

Navam Jascent Pearl G.

Midterm Exam

① Define a Markov Decision Process (MDP). List its key components.

→ It is an environment in which all states succeed in retaining all relevant information. It's a Markov Reward process with decisions, because MDP policies depend on the current state to fully define the behaviour of the agent.

→ Key Components: Policies, state-value function, Action-value function, Bellman Expectation Equation for MDP, Optimal state-value function, optimal action-value function, optimal value function, optimal policy.

② What does it mean for a process to satisfy the Markov property?

→ It succeeds in retaining all relevant information. It summarizes everything important about the complete sequence that led to it. It depends only on the present state, not on the past.

③ Explain the difference between a policy and a value function.

→ Policy is the actions that fully defines the behavior of the agent.

→ Value function is the expected return or the long-term value of a state.

④ What is the role of the discount factor (γ) in an MDP?

• What happens when $\gamma = 0$ and when $\gamma \rightarrow 1$?

→ The role of discount factor in an MDP is to ensure the math stays well-behaved. Without discounting, if reward keeps coming, the sum could be infinite.

→ When the $\gamma = 0$, it will return a 0 value.

→ When the $\gamma = 1$, it will return a same value.

⑤ (a) Compute the average expected reward for sunny.

$$r_{\pi} = 0.5 \times (2) + 0.5 \times (0) = 1 + 0 = 1$$

(b) compute the average expected reward for rainy.

$$r_{\pi} = 0.5 \times (1) + 0.5 \times (-3) = 0.5 - 1.5 = -1$$

(c) Using the Bellman expectation equation, solve for V_{π} (sunny).

$$V_1 = 1 + 0.5(0.5V_1 + 0.5V_2)$$

$$V_1 = 1 + 0.5V_1 + 0.5V_2$$

$$V_1 - 0.5V_1 - 0.5V_2 = 1$$

$$0.5V_1 - 0.5V_2 = 1$$

$$V_1 = 1 + 0.5V_2$$

$$V_1 = 1 + 0.5(3.33)$$

$$V_{\pi}(\text{sunny}) = 2.665$$

MDP (sunny):

$$r_{\pi}(1,1) = 0.5 \times 2.0 + 0.5 \times 0.0 =$$

$$0.0 + 0.0 = 0$$

$$r_{\pi}(1,2) = 0.5 \times 1.0 + 0.5 \times 1.0 =$$

$$0.5 + 0.5 = 1.0 = (A)$$

$$1 + 1.665 = 2.665 = (A)$$

(d) Using the Bellman expectation equation, solve for $V_{\pi}(\text{Rainy})$.

$$V_2 = 2 + 0.5(1V_1 + 0V_2)$$

$$V_2 = 2 + 0.5V_1 + 0V_2 \quad | \quad V_1 - 0.5V_2 = 1$$

~~$$V_2 = 2 + 0.5V_1 + 0V_2$$~~

~~$$V_2 = 2 + 0.5V_1 + 0V_2$$~~

$$-0.5V_1 + 1V_2 = 2$$

$$V_1 = 1 + 0.5V_2$$

$$= 0.5(1 + 0.5V_2) + 1V_2 = 2$$

$$= (0.5 + 1)V_2 + 0.5 = 2$$

$$= 0.5 + 1V_2 = 2 \quad \Rightarrow \quad 1V_2 = 1.5$$

$$= 0.5 + 1.5 = 2$$

$$V_2 = 2 - 0.5$$

$$V_{\pi}(\text{Rainy}) = \frac{2 - 0.5}{1} = 1.5$$

$$= -0.5(1 + 0.5V_2) + 1V_2 = 2$$

$$= (-0.5 \times 1) + (-0.5 \times 0.5V_2) + 1V_2 = 2$$

$$= -0.5 - 0.25V_2 + 1V_2 = 2$$

$$= -0.5 + (-0.25V_2 + 1V_2) = 2$$

$$= -0.5 + 0.75V_2 = 2$$

$$= 0.75V_2 = 2 + 0.5$$

$$V_{\pi}(\text{Rainy}) = \frac{2 + 0.5}{0.75} = \frac{2.5}{0.75} = 3.33$$

for (unny):

