Lesson Review: Module Summary

Response to and Request for feedback

If you are resubmitting, include a statement outlining the changes you have made to your submission. This section can be short but should be precise. It is a good idea to quote the feedback you are responding to.

If this is your first submission, include a statement about what part of the lesson review you would most like to receive feedback (and why). Your tutor will take this in consideration when reviewing your work, although they may choose to give you feedback on a different thing if they think it's more appropriate.

Module Learning Objectives

I certify that I achieved the following learning objectives for the module (these objectives can be found in the introduction of the module):

Summarising the content:

- Identify the key terms and concepts in the module. For each of these terms and concepts:
  - Define the term and explain the concept **in your own words** (beware plagiarism – this is an assessment task).
  - Summarise the most important results related to these concepts, including theorems and propositions, algorithms and procedures, etc.
  - You can provide examples, figures, diagrams, but only if they help illustrate your point. This is a summary, so it's best to restrict the explanations to the main points.
  - Make sure you include references to the Module Learning Objectives.

PREDICATE LOGIC:

PREDICATES- A statement that can be assigned a Truth value based upon values of the variables, within the statement, is called a predicate

They can be denoted by p(x).

P(x) is the propositional function P at x .

Example of such statement can be 1. X>13 2. X+y = 4 In any predicate p (x)

1. x is the subject

2. P is the predicate

3. Domain is the ensemble of values that the subject can take.

For example: : p (s) = my name is jasveena (true)

My name is xyz (false)

The predicate is " is jasveena"]

For example:

: Predicate: "x is divisible by 4" Domain: Integers Justification: Integers include all whole numbers, both positive and negative, and zero, which allows for the consideration of divisibility by 4 for any integer value. Truth values:

- When $x = 8$, the predicate is true because 8 is divisible by 4.
- When $x = 5$, the predicate is false because 5 is not divisible by 4.

For example:

: Predicate: "y is a multiple of 3" Domain: Positive integers Justification: Positive integers are whole numbers greater than zero, which are relevant for discussing multiples. Since divisibility by 3 is the focus, positive integers cover all numbers that could potentially be multiples of 3. Truth values:

- When $y = 6$, the predicate is true because 6 is a multiple of 3.
- When $y = 5$, the predicate is false because 5 is not a multiple of 3.

QUANTIFIERS- some compound propositions can be expressed briefly with the help of predicates when they repeat the same predicate over the whole domain.

- Every element in the domain satisfies the predicate

- To express that one or more elements in the domain satisfy the predicate.

- Another example of domain is when it is infinite we can express more than that.

UNIVERSAL QUANTIFER

The universal quantifier (∀) is used to denote that a statement is true for all elements in a domain.

For Example: Consider the statement: "For all natural numbers n, n is greater than 0." Symbolically, this statement can be represented as $\forall n \in N\ ,n>0$

In this example:

- The universal quantifier (∀) indicates that the statement applies to all natural numbers.
- The domain is the set of natural numbers N
- The predicate is "n is greater than 0."

. EXISTENTIAL QUANTIFIER

The existential quantifier (∃) is used to denote that there exists at least one element in a domain that satisfies a given condition.

For Example: Consider the statement: "There exists a prime number p such that p is greater than 10." Symbolically, this statement can be represented as $\exists p \in N, p>10$

In this example:

- The existential quantifier (∃) indicates that the statement asserts the existence of at least one prime number greater than 10.
- The domain is the set of natural numbers (N)
- The predicate is "p is greater than 10" and "p is a prime number."

NESTED QUANTIFIERS

Nested quantifiers involve using quantified predicates within other quantified predicates to construct new propositions.

Example: Consider the statement: "For all x, there exists a y such that y equals x." Symbolically, this statement can be represented as ∀x ∃y (y=x)

In this example:

- The outer quantifier is ∀ (for all), indicating that the statement applies to all elements in the domain.
- The inner quantifier is ∃ (there exists), indicating the existence of at least one element satisfying a condition.
- The predicate is "y equals x."

