course objectives

1. To describe and explain the basic principles of digital image processing.
2. To discuss various image processing techniques

course outcomes.

1. The students will be able to illustrate fundamental steps in digital image processing.
2. Make use of appripriate digital Image Enhancement techniques in spatial domain for real world problems.
3. Apply suitable image segmentation and compression techniques for an application.
4. Demonstrate colour Image processing techniques
5. Summarize different reshaping techniques/operations on the Image and they practical applications.

Unit-I - Introduction

Unit-II - Image Enhancement in spatial domain

Unit-III - Colour Image processing.

Unit-IV - Image compresion

Unit-V - Morphology and Segmentation

Unit-I

Applications of Digital Image processing

1. Nuclear medicine
2. Digital Radio graphy
3. Angio graphy
4. visual inspection of Manfactureгoods
5. used in MRI
6. Astrinomical study
7. Remote sensing

Three types of processes in Image processing

1. Low Level processing
2. Mid Level Processing
3. High level processing

→ Gamma-Ray Imaging:

used in PET scan

→ X-ray Imaging

Fundamental steps in Digital Image Processing

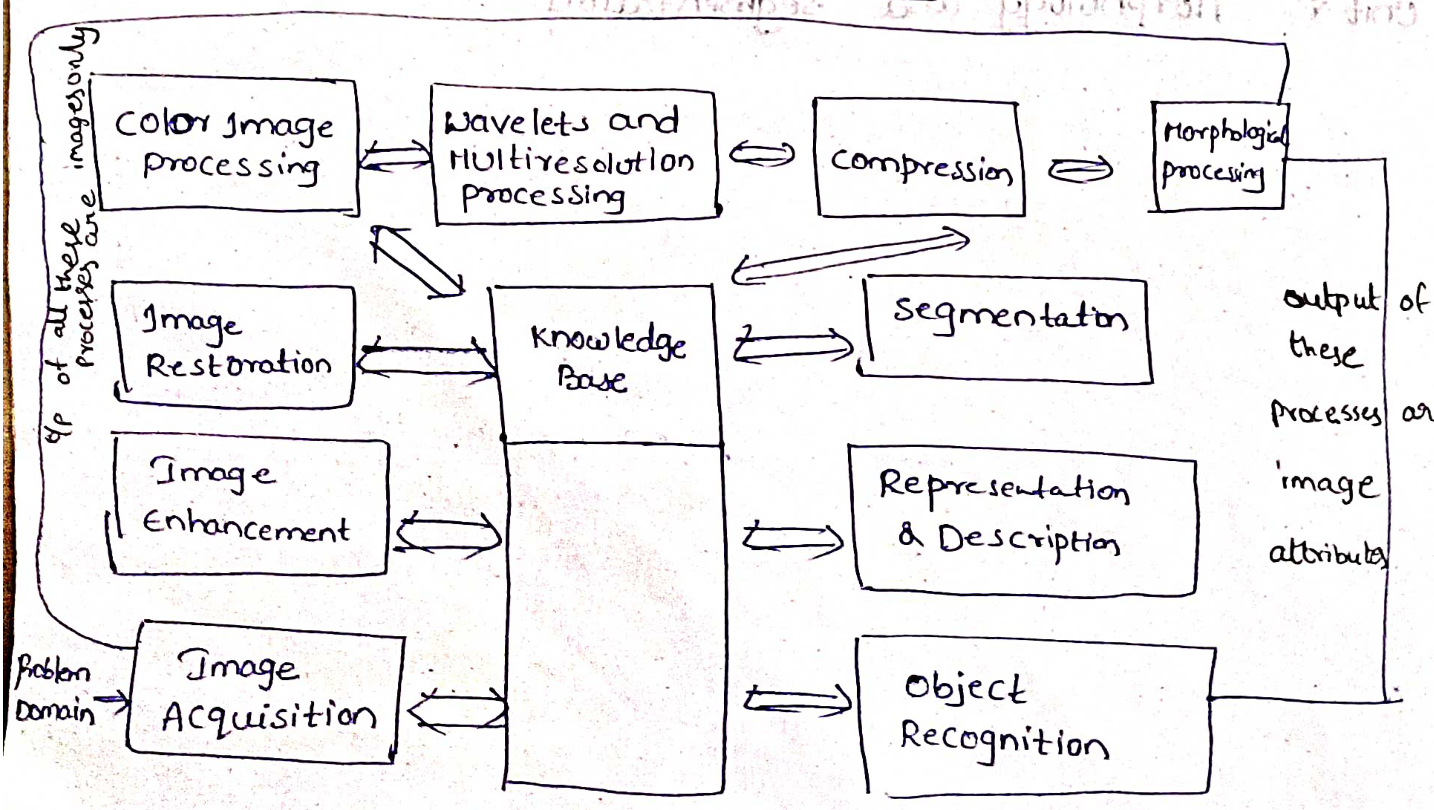


Image Acquisition

→ This is simple when image is already in digital form
→ This strange involves preprocessing such as scaling

Image Enhancement

→ This is used to bringout detail that is hidden (or) simply to highlight certain feautures of interest in an image.
→ It is subjective area of image processing

Ex: Increasing contrast of an Image Looks Better

Image Restoration

→ It is based on Mathemeatical (or) probablistic Models of image degradation
→ It also deals with Improving the apperance of an Image
→ It is objective area of image processing.

Colour Image processing

→ It has become important because significant Increase of digital images over the Internet.

→ It is used for extracting feautures of interest in an image.

Wavelets

→ It is used for, these are the tools for representing images in various degress of resolution

Compression

→ It deals with techniques for reducing storage required to store | save an Image (or) Bandwidth required to transmit it

Morphological processing

→ It deals with the tools for extracting image components that are useful in the representation and description of the shape.

Segmentation

→ These processors partioned the entire image into it's objects (or) parts

Representation & Description

→ It almost follow the output of Segmentation which is usually raw pixel data. That may be either boundary of a region (or) the point in the region itself, In either case converting the data into a form suitable for computer processing is necessary.

→ Boundary representation is appropriate when the focus is on external shape charcterstics such as corners, inflectio

→ Regional representation is appropriate when the focus is on internal properties such as texture,

→ Description deals with extracting attributes from the region of interest in an image.

→ Description can also be called feature selection and it can used to differentiate one object from another.

Object Recognition

→ It is the process that assigns a Label to an object based on its descriptors

Ex:- Vehicle, Tree, etc...

Knowledge Base

Knowledge about the problem domain is coded into the form suitable for computer processing and is stored in knowledge Base.

→ It controls the Interaction b/w models, It Limits the search that has to be conducting in seeking the Information

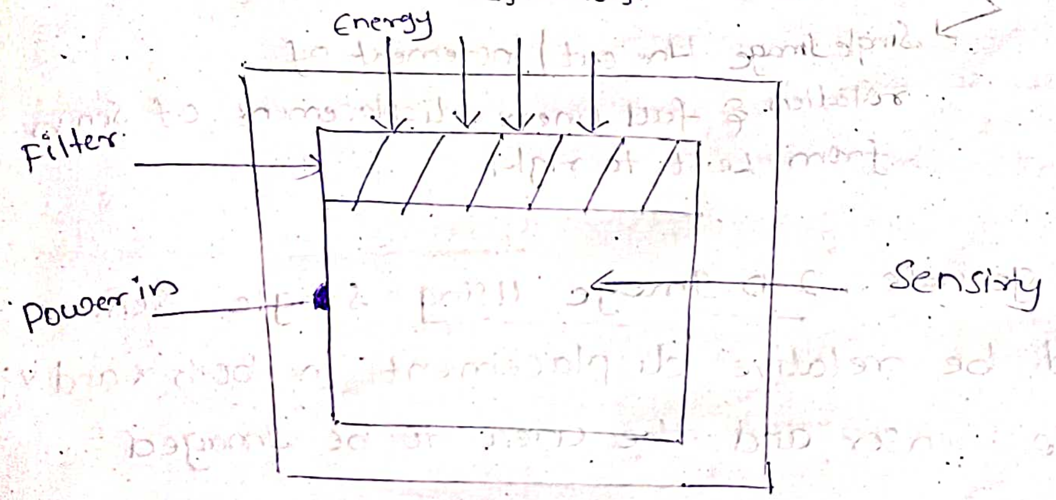
Image Sensing & Acquisition

Most of the images are generated by the combination of illumination $i(x,y)$ and reflection of the energy from the source by the object of the seen being imaged

$$f(x,y) = \underbrace{i(x,y)}_{\text{illumination}}\ \underbrace{r(x,y)}_{\text{reflection}}$$

$$\underset{\text{no light}}{0} < i(x,y) < \underset{\text{infinity light is focused}}{\infty}$$

$$\underset{\text{Total absorption}}{0} < r(x,y) < \underset{\text{Total reflection}}{1}$$



→ The incoming energy is transformed into a voltage by the combination of input electrical power and sensing material that is responsed to particular type of energy being detected.

→ The use of filter improves selectivity

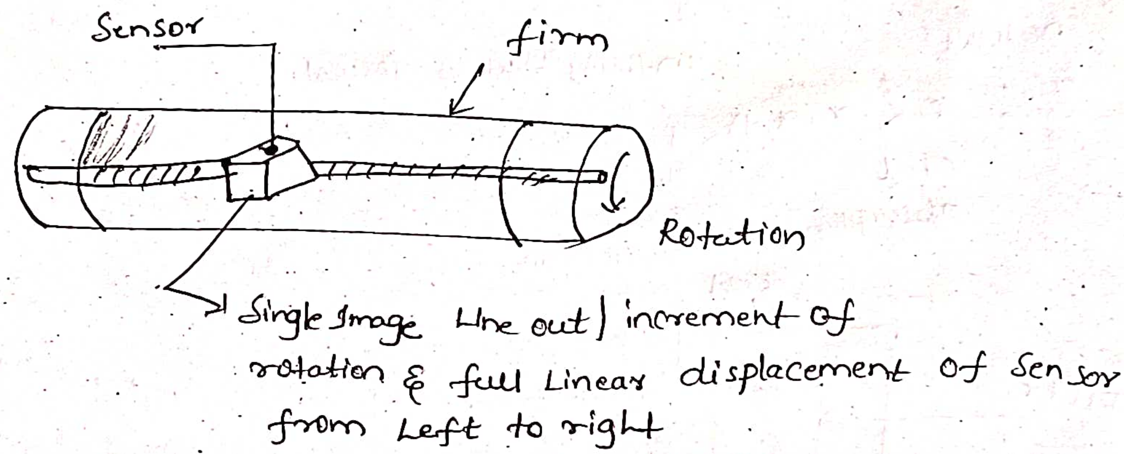
Ex:- A green filter infront of sensing material favours green color light.

The o/p waveform Oltage is a response of sensor and a digital quantity is obtained by digitising it.

(i) Single Imaging Sensor

Generation of 2-D Image using Single Sensor

→ In order to generate a 2-D image using



In order to generate 2-D Image using single sensor there should be relative displacement in both x and y director b/w sensor and the area to be Imaged

→ A film Negative is mounted on to a drum whose Mechanical rotation provides displacement in one direction, the single sensor is mounted on a screw that provides displacement in perpendicular direction.

Image Acquisition using Sensor Strip

It consist of an inline arrangement of sensors in the form of sensor strip, the strip provides imaging elements in one direction.

