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I

(a)

$$f = 1$$

$$x_L - x_R = 8 - 1 = 7$$

$$d = (25 - (-25)) = 50$$

With respect to left camera.

$$X = \frac{d f x_L}{x_L - x_R} = \frac{50 \cdot 1 \cdot 8}{7}$$

$$Y = \frac{d f y_L}{x_L - x_R} = \frac{50 \cdot 1 \cdot 8}{7}$$

$$Z = \frac{df}{x_L - x_R} = \frac{50 \cdot 1}{7}$$

Actual points.

$$X = \frac{50 \cdot 1 \cdot 8}{7} - 25 = \frac{225}{7}$$

$$Y = \frac{400}{7} \; ; \; Z = \frac{50}{7}.$$

(b)

$X + Z = 0$ —— ⓪

$X(x_L - x_R) = d f x_L$ —— ①

$Z(x_L - x_R) = d f$ —— ②

Disparity, $d = x_L - x_R$

$\Rightarrow x_L = d + x_R$

Given $Z > 1$

Dividing ① & ② and using ⓪

$\Rightarrow -1 = x_L$

$\Rightarrow -1 = d + x_R$

$\Rightarrow -1 - x_R = d$

Replacing x_R with 'u'.

$\Rightarrow \boxed{d = -1 - u}$

2

(a)

$R = I \quad , \quad t = [t_x, 0, 0]^T$

Let camera matrix, $M = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$
with focal length f

Fundamental Matrix, $F = M^{-T} E M'^{-1}$ ($M = M'$ in our case)

$$\Rightarrow \quad F = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} [t_\times] R \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{pmatrix} \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x/f \\ 0 & t_x/f & 0 \end{pmatrix}$$

If $P_A = (x_A \ y_A \ 1)^T$ & $P_B = (x_B \ y_B \ 1)^T$

Then

$$P_A^T F P_B = 0$$

Thus,

Equation of line in Image 2 corresponding to point A in Image 1 is

Solving the equation

$$\boxed{Y_A = Y_B.}$$

$$\cancel{l_B : \; P_A \cdot F}$$

(b) Similar to part (a) only $[t_\times]R$ changes.

$$F = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{pmatrix} \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & \frac{t_y}{f} \\ 0 & 0 & -t_x/f \\ -t_y & t_x & 0 \end{pmatrix} \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & t_y/f \\ 0 & 0 & -t_x/f \\ -t_y/f & \frac{t_x}{f} & 0 \end{pmatrix}$$

$$P_A^T F P_B = 0$$

$$\Rightarrow (x_A \;\; y_A \;\; 1) \begin{pmatrix} t_y/f \\ -\frac{t_x}{f} \\ -\frac{t_y}{f} x_B + \frac{t_x}{f} y_B \end{pmatrix} = 0$$

$$\Rightarrow x_A \frac{t_y}{f} - y_A \frac{t_x}{f} = \frac{t_y}{f} x_B - \frac{t_x}{f} y_B.$$

$$\Rightarrow y_B = \frac{t_y}{t_x} x_B + \left(y_A - \frac{t_y}{t_x} \right) x_A$$

Thus line have constant slope & intercept for fixed Y_A, t_y & t_x.

(c)

$$t = (0, 0, t_z)^T, \quad R = I$$

$$F = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow F = \begin{pmatrix} 0 & \frac{-t_z}{f^2} & 0 \\ \frac{t_z}{f^2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$P_A^T F P_B = 0$$

$$\Rightarrow x_A y_B t_z = y_A x_B t_z$$

$$\Rightarrow \boxed{y_B = \frac{y_A}{x_A} x_B}$$

(d) $T = [0, 0, 0)$

R = Random.

$E = [t_\times] R = [0]$ matrix.

Thus $F = [0]$

Degenerate case, Rectification not possible

(3)

$$NCC = \sum_{ij} \hat{w}_1(i,j) \cdot \hat{w}_2(i,j)$$

$$NSSD = \sum_{ij} \left| \tilde{w}_1(i,j) - \tilde{w}_2(i,j) \right|^2$$

$$= \sum_{ij} \left(\tilde{w}_1(i,j) - \tilde{w}_2(i,j) \right)^T \left(\tilde{w}_1(i,j) - \tilde{w}_2(i,j) \right)$$

$$= \sum_{ij} \tilde{w}_1^T w_1 + \sum_{ij} \tilde{w}_2^T w_2 - 2 \sum_{ij} \tilde{w}_1(i,j) \cdot w_2^T(i,j)$$

$$= 1 + 1 - 2 \sum_{ij} \hat{w}_1(i,j)\, w_2^T(i,j)$$

$$= 2 - 2\, NCC)$$

Thus, Maximizing NCC is eventually minimizing NSSD.