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# Math for Machine Learning

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## Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix

# W2 Lesson 1



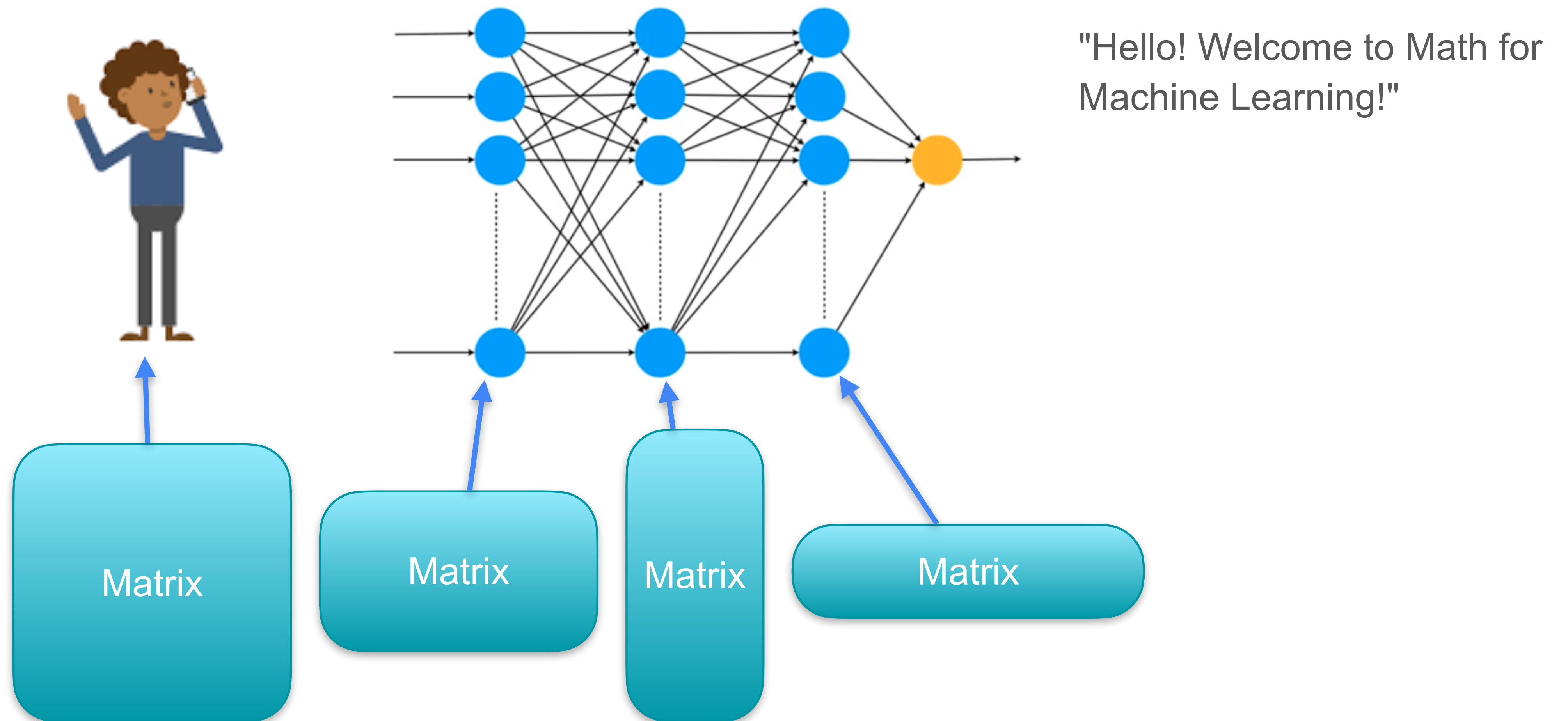
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# Solving System of Linear Equations

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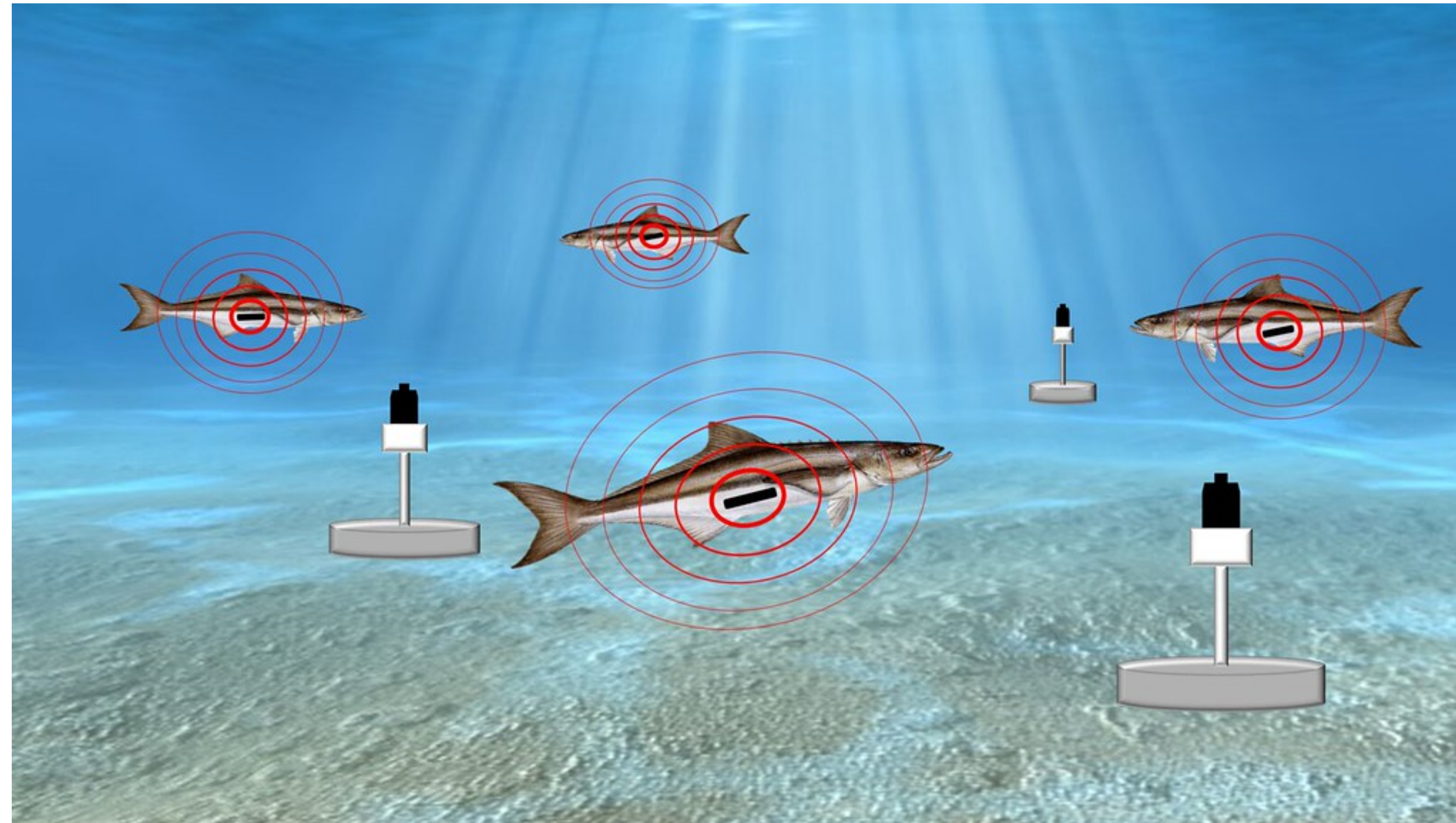
**Machine learning motivation**