→ 4000 inline Sensors (or) more can be placed in a Sensor Strip that are used in aircraft application inwhich the imaging system is mounted on an aircraft that flies at a constant speed and altitude over the giographical area to be Imaged

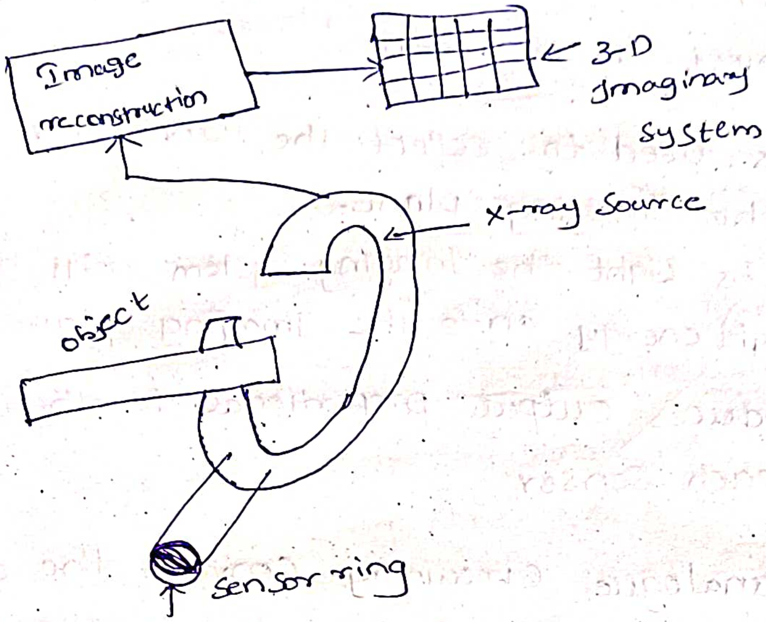


Fig: 3-D Image Acquisition using Sensor Strips

→ Sensor Strips are mounted on a Sensor ring that are used in Medical imaging to obtain 3-D Images

→ A rotating x-ray source provides illumination and Sensors opposite to the source collect x-ray energy that passes through the object

→ The output of Sensors must be processed by reconstruction algorithms whose objective is to transform the sensed data into meaningful 3-D Image.

Image Acquisition using Sensor Arrays

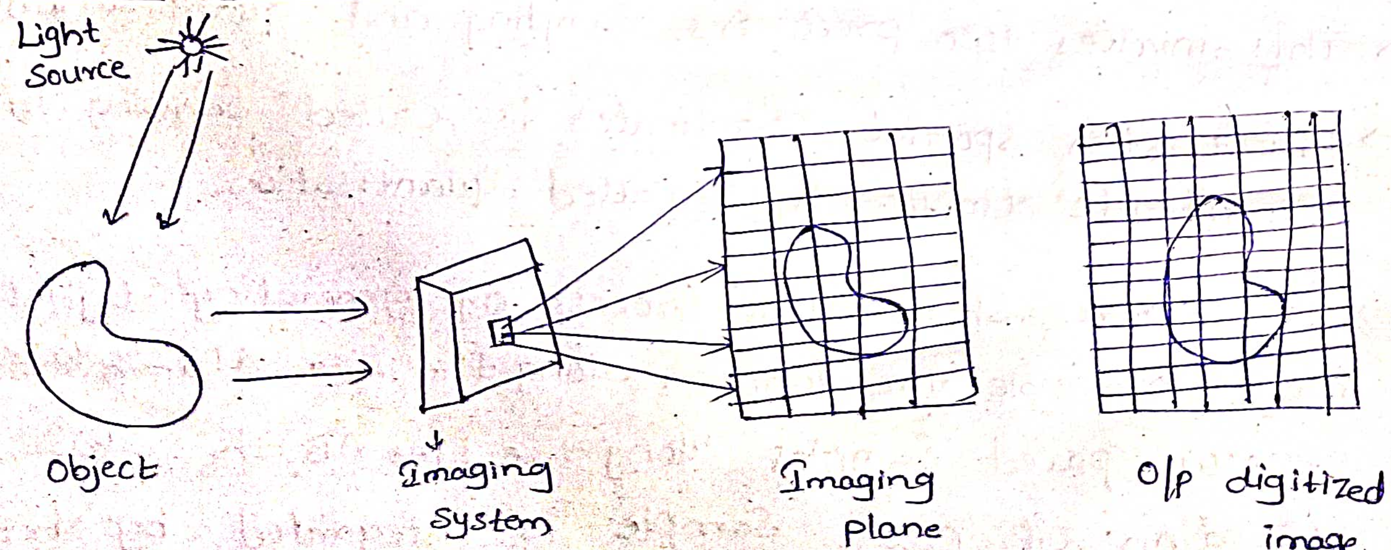


Fig: Digital image acquisition using Sensor array

-> Fig shows the energy from illumination source being reflected from a object in a Scen.

-> Imaging system is used to collect the Incoming energy and focus it on to the Imaging plane.

-> If the Illumination is Light the Imaging System will have a Lens to focus the Light energy on to the Imaging plane.

-> The Sensor array produces output proportional to the integral of Light received at each Sensor

-> The digital and analogue circuitory Converts the analog Signal into digital form by using other Sections of Imaging system.

Image Sampling and quantisation

Illustration of the process of Image Sampling and quantisation

-> The output of most Sensors is analogue voltage wave form whose amplitude and spatial behaviour are to be converted into digital form

-> To Create a digital image we need to convert analogue data into digital form.

-> This involves two processors Sampling and quantisation

-> Digitisation spaitial co-ordinates is called Sampling and amplitude digitisation is called quantisation.

-> The Figure shows the process of Generating digital image. to Sample the function along the Line AB, we take equally spaced Samples along the Line AB, the spatial Location of each Sample is integrated by vertical tips each along the Line AB in the bottom part of the figure.

→ The Samples are shown with white ~~space~~ squares at each Super-imposed on the function itself.

→ The set of these discrete Locations gives the Sampled functions.

→ The value of Samples span range of intensity Levels inorder to form a digital function, the intensity value must also be converted into digital quantities.

→ The Intensity Scale is divided into eight Levels ranging from Black to white, the analogue intensity Levels are quantised by Assiging one code to each discrete intensity Level

→ The digital Samples resulting from Sampling and quantisation are shown in the Last figure forms a digital Image

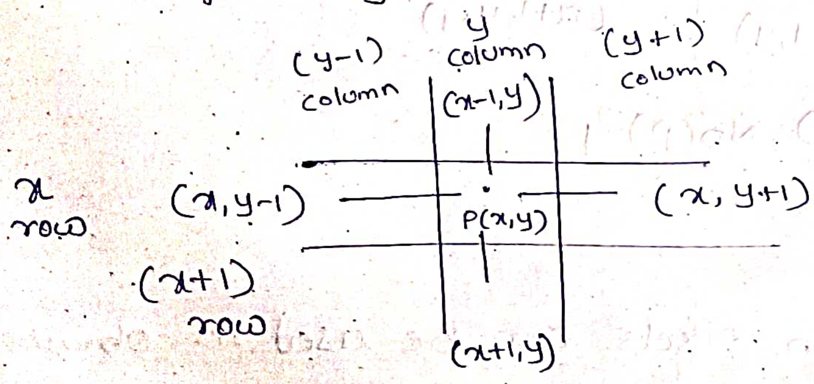
Relationship between pixels

(i) Neighbors of a pixel

A pixel 'p' at co-ordinates (x,y) has four horizontal and vertical neighbors whose co-ordinates are given by

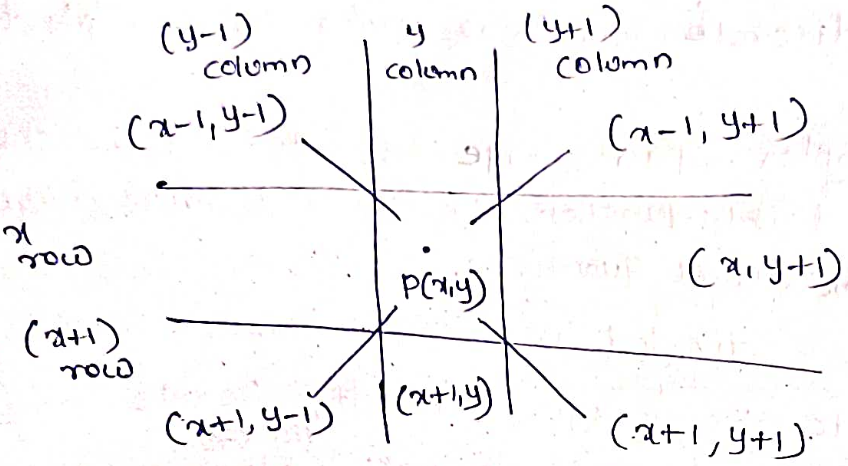
$$N_4(P) = \{(x, y-1), (x-1, y), (x, y+1), (x+1, y)\}$$ are

Called four-neighbors of p denoted by $N_4(P)$



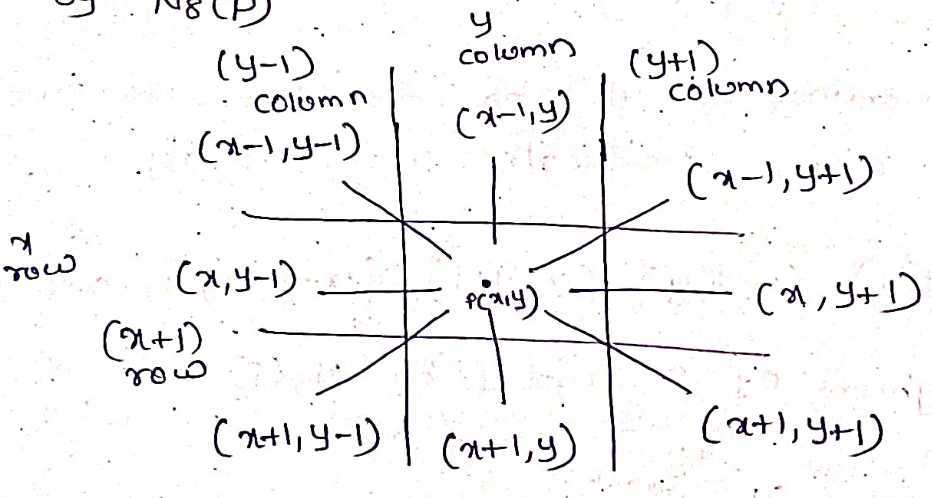
Each pixel is a unit distance from $P(x,y)$ and some of the Neighbor locations of P Lie outside the digital image if (x,y) is on the border of the Image.

→ The four diagonal neighbors of P has co-ordinates



$N_D(P) = \{(x-1, y-1), (x-1, y+1), (x+1, y-1), (x+1, y+1)\}$

→ The four neighbors of P together with Diagonal neighbors of P are said to be Eight-neighbors of P and is denoted by $N_8(P)$



$N_8(P) = \{N_4(P), N_D(P)\}$

(ii) <u>connectivity</u>

connectivity between pixels can be used in obtaing boundaries of objects and components of regions in an image.

→ Let 'V' be the set of grey level values used to define connectivity in a binary image $V = \{1\}$

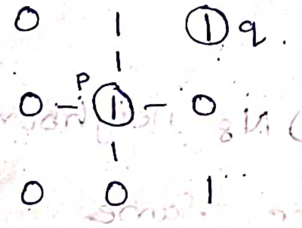
In a binary image $V = \{1\}$ with connectivity of value 1

→ V have three types of connectivity

a) 4-connectivity

Two pixels p and q with values from 'V' are four connected if q is in the set $N_4(P)$

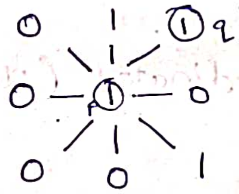
$V = \{1\}$



q is not in the set of four neighbors of p

p, q are not said to be 4-connected

Two pixel p and q with values from 'V' are eight connected if q is in the set $N_8(P)$



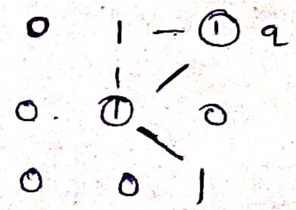
q is in the set of eight-neighbors of p and hence p, q are said to be 8-connected

c) mixed connectivity (m-connectivity)

Two pixels p and q with values from 'v' are 'm' connected if

(i) q is in $N_4(P)$ (or)

(ii) q is in $N_D(P)$ and $N_4(P) \cap N_4(q) = \phi$



Adjacency

A pixel 'p' is adjacent to pixel 'q' if they are connected.

Two Image subsets S_i and S_j are said to be adjacent if the pixel 'p' in S_i and pixel 'q' in S_j are adjacent to each other.