$$P_{\pi}(2,1) = 0.5 \times 1.0 + 0.5 \times 1.0 =$$

$$0.5 + 0.5 = 1$$

$$P_{\pi}(2,2) = 0.5 \times 0.0 + 0.5 \times 0.0 =$$

$$0.0 + 0.0 = 0$$

$V_{k+1}(s)$

A	-3
B	-3
C	-2.94
D	-3
F	-2.375
G	-2.94
A	-2.375

(b) (a) Using dynamic programming, compute the optimal state-value function $V_{\pi}(s)$ for all non-terminal states.

$$V_{k+1}(A) = \frac{1}{4} [(-1 + V(A)) + (-1 + V(B)) + (-1 + V(D)) + (-1 + V(A))]$$

$$V_{k+1}(A) = \frac{1}{4} [(-1 + 0) + (-1 + 0) + (-1 + 0) + (-1 + 0)]$$

$$V_{k+1}(A) = -1$$

$$V_{k+1}(B) = \frac{1}{4} [(-1 + V(A)) + (-1 + V(C)) + (-1 + V(B)) + (-1 + V(B))]$$

$$= \frac{1}{4} [(-1 + 0) + (-1 + 0) + (-1 + 0) + (-1 + 0)]$$

$$V_{k+1}(B) = -1$$

	$V_k(s)$	$V_{k+1}(s)$	$V_{k+2}(s)$
A	0	-1	-2
B	0	-1	-2
C	0	-1	-2
D	0	-1	-2
F	0	-1	-1.75
G	0	-1	-2
A	0	-1	-1.75

$$v_{k+1}(c) = \frac{1}{4} [(-1+v(b)) + (-1+v(c)) + (-1+v(f)) + (-1+v(c))] = -1$$

7(c)

$$= \frac{1}{4} [(-1+0) + (-1+0) + (-1+0) + (-1+0)] = -1$$

9(c)

$$\boxed{v_{k+1}(c) = -1}$$

$$v_{k+1}(d) = \frac{1}{4} [(-1+v(d)) + (-1+v(d)) + (-1+v(g)) + (-1+v(a))] = -1$$

$$= \frac{1}{4} [(-1+0) + (-1+0) + (-1+0) + (-1+0)] = -1$$

$$\boxed{v_{k+1}(d) = -1}$$

10(a)

$$v_{k+1}(f) = \frac{1}{4} [(-1+v(f)) + (-1+v(f)) + (-1+v(i)) + (-1+v(c))] = -1$$

$$= \frac{1}{4} [(-1+0) + (-1+0) + (-1+0) + (-1+0)] = -1$$

$$\boxed{v_{k+1}(f) = -1}$$

$$v_{k+1}(g) = \frac{1}{4} [(-1+v(g)) + (-1+v(h)) + (-1+v(g)) + (-1+v(d))] = -1$$

$$= \frac{1}{4} [(-1+0) + (-1+0) + (-1+0) + (-1+0)] = -1$$

$$\boxed{v_{k+1}(g) = -1}$$

$$v_{k+1}(h) = \frac{1}{4} [(-1+v(h)) + (-1+v(i)) + (-1+v(h)) + (-1+v(h))] = -1$$