# Neural networks - Matrix operations





# Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.



# Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.



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# Solving System of Linear Equations



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**Solving non-singular system  
of linear equations**



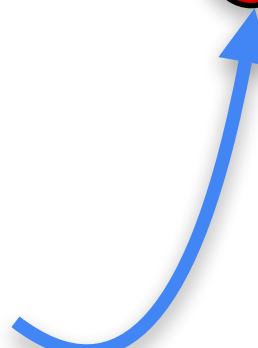






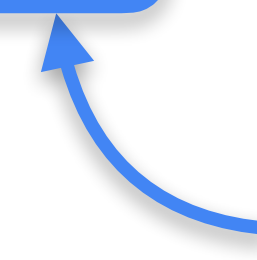
# Solving systems of equations

## System

- $a + b = 10$   
 

- $a + 2b = 12$   
 

 +  = \$10  
\$8  \$2 

 +  +  = \$12  
 \$2

# Solving systems of equations

**System**

- $a + b = 10$



- $a + 2b = 12$



**Some process**

**Manipulating equations**

**Swapping equations**

**Adding equations**

**Multiplying equations by a constant**

**Solved system**

- $a = 8$



- $b = 2$



# Solving systems of equations

## System

- $a + b = 10$



- $a + 2b = 12$



Eliminate 'a' from this equation

## Solved system

- $a = 8$



- $b = 2$



# Manipulating equations

## Multiplying by a constant

$$\begin{array}{r} a + b = 10 \\ \times \qquad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$\begin{array}{r} a + b = 10 \\ + \quad 2a + 3b = 22 \\ \hline 3a + 4b = 32 \end{array}$$



# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate 'a'  
from this equation

## Divide by coefficient of a

- $a + 0.2b = 3.4$
- $a - 0.75b = 1.5$

## Subtract equation 1 from equation 2

$$a - 0.75b = 1.5$$

$$\begin{array}{r} a - 0.75b = 1.5 \\ - (a + 0.2b = 3.4) \\ \hline \end{array}$$

$$0a - 0.95b = -1.9$$

$$-0.95b = -1.9$$

$$b = 2$$

## Solved system

- $a = ?$  3
- $b = ?$  2

$$a + 0.2(2) = 3.4$$

$$a + 0.4 = 3.4$$

$$a = 3$$

# What if one of the coefficients of a is zero?



# Quiz

- Solve the following system of equations

## System

- $2a + 5b = 46$
- $8a + b = 32$

# Solution

- Solve the following system of equations

## System

- $2a + 5b = 46$
- $8a + b = 32$

## Solution

- $a = 3$
- $b = 8$





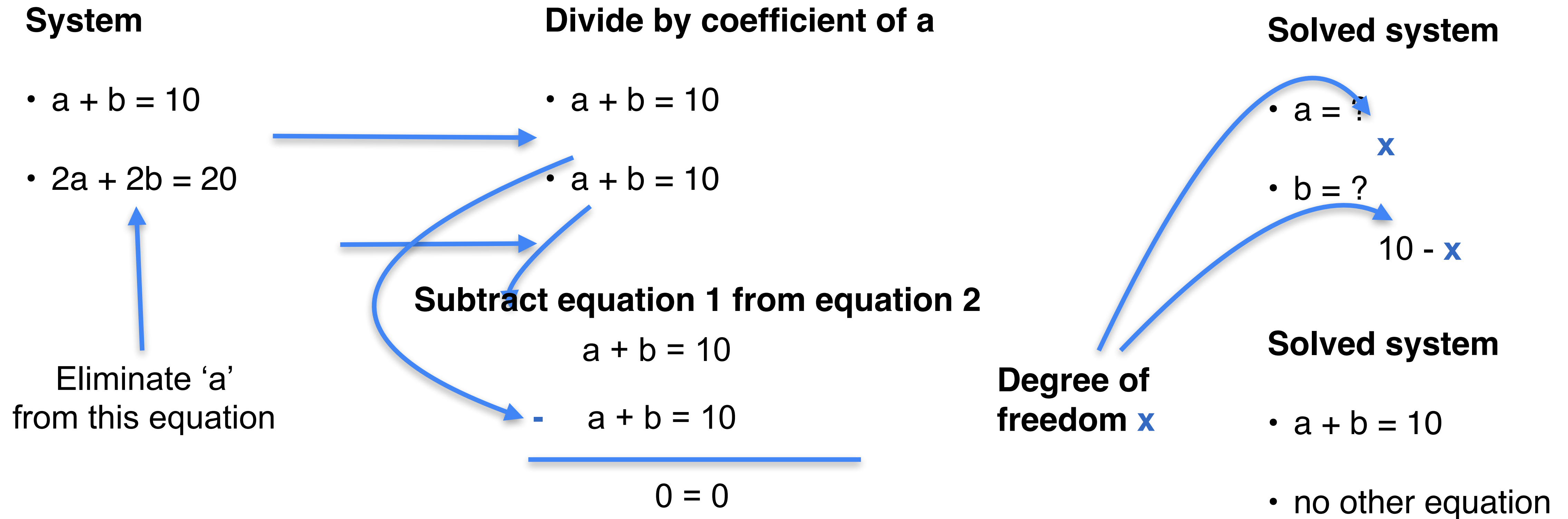
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# Solving System of Linear Equations