→ Two pixels are connected if they are adjacent in the sense if

(i) They are N_4 (or) N_D (or) N_8 neighbors and

(ii) Their gray levels are same

Path

A path from pixel 'p' with the co-ordinates (x,y) to the pixel 'q' with the co-ordinates (s,t) is a sequence of different pixels with co-ordinates

→ Sequences of pixels of co-ordinates $(x_0,y_0),(x_1,y_1),(x_2,y_2)$ ---- (x_n,y_n)

→ If $(x_0,y_0)=(x_n,y_n)$ it is said to be closed path

→ Distance Measures

For pixels p, q and z with the co-ordinates (x,y), (s,t) and (u,v) respectively and D is the distance measure or metric if

a) $D(p,q) \geq 0$

b) $D(p,q) = D(q,p)$

c) $D(p,z) = D(p,q) + D(q,z)$

Euclidean distance

Euclidean distance between P and q is defined as

$D_e(P,q) = \left[(x-s)^2 + (y-t)^2 \right]^{1/2}$

→ for this distance Measure the pixels having a distance Lessthan (or) equal to (≤) Some value 'r' from (x, y) are the points Contained in a disk of radius 'r'. Centered at (x, y)

City block distance

cityblock distance between P and q is defined as

$D_4(P,q) = |x-s| + |y-t|$

It Can be also called as D4 distance.

In this case the pixels having D4 distance from (x, y) ≤ r form a diamond Centered at (x, y)

Chess board distance (d8 distance)

Chess board distance between P and q is defined as

$D_8(P,q) = \max\left[|x-s|, |y-t| \right]$

→ In this case the pixels with d8 distance from (x, y) ≤ r form's a Square Centered at (x, y)

→ Consider the two Image Subsets S_1 and S_2 with $V = \{1\}$ determine whether these two subsets are

a. 4-adjacent
b. 8-adjacent
c. m- adjacent

	S_1				S_2		
0	0	0	0	0	0	1	1
0	0	1	0	0	1	0	0
0	0	1	0	1 q	1	0	0
0	1	1	1 p	0	0	0	0

a) since q is not the 4-neighbors of P even though their gray Levels are Same the Sets S_1 and S_2 are not four adjacent.

b) q is in 8-neighbors of P their gray Level are same, so the Sets S_1 and S_2 are 8-adjacents.

c) q is in $N_D(P)$, and $N_4(P) \cap N_4(q) = \phi$

Since these two conditions are Satisfied, P and q are m-connected

∴ The Subsets S_1 and S_2 m adjacent as they are said to be m-adjacent.

→ Consider the Image Segment Shown

a) Let $V = \{0, 1\}$ and compute the Lengths of Shortest 4, 8 & m-path between P and q, if a Particular path does not exist between these two pixles explain why

4-path

3	1	2	1 (q)
2	2	0	2
1	2	1	1
(P) 1	0	1	2

There is no path is Establishing P and q from '0' there is no possible connection using 4-connectivity to reach 'q'

8-path

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1 →	0	1	2

Shortes path - 4

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1 →	0 →	1	2

Short path - 5

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1 →	0 →	1	2

shortest path - 5

	3	1	2	1 (q)
	2	2	0	2
	1	2	1 →	1
(p)	1 →	0 →	1	2

Shortest Path - 6

∴ The Shortest 8-path Length is 4

V = {1,2}

m-path

	3	1	2 $\xrightarrow{5}$	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1 $\xrightarrow{1}$	0 $\xrightarrow{2}$	1	2

Length = 5

Let $V = \{1, 2\}$

4-path

$$\begin{array}{ccccc} & 3 & 1 & 2 & 1 \; (q) \\ & 2 & 2 & 0 & 2 \\ & 1 & 2 & 1 & 1 \\ (p) & 1 & 0 & 1 & 2 \end{array}$$

There is no path establishing between p and q from 1 to 2 using 4-connectivity.

Let $V = \{1, 2\}$

8-path

$$\begin{array}{ccccc} & 3 & 1 & 2 \rightarrow & 1 \; (q) \\ & 2 & 2 & 0 & 2 \\ & 1 & 2 & 1 & 1 \\ (p) & 1 & 0 & 1 & 2 \end{array}$$

Length = 4

$$\begin{array}{ccccc} & 3 & 1 \rightarrow & 2 \rightarrow & 1 \; (q) \\ & 2 & 2 & 0 & 2 \\ & 1 & 2 & 1 & 1 \\ (p) & 1 & 0 & 1 & 2 \end{array}$$

Length = 5

$$\begin{array}{ccccc} & 3 & 1 \rightarrow & 2 \rightarrow & 1 \; (q) \\ & 2 & 2 & 0 & 2 \\ & 1 & 2 & 1 & 1 \\ (p) & 1 & 0 & 1 & 2 \end{array}$$

Length = 5

$$\begin{array}{cccc} 3 & 1 & 2 \rightarrow & 1 \; q \\ 2 & 2 & 0 & 2 \\ 1 & 2 \rightarrow & 1 \rightarrow & 1 \\ p \; 1 & 0 & 1 & 2 \end{array}$$

Length = 6

∴ The Shortest Length = 4

4-Path

$$\begin{array}{cccc} 3 & 1 \rightarrow & 2 \rightarrow & 1 \; q \\ 2 \rightarrow & 2 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ p \; 1 & 0 & 1 & 2 \end{array}$$

Length = 6

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Length = 6

m-path

$\{1, 2\}$

q is in $N_4(P)$ or
q is in $N_D(P)$ and $N_4(P) \cap N_4(q) = \phi$

$$
\begin{array}{cccc}
3 & 1 & 2 & 1 \; (q) \\
2 & 2 & 0 & 2 \\
1 & 2 & 1 & 1 \\
(P)\; 1 & 0 & 1 & 2
\end{array}
$$

Since we can perform a path for pixels P and q by m-Connectivity. So Pixels P and q have m-path

Digital image Representations

A digital image is represented by a 2-D functions of the form $f(x,y)$, when an Image is generated from a physical process, it's intensity values are proportional to Energy radiated by a physical Source. and $f(x,y)$ must be a non-zero and finite $(0 < f(x,y) < \infty)$

→ $f(x,y)$ can be charcterized by the two components illumination and reflection and is represented by

$$f(x,y) = i(x,y)\, r(x,y)$$

$$0 < i(x,y) < \infty$$

$$0 < r(x,y) < 1$$

Total (absorption) — Total reflection

Gray Scale Images

→ Let the Intensity Level (or) value of a Monochrome Image at any co-ordinates is given by

$$l = f(x_0, y_0)$$

l lies in the range $L_{min} < l < L_{max}$ where L_{min} is equal to

$$L_{min} = i_{min} \cdot r_{min}$$

$$L_{max} = i_{max} \cdot r_{max}$$

→ In common we represent the range of gray Levels with the interval $[0, \underbrace{L-1}_{\text{max}}]$ gray level

$\uparrow$ min dark — Bright

Where the grey Level '0' represent dark / Black
'1' represend white

→ Suppose if $f(x,y)$ is represented with a 2-D array that has m-rows and N-columns and is given by

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & f(1,2) & & \\ \vdots & & & & \\ f(M-1,0) & f(M-1,1) & f(M-1,3) & \cdots & f(M-1,N-1) \end{bmatrix}$$

→ Each Element of this Matrix is called Image element. (or) picture Element (or) pixel (or) pel.

→ The Number of intensity Levels in the Image is an Integer powers of '2' i.e. $L = 2^k$.

2^{k} → no. of bits used to represent each pixel

$L = 8$ $(0, 7)$

→ The Number of bits required to store a digital Image is $b = M \times N \times K$

when $M = N$ then b is equal to N^2K an Image can have 2^k gray Levels and we refer that Image as a k-bit Image.

What is the storage requirement of

UNIT - II

Image Enhancement in Spatial domain

The Main Objective of Image Enhancement is to process an Image such that the result is more suitable than the original Image for a specific application.

→ There are two Image Enhancement categories

i) Time Domain | spatial Domain

ii) Frequential Domain

Spatial Domain Techniques

→ It is based on direct manipulation pixels in an Image.

Frequency Domain Technique

→ These are based on modifying fourier transform of an Image.

Spatial Domain Techniques

Image processing functions in the spatial domain can be Expressed as $g(x,y) = T[f(x,y)]$ where

$f(x,y)$ is the input Image

$g(x,y)$ is the processed Image

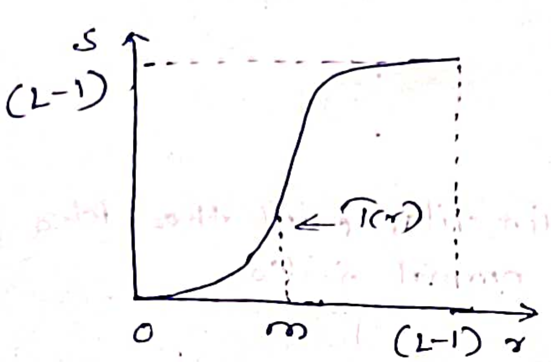
T is operator on f defined over some neighbourhood of (x,y)

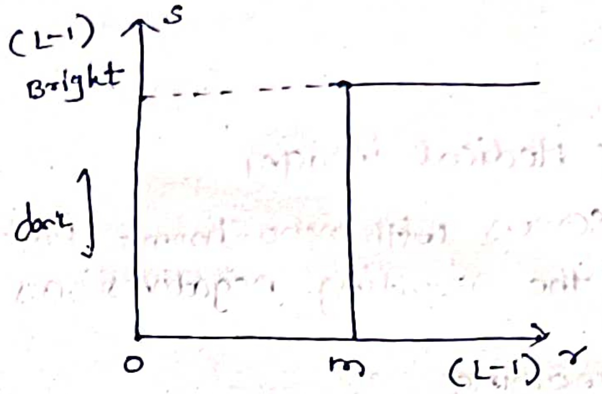
→ The simplest form T can be represented as

$$s = T(r)$$

r denotes the gray level value $f(x,y)$ and 's' denotes gray level of $g(x,y)$ at a point (x,y)

→ The effect of this transformation function is to produce an Image with high contrast than the original by darkening the vowels below 'm' and brightening the levels above 'm' in the original Image.





→ The Values 'r' below 'm' are compressed by the transformation function 's' towards black and the Values 'r' above m are expanded towards white. That produces Binary Images as shown in fig.

→ The alternative approach is based on the use of mask (or) filter (or) Tamplet (or) Window

→ A Mask is a 2-D array in which the values of filter coefficents determined the nature of process such as smoothing and sharpening etc..

→ Image Enhancement by point processing (or) Basic gray Level transfermations

1. If the enhancement at any point in an Image depends only on the gray Level at the point (Independent of nature of it's neighbours) The technique refers to point processing.

1. Image Negatives

The Negative of the digital Image can be obtained by using the transformation function.

$$s = L-1-r$$

If $r=0$ (Black) it will be appeared as in the o/p Image with gray Level

$$s = L-1-0$$

$$s = L-1 \text{ (White)}$$

If $r = L-1$ (white), then

$$s = L-1-(L-1) = 0 \text{ (Black)}$$

i-e reversing the intensity values of an Image produces the equivalent of photographic

Applications

1. Display of Medical images
2. photograph screen with mono Chrome positive film with the idea of using the resulting negatives as normal slides.

Contrast Strecthing

Low contrast Images can result from poor illumination (or) wrong setting of Lens. during Image acquisition the Idea behind contrast streching is to increase the dynamic range of gray Levels in the Image being processed.