. PRASING NESTED QUANTIFIERS Propositions with multiple quantifiers are not fundamentally different from each other quantified propositions: these are composed of one quantified and one predicate

PROVING NESTED QUANTIFIERS

Here ∀ x is a universal quantifier:

- When true, we cannot give an example

- We need to make an argument that works for all x.

- When false, we give only one value of x that falsifies the predicate.

One proposition is true : so our argument has to hold for any x

- P (x) = ∀ y = (x>2) ^ (y>2) – (xy >4).

- Ay is a universal quantifier, so for p(x) to be true our argument has to hold for any y.

- If x>2 and y >2, xy>2y>2 . 2>4 is an algebraic arguemt and since we did not assume any value for x and y , our argument is correct.

NEGATING QUANTIFIERS

I would like to explain this with an example,

- Consider the predicate $P(x) \equiv$ "x is prime."

Consider the proposition: $\neg\forall x : P(x)$.

= The negation of $\forall x P(x)$ is $\neg\forall x P(x)$.

• "not for all x, $P(x)$"; "$P(x)$ is not true for every single x." e.g. $\neg\forall x : x > 3$ (for x a real number) is equivalent to saying "it is not true that all real numbers are greater than 3."

• An equivalent statement is: $\exists x, \neg(P(x))$. i.e. "there exists at least one real number x such that x is not greater than 3

. Number Sets

• $\mathbb{N}$ the set of all natural numbers, { 1,2,3,… }

• $\mathbb{Z}$ the set of all integers, { 0,±1,±2,… }

• $\mathbb{Q}$ is the set of all rational numbers, defined as numbers that can be expressed as fractions of integers, where a and b are integers and $b \neq 0$.

• Irrational numbers are numbers that are not rational numbers. Irrational numbers have decimal expansions that neither terminate nor become periodic. Famous irrational numbers include: √2, √3, and π.

• ℝ the set of all real numbers, which includes all of the above.

• ℂ The set of all complex numbers, written as $x + iy$, where x,y are real, and $i^2 = -1$

Reflecting on the content:

- What is the most important thing you learnt in this module?

  In this module, the most important thing I learned is the formalization of mathematical statements using predicate logic and quantifiers. Understanding how to express propositions, predicates, and quantifiers in a precise and logical manner is crucial for various fields, including mathematics, computer science, and philosophy.

- How does this relate to what you already know?

  This module relates to what I already know by providing a structured framework for expressing mathematical concepts and reasoning about them rigorously. It builds upon my existing knowledge of mathematical logic and expands my ability to analyze and prove statements using formal methods.

- Why do you think your course team wants you to learn the content of this module for your degree?

  I believe my course team wants me to learn the content of this module for my degree because it forms the foundation for advanced topics in mathematics and computer science. Predicate logic and quantifiers are fundamental tools used in various fields of study, including mathematical reasoning, proof techniques, programming languages, and artificial intelligence. Mastering these concepts enables me to think critically, solve complex problems, and communicate ideas effectively in my chosen field of study.

ADDITIONAL QUESTION:

Determine if the following proposition is true. Clearly provide working to support or a counter-example to disprove. There exists n in N for all m in N such that n is strictly greater than 2^m. Note that N represents the natural numbers.

The given proposition can be written mathematically as:

$\exists n \in N \ , \forall m \in N \ , n > 2^m$

To determine if this proposition is true, we need to either provide a valid argument to support it or provide a counter-example to disprove it.

To support the proposition: Let's choose a specific value for n and show that it satisfies the condition for all possible values of mmm. If we let n=4n = 4n=4, then for any value of mmm greater than 2, $4 > 2^m$ holds true. For example, when m=3,$4 > 2^3$ · since 4>8

To disprove the proposition: We need to find a counter-example where the proposition does not hold true. If we choose n=2, then for m=1, 2 is not strictly greater than 2^1 since $2 \leq 2^1$

Therefore, the proposition is false, as we can provide a counter-example: n=2 and m=1 , , where n is not strictly greater than 2^m