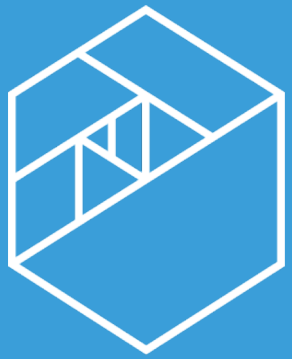

Lesson 8: Integrals

Introduction

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Lecture Overview:

Goals of the lecture:

1. Understand the anti-derivatives (Integral) and its properties

Integrals

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Indefinite Integrals

Definition:

Indefinite integrals are the anti-derivatives

$$\int f(x)dx = F(x) + C$$

$$\frac{d}{dx}(F(x) + C) = f(x)$$

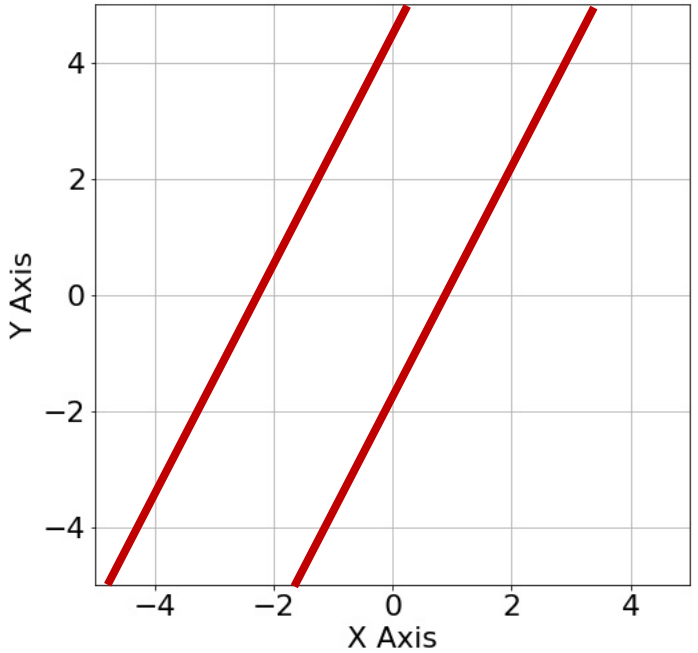
Constant C

Definition:

Indefinite integrals are the anti-derivatives

$$\int f(x)dx = F(x) + C$$

$$\frac{d}{dx}(F(x) + C) = f(x)$$



Integrals (Cheat Sheet)

Polynomials

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

Radicals

$$\int \sqrt[m]{x^n} \, dx = \int x^{\frac{n}{m}} \, dx = \frac{x^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C$$

Exponentials

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln(a)} + C$$

Logarithms

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

Integrals (Cheat Sheet)

Trigonometric

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \csc^2(x)\,dx = -\cot(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

Integrals (Cheat Sheet)

Inverse Trigonometric

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

Example: Indefinite Integrals

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int x^{20} dx = \frac{x^{20+1}}{20+1} + C = \frac{x^{21}}{21} + C$$

Rules for Integrals

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Rules for Integrals

Definition:

Addition:

$$\int f(x) \pm g(x)\, dx = \int f(x)dx \pm \int g(x)dx$$

Multiplication:

$$\int cf(x)dx = c\int f(x)dx$$

Rules for Integrals (Advanced)

Definition:

Integration by parts:

$$\int u(x)v(x)\,dx = u(x)\int v(x)dx - \int u'(x)\left(\int v(x)dx\right)dx$$

Integration by substitution:

$$\int f(u)du = \int f(g(x))g'(x)dx$$

Problem 1:

Problem 1: Calculate the integral.

$$\int x^3 - 2x \, dx$$

Problem 1:

Problem 1: Calculate the integral.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int x^3 - 2x \, dx$$

$$\int x^3 \, dx = \frac{x^{3+1}}{3+1} = \frac{x^4}{4}$$

$$\int -2x \, dx = -2 \int x \, dx =$$

$$-2 \frac{x^{1+1}}{1+1} = -2 \frac{x^2}{2} = -x^2$$

$$\int x^3 - 2x \, dx = \frac{x^4}{4} - x^2 + C$$

QUESTIONS?