$$= \frac{1}{4} [(-1+0) + (-1+0) + (-1+0) + (-1+0)] = -1$$

$$\boxed{v_{k+1}(h) = -1}$$

$$q(A, \text{left}) = -1 + v(A)$$

$$= -1 + (-1)$$

$$\underline{q(A, \text{left}) = -2}$$

$$q(A, \text{up}) = -1 + v(B)$$

$$= -1 + (-1)$$

$$\underline{q(A, \text{up}) = -2}$$

$$q(A, \text{up}) = -1 + v(A)$$

$$= -1 + (-1)$$

$$\underline{q(A, \text{up}) = -2}$$

$$q(A, \text{down}) = -1 + v(b)$$

$$= -1 + (-1)$$

$$\underline{q(A, \text{down}) = -2}$$

$$\pi(A) = \begin{matrix} \updownarrow \\ \leftarrow \rightarrow \end{matrix}$$

$$q(B, \text{LEFT}) = -1 + v(A) \\ = -1 + (-1)$$

$$q(B, \text{RIGHT}) = -1 + v(C) \\ = -1 + (-1)$$

$$q(B, \text{DOWN}) = -1 + v(B) \\ = -1 + (-1)$$

$$q(B, \text{LEFT}) = -2$$

$$q(B, \text{RIGHT}) = -2$$

$$q(B, \text{DOWN}) = -2$$

$$q(B, \text{UP}) = -1 + v(B) \\ = -1 + (-1)$$

$$\pi(B) = \begin{array}{c} \updownarrow \\ \leftarrow \rightarrow \end{array}$$

$$q(B, \text{UP}) = -2$$

$$q(C, \text{LEFT}) = -1 + v(B) \\ = -1 + (-1)$$

$$q(C, \text{RIGHT}) = -1 + v(D) \\ = -1 + (-1)$$

$$q(C, \text{DOWN}) = -1 + v(E) \\ = -1 + (-1)$$

$$q(C, \text{LEFT}) = -2$$

$$q(C, \text{RIGHT}) = -2$$

$$q(C, \text{DOWN}) = -2$$

$$q(C, \text{UP}) = -1 + v(C) \\ = -1 + (-1)$$

$$\pi(C) = \begin{array}{c} \updownarrow \\ \leftarrow \rightarrow \end{array}$$

$$q(C, \text{UP}) = -2$$

$$q(D, \text{LEFT}) = -1 + v(C) \\ = -1 + (-1)$$

$$q(D, \text{RIGHT}) = -1 + v(E) \\ = -1 + (-1)$$

$$q(D, \text{DOWN}) = -1 + v(F) \\ = -1 + (-1)$$

$$q(D, \text{LEFT}) = -2$$

$$q(D, \text{RIGHT}) = -2$$

$$q(D, \text{DOWN}) = -2$$

$$q(D, \text{UP}) = -1 + v(D) \\ = -1 + (-1)$$

$$\pi(D) = \begin{array}{c} \updownarrow \\ \leftarrow \rightarrow \end{array}$$

$$q(D, \text{UP}) = -2$$

$$q(E, \text{LEFT}) = -1 + v(D) \\ = -1 + (-1)$$

$$q(E, \text{RIGHT}) = -1 + v(F) \\ = -1 + (-1)$$

$$q(E, \text{DOWN}) = -1 + v(G) \\ = -1 + (-1)$$

$$q(E, \text{LEFT}) = -2$$

$$q(E, \text{RIGHT}) = -2$$

$$q(E, \text{DOWN}) = -2$$

$$q(E, \text{UP}) = -1 + v(E) \\ = -1 + (-1)$$