The Locations of points (r_1, s_1) and (r_2, s_2) controlls the shape of transfer function.

if $r_1 = s_1$ and $r_2 = s_2$ the transformation function is a Linear function that produces no change in the gray Levels of the output Image.

if $r_1 = r_2$, $s_1 = 0$, $s_2 = (L-1)$ the transformation is a threshholding function that creates binary Image

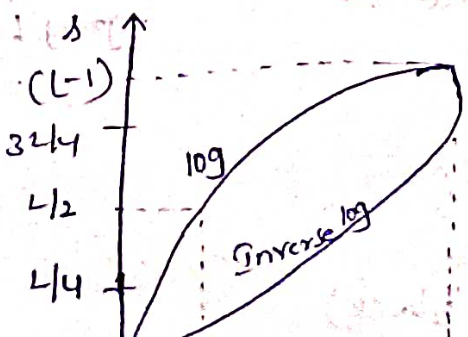
→ Log transformations

It is a Non-Linear transformation that transforms the input image into the output Image using transformation function.

$$s = c \log(1+r)$$

c is a Scaling constant

r is the gray Level of Input Image



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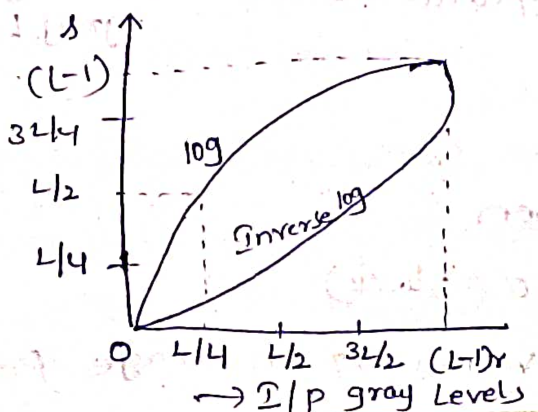


Log transformations are used in the situation when small change in the input gray Level will cause Large in the output gray Level

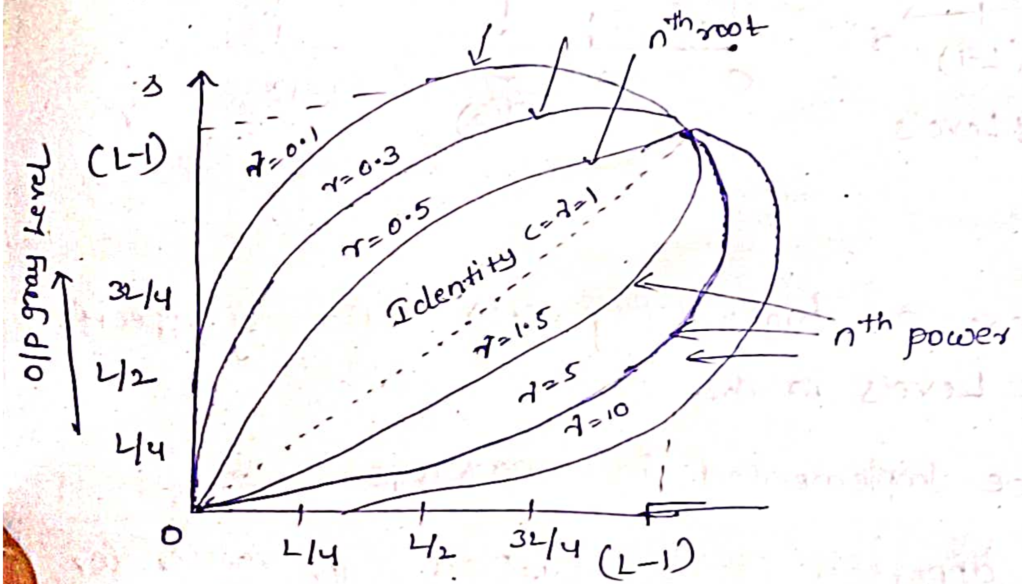
→ wide change in the input gray Level causes narrow changes in the gray Level.

Power Law transformations (or) Gamma Correction

It is also a non-linear transformation that transforms the input Image into the output Image according to the relation $\boxed{s = c \cdot r^{\gamma}}$

where C and γ are positive constants in this transformation the quality of processed Image can be varied by varing the value of γ

hence, the enhancement using power law transformation can also be called as γ connection.



when $\gamma > 1$ it will result narrow ~~wide~~ change in the output gray Level for wide ~~narrow~~ change input gray Level

when $\gamma < 1$ it will result wide change in the output gray Level for narrow change input gray Level.

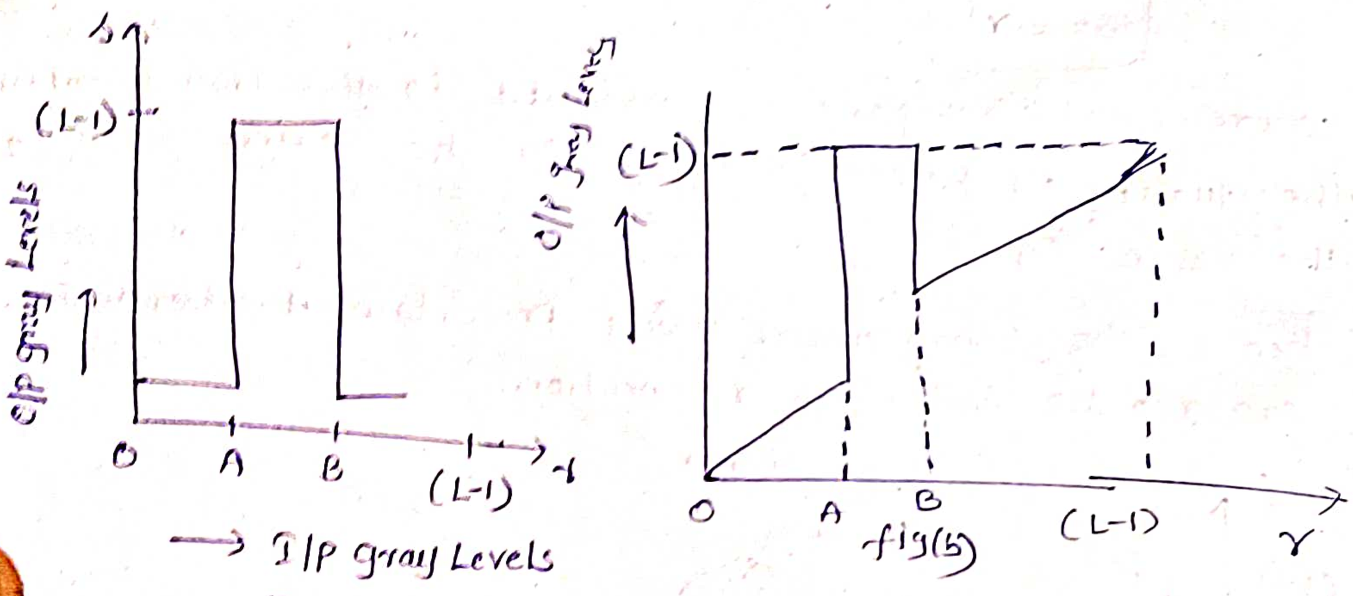
when $c = \gamma = 1$ the power ~~tra~~ Law transformation becomes Identity (or) Linear.

Applications

1. Used for Image capturing devices.
2. printing devices.
3. Display devices.
4. Scanners.
5. CRT displays

Gray Level Slicing

Highlighting the graylevel values.



fig(a) fig(b)

→ It is the process of highlighting (or) extracting specific range of gray Levels in the Image

This can be Implemented in two ways.

a. one approach is to display high value for all the gray Levels within the desired range AB and Low value for all other gray Levels as shown in fig(a).

b. The Second approach is based on highlighting the desired range of gray Levels between A and B but preserves the background.

Applications

1. Masses Of Water in Satellite Imagary
2. Enhancing flaws in x-ray Images
3. Bitplane slicing

Bit-plane Slicing

Instead of highlighting all the Intensity ranges. highlighting the contribution Made to the total Image appearence by specific bits may be sufficient.

→ suppose each pixel in an Image is represented by eight bits, this Image is Composed Of 8-1 bit plane ranging from plane '0' to plane '7'
bit plane '0' represents Least significant data present in the Image and the first five bitplane holds most significant data holds.
bit plane slicing is used for Image compression.

Advantage

1. By representing the pixel gray Level ~~Imag~~ values using bit planes we can Identify the contribution made by each and every bit plane in the Image.
This helps eliminating redundant information in the Image called as compressed Image.

(4M) **

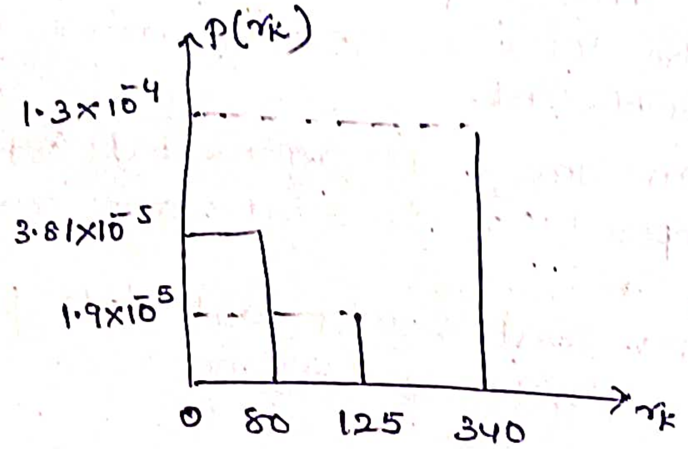
Histogram processing

Histogram
→ It is a plot between gray Levels Of the Image and their probabilities

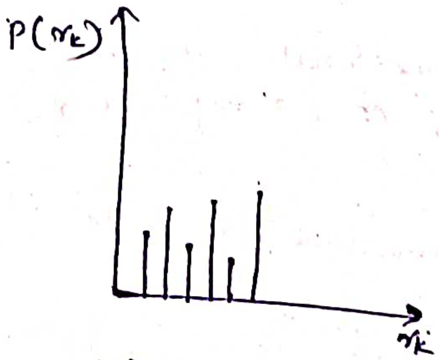
for Ex: if in a 1024x1024 image r_k represents gray Level of pixels in the Image
n_k represents no. of pixels with the gray Level r_k then

$P(r_k) = \frac{n_k}{n}$ represents the probability Of occurence Of r_k in the given Image.

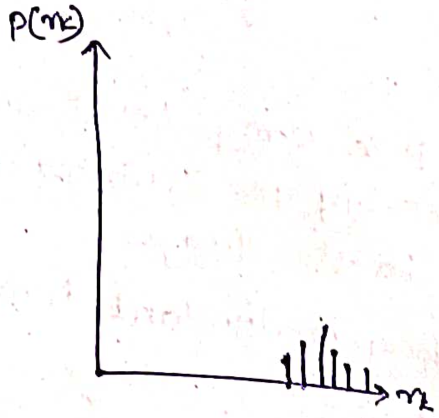
r_k	n_k	$P(r_k) = \frac{n_k}{n}$
80	4	$4/(1024 \times 1024) = 3.81 \times 10^{-6}$
125	20	$20/(1024 \times 1024) = 1.907 \times 10^{-5}$
340	40	$40/(1024 \times 1024) = 1.3 \times 10^{-4}$



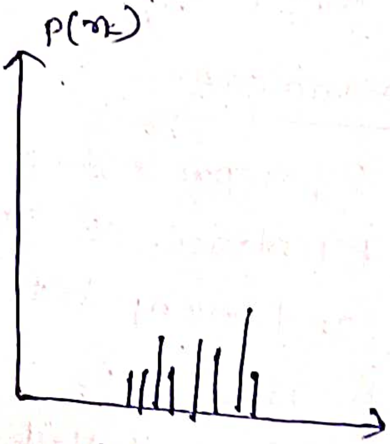
n - is the total number of pixels in the Image



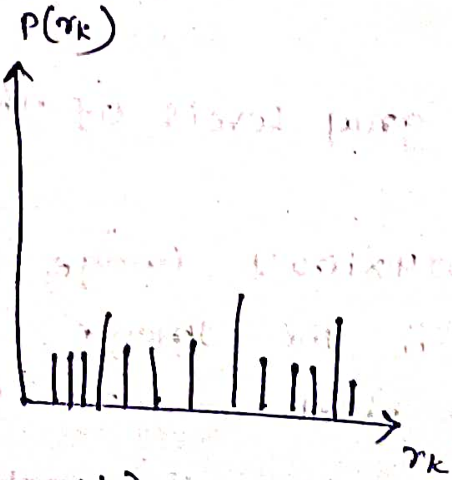
(a) Dark Image



(b) Bright Image



(c) Low contrast Image



(d) High contrast Image

→ In the dark Image the components of histogram are concentrated on the Low intensity scale this corresponds to an Image with overall dark charcteristics.

→ ~~This corresponds to any~~

→ In the bright Image the components of histogram are concentrated towards the high intensity Scale this corresponds to bright Image charcteristics.