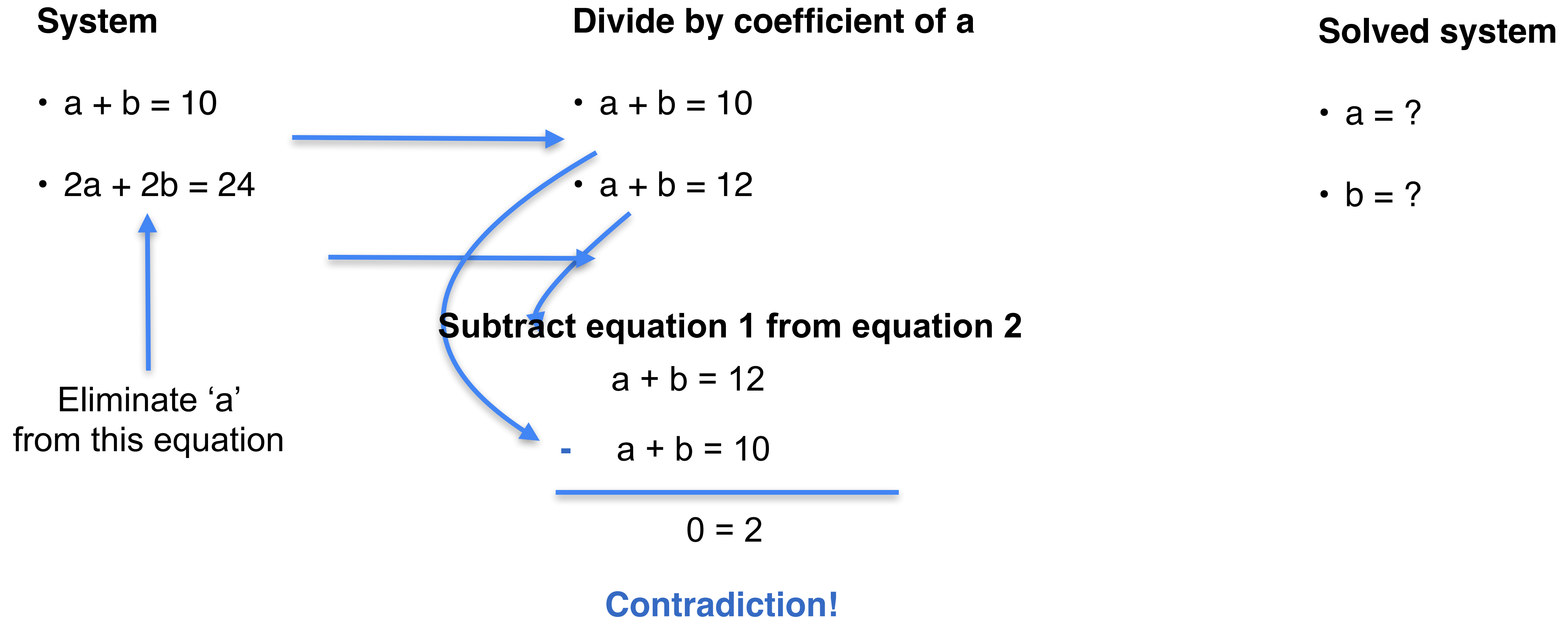
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**Solving singular system of  
linear equations**

# What if the system is singular (redundant)?



# What if the system is singular (contradictory)?



# Quiz

- Solve the following system of equations

## System

- $5a + b = 11$
- $10a + 2b = 22$



# Solution

- Solve the following system of equations

## System

- $5a + b = 11$
- $10a + 2b = 22$

**Solution:** If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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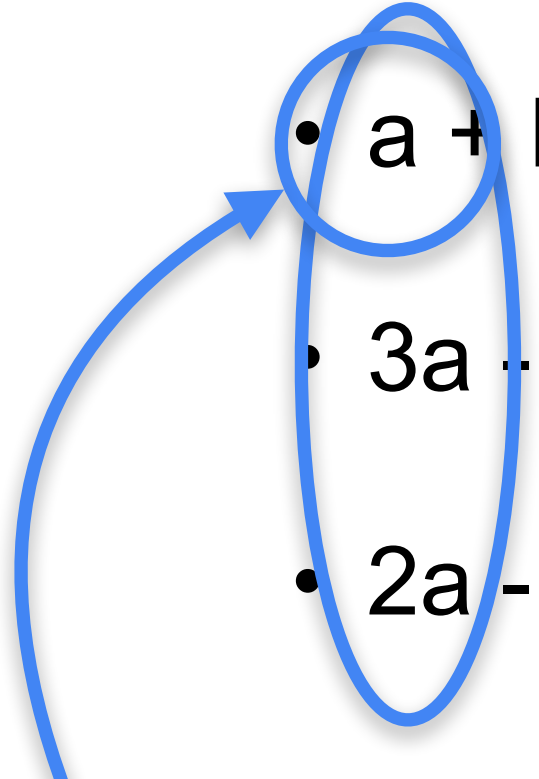
# Solving System of Linear Equations

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**Solving system of equations  
with more variables**

# Elimination method

## System



- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

Leave 'a' by  
itself

Divide each row  
by the  
coefficient of 'a'



- $a + b + 2c = 12$

- $a - b - \frac{1}{3}c = 1$

- $a - \frac{b}{2} + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others



- $a + b + 2c = 12$

- $-2b - \frac{7}{3}c = -11$

- $-\frac{3}{2}b + c = 0$

Isolated 'a'

Solve this new  
system of 2  
equations

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - \frac{7}{3}c = -11$

- $-\frac{3}{2}b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$

- $b + \frac{7}{6}c = \frac{11}{2}$

- $b - \frac{2}{3}c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$

- $b + \frac{7}{6}c = \frac{11}{2}$

- $-\frac{11}{6}c = -\frac{11}{2}$

Isolated 'b'

$c = 3$



# Elimination method

## System

- $a + b + 2c = 12$  
- $b + \frac{7}{6}c = \frac{11}{2}$  
- $c = 3$

$a + 2 + 6 = 12$   
 $a = 4$

$b + \frac{7}{2} = \frac{11}{2}$   
 $b = 2$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

The solution is  
 $a = 4$   
 $b = 2$   
 $c = 3$



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# Solving System of Linear Equations

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## **Matrix row reduction**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

