_\_\_*, the system must fill in the word *Norway*. Or more syntactically-oriented examples: given *mouse*, *mice*, and *dollar*

the system must return *dollars*. Large sets of such tuples have been created (Mikolov et al. 2013, Mikolov et al. 2013b).

## 6.13 Summary

- In vector semantics, a word is modeled as a vector—a point in high-dimensional space, also called an **embedding**.
- Vector semantic models fall into two classes: **sparse** and **dense**. In sparse models like **tf-idf** each dimension corresponds to a word in the vocabulary  $V$ ;
- Cell in sparse models are functions of **co-occurrence counts**. The **term-document** matrix has rows for each word (**term**) in the vocabulary and a column for each document.
- The **word-context** matrix has a row for each (target) word in the vocabulary and a column for each context term in the vocabulary.
- A common sparse weighting is **tf-idf**, which weights each cell by its **term frequency** and **inverse document frequency**.
- Word and document similarity is computed by computing the **dot product** between vectors. The cosine of two vectors—a normalized dot product—is the most popular such metric.
- **PPMI** (pointwise positive mutual information) is an alternative weighting scheme to tf-idf.
- Dense vector models have dimensionality 50-300 and the dimensions are harder to interpret.
- The **word2vec** family of models, including **skip-gram** and **CBOW**, is a popular efficient way to compute dense embeddings.
- Skip-gram trains a logistic regression classifier to compute the probability that two words are ‘likely to occur nearby in text’. This probability is computed from the dot product between the embeddings for the two words,
- Skip-gram uses stochastic gradient descent to train the classifier, by learning embeddings that have a high dot-product with embeddings of words that occur nearby and a low dot-product with noise words.
- Other important embedding algorithms include **GloVe**, a method based on ratios of word co-occurrence probabilities, and **fasttext**, an open-source library for computing word embeddings by summing embeddings of the bag of character n-grams that make up a word.

## Bibliographical and Historical Notes

The idea of vector semantics arose out of research in the 1950s in three distinct fields: linguistics, psychology, and computer science, each of which contributed a fundamental aspect of the model.

The idea that meaning was related to distribution of words in context was widespread in linguistic theory of the 1950s, among distributionalists like Zellig Harris, Martin Joos, and J. R. Firth, and semioticians like Thomas Sebeok. As Joos (1950) put it,

the linguist’s “meaning” of a morpheme... is by definition the set of conditional probabilities of its occurrence in context with all other morphemes.

The idea that the meaning of a word might be modeled as a point in a multi-dimensional semantic space came from psychologists like Charles E. Osgood, who had been studying how people responded to the meaning of words by assigning values along scales like *happy/sad*, or *hard/soft*. Osgood et al. (1957) proposed that the meaning of a word in general could be modeled as a point in a multidimensional Euclidean space, and that the similarity of meaning between two words could be modeled as the distance between these points in the space.

mechanical  
indexing

A final intellectual source in the 1950s and early 1960s was the field then called **mechanical indexing**, now known as **information retrieval**. In what became known as the **vector space model** for information retrieval (Salton 1971, Sparck Jones 1986), researchers demonstrated new ways to define the meaning of words in terms of vectors (Switzer, 1965), and refined methods for word similarity based on measures of statistical association between words like mutual information (Giuliano, 1965) and idf (Sparck Jones, 1972), and showed that the meaning of documents could be represented in the same vector spaces used for words.

semantic  
feature

More distantly related is the idea of defining words by a vector of discrete features, which has a venerable history in our field, with roots at least as far back as Descartes and Leibniz (Wierzbicka 1992, Wierzbicka 1996). By the middle of the 20th century, beginning with the work of Hjelmslev (Hjelmslev, 1969) and fleshed out in early models of generative grammar (Katz and Fodor, 1963), the idea arose of representing meaning with **semantic features**, symbols that represent some sort of primitive meaning. For example words like *hen*, *rooster*, or *chick*, have something in common (they all describe chickens) and something different (their age and sex), representable as:

<i>hen</i>	+female, +chicken, +adult
<i>rooster</i>	-female, +chicken, +adult
<i>chick</i>	+chicken, -adult

The dimensions used by vector models of meaning to define words, however, are only abstractly related to this idea of a small fixed number of hand-built dimensions. Nonetheless, there has been some attempt to show that certain dimensions of embedding models do contribute some specific compositional aspect of meaning like these early semantic features.