→ An Image with Low Contrast has narrow histogram Located at the middle of intensity scale as shown in fig(c).

→ The components of histogram in the high contrast image cover wide range of intensity scale.

Rules for Histogram processing

1. $T(r)$ should be monotonically increasing in between $0 \leq r \leq (L-1)$
2. if $0 \leq r \leq (L-1)$ then $0 \leq T(r) \leq (L-1)$

Histogram processing Techniques

1. Histogram equilization
2. Histogram Specification (or) Histogram Matching

Histogram equilization

→ It is a histogram processing technique which distributes the component of an histogram uniformly throughout the gray scale of an Image.

→ The gray Level values of histogram equilized Image can be calculated by

$$s_K = (L-1) \sum_{i=0}^{K} P_i(r_i)$$

Procedure for Histogram equilization

1. ~~for~~ the gray level values of input image 'r_K' calculate the probabilities $P(r_K)$

~~2. plot for the gray~~

2. Plot the graph between r_K and $P(r_K)$ to produce histogram of the input image

3. Calculate the gray level values of histogram equilized Image using

$$s_K = (L-1) \sum_{i=0}^{K} P(r_i)$$

4. Plot a graph between the input gray values r_K and Output gray level values s_K to check for Monotonicity

5. Round off the gray level values s_K to the nearst integer

6. Map the input gray level values r_K with the round off gray levels s_K to calculate the probabilities of s_K

7. Plot the graph between s_K and $P(s_K)$ to produce histogram of equalized Image.

→

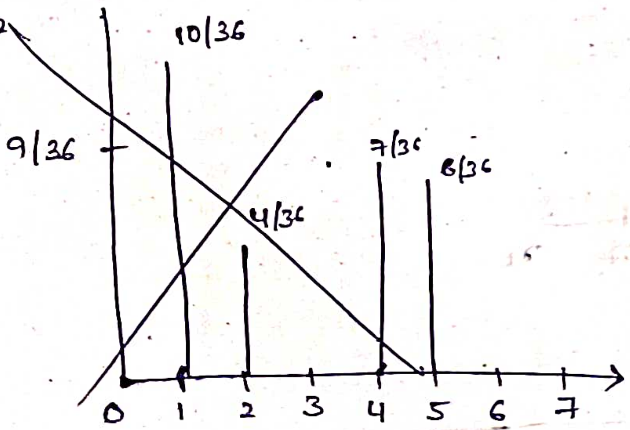
r_K	n_K
0	9
1	10
2	4
3	7
4	6
5	0
6	0
7	0

Perform Histogram equilization for the given data.

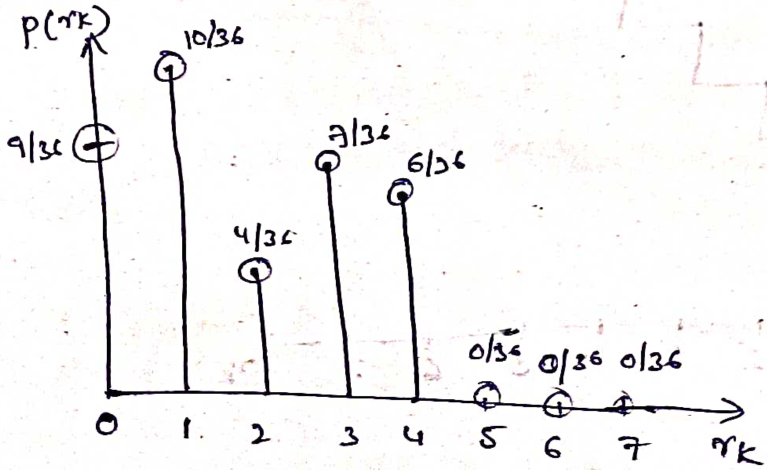
step-1

r_K	n_K	$P(r_K) = \frac{n_K}{n}$
0	9	9/36
1	10	10/36
2	4	4/36
3	7	7/36
4	6	6/36
5	0	0/36
6	0	0/36
7	0	0/36

step-2



Step-2



Step-3

$$S_K = (L-1)\sum_{i=1}^{K} P(r_i)$$

$$S_0 = (8-1)\,P(r_0) = 7\times\frac{9}{36} = 1.75$$

$$S_1 = (8-1)\sum_{i=0}^{1} P(r_i) = 7\left[\frac{9}{36}+\frac{10}{36}\right] = 3.69$$

$$S_2 = (8-1)\sum_{i=0}^{2} P(r_i) = 7\left[\frac{9}{36}+\frac{10}{36}+\frac{4}{36}\right] = 4.47$$

$$S_3 = (8-1)\sum_{i=0}^{3} P(r_i) = 7\left[\frac{9}{36}+\frac{10}{36}+\frac{4}{36}+\frac{7}{36}\right] = 5.8$$

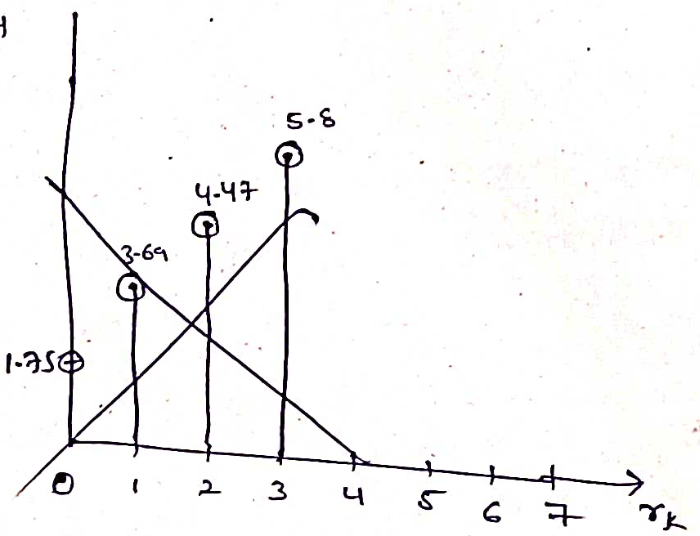
$$S_4 = (8-1)\sum_{i=0}^{4} P(r_i) = 7\left[\frac{9}{36}+\frac{10}{36}+\frac{4}{36}+\frac{7}{36}+\frac{6}{36}\right] = 7.$$

$$S_5 = (8-1)\sum_{i=0}^{5} P(r_i) = 7\left[\frac{9}{36}+\frac{10}{36}+\frac{4}{36}+\frac{7}{36}+\frac{6}{36}+\frac{0}{36}\right] = 7$$

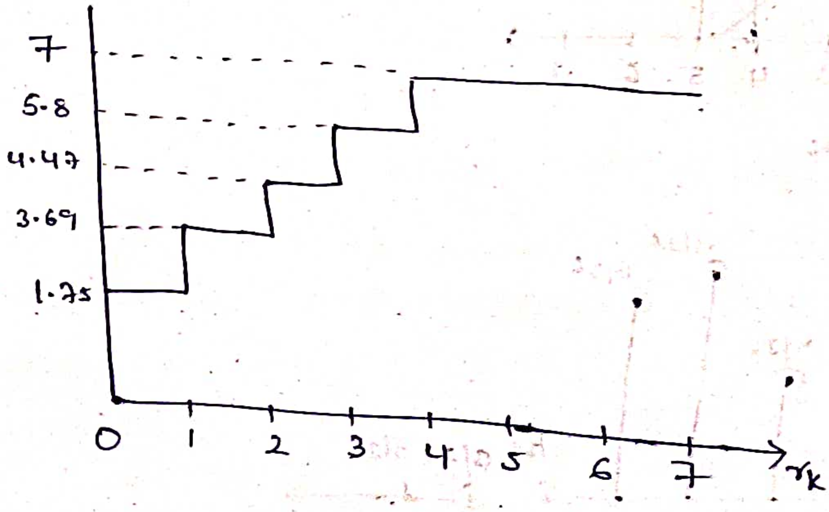
$$S_6 = (8-1)\sum_{i=0}^{6} P(r_i) = 7\left[\frac{9}{36}+\frac{10}{36}+\frac{4}{36}+\frac{7}{36}+\frac{6}{36}+\frac{0}{36}+\frac{0}{36}\right] = 7$$

$$S_7 = (8-1)\sum_{i=0}^{7} P(r_i) = 7\left[\frac{9}{36}+\frac{10}{36}+\frac{4}{36}+\frac{7}{36}+\frac{6}{36}+\frac{0}{36}+\frac{0}{36}+\frac{0}{36}\right] = 7$$

step-4



step-4



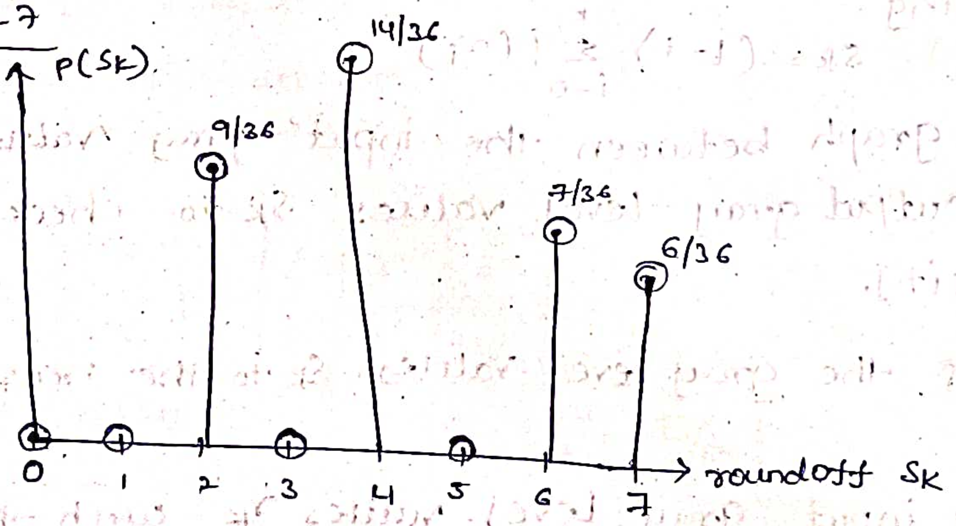
Step-5

S_K	Roundoff S_K
S_0	2
S_1	4
S_2	4
S_3	6
S_4	7
S_5	7
S_6	7
S_7	7

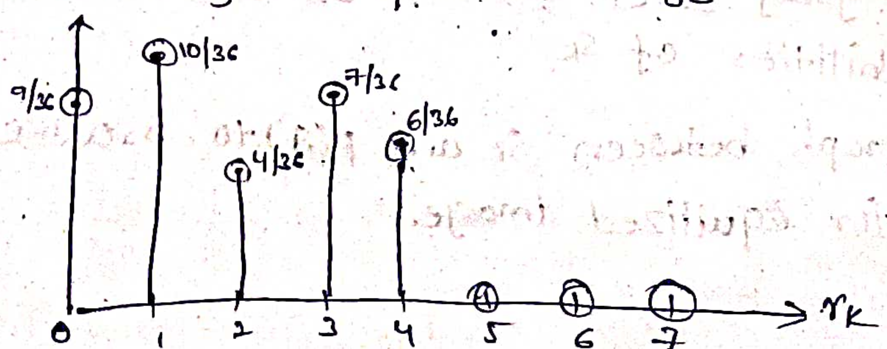
Step-6

r_k	0	1	2	3	4	5	6	7
$P(r_k)$	9/36	10/36	4/36	7/36	6/36	0	0	0
round off S_K	~~9/36~~ 2	~~14/36~~ 4	~~14/36~~ 4	~~7/36~~ 6	~~6/36~~ 7	~~6/36~~ 7	~~6/36~~ 7	~~6/36~~ 7
$P(S_K)$	9/36	14/36	14/36	7/36	6/36	6/36	6/36	6/36

Step-7



P(SK) Histogram equalized Image



If it is ask for HM → write two techniq
a. equilization
b. specification

drawback

1. Histogram equilization produces uniform distribution of Histograms throughout the gray scale Irrespective of input Image histogram.

But most of the applications in Image processing does not require uniformly distributed histogram for such cases we go for histogram specification.