$$\pi(E) = \begin{array}{c} \updownarrow \\ \leftarrow \rightarrow \end{array}$$

$$q(E, \text{UP}) = -2$$

$$q(F, \text{LEFT}) = -1 + v(E) \\ = -1 + (-1)$$

$$q(F, \text{RIGHT}) = -1 + v(G) \\ = -1 + (-1)$$

$$q(F, \text{DOWN}) = -1 + v(H) \\ = -1 + (-1)$$

$$q(F, \text{LEFT}) = -2$$

$$q(F, \text{RIGHT}) = -2$$

$$q(F, \text{DOWN}) = -2$$

$$q(F, \text{UP}) = -1 + v(F) \\ = -1 + (-1)$$

$$\pi(F) = \begin{array}{c} \updownarrow \\ \leftarrow \rightarrow \end{array}$$

$$q(A, \text{LEFT}) = -1 + v(G)$$

$$= -1 + (-1)$$

$$q(A, \text{RIGHT}) = -1 + v(I)$$

$$= -1 + (0)$$

$$q(A, \text{DOWN}) = -1 + v(B)$$

$$= -1 + (-1)$$

$$q(A, \text{LEFT}) = -2$$

$$q(A, \text{RIGHT}) = -1$$

$$q(A, \text{DOWN}) = -2$$

$$q(H, \text{UP}) = -1 + v(H)$$

$$= -1 + (-1)$$

$$q(H) = \rightarrow$$

$$q(H, \text{UP}) = -2$$

$$v_{k+2}(A) = \frac{1}{4} [(-1 + v_{k+1}(A)) + (-1 + v_{k+1}(B)) + (-1 + v_{k+1}(D)) + (-1 + v_{k+1}(A))]$$

$$= \frac{1}{4} [(-1-1) + (-1-1) + (-1-1) + (-1-1)]$$

$$(a) \quad v_{k+2}(A) = -2$$

$$v_{k+2}(B) = \frac{1}{4} [(-1 + v_{k+1}(A)) + (-1 + v_{k+1}(C)) + (-1 + v_{k+1}(B)) + (-1 + v_{k+1}(B))]$$

$$= \frac{1}{4} [(-1-1) + (-1-1) + (-1-1) + (-1-1)]$$

$$v_{k+2}(B) = -2$$

$$v_{k+2}(C) = \frac{1}{4} [(-1 + v_{k+1}(B)) + (-1 + v_{k+1}(C)) + (-1 + v_{k+1}(F)) + (-1 + v_{k+1}(C))]$$

$$= \frac{1}{4} [(-1-1) + (-1-1) + (-1-1) + (-1-1)]$$

$$v_{k+2}(C) = -2$$

$$v_{k+2}(D) = \frac{1}{4} [(-1 + v_{k+1}(D)) + (-1 + v_{k+1}(D)) + (-1 + v_{k+1}(G)) + (-1 + v_{k+1}(A))]$$

$$= \frac{1}{4} [(-1-1) + (-1-1) + (-1-1) + (-1-1)]$$

$$v_{k+2}(D) = -2$$

$$v_{k+2}(F) = \frac{1}{4} [(-1 + v_{k+1}(F)) + (-1 + v_{k+1}(F)) + (-1 + v_{k+1}(I)) + (-1 + v_{k+1}(C))]$$

$$= \frac{1}{4} [(-1-1) + (-1-1) + (-1+0) + (-1-1)]$$

$$v_{k+2}(F) = -1.75$$

$$v_{k+2}(G) = \text{avg} \frac{1}{4} [(-1+v_k(H)) + (-1+v_k(I)) + (-1+v_k(J)) + (-1+v_k(K))] + 1 = (1+1)/2$$

$$= \frac{1}{4} [(-1-1) + (-1-1) + (-1-1) + (-1-1)]$$

$$v_{k+2}(G) = -2$$

$$v_{k+2}(H) = \frac{1}{4} [(-1+v_k(G)) + (-1+v_k(I)) + (-1+v_k(J)) + (-1+v_k(K))] + 1$$