Row echelon form

## Diagonal matrix

1	0
0	1

Reduced row echelon form

# Systems of equations to matrices

## Original system

- $a + b = 10$
- $2a + 2b = 20$

## Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

## Original matrix

1	1
2	2

## Upper diagonal matrix

1	1
0	0

Row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$

## Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

## Original matrix

5	1
10	2

## Upper diagonal matrix

1	0.2
0	0

Row echelon form

# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

## Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

## Original matrix

0	0
0	0

## Upper diagonal matrix

0	0
0	0

Row echelon form



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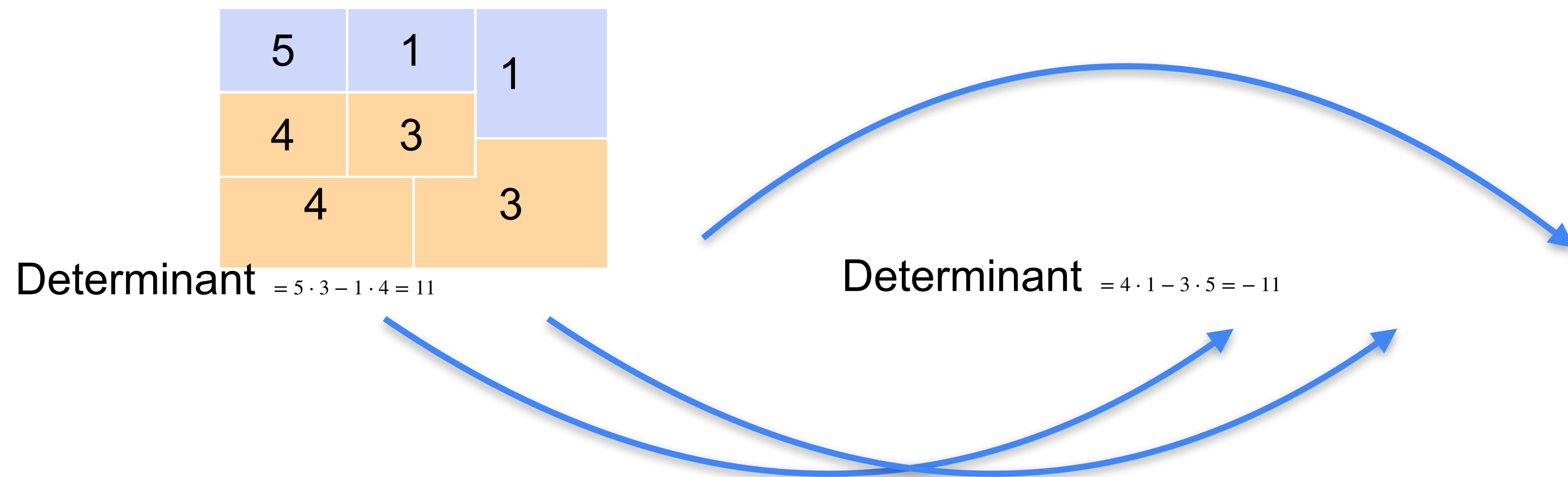
# Solving System of Linear Equations

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**Row operations that preserve  
singularity**



# Switching rows



# Multiplying a row by a (non-zero) scalar

5	1	1
4	3	
4		-3

Determinant  $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

$\times 10 =$

50	10
----	----

Determinant  $= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$

$= 10 \cdot 11$

# Adding a row to another row



A 3x3 matrix is shown with blue and orange blocks. The top row has three blue blocks with values 5, 1, and 1. The middle row has two orange blocks with values 4 and 3, followed by a blue block with value 1. The bottom row has two orange blocks with values 4 and 3.

Determinant  $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

+

---



A 2x2 matrix is shown with teal blocks. The top row has two teal blocks with values 9 and 4. The bottom row has two teal blocks with values 4 and 3.

Determinant  $= 9 \cdot 3 - 4 \cdot 4$

$= 11$

# W2 Lesson 2



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# Solving System of Linear Equations

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## **Rank of a matrix**

# Compressing Images - Reducing rank

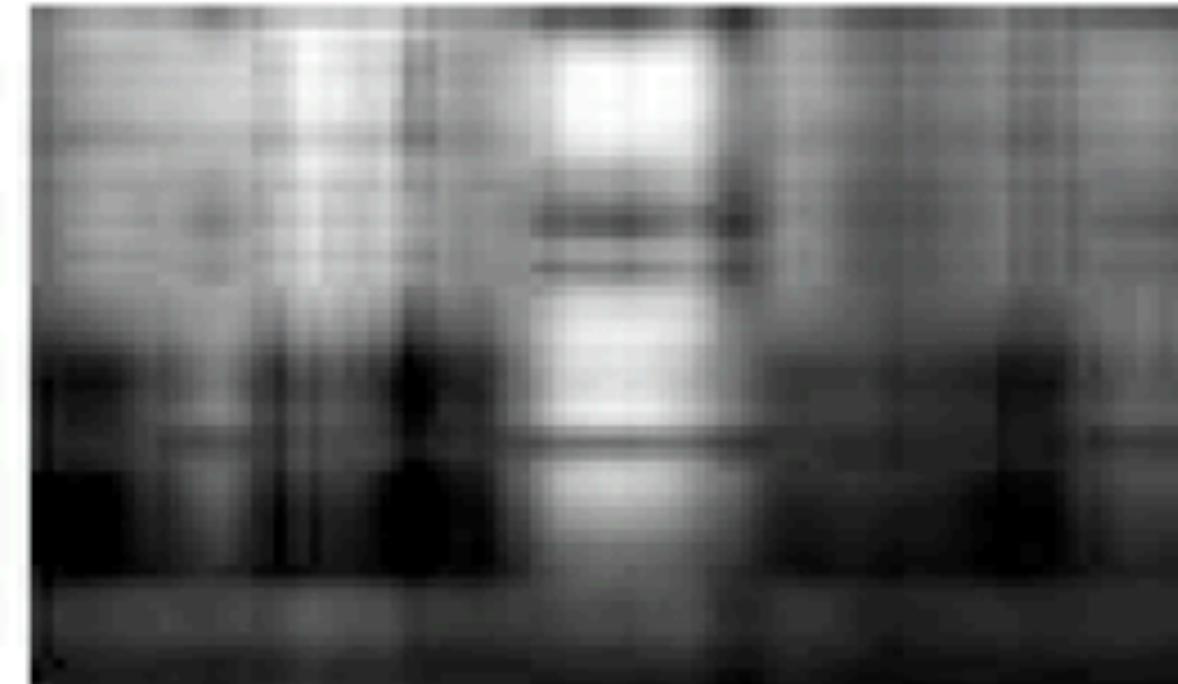
Original (Rank 200)