Histogram specification | Histogram Matching

1. It is an Image enhancement technique that produces Output Image with the histogram as specified by the user.

2. procedure for histogram specification

1.(a) for the gray level values of the input image r_k find the probabitities of $P(r_k)$ using $\frac{n_k}{n}$

2.(b) plot the histogram of the Input Image r_k Vs $P(r_k)$

3.(c) Calculate the gray levels of histogram equilized Image using

$$S_k = (L-1)\sum_{i=0}^{k} P(r_i)$$

4.(d) plot a graph between the input gray values r_k and Output gray level values S_k to check for Monotonicity.

5.(e) Round off the gray level values S_k to the nearst integer

6.(f) Map the input gray level values r_k with the round off gray Level values S_k to calculate the probabilities of S_k

7.(e) plot the graph between S_k and $P(S)$ to produce histogram for equilized image.

8.(g) Specify the histogram which is required z_k & $P(z_k)$

9.(h) From the specified histogram calculate $G(z_k) = (L-1)\sum_{i=0}^{k} P(z_i)$.

$$G(z_k) = (L-1)\sum_{i=0}^{k} P(z_i)$$

10.(i) ~~from~~ plot the graph between z_k and $G(z_k)$ to check for Monotonicity.

11.(j) Round off $G(z_k)$ to the nearst integer

12.(k) Map the rounded of values of $G(z_k)$ to the gray Levels of equilized Image s_k such that non-zero values of $G(z_k)$ must be mapped to the nearst values of s_k.

13.(L) From the values of $G(z_k)$ estimate the gray Levels of z_k.

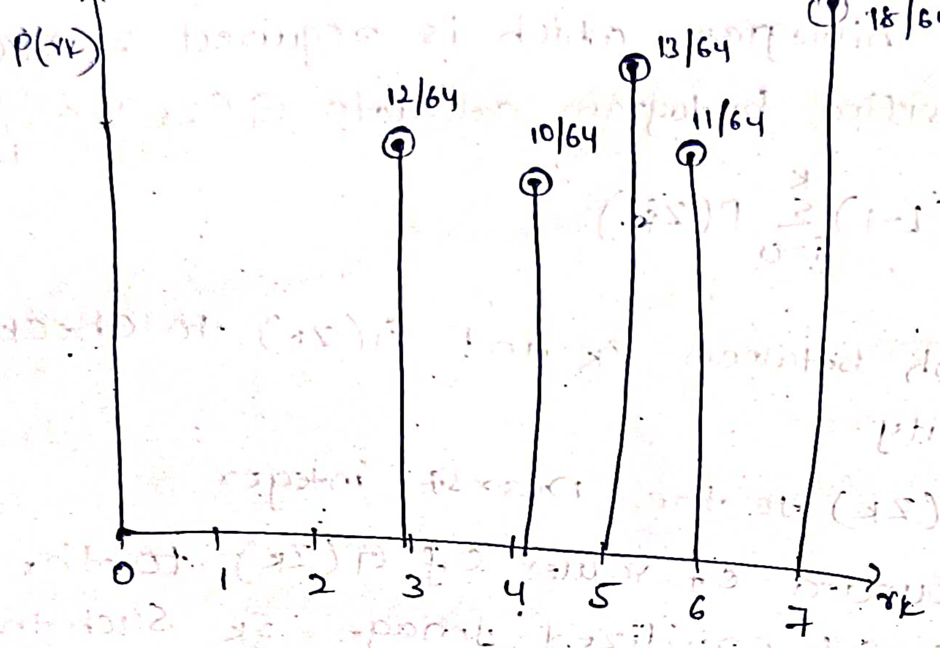
14.(M) Map the gray Levels z_k to the gray levels s_k to calculate their probabilities. $P(z_k)$. by Mapping ~~zk~~

15.(N) plot the graph between z_k and $P(z_k)$ to produce a specified histogram.

→ Let us consider a 3-bit Image of size 8×8

Ah Since it is an 8×8 Image, the total no. of pixels in that Image are 64

r_k	0	1	2	3	4	5	6	7
n_k	0	0	0	12	10	13	11	18
$P(r_k) = \frac{n_k}{n}$	$\frac{0}{64}$	$\frac{0}{64}$	$\frac{0}{64}$	$\frac{12}{64}$	$\frac{10}{64}$	$13/64$	$11/64$	$18/64$



$$s_k = (L-1)\sum_{i=1}^{k} P(r_i)$$

$$s_0 = (8-1)\sum_{i=0} P(r_0) = 7\times\frac{0}{36} = 0$$

$$s_1 = 0$$

$$s_2 = 0$$

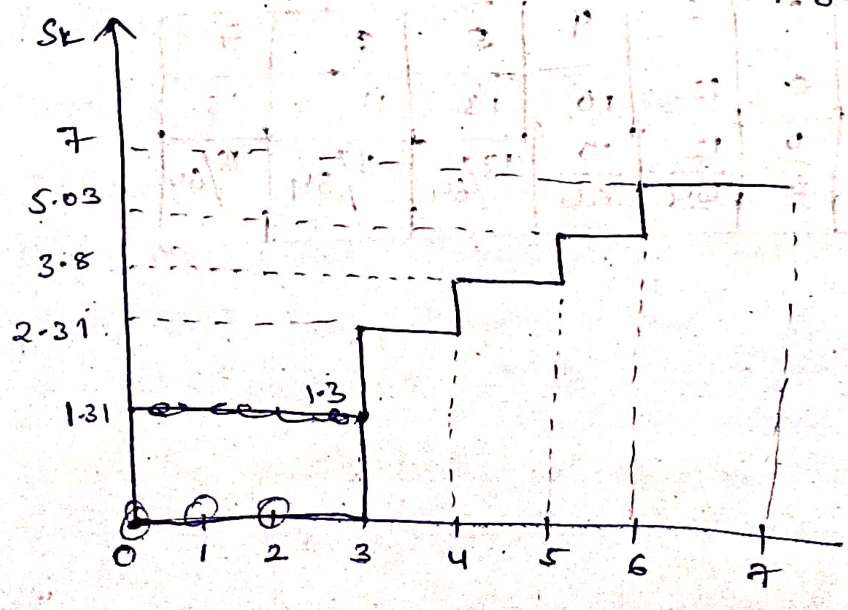
$$s_3 = (8-1)\sum_{i=0}^{3} P(r_i) = 7\times\left[\frac{12}{64}\right] = 1.3125$$

$$s_4 = (8-1)\sum_{i=1}^{4} P(r_i) = 7\times\left[\frac{12}{64}+\frac{10}{64}\right] = 2.406$$

$$s_5 = (8-1)\sum_{i=1}^{5} P(r_i) = 7\times\left[\frac{12}{64}+\frac{10}{64}+\frac{13}{64}\right] = 3.828$$

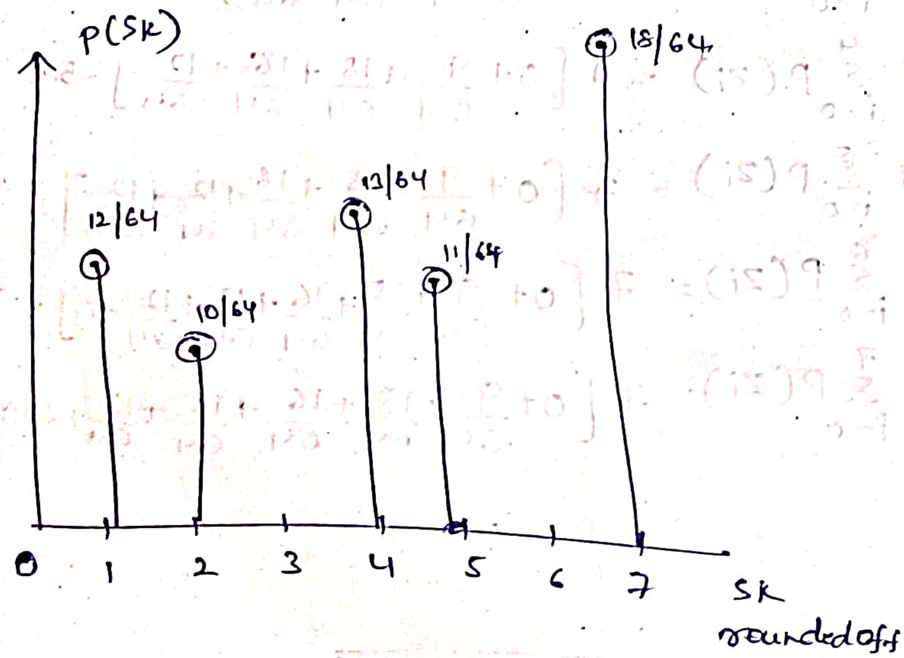
$$s_6 = (8-1)\sum_{i=1}^{6} P(r_i) = 7\times\left[\frac{12}{64}+\frac{10}{64}+\frac{13}{64}+\frac{11}{64}\right] = \cancel{0.71835}\ 5.03125$$

$$s_7 = (8-1)\sum_{i=1}^{6} P(r_i) = 7\times\left[\frac{12}{64}+\frac{10}{64}+\frac{13}{64}+\frac{11}{64}+\frac{18}{64}\right] = 7$$

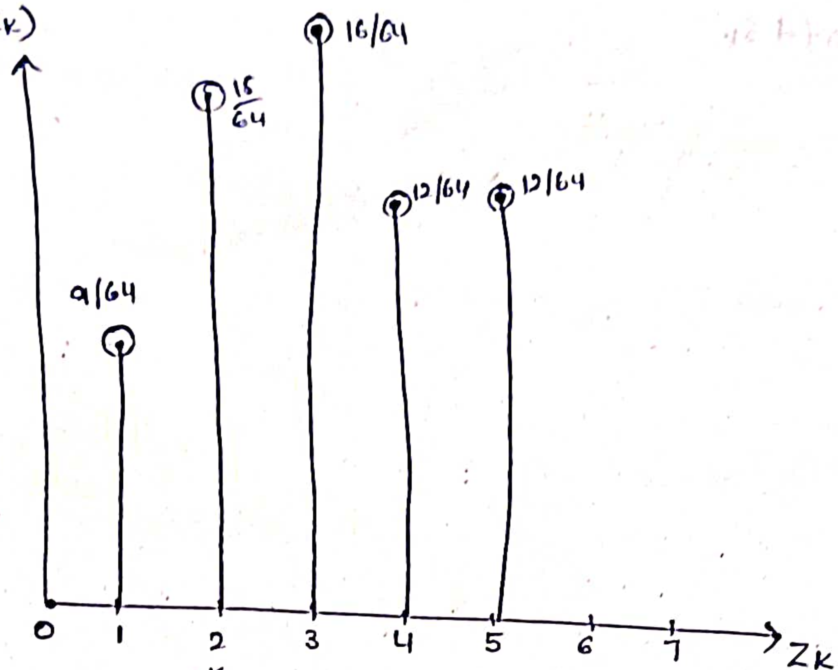


S_k	Roundoff of S_k
S_0	0
S_1	0
S_2	0
S_3	1
S_4	2
S_5	4
S_6	5
S_7	7

r_k	0	1	2	3	4	5	6	7
$P(r_k)$	0	0	0	12/64	10/64	13/64	11/64	18/64
rounded off S_k	0	0	0	1	2	4	5	7
$P(S_k)$	0/64	0/64	0/64	12/64	10/64	13/64	11/64	18/64



8.



