Problem #1

Problem 1.

You are playing a version of the roulette game, where the pockets are from 0 to 14 and even numbers are red and odd numbers are black (0 is green). You spin 3 times and add up the values you see. What is the probability that you get a total of 17 given on the first spin you spin a 4? What about a 5?
Solve by simulation and analytically.

red = {2, 4, 6, 8, 10, 12, 14}
black = {1, 3, 5, 7, 9, 11, 13}
green = {0}

↳ color doesn't matter?

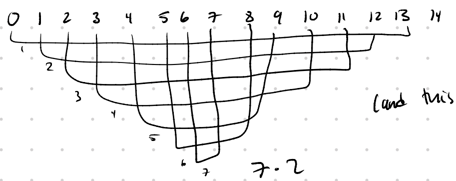
a) 1st spin is 4

→ 17 total − 4 = 13 left

→ P(three spins sum is 17 | first spin is 4) = P(last two spins sum to 13)

→ By multiplication rule the total # possible outcomes for last two spins
= (# possible outcomes in second spin)·(# possible outcomes in third spin)
= (15) · (15)
= 225

→ # times last two spins can sum to 13 = 7



(and this flipped)

7 · 2

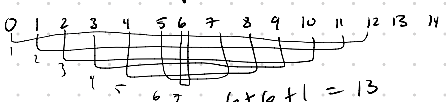
↳ ∴ P(three spins sum to 17 | first spin is 4) = $\boxed{\frac{14}{225}}$

b) first spin is 5

→ 17 total − 5 = 12 left

→ P(three spins sum to 17 | first spin 5) = P(last two spins sum 12)

→ # times last two spins can sum to 12 = 7



6 + 6 + 1 = 13

↳ P(three spins sum 17 | first spin 5) = $\boxed{\frac{13}{225}}$

Problem 2

\# messages left to send in Epoch 1 ≠ # Epoch 2

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Problem 2.(Matloff)
Consider the ALOHA example. Suppose it is known that $X_1 \neq X_2$. Find the probability that there were 0, 1 or 2 collisions during those two epochs analytically and confirm via R simulation.

Epoch 1: Send or Not Send
↳ Epoch 2: follows Epoch 2

Epoch 1 $x_{total} = 2$

	node 1	node 2	x_1	c	probability
Case 1 A	S	S	$\boxed{2}$	$\boxed{1}$	p·p P(A)
Case 2 B	S	N	1	0	p(1-p)
Case 3 C	N	S	1	0	(1-p)(p)
Case 4 D	N	N	2	0	(1-p)(1-p)

p: probality to send
1-p: probability to not send
Node still active if still has messages to send
NA: Not Active
AS: Activating Sending

q: probability of activating
(1-q): not ↑
AN: Activating Not Send

Epoch 2A: (A occurred in Epoch 1)

$x_1 = 2$ − +

	node 1	node 2	$\boxed{x_2}$	$\boxed{c}$	probability
A	S	S	2	2	p·p P(C=2|A)
B	S	N	1	1	p(1-p)
C	N	S	1	1	(1-p)(p)
D	N	N	2	1	(1-p)(1-p)

Epoch 2B: B occurred in Epoch 1

P(C=1 AND Case 2)

	node 1	node 2	x_2	c	probability
A	NA	S	0	0	(1-q)(p)
B	NA	N	1	0	(1-q)(1-p)
C	AS	S	2	1	(q)(p)(p)
D	AS	N	1	0	qp(1-p)
E	AN	S	1	0	(q)(1-p)(p)
F	AN	N	2	0	(q)(1-p)(1-p)

Epoch 2C: (C occurred in Epoch 1)

	node 1	node 2	x_2	c	probability
A	S	NA	0	0	(p)(1-q)
B	N	NA	1	0	(1-p)(1-q)
C	S	AS	2	1	(p)(q)(p)
D	N	AS	1	0	(1-p)qp
E	S	AN	1	0	(1-p)(p)(q)
F	N	AN	2	0	(1-p)(1-p)(q)

Epoch 2D: D occurred in Epoch 1

	node 1	node 2	x_2	c	probability
A	S	S	2	1	p·p
B	S	N	1	0	p(1-p)
C	N	S	1	0	(1-p)(p)
D	N	N	2	0	(1-p)(1-p)

C=2

Given: $p = .4 \quad q = .8$

$1-p = .6 \quad 1-q = .2$

$P(x_1 \neq x_2) = P(x_1 \neq x_2 \text{ AND } 2A) = P(x_1 \neq x_2 \mid A) \cdot P(A) = [p(1-p) + (1-p)(p)] \cdot p^2 = [2p(1-p)]p^2 = (2)(.4)(.6)(.4)^2 = .0768$

$+$

$P(x_1 \neq x_2 \text{ AND } 2B) = P(x_1 \neq x_2 \mid B) \cdot P(B) = [(1-q)(p) + (qp^2) + (q)(1-p)^2] \cdot p(1-p) = (.8)(.4) + (.8)(.4)^2 + (.8)(.6)^2 = .736$

$+$

$P(x_1 \neq x_2 \text{ AND } 2C) = P(x_1 \neq x_2 \mid C) \cdot P(C) = [(p)(1-q) + p^2q + (1-p)^2 q] \cdot (1-p)(p) = .736$

$+$

$P(x_1 \neq x_2 \text{ AND } 2D) = P(x_1 \neq x_2 \mid D) \cdot P(D) = [p(1-p) + p(1-p)] \cdot (1-p)^2 = [(.4)(.6) + (.4)(.6)] \cdot (.6)^2 = .1728$

$= {}^\wedge$ sum $= .9856$

$P(C=2 \mid x_1 \neq x_2) = \dfrac{P(C=2 \text{ AND } x_1 \neq x_2)}{P(x_1 \neq x_2)} = \dfrac{0}{P(x_1 \neq x_2)} = \boxed{0}$ because no case where $x_1 \neq x_2$ AND $C=2$

$P(C=1 \mid x_1 \neq x_2) = \dfrac{P(C=1 \text{ AND } x_1 \neq x_2)}{P(x_1 \neq x_2)} = \dfrac{P(C=1 \mid 2A) \cdot P(A) + P(C=1 \mid 2B) \cdot P(B) + P(C=1 \mid 2C) \cdot P(C)}{P(x_1 \neq x_2)} = \dfrac{\overset{[(.4)(.6)(.4)^2]^2 + (.8)(.4)^2(.4)(.6) + (.8)(.4)^3(.6)}{[p(1-p) \cdot p^2]^2 + (qp^2)(p)(1-p) + (qp^2)(1-p)p}}{.9856}$

$= \boxed{0.0034}$

$P(C=0 \mid x_1 \neq x_2) = \dfrac{P(C=0 \text{ AND } x_1 \neq x_2)}{P(x_1 \neq x_2)} = \dfrac{P(C=0 \mid 2B) \cdot P(B) + P(C=0 \mid 2C) \cdot P(C) + P(D=0 \mid 2D) P(D)}{P(x_1 \neq x_2)} =$

$= \dfrac{\overset{(.2)(.4)(.8)(.6)^2 + (.4)(.2)(.6)^2(.8) + (.4)^2(.6)^2}{(1-q)(p)(q)(1-p)^2 + (p)(1-q)(1-p)^2(q) + p^2(1-p)^2}}{.9856}$

$= \boxed{.1052}$