Problem 3

Problem 3.

Suppose we deal a 5-card hand from a regular 52-card deck. Which is larger, P(exactly 2 access) or P(exactly 4 diamonds)?

Before continuing, take a moment to guess which one is more likely

Please solve both analytically and use R to simulate and provide probabilities.

My guess: 2 aces :)

P(exactly 2 aces)

* Choosing R items out of n where order doesn't matter → Combinations
* Each card can only be drawn once

52 cards total

5 cards dealt

A A x x x
1 2 3 4 5

$$nCr = \frac{n!}{(n-r)!\,r!}$$

→ Exactly 4 aces in a deck of 52 cards
↳ Only want 2 of them

$$\therefore\ 4C2 = \frac{4!}{2!\,2!} = \frac{24}{4} = 6$$

→ Excluding aces, there are 48 cards total
↳ Only want 3 of them

$$\therefore\ 48C3 = \frac{48!}{45!\,3!} = 17296$$

→ Total combination of 5 hand draw

$$52C5 = \frac{52}{47!\,5!} = 2598960$$

$$P(\text{exactly two aces}) = \frac{(2\text{ Aces})\text{ AND }(3\text{ Non Aces})}{(\text{total \# outcomes})} = \frac{(4C2)\cdot(48C3)}{(52C5)}$$

$$\boxed{= .0399 \approx .04}$$

P(exactly 4 diamonds)

* Choosing R items out of n where order doesn't matter → Combinations
* Each card can only be drawn once

52 cards total

5 cards dealt

D D D x x
1 2 3 4 5

$$nCr = \frac{n!}{(n-r)!\,r!}$$

→ Exactly 13 diamonds in a deck of 52 cards
↳ Only want 4 of them

$$\therefore\ 13C4 = \frac{13!}{9!\,4!} = 715$$

→ Excluding aces, there are 39 cards total
↳ Only want 1 of them

$$\therefore\ 39C1 = \frac{39!}{38!\,1!} = 39$$

→ Total combination of 5 hand draw

$$52C5 = \frac{52}{47!\,5!} = 2598960$$

$$P(\text{exactly four diamonds}) = \frac{(\text{four diamonds})\text{ AND }(\text{one non diamond})}{(\text{total \# outcomes})} = \frac{(13C4)(39C1)}{(52C5)}$$

$$\boxed{= .01}$$

Problem 4

Problem 4.

You are running a camp of 30 students, including John and Jane.

3a.) What is the total possible ways you can arrange 2 focus groups of students one group being size 5, and the other size 6.

3b.) What is the probability that John and Jane are not in the same group (so either not chosen or are chosen but not in the same group).

a) 2 focus groups → Group 1: Pick 5 from all 30 students (no repeating)

→ Group 2: Pick 6 from the remaining 25 students (also no repeats)

$\therefore (30C5) \cdot (25C6)$

↳ mult because it's AND

$$\boxed{= \left(\frac{30!}{25!\,5!}\right) \cdot \left(\frac{25!}{19!\,6!}\right) = (142506)(177100) = 252\,378\,126\,00}$$

b) John and Jane not in same group

Possibilities: Both in different groups, one in group and one not, the other in group and the other not

P(John and Jane not in same group) = 1 − P(both in same group)

P(both in same group) = P(Both John & Jane in 5 person group) + P(Both John and Jane in 6 person group)

By definition

$$↳ P(\text{Both John \& Jane in Group of 5}) = \frac{\overbrace{(28C3)}^{\text{Excluding John \& Jane}} \cdot \overbrace{(25C6)}^{\text{other 6 person group}}}{\underbrace{(30C5) \cdot (25C6)}_{\text{total possibilities}}}$$

$$↳ P(\text{Both John \& Jane in Group of 6}) = \frac{\overbrace{(28C4)}^{\text{Excluding John \& Jane}} \overbrace{(24C5)}^{\text{other 5 person group}}}{\underbrace{(30C5) \cdot (25C6)}_{\text{total possibilities}}}$$

$$\boxed{P(\text{John \& Jane not in same group}) = 1 - \frac{(28C3) \cdot (25C6)}{(30C5) \cdot (25C6)} + \frac{(28C4)(24C5)}{(30C5) \cdot (25C6)}}$$

Problem 5.

You are a doctor. You have a medical test that 98% of patients with cancer have a high protein level (X=true).
Given the person is Healthy the test sees the protein X as high(X=true) in 11% of patients.
Additionally, it is known that 12% of the population is found to have cancer at the time of being screened.

A new patient with high levels of protein X wants to know how probable is it that he has cancer.

Hint (Bayes Theorem) healthy ⇒ doesn't have cancer

$$P(B|A) = \frac{P(A \text{ AND } B)}{P(A)}$$

Given statistics: $P(x=\text{True} \mid \text{has cancer}) = 98\%$

$P(x=\text{True} \mid \text{doesn't have cancer}) = 11\%$

$P(\text{has cancer}) = 12\%$

$P(\text{doesn't have cancer}) = 88\%$

Solve: $P(\text{has cancer} \mid x=\text{True}) = \frac{P(\text{has cancer AND } x=\text{True})}{P(x=\text{True})}$ By definition

↳ $P(\text{has cancer AND } x=\text{TRUE}) = P(x=\text{True} \mid \text{has cancer}) \cdot P(\text{has cancer})$ → By defn

$= (.98)(.12)$

↳ $P(x=\text{True}) = \underline{P(x=\text{True} \mid \text{has cancer}) \cdot P(\text{has cancer})} + \underline{P(x=\text{True} \mid \text{doesn't have cancer}) \cdot P(\text{no cancer})}$

↳ Both fully disjoint (ie no intersection)

$= (.98)(.12) + (.11)(.88)$

→ $P(\text{has cancer} \mid x=\text{True}) =$

$$\boxed{\frac{(.98)(.12)}{(.98)(.12) + (.11)(.88)}}$$

$\approx 55\%$