Rank 1



Rank 2



Rank 5



Rank 15





Rank 50



# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

**Two sentences**

**Two pieces of information**

**Rank = 2**

## System 2


 The dog is **black**  
 The dog is **black**

**Two sentences**

**One piece of information**

**Rank = 1**

## System 3

 The dog  
 The dog

**Two sentences**

**Zero pieces of information**

**Rank = 0**



# Systems of equations

System 1

$a + b = 0$   
   
 $a + 2b = 0$   
 

	
1	1
1	2





Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$   
   
 $2a + 2b = 0$   
 

	
1	1
2	2

Rank = 1



Two equations

One piece of information

Rank = 1

System 3

$0a + 0b = 0$   
 $0a + 0b = 0$

	
0	0
0	0

Rank = 0

Two equations

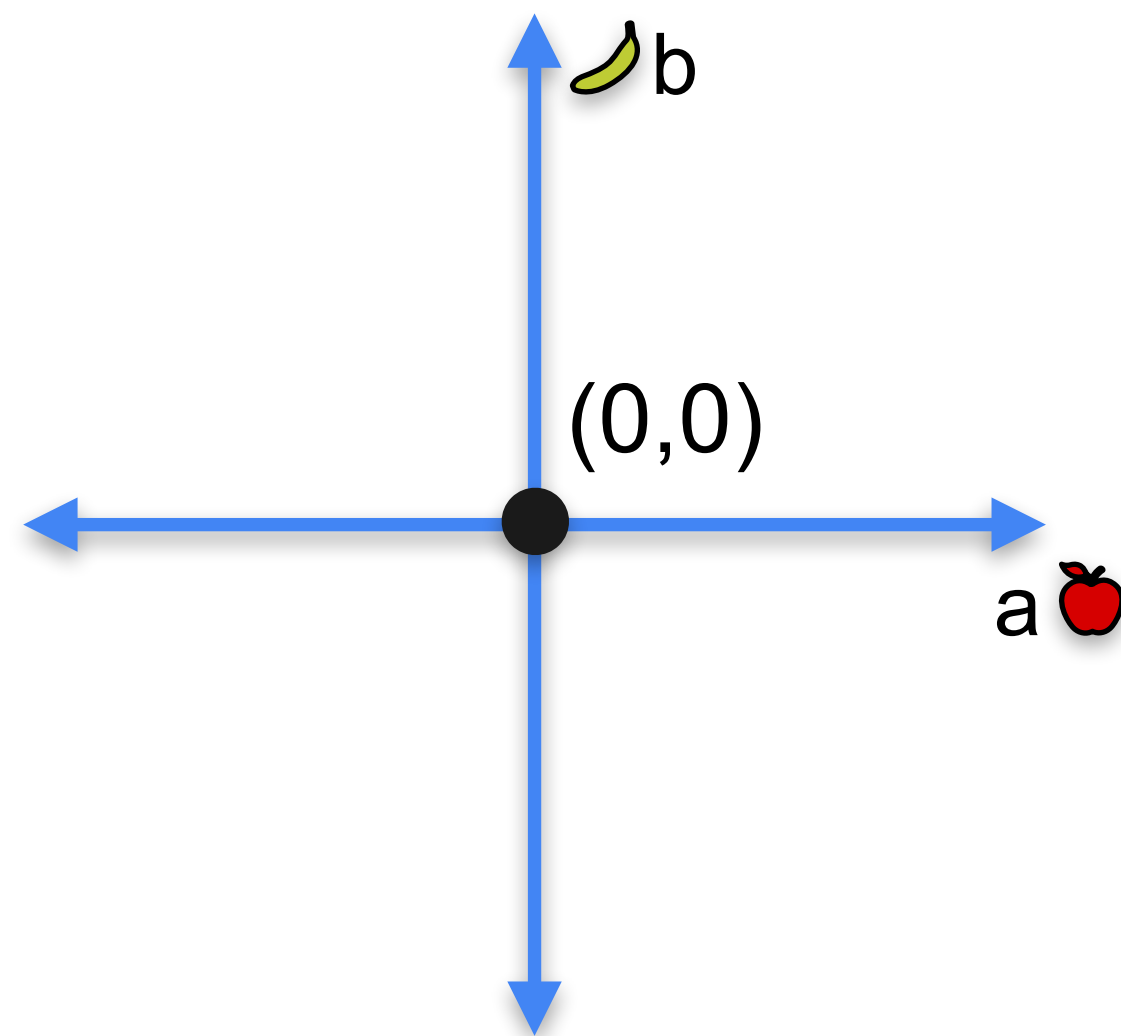
Zero pieces of information



Rank = 0

# Rank and solutions to the system

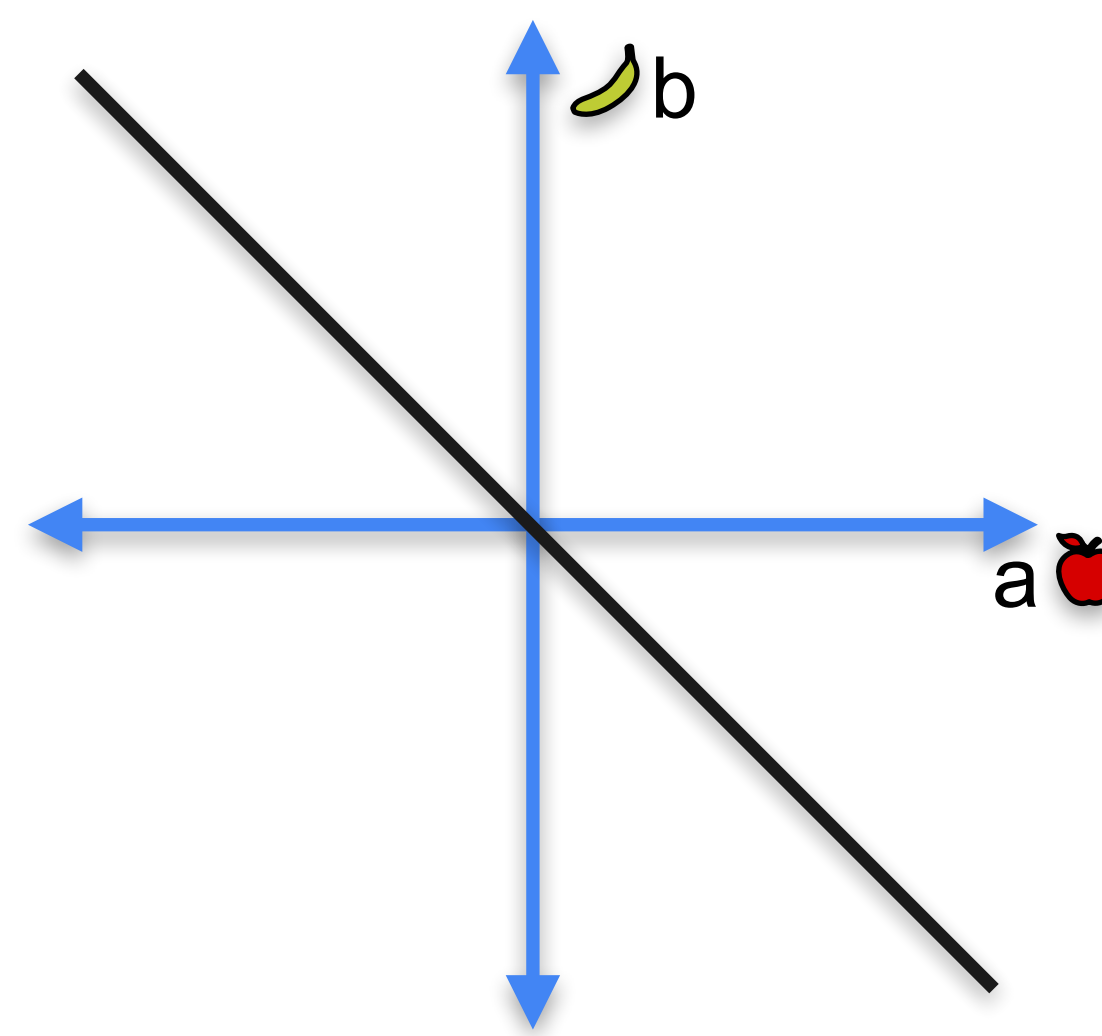
	
1	1
<sup>1</sup> Rank = 2	2

Dimension of solution space = 0



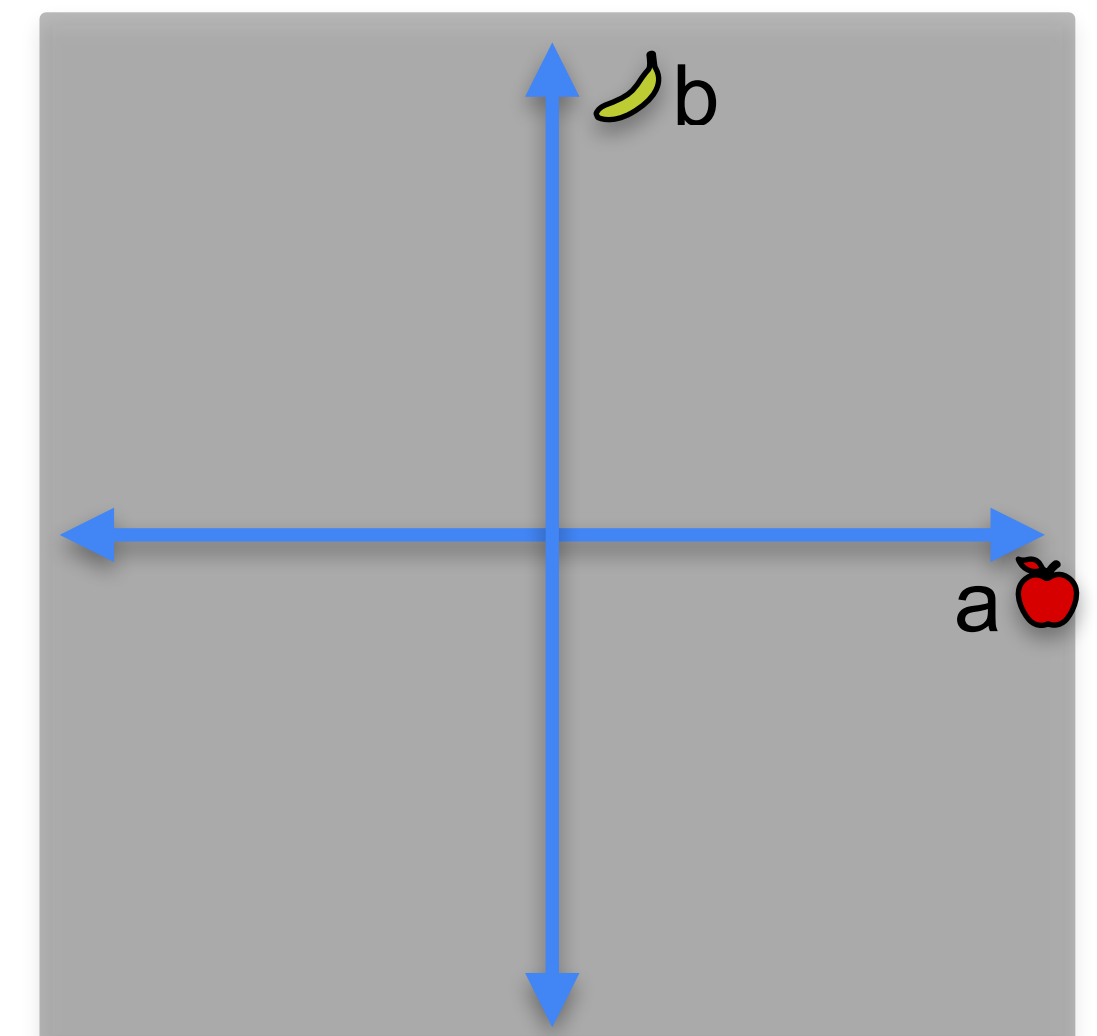
	
1	1
<sup>2</sup> Rank = 1	2

Dimension of solution space = 1



	
0	0
<sup>0</sup> Rank = 0	0

Dimension of solution space = 2



# Rank of a matrix

	
1	1
<sup>1</sup> Rank = 2	2

Dimension of solution space = 0

	
1	1
<sup>2</sup> Rank = 1	2

Dimension of solution space = 1

	
0	0
<sup>0</sup> Rank = 0	0



Dimension of solution space = 2

$$\text{Rank} = 2 - (\text{Dimension of solution space})$$

# Rank and singularity

	
1	1
<sup>1</sup> Rank = 2	2

Non-singular

	
1	1
<sup>2</sup> Rank = 1	2

Singular

	
0	0
<sup>0</sup> Rank = 0	0

Singular

# Quiz: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1**

5	1
-1	3

**Matrix 2**

2	-1
-6	3

# Solutions: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1:** Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

**Matrix 2:** Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3



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# Solving System of Linear Equations

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**Rank of a matrix:  
General case**



# Rank for matrices

## System 1

$a + b + c = 0$  ✓  
 $a + 2b + c = 0$  ✓  
 $a + b + 2c = 0$  ✓

3 Equations  
3 Pieces of information

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$a + b + c = 0$  ✓  
 $a + b + 2c = 0$  ✗  
 $a + b + 3c = 0$  ✓