The first use of dense vectors to model word meaning was the **latent semantic indexing** (LSI) model (Deerwester et al., 1988) recast as **LSA (latent semantic analysis)** (Deerwester et al., 1990). In LSA SVD is applied to a term-document matrix (each cell weighted by log frequency and normalized by entropy), and then using the first 300 dimensions as the embedding. LSA was then quickly widely applied: as a cognitive model Landauer and Dumais (1997), and tasks like spell checking (Jones and Martin, 1997), language modeling (Bellegarda 1997, Coccoaro and Jurafsky 1998, Bellegarda 2000) morphology induction (Schone and Jurafsky 2000, Schone and Jurafsky 2001), and essay grading (Rehder et al., 1998). Related models were simultaneously developed and applied to word sense disambiguation by Schütze (1992). LSA also led to the earliest use of embeddings to represent words in a probabilistic classifier, in the logistic regression document router of Schütze et al. (1995). The idea of SVD on the term-term matrix (rather than the term-document matrix) as a model of meaning for NLP was proposed soon after LSA by Schütze (1992). Schütze applied the low-rank (97-dimensional) embeddings produced by SVD to the task of word sense disambiguation, analyzed the resulting semantic



space, and also suggested possible techniques like dropping high-order dimensions. See [Schütze \(1997\)](#).

A number of alternative matrix models followed on from the early SVD work, including Probabilistic Latent Semantic Indexing (PLSI) ([Hofmann, 1999](#)) Latent Dirichlet Allocation (LDA) ([Blei et al., 2003](#)). Nonnegative Matrix Factorization (NMF) ([Lee and Seung, 1999](#)).

By the next decade, [Bengio et al. \(2003\)](#) and [Bengio et al. \(2006\)](#) showed that neural language models could also be used to develop embeddings as part of the task of word prediction. [Collobert and Weston \(2007\)](#), [Collobert and Weston \(2008\)](#), and [Collobert et al. \(2011\)](#) then demonstrated that embeddings could play a role for representing word meanings for a number of NLP tasks. [Turian et al. \(2010\)](#) compared the value of different kinds of embeddings for different NLP tasks. [Mikolov et al. \(2011\)](#) showed that recurrent neural nets could be used as language models. The idea of simplifying the hidden layer of these neural net language models to create the skip-gram and CBOW algorithms was proposed by [Mikolov et al. \(2013\)](#). The negative sampling training algorithm was proposed in [Mikolov et al. \(2013a\)](#).

Studies of embeddings include results showing an elegant mathematical relationship between sparse and dense embeddings ([Levy and Goldberg, 2014c](#)), as well as numerous surveys of embeddings and their parameterizations. ([Bullinaria and Levy 2007](#), [Bullinaria and Levy 2012](#), [Lapesa and Evert 2014](#), [Kiela and Clark 2014](#), [Levy et al. 2015](#)).

There are many other embedding algorithms, using methods like non-negative matrix factorization ([Fyshe et al., 2015](#)), or by converting sparse PPMI embeddings to dense vectors by using SVD ([Levy and Goldberg, 2014c](#)). The most widely-used embedding model besides word2vec is GloVe ([Pennington et al., 2014](#)). The name stands for Global Vectors, because the model is based on capturing global corpus statistics. GloVe is based on ratios of probabilities from the word-word co-occurrence matrix, combining the intuitions of count-based models like PPMI while also capturing the linear structures used by methods like word2vec.

fasttext

An extension of word2vec, **fasttext** ([Bojanowski et al., 2017](#)), deals with unknown words and sparsity in languages with rich morphology, by using subword models. Each word in fasttext is represented as itself plus a bag of constituent n-grams, with special boundary symbols < and > added to each word. For example, with  $n = 3$  the word *where* would be represented by the character n-grams:

<wh, whe, her, ere, re>

plus the sequence

<where>

Then a skipgram embedding is learned for each constituent n-gram, and the word *where* is represented by the sum of all of the embeddings of its constituent n-grams. A fasttext open-source library, including pretrained embeddings for 157 languages, is available at <https://fasttext.cc>.

See [Manning et al. \(2008\)](#) for a deeper understanding of the role of vectors in information retrieval, including how to compare queries with documents, more details on tf-idf, and issues of scaling to very large datasets.

[Cruse \(2004\)](#) is a useful introductory linguistic text on lexical semantics.



## Exercises

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