The Bondareva-Shapley Theorem

Felix Fritz

May 11, 2018

Contents

1 Introduction

Before diving into the concept of balanced collections, balanced games and the Bondareva-Shapley Theorem, I want to establish a basic understanding of cooperative games, imputations and the core, mainly as it is described in Robert P. Gilles book [1], pp. 12–13, 18–20 and 29–35.

1.1 Cooperative Games and the Core

First we introduce a formal definition of the core.

Definition 1.1. The pair (N, v) is a *cooperative game* if N is a finite player set and $v : 2^N \to \mathbb{R}$ is a characteristic function that assigns to every coalition $S \subset N$ an attainable payoff $v(S)$ such that $v(\emptyset) = 0$.

2 Balanced Games and the Bondareva-Shapley Theorem

In the following chapter we will discuss the properties of cooperative games with a non-empty core and introduce the notion of balancedness, which leads us to the Bondareva-Shapley Theorem.

3 Market Games with nonempty cores

Market games can be proven to have a non-empty core using the Bondareva-Shapley Theorem. We will take a closer look on how this can be achieved in the third chapter.

4 Proving the Theorem using Linear Programming

Finally, we will show that the Bondareva-Theorem holds true with a prove using the Duality Problem in Linear Programming.

References

[1] Robert P. Gilles. *The Cooperative Game Theory of Networks and Hierarchies*, volume 44 of *Series C: Game Theory, Mathematical Programming and Operations Research*. Springer, 2010.