3 Equations  
2 Pieces of information

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$a + b + c = 0$  ✓  
 $2a + 2b + 2c = 0$  ✗  
 $3a + 3b + 3c = 0$  ✗

3 Equations  
1 Piece of information

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$0a + 0b + 0c = 0$  ✗  
 $0a + 0b + 0c = 0$  ✗  
 $0a + 0b + 0c = 0$  ✗

3 Equations  
0 Pieces of information

## Rank 0

0	0	0
0	0	0
0	0	0

# Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



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# Solving System of Linear Equations

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## Row echelon form

# Row echelon form of a matrix

Original matrix

5	1
4	-3
5	1
10	2
0	0
0	0

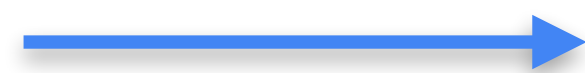
Row echelon form

1	0.2
0	1
1	1
0	0
0	0
0	0

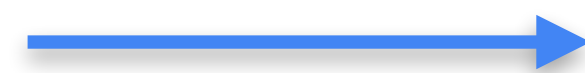
# Row echelon form

Original matrix

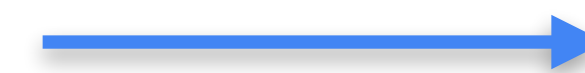
5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Row echelon form

1	0.2
0	1

Divide each row by  
the leftmost coefficient

-	1	-0.75
	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by  
the leftmost coefficient

1	0.2
1	0.2

Row echelon form

1	0.2
0	0

Divide the second row by  
the leftmost non-zero coefficient

1	0.2
?	?

-	1	0.2
	1	0.2
<hr/>		
	0	0

# Row echelon form for singular matrices

Row echelon form

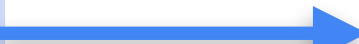
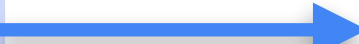
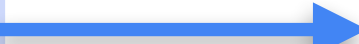
Original matrix



Divide each row by  
the leftmost coefficient



# Row echelon form, singularity, and rank

Non-singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>-3</td></tr></table> 	5	1	4	-3	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>1</td></tr></table>	1	0.2	0	1	<b>Rank 2</b> 2 ones in the diagonal
5	1										
4	-3										
1	0.2										
0	1										
Singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>10</td><td>2</td></tr></table> 	5	1	10	2	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>0</td></tr></table>	1	0.2	0	0	<b>Rank 1</b> 1 one in the diagonal
5	1										
10	2										
1	0.2										
0	0										
Singular matrix	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table> 	0	0	0	0	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table>	0	0	0	0	<b>Rank 0</b> 0 ones in the diagonal
0	0										
0	0										
0	0										
0	0										



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# Solving System of Linear Equations

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**Row echelon form:  
General case**

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

## Matrix

1	1	2
3	-3	-1
2	-1	6



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

## Row echelon form matrix

1	1	2
0	-6	7
0	0	6

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

**Rank 5**

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

**Rank 3**

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

# Row echelon form

<b>3</b>	*	*	*	*
0	0	<b>1</b>	*	*
0	0	0	<b>-4</b>	*
0	0	0	<b>0</b>	0
0	0	0	0	<b>0</b>

$\div 3$

$\div 1$

$\div (-4)$

<b>1</b>	*	*	*	*
0	0	<b>1</b>	*	*
0	0	0	<b>1</b>	*
0	0	0	<b>0</b>	0
0	0	0	0	<b>0</b>

## Note:

- In general, pivots different than 1 are allowed
- For this class, pivots are 1. This makes no mathematical difference.

# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

**Row echelon form**

1	1	1
0	1	0
0	0	1

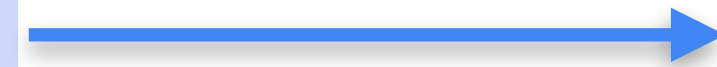
Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

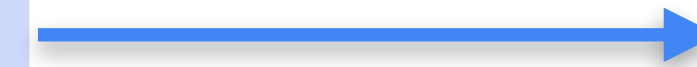
1	1	1
1	1	2
1	1	3

Subtract the first row  
from the second and  
the third ones



1	1	1
0	0	1
0	0	2

Subtract twice the  
second row from the  
third one



**Row echelon form**

1	1	1
0	0	1
0	0	0

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the first  
row from the second  
row



1	1	1
0	0	0
3	3	3

Subtract three times  
the first row from the  
third row



**Row echelon form**

1	1	1
0	0	0
0	0	0



# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Rank = 1**

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Rank = 0**

## Row echelon forms

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**



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# Solving System of Linear Equations

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## **Reduced row echelon form**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

Row echelon form

## Diagonal matrix

1	0
0	1

Reduced row echelon form

# Reduced row echelon form

Row echelon form

1	0.2
0	1

0	1
---	---

x                      0.2

---

0	0.2
---	-----

Reduced row echelon form

1	0.2
0	1

1	0.2
0	0.2

-

---

1	0
---	---

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Rank 5

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by the  
value of the pivot

Reduced row  
echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a  
pivot to 0

# Reduced row echelon form

**Row echelon form**

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one

1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one

1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one

**Reduced row echelon form**

1	0	0
0	1	0
0	0	1



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# Solving System of Linear Equations

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## **The Gaussian Elimination Algorithm**



# Augmented matrix

$$\begin{array}{rcl} 2a - b + c & = & 0 \\ 2a + 2b + 4c & = & 0 \\ 4a + b & = & 0 \end{array}$$



**Augmented matrix**

2	-1	1
2	2	4
4	1	0

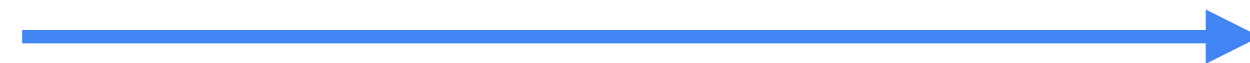
**Proceed with the elimination method**

# Augmented matrix

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$



$R_1$	2	-1	1		1
$R_2$	2	2	4		-2
$R_3$	4	1	0		-1



# Pivoting

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$



$R_1$	2	-1	1		1
$R_2$	2	2	4		-2
$R_3$	4	1	0		-1

$$R_1 \leftarrow \frac{1}{2}R_1$$

$$R_1 \leftarrow \frac{1}{2}$$

=	1	-1/2	1/2		1/2
---	---	------	-----	--	-----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$