9.

$$G(z_k) = (L-1)\sum_{i=0}^{k} P(z_i)$$

$$G(z_0) = (8-1)\sum_{i=0}^{0} P(z_0) = 7(0) = 0$$

$$G(z_1) = 7\sum_{i=0}^{1} P(z_i) = 7\left[0 + \frac{9}{64}\right] = 0.98$$

$$G(z_2) = 7\sum_{i=0}^{2} P(z_i) = 7\left[0 + \frac{9}{64} + \frac{15}{64}\right] = 2.6$$

$$G(z_3) = 7\sum_{i=0}^{3} P(z_i) = 7\left[0 + \frac{9}{64} + \frac{15}{64} + \frac{16}{64}\right] = 4.3$$

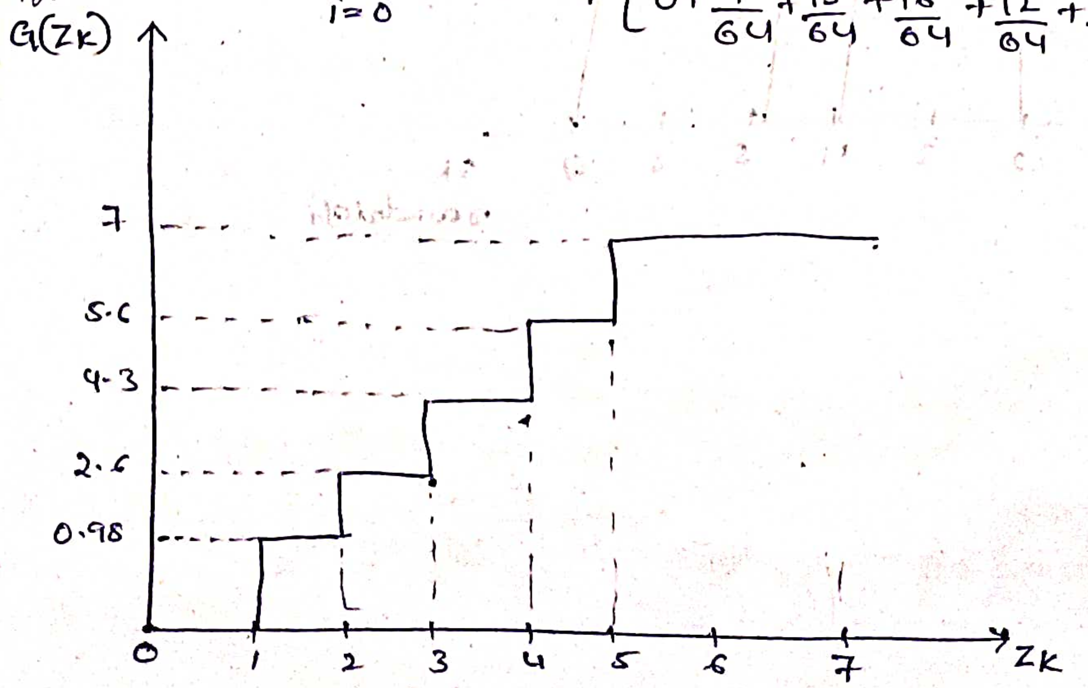
$$G(z_4) = 7\sum_{i=0}^{4} P(z_i) = 7\left[0 + \frac{9}{64} + \frac{15}{64} + \frac{16}{64} + \frac{12}{64}\right] = 5.6$$

$$G(z_5) = 7\sum_{i=0}^{5} P(z_i) = 7\left[0 + \frac{9}{64} + \frac{15}{64} + \frac{16}{64} + \frac{12}{64} + \frac{12}{64}\right] = 7$$

$$G(z_6) = 7\sum_{i=0}^{6} P(z_i) = 7\left[0 + \frac{9}{64} + \frac{15}{64} + \frac{16}{64} + \frac{12}{64} + \frac{12}{64} + 0\right] = 7$$

$$G(z_7) = 7\sum_{i=0}^{7} P(z_i) = 7\left[0 + \frac{9}{64} + \frac{15}{64} + \frac{16}{64} + \frac{12}{64} + \frac{12}{64} + 0 + 0\right] = 7$$

10.



11

$G(z_k)$	roundoff $G(z_k)$
z_0	0
z_1	1
z_2	3
z_3	4
z_4	6
z_5	7
z_6	7
z_7	7

12.

for Roundoff $G(z_k)$	Roundedoff S_k	$P(S_k)$	$P(z_k)$
0	0	0/64	0/64
1	0	0/64	0/64
3	0	0/64	0/64
4	1	12/64	12/64
6	2	10/64	10/64
7	4	13/64	13/64
7	5	11/64	11/64
7	7	18/64	18/64

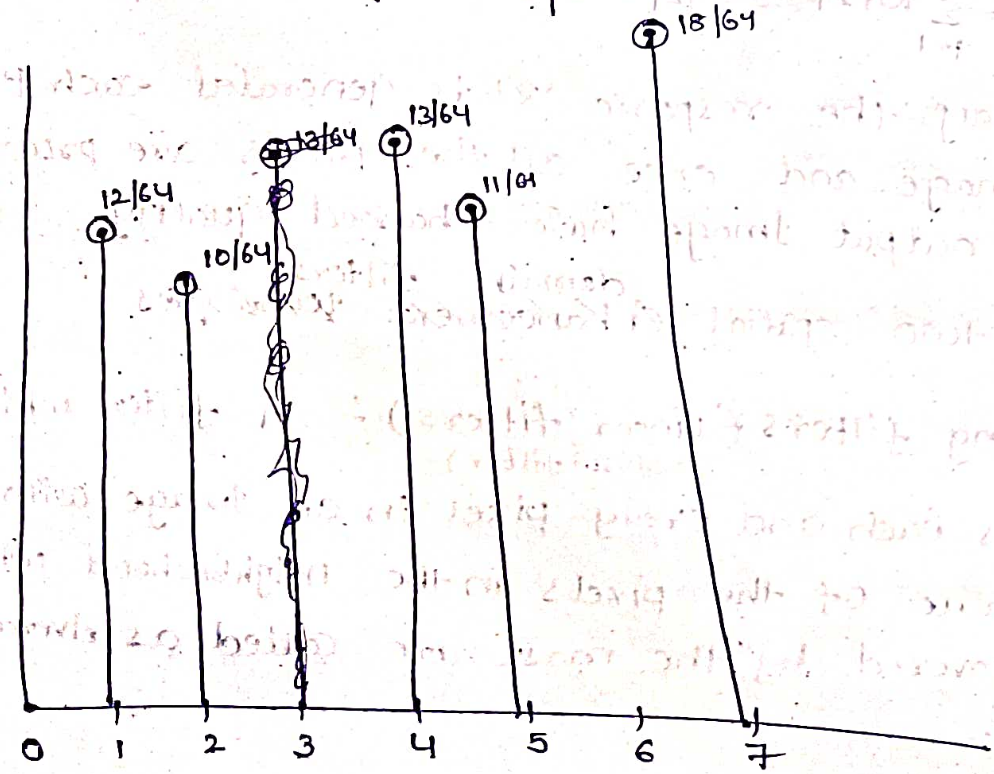


Image enhancement using arithmetic & Logical operations:

a) Image addition

Basics of Spatial filters

Spatial :-

It is a technique in which the processing of any pixels is done by considering the neighborhood of the pixel along with the pixel to be processed is called Spatial.

→ Inorder to process a pixel, Project a mask such that the pixel to be processed occupy the centre of the mask and then create partial products by Multiplying filter coefficents with the image gray level values.

finally the response 'R' can be generated by adding all the partial products and replace the gray level of the pixel with the value 'R'

$$R = \sum_{i=1}^{9} w_i z_i = w_1 z_1 + w_2 z_2 + \cdots w_9 z_9$$

In this way the response 'R' is generated each pixel in the Image. and once all the pixels are processed and the output Image have enhanced quality.

There are two spatial domain filters techniques

(i) Smoothing filters (Linear filters) :- A filter which (Lowpass filter)

(ii) replaces each and every pixel in an Image with the Avg value of the pixels in the neighborhood which are covered by the mask are called as Averaging filter.

due to averaging operation small variations in the gray levels in the Image are preserved where as sharp variation in the gray levels values are eliminated.

→ Application of Smoothing filter is noise reduction.

(i) Mean filter

(a) arithmetic mean :-

Let S_{xy} denotes set of co-ordinates in a rectangular subImage of size $m \times n$, centred at point (x,y) the arithmetic mean filter computes the Avg value of corrupted Image $g(x,y)$ in the area defined by S_{xy}.

→ This operation can be implemented using arithmetic mean filter that will smooth local variations in an Image and noise is reduced as a result of blur Image

Geometric mean filter

An Image can be restored by Geometric mean filter can be given by

$$\hat{f}(x,y) = \left[\prod_{s,t \in S_{xy}} g(s,t) \right]^{1/mn}$$

→ Each restored given pixel is given by the product of pixels in the SubImage window raised to the power of $1/mn$.. This filter achieve smoothing comparable to arithmetic mean filter with loss-Less Image details in the process

Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{s,t \in S_{xy}} \frac{1}{g(s,t)}}.$$

Contra harmonic

$$\text{for } Q=0, \; \hat{f}(x,y) = \frac{\sum_{s,t \in S_{xy}} g(s,t)^{Q+1}}{mn}$$

for $Q=-1$, $\hat{f}(x,y) = \dfrac{\sum_{s,t \in S_{xy}} g(s,t)^{(Q+1)}}{\sum_{s,t \in S_{xy}} \frac{1}{g(s,t)}}$

Ex:- I/P Image $\begin{bmatrix} 30 & 10 & 20 \\ 10 & 250 & 25 \\ 20 & 25 & 30 \end{bmatrix}$

Arithmetic mean filter $\hat{f}(x,y) = \frac{1}{9}\left[30+10+20+10+250+25+20+25+30\right]$ ← $\angle$int

$= 46.7 \cong 47$

Geometric mean filter $\hat{f}(x,y) = [30 \times 10 \times 20 \times 10 \times 250 \times 25 \times 20 \times 25 \times 30$

$= 6.25 \times 10^{11}$

Harmonic Mean $\hat{f}(x,y)$

$= \dfrac{9}{\frac{1}{30}+\frac{1}{10}+\frac{1}{20}+\frac{1}{10}+\frac{1}{250}+\frac{1}{25}+\frac{1}{20}+\frac{1}{25}+\frac{1}{30}} = 19.97 \cong 20$

contra Harmonic : $\hat{f}(x,y) = \frac{1}{9} \sum g(s,t)^{1}$

$Q=0$, $\hat{f}(x,y) = \frac{1}{mn} \sum g(s,t)^{1} = 47$

$Q=-1$, $\hat{f}(x,y) = \dfrac{1}{\frac{1}{30}+\frac{1}{10}+\frac{1}{20}+\frac{1}{10}+\frac{1}{250}+\frac{1}{25}+\frac{1}{20}+\frac{1}{25}+\frac{1}{30}} = 2.2$

Order-statistic filters (Non-Linear)

(α) → This technique depends on ordering the values of pixels contain in the Image area covered by the filter.

(a) Median filter

$$\hat{f}(x,y) = \underset{s,t \in S_{xy}}{\text{median}} \{g(s,t)\}$$

(b) Max and Min filters

$$\hat{f}(x,y) = \max_{s,t \in S_{xy}} \{ g(s,t) \}$$

Min filter

$$\hat{f}(x,y) = \min_{s,t \in S_{xy}} \{ g(s,t) \}$$

c) mid-point filter

$$\hat{f}(x,y) = _{s,t \in S_{xy}} \frac{1}{2} \left[\max[g(s,t)] + \min[g(s,t)] \right]$$

→ Let There are $m \times n$ number of pixels in S_{xy} remove the $d/2$ lowest and $d/2$ highest gray Level Values which are no.of remaining pixels are $(mn-d)$ represented by $g_r(s,t)$

→ Restored Image using alpha trimmed Mean filter is given by

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{s,t \in S_{xy}} g_r(s,t)$$

d lies in the range 0 to $mn-1$ i.e $0 < d < (mn-1)$

for $d=0$ $\hat{f}(x,y) = \frac{1}{mn} \sum_{s,t \in S_{xy}} g_r(s,t)$ is similar.

for $d = \frac{mn-1}{2}$ alpha trimmed Mean filter works as Median

↓

$$\hat{f}(x,y) = \frac{2}{mn+1}$$

Sharpning filters

→ The main aim of Sharpning filter is to preserves sharp Variations in gray levels of image & Eliminates

This filter Exhibits max positive value at the center and negative Values at other places, such that the sum of all filter coefficients = 0 is called Sharpning filter.

w_1 -1	w_2 -1	w_3 -1
w_4 -1	w_5 8	w_6 -1
w_7 -1	w_8 -1	w_9 -1

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$R = \sum_{i=1}^{9} w_i z_i$$

$$= -z_1 - z_2 - z_3 - z_4 + 8z_5 - z_6 - z_7 - z_8 - z_9$$

$$R = 8z_5 - (z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9)$$

→ Since this filter preserves sharp variations The effect of noise is more.