$$= \frac{1}{4} [(-1-1) + (-1+0) + (-1-1) + (-1-1)]$$

$$v_{k+2}(H) = -1.75$$

$$q(A, \text{LEFT}) = -1 + v(L)$$

$$= -1 + (-2)$$

$$q(A, \text{RIGHT}) = -1 + v(B)$$

$$= -1 + (-2)$$

$$q(A, \text{DOWN}) = -1 + v(D)$$

$$= -1 + (-2)$$

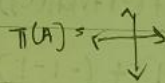
$$q(A, \text{LEFT}) = -3$$

$$q(A, \text{RIGHT}) = -3$$

$$q(A, \text{DOWN}) = -3$$

$$q(A, \text{UP}) = -1 + v(A)$$

$$= -1 + (-2)$$



$$q(A, \text{UP}) = -3$$

$$q(B, \text{LEFT}) = -1 + v(A)$$

$$= -1 + (-2)$$

$$q(B, \text{RIGHT}) = -1 + v(C)$$

$$= -1 + (-2)$$

$$q(B, \text{DOWN}) = -1 + v(B)$$

$$= -1 + (-2)$$

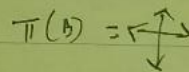
$$q(B, \text{LEFT}) = -3$$

$$q(B, \text{RIGHT}) = -3$$

$$q(B, \text{DOWN}) = -3$$

$$q(B, \text{UP}) = -1 + v(B)$$

$$= -1 + (-2)$$



$$q(B, \text{UP}) = -3$$

$$q(C, \text{LEFT}) = -1 + v(B)$$

$$= -1 + (-2)$$

$$q(C, \text{RIGHT}) = -1 + v(D)$$

$$= -1 + (-2)$$

$$q(C, \text{DOWN}) = -1 + v(E)$$

$$= -1 + (-1.75)$$

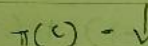
$$q(C, \text{LEFT}) = -3$$

$$q(C, \text{RIGHT}) = -3$$

$$q(C, \text{DOWN}) = -2.75$$

$$q(C, \text{UP}) = -1 + v(C)$$

$$= -1 + (-2)$$



$$q(C, \text{UP}) = -3$$

09. 24. 2029

Essay

$$q(D, LEFT) = -1 + v(D) \\ = -1 + (-2)$$

$$q(D, LEFT) = -3$$

$$q(D, RIGHT) = -1 + v(D) \\ = -1 + (-2)$$

$$q(D, RIGHT) = -3$$

$$q(D, DOWN) = -1 + v(D) \\ = -1 + (-2)$$

$$q(D, DOWN) = -3$$

$$q(D)$$

$$q(D, UP) = -1 + v(A)$$

$$= -1 + (-2)$$

$$q(D, UP) = -3$$

$$\pi(D) = \begin{matrix} \swarrow & \searrow \\ \uparrow & \downarrow \end{matrix}$$

$$q(D)$$

$$q(G, LEFT) = -1 + v(G) \\ = -1 + (-2)$$

$$q(G, LEFT) = -3$$

$$q(G, RIGHT) = -1 + v(A)$$

$$= -1 + (-1.75)$$

$$q(G, RIGHT) = -2.75$$

$$q(G, DOWN) = -1 + v(G)$$

$$= -1 + (-2)$$

$$q(G, DOWN) = -3$$

$$q(G, UP) = -1 + v(D)$$

$$= -1 + (-2)$$

$$q(G, UP) = -3$$

$$\pi(G) = \rightarrow$$

(a)

$$vk_3(A) = \frac{1}{4} [(-1 + vk_2(A)) + (-1 + vk_2(B)) + (-1 + vk_2(D)) + (-1 + vk_2(A))] \\ = \frac{1}{4} [(-1 - 2) + (-1 - 2) + (-1 - 2) + (-1 - 2)]$$

$$vk_3(A) = -3$$

$$vk_3(B) = \frac{1}{4} [(-1 + vk_2(A)) + (-1 + vk_2(C)) + (-1 + vk_2(D)) + (-1 + vk_2(B))] \\ = \frac{1}{4} [(-1 - 2) + (-1 - 2) + (-1 - 2) + (-1 - 2)]$$

$$(b) \quad vk_3(B) = -3$$

$$vk_3(C) = \frac{1}{4} [(-1 + vk_2(B)) + (-1 + vk_2(C)) + (-1 + vk_2(D)) + (-1 + vk_2(C))] \\ = \frac{1}{4} [(-1 - 2) + (-1 - 2) + (-1 - 1.75) + (-1 - 2)]$$

$$vk_3(C) = -2.94$$

$$vk_3(D) = \frac{1}{4} [(-1 + vk_2(D)) + (-1 + vk_2(D)) + (-1 + vk_2(G)) + (-1 + vk_2(A))] \\ = \frac{1}{4} [(-1 - 2) + (-1 - 2) + (-1 - 2) + (-1 - 2)]$$

