Intelligent Systems (CSE-523)
Mid Semester Examination
(Odd Semester 2024-25)

Full Marks: 60 **Time: 2 hours**

[Question paper has 2 pages. Give ***precise*** *answers to the questions. All parts of a question must be answered together. Use of calculator is permitted]*

1. **(a)** What are the major limitations of Mc-Culloch-Pitts (M-P) neuron model?
(b) Consider the following table showing the dependence of a variable Y on independent variables X1, X2, X3.

X1	0	0	0	0	1	1	1	1
X2	0	0	1	1	0	0	1	1
X3	0	1	0	1	0	1	0	1
Y	1	1	0	1	0	1	0	0

Suppose we use aim to realize the relationship of Y with X1, X2, X3 using an M-P model.
(i) Draw the structure of the required M-P model, clearly showing the number of neurons required and the inputs to each neuron.
(ii) Considering a suitable threshold function, check which inputs will be excitatory and which inputs will be inhibitory. **[2+(2+8) = 12 marks]**

2. **(a)** Consider the following function $F(x) = \begin{cases} |x|, & -40 \le x < 5 \\ (x-5)^2 + 5, & 5 \le x < 10 \\ \left|\frac{31}{20}(30-x)\right| - 1 & 10 \le x \le 40 \end{cases}$ defined over x ∈ [-40,40].
Our goal is to find the minimum value of F(x) in the range [-40, 40]. Among Genetic Algorithm (GA) and Method of Steepest Descent, which optimization technique would you prefer to solve this problem? Justify your answer.
(b) For *Simulated Binary Crossover* (SBX) in real-coded GA, the probability distribution of the spread factor (β) is given by the PDF:

$$P(\beta) = \begin{cases} 0, & if\ \beta \le 0 \\ 0.5(n+1)\beta^n, & if\ 0 < \beta \le 1 \\ 0.5(n+1)\frac{1}{\beta^{n+2}}, & if\ \beta > 1 \end{cases}$$

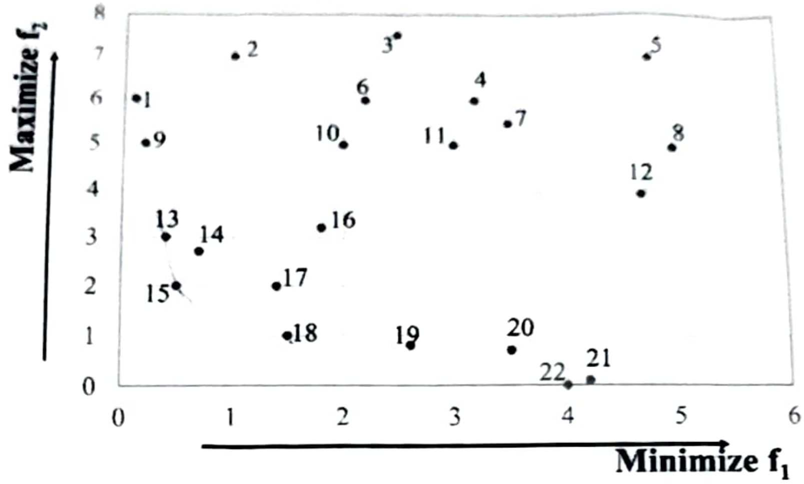
, where n ≥ 0 is the SBX crossover distribution factor.

(i) Derive the expression/s for β for any number (u) representing the area under the PDF curve, $(0 \le u \le 1)$.
(ii) How β is used to compute two offspring chromosomes from two parent chromosomes? Write the expressions only.
(iii) From (i) and (ii), conclude what are the effects of large and low values of n on the offsprings generated after SBX crossover. **[2+(6+2+2) = 12 marks]**

3. Suppose GA with real-value encoding using 4 bit binary codes is used to find a straight line that best fits the set of 2D points: {(-8,5), (-4,3), (0,-1), (1,1), (3,-2)} with the following constraints:
(i) The line must pass through origin.
(ii) The slope of the line to be determined through GA should be in the interval [-1 -0.5], i.e., encoding of -1 is 0000 and encoding of -0.5 is 1111.
The following four chromosomes form the population: 0100, 1001, 1100, 0001.
(a) Determine a suitable fitness function to evaluate the fitness value of each chromosome.
(b) Compute the fitness values of the four chromosomes based on the chosen function.
(c) Suppose Roulette Wheel Selection on fitness values is used to select two chromosomes from the above population into a mating pool. What is the angle subtended at the center of the wheel for each chromosome in the population?
(d) Assume two randomly generated numbers at each iteration of the selection process and show which two chromosomes will be selected in the two iterations. **[2+4+4+2 = 12 marks]**

5. (a) Explain with diagram why for non-convex objective space, the Single-Objective Evolutionary Algorithm (SOEA) method to solve MOOP cannot produce all the solutions on the pareto-optimal front by varying the weights of the different objectives.

(b) Consider the following figure that shows the objective space for a two-objective optimization problem, where f_1 is minimization objective and f_2 is maximization objective.



Pick up the pareto-optimal solutions among the 22 different solutions shown in the objective space.

(c) Consider the following objective functions: Minimize $f_1(x) = x_4 - x_3$, Minimize $f_2(x) = 1 + x_2^2 - x_1^2$, Maximize $f_3(x) = x_1 + x_3$. where $x=[x_1\ x_2\ x_3\ x_4]^T$. Further consider the following constraints: $-7 \le x_1 \le 5$, $-2 \le x_2 \le 2$, $0 \le x_3 \le 1$, $-2 \le x_4 \le 2$. Use Lexicographic Ordering to find the optimal solution x^* to the above multi-objective problem assuming $f_2 < f_3 < f_1$, where '<' means "has a higher priority than". *[Note that the individual objectives can be solved using any conventional optimization technique]*.

[6+2+4=12 marks]

6. Consider a cost function $C(w) = \frac{1}{2}\sigma^2 - r_{xd}^T w + \frac{1}{2} w^T R_X w$, where σ^2 is some constant. $r_{xd} = \begin{bmatrix} 0.8182 \\ 0.3540 \end{bmatrix}$ and $R_X = \begin{bmatrix} 1 & 0.8182 \\ 0.8182 & 1 \end{bmatrix}$. Clearly w has two components.

(a) Derive the expression of the gradient vector from the above cost function.

(b) Considering a learning rate 0.5 and initial weight vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, apply the Method of Steepest Descent for two iterations. What are the updated weights after these two iterations?

[4+8 = 12 marks]