High boost filter

→ A filter which preserve which preserves sharp Varitions along with small variations in the gray level values in the Image called High boost filter.

Original image = Low pass image + High pass image

High pass image = original image - Low pass Image

High boost image = A (original Image) - low pass image

= (A+1-1) (original image) - low pass image

= (A-1) original image + original image - Low pass image

High boost image = (A-1) original image + High pass image.

Derivative filters

Averaging of filters over a region results blurring of details in an Image as averaging is similar to Integration and differentation exhibits sharpning the image.

There are two types of derivative filter

(i) First order derivative filter

(ii) Second order derivative filter

First order derivative filter

It must be zero in the area of constant gray level

It must be non-zero for different gray levels either step change (or) ramp change.

If $f(x,y)$ is an image then the first order derivative of $f(x,y)$ called as gradient can be written as Sum of the partial derivative of $f(x,y)$ along x-direction and partial derivative $f(x,y)$ along y-direction.

$$\text{gradient of } f(x,y) = \nabla f(x,y) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

$$\nabla f(x,y) = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}$$

along x-direction $\quad \frac{\partial f(x,y)}{\partial x} = f(x+1,y) - f(x,y)$

$$\frac{\partial f(x,y)}{\partial y} = f(x,y+1) - f(x,y)$$

$$\nabla f(x,y) = f(x+1,y) - f(x,y) + f(x,y+1) - f(x,y)$$

$$= f(x+1,y) + f(x,y+1) - 2f(x,y)$$

	y-1	y	y+1
x-1	0	0	0
x	0	-2	1
x+1	0	1	0

↑ gradient

Robert's Mask

0	-1
+1	0

-1	0
0	+1

Sobel mask

-1	-2	-1
0	0	0
1	2	1

1	0	-1
2	0	-2
1	0	-1

Prewitt's mask

-1	-1	-1
0	0	0
1	1	1

1	0	-1
1	0	-1
1	0	-1

∇f using cross differences

$$\nabla f = |z_5 - z_9| + |z_6 - z_8|$$

(or)

$$|z_5 - z_8| + |z_5 - z_6|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

2nd order derivative filters (Laplacian)

$$\frac{\partial f(x,y)}{\partial x^2} + \frac{\partial f(x,y)}{\partial y^2}$$

2nd order derivative of $f(x,y)$ along x-direction

$$\frac{\partial f(x,y)}{\partial x^2} = \left[f(x+1,y) - f(x,y)\right] + \left[f(x-1,y) - f(x,y)\right]$$

$$\frac{\partial f(x,y)}{\partial y^2} = [f(x,y+1) - f(x,y)] + [f(x,y-1) - f(x,y)]$$

$$\frac{\partial f(x,y)}{\partial x^2} + \frac{\partial f(x,y)}{\partial y^2} = f(x+1,y) - 4f(x,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)$$

	$y-1$	y	$y+1$
$x-1$	0	1	0
x	1	-4	1
$x+1$	0	1	0

Histogram Equilization

1. Find the Histogram of gray Levels of the Input Image by making the table of r_K (gray levels) and No. of Pixels with that gray Level n_K

2. For Each input gray level Calculate the cumulative sum $\sum_{i=0}^{K} n_i$ by adding no. of pixels for Each gray Level

3. Divide the cumulative Sum by the total no. of Pixels in the Image. $\frac{\sum_{i=0}^{K} n_i}{n}$

4. Scale the output by Multiplying the Value obtained in Step 3 with the Maximum grey Level Value.

5. Round off the output of Step-4 to the nearst integer Value.

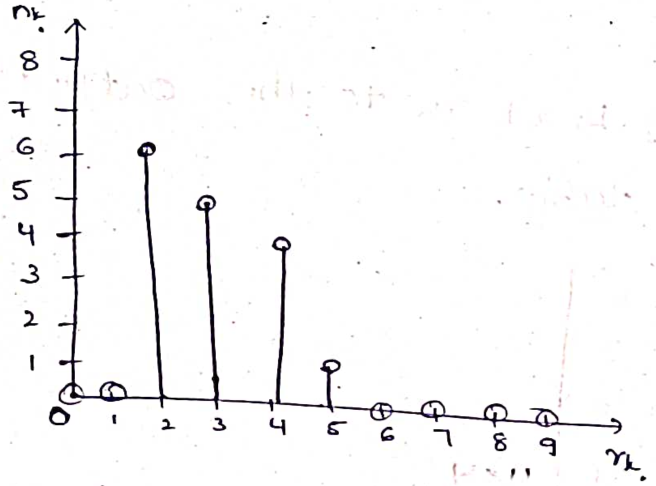
6. Map the input gray Level r_K to the output level S_K and draw the table.

$$\begin{bmatrix} 2 & 3 & 3 & 2 \\ 4 & 2 & 4 & 3 \\ 3 & 2 & 3 & 5 \\ 2 & 4 & 2 & 4 \end{bmatrix}_{4\times4}$$

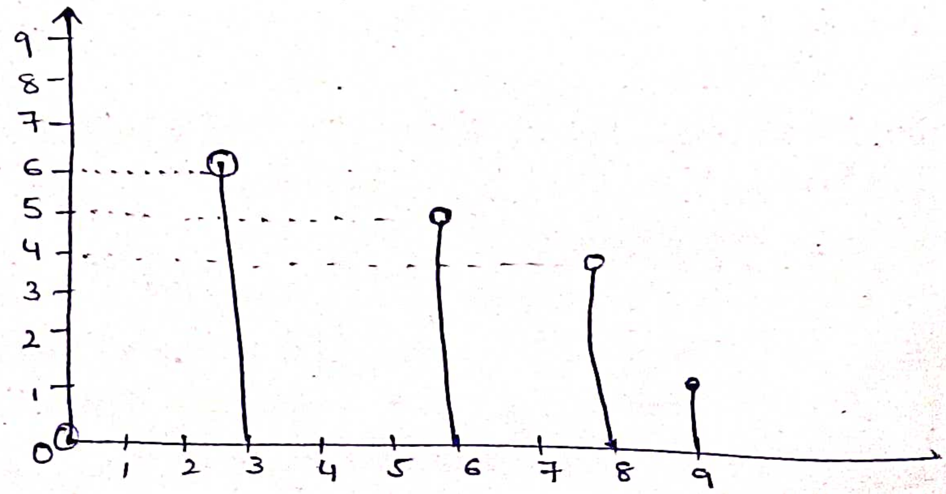
For the given 4x4 Image having the gray levels between [0,9] plot histogram of the Equalized Image.

r_k	n_k	$\sum_{i=0}^{k} n_i$	$\sum_{i=0}^{k} \frac{n_i}{n}$	$\left[\sum_{i=0}^{k} \frac{n_i}{n}\right] * r_{k(max)}$	(6) round off s_k
0	0	0	$\frac{0}{16}=0$	0	0
1	0	0	$\frac{0}{16}=0$	0	0
2	6	6	$\frac{6}{16}=0.37$	$0.37 \times 9 = 3.33$	3
3	5	11	$\frac{11}{16}=0.68$	$0.68 \times 9 = 6.12$	6
4	4	15	$\frac{15}{16}=0.93$	$0.93 \times 9 = 8.37$	8
5	1	16	$\frac{16}{16}=1$	$1 \times 9 = 9$	9
6	0	16	$\frac{16}{16}=1$	$1 \times 9 = 9$	9
7	0	16	$\frac{16}{16}=1$	$1 \times 9 = 9$	9
8	0	16	$\frac{16}{16}=1$	$1 \times 9 = 9$	9
9	0	16	$\frac{16}{16}=1$	$1 \times 9 = 9$	9

step-2



r_K	0	1	2	3	4	5	6	7	8	9
S_K	0	0	3	6	8	9	9	9	9	9

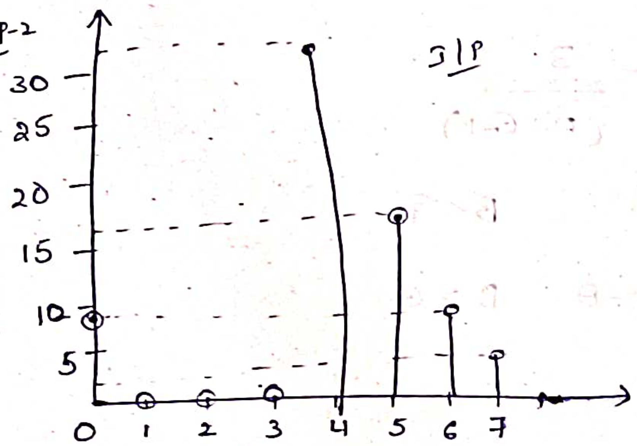


②

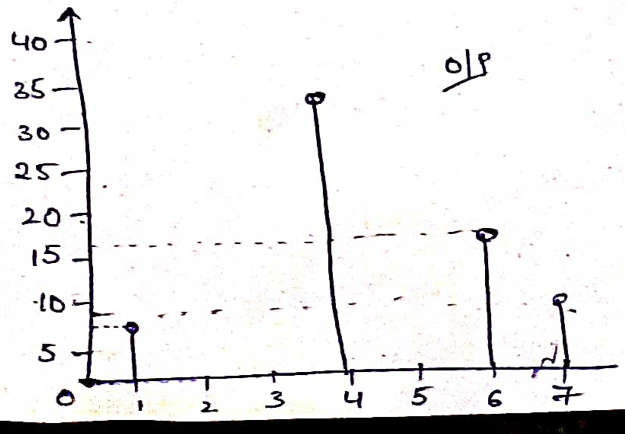
r_k	0	1	2	3	4	5	6	7	(0,7)
n_k	8	0	0	0	31	16	8	1	→ n=64

r_k	0	1	2	3	4	5	6	7
n_k	8	0	0	0	31	16	8	1
$\sum_{i=0}^{k} n_i$	8	8	8	8	39	55	63	64
$\sum_{i=0}^{k} \frac{n_i}{n}$	$\frac{8}{64}=0.11$	$\frac{8}{64}=0.11$	$\frac{8}{64}=0.11$	$\frac{8}{64}=0.11$	$\frac{39}{64}=0.61$	$\frac{59}{64}=0.86$	$\frac{63}{64}=0.98$	$\frac{64}{64}=1$
$\sum_{i=0}^{k} \frac{n_i}{n} \times r_k(\max)$	0.87	0.87	0.87	0.87	4.27	6.02	6.86	7
rounded off S_k	1	1	1	1	4	6	7	7

Step-2



r_k	0	1	2	3	4	5	6	7
S_k	1	1	1	1	4	6	7	7



HSI to RGB

(i) RG Sector

$0^\circ \le H < 120^\circ$

$H = H - 0^\circ$

$B = I(1-S)$

$$R = I\left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$$

$G = 3I - (R+B)$

(ii) GB Sector

$120^\circ \le H < 240^\circ$

$H = H - 120^\circ$

$R = I(1-S)$

$$G = I\left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$$

$B = 3I - (R+G)$

(iii) BR Sector

$240^\circ \le H \le 360^\circ$

$H = H - 240^\circ$

$G = I(1-S)$

$$B = I\left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$$

$R = 3I - (G+B)$

RGB to HSI

$$R = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$Z = \frac{Z}{X+Y+Z}$$

$X+Y+Z = 1$

$$I = \frac{1}{3}(R+G+B)$$

$$S = 1 - \frac{3}{(R+G+B)}$$

$$H = \begin{cases} \theta & B < G \\ 360 - \theta & B \ge G \end{cases}$$

$$\theta = \cos^{-1}\left[\frac{\frac{1}{2}(R-G) + (R-B)}{\sqrt{(R-G)^2 + (R-B)(G-B)}}\right]$$

RGB $\rightarrow$ HSI

(29, 104, 215)

X Y Z

$x = R$

$y = G$

$z = B$

$$R = \frac{29}{29+104+215} = \frac{29}{348} = 0.083$$

$$G = \frac{104}{29+104+215} = \frac{104}{348} = 0.299$$

$$B = \frac{215}{29+104+215} = \frac{215}{348} = 0.618$$