$$vk_3(D) = -3$$

$$v_{kt3}(F) = \frac{1}{4} [(-1 + v_{kt2}(F)) + (-1 + v_{kt2}(F)) + (-1 + v_{kt2}(1)) + (-1 + v_{kt2}(0))] = \frac{1}{4} [(-1 - 1.75) + (-1 - 1.75) + (-1 + 0) + (-1 - 2)]$$

$$v_{kt3}(F) = -2.375$$

$$v_{kt3}(G) = \frac{1}{4} [(-1 + v_{kt2}(G)) + (-1 + v_{kt2}(H)) + (-1 + v_{kt2}(G)) + (-1 + v_{kt2}(D))] = \frac{1}{4} [(-1 - 2) + (-1 - 1.75) + (-1 - 2) + (-1 - 2)]$$

$$v_{kt3}(G) = -2.94$$

$$v_{kt3}(H) = \frac{1}{4} [(-1 + v_{kt2}(G)) + (-1 + v_{kt2}(1)) + (-1 + v_{kt2}(A)) + (-1 + v_{kt2}(4))] = \frac{1}{4} [(-1 - 2) + (-1 + 0) + (-1 - 1.75) + (-1 - 1.75)]$$

$$v_{kt3}(H) = -2.375$$

$$q(A, \text{LEFT}) = -1 + v(A) = -1 + (-3) = -4$$

$$q(A, \text{RIGHT}) = -1 + v(B) = -1 + (-3) = -4$$

$$q(A, \text{DOWN}) = -1 + v(D) = -1 + (-3) = -4$$

$$q(A, \text{LEFT}) = -4$$

$$q(A, \text{RIGHT}) = -4$$

$$q(A, \text{DOWN}) = -4$$

$$q(A, \text{UP}) = -1 + v(A) = -1 + (-3) = -4$$

$$q(A, \text{UP}) = -4$$

$$\pi(A) = \begin{matrix} \uparrow \\ \rightarrow \\ \downarrow \end{matrix}$$

$$q(B, \text{LEFT}) = -1 + v(A) = -1 + (-3) = -4$$

$$q(B, \text{RIGHT}) = -1 + v(C) = -1 + (-2.94) = -3.94$$

$$q(B, \text{DOWN}) = -1 + v(D) = -1 + (-3) = -4$$

$$q(B, \text{LEFT}) = -4$$

$$q(B, \text{RIGHT}) = -3.94$$

$$q(B, \text{DOWN}) = -4$$

$$q(B, \text{UP}) = -1 + v(B) = -1 + (-3) = -4$$

$$q(B, \text{UP}) = -4$$

$$\pi(B) = \rightarrow$$

$$q(B, \text{UP}) = -4$$

$$q(0)^{\text{LEFT}} = -1 + v(0) \\ = -1 + (-3)$$

$$\underline{q(0)^{\text{LEFT}} = -4}$$

$$q(0, \text{RIGHT}) = -1 + v(0) \\ = -1 + (-3)$$

$$\underline{q(0, \text{RIGHT}) = -4}$$

$$q(0, \text{UP}) = -1 + v(A) \\ = -1 + (-3)$$

$$\underline{q(0, \text{UP}) = -4}$$

$$q(0, \text{DOWN}) = -1 + v(B) \\ = -1 + (-2.94)$$

$$\underline{q(0, \text{DOWN}) = -3.94}$$

$$\pi(0) = \downarrow$$

(a)	$v_k(s)$	$v_{k+1}(s)$	$v_{k+2}(s)$	$v_{k+3}(s)$
A	0	-1	-2	-3
B	0	-1	-2	-3
C	0	-1	-2	-2.94
D	0	-1	-2	-3
F	0	-1	-1.75	-2.375
G	0	-1	-2	-2.94
H	0	-1	-1.75	-2.375

(b)

$$\pi(A) = \begin{array}{c} \updownarrow \\ \leftarrow \rightarrow \end{array}$$

$$\pi(B) = \downarrow$$

$$\pi(H) = \rightarrow$$

$$\pi(D) = \rightarrow$$

$$\pi(F) = \downarrow$$

$$\pi(C) = \downarrow$$

$$\pi(G) = \rightarrow$$