$R_1$	1	-1/2	1/2	1/2
$R_2$	2	2	4	-2
$R_3$	4	1	0	-1

$$R_2 \leftarrow R_2 - 2R_1$$

	2	2	4	-2
-2	1	-1/2	1/2	1/2
<hr/>				
$R_2 \leftarrow$	0	3	3	-3

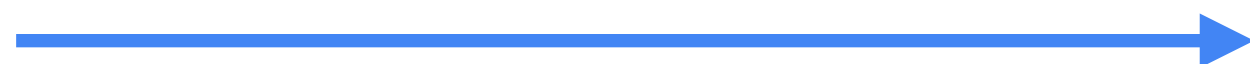
$$R_3 \leftarrow R_3 - 4R_1$$

	4	1	0	-1
-4	1	-1/2	1/2	1/2
<hr/>				
$R_3 \leftarrow$	0	3	-2	2

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$
$$3b + 3c = -3$$

$$3b - 2c = 2$$



$R_1$	1	-1/2	1/2	1/2
$R_2$	0	3	3	-3
$R_3$	0	3	-2	2

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$3b + 3c = -3$$

$$3b - 2c = 2$$



$R_1$	1	-1/2	1/2		1/2
$R_2$	0	3	3		-3
$R_3$	0	3	-2		2

$$R_2 \leftarrow \frac{1}{3}R_2$$

$$R_2 \leftarrow \frac{1}{3}$$

0	3	3		-3
---	---	---	--	----

=

0	1	1		-1
---	---	---	--	----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$

 $R_1$ 

1	-1/2	1/2	1/2
---	------	-----	-----

 $R_2$ 

0	1	1	-1
---	---	---	----

 $R_3$ 

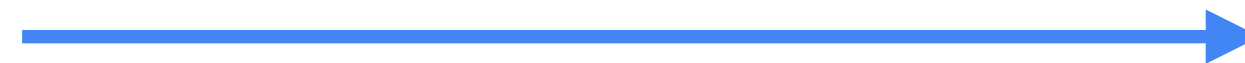
0	3	-2	2
---	---	----	---

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$



$R_1$	1	-1/2	1/2		1/2
$R_2$	0	1	1		-1
$R_3$	0	3	-2		2

$$R_3 \leftarrow R_3 - 3R_2$$

	0	3	-2		2
-3	0	1	1		-1
<hr/>					
$R_3 \leftarrow$	0	0	-5		5



# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$-5c = 5$$


 $R_1$ 

1	-1/2	1/2		1/2
---	------	-----	--	-----

 $R_2$ 

0	1	1		-1
---	---	---	--	----

 $R_3$ 

0	0	-5		5
---	---	----	--	---

$$R_3 \leftarrow -\frac{1}{5}R_3$$

$$R_3 \leftarrow -\frac{1}{5}$$

0	0	-5		5
---	---	----	--	---

 $=$ 

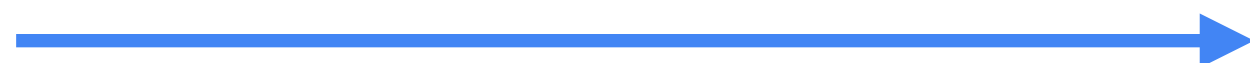
0	0	1		-1
---	---	---	--	----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 $R_1$ 

1	-1/2	1/2	1/2
---	------	-----	-----

 $R_2$ 

0	1	1	-1
---	---	---	----

 $R_3$ 

0	0	1	-1
---	---	---	----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 $R_1$  $R_2$  $R_3$ 

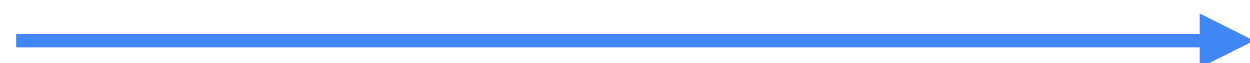
1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 $R_1$ 

1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

 $R_2$  $R_3$

# Back substitution

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

$R_1$	1	-1/2	1/2		1/2
$R_2$	0	1	1		-1
$R_3$	0	0	1		-1

$$R_2 \leftarrow R_2 - R_3$$

	0	1	1		-1
-	0	0	1		-1

$$R_2 \leftarrow$$

0	1	0		0
---	---	---	--	---

$$R_1 \leftarrow R_1 - \frac{1}{2}R_3$$

$$-\frac{1}{2}$$

$$R_1 \leftarrow$$


0	0	1		-1
1	-1/2	1/2		1/2
1	-1/2	0		1

# Back substitution

$$a - \frac{1}{2}b = 1$$

$$b = 0 \quad -1$$

$$c = -1$$



$R_1$	1	-1/2	0		1
$R_2$	0	1	0		0
$R_3$	0	0	1		-1

$$R_1 \leftarrow R_1 + \frac{1}{2}R_2$$

$$+\frac{1}{2}$$

---

$R_1 \leftarrow$	1	0	0		1
------------------	---	---	---	--	---

# The result

$a = 1$   
 $b = 0$   
 $c = -1$



$R_1$

1	0	0	1
---	---	---	---

$R_2$

0	1	0	0
---	---	---	---

$R_3$

0	0	1	-1
---	---	---	----

Solution to the system

Identity matrix

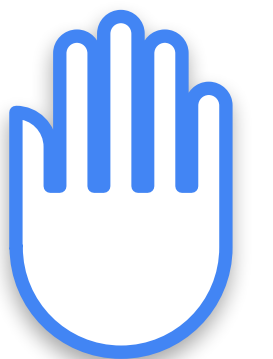
# What if the system is singular?

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



1	2	-1	5
0	0	-7	9
0	0	0	0



There is no need to worry!



# Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



Look at the column of constants

1	2	-1	5
0	0	-7	9
0	0	0	0



$$0a + 0b + 0c = 0$$

Infinitely many  
solutions

# Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	10

After row reduction...



Look at the column of constants

1	2	-1	5
0	0	-7	9
0	0	0	0
0	0	0	4

$$0a + 0b + 0c = 4$$

The system has  
no solutions

# Checking if it has infinitely many or no solutions

- Row full of zeroes in row echelon form
- Constant in that row is zero
- **Infinitely many solutions**

1	2	-1	5
0	0	-7	9
0	0	0	0

- Row full of zeroes in row echelon form
- Constant in that row is not zero
- **No solutions**

1	2	-1	5
0	0	-7	9
0	0	0	4

# Gaussian Elimination - Summary

1. Create the augmented matrix
2. Get the matrix into reduced row echelon form
3. Complete back substitution
4. Stop if you encounter a row of 0s



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# Solving System of Linear Equations

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## Conclusion