**Circular Diffusion Models**

We test three alternative versions of the circular diffusion model that express different hypotheses about the process of memory retrieval. The first of these was designed to be analogous to the continuous model of source memory presented in Harlow and Donaldson (2013) and was implemented as a circular diffusion process with trial-to-trial drift rate variability. This model variant will subsequently be referred to as the *continuous diffusion* model. Drift rate variability was set to be equal in both dimensions of the 2-dimensional space, but different between imageability conditions. There were four parameters for mean drift rates (*μ1x*, *μ2x*, *μ1y*, *μ2y*),which characterize the components of the drift rate in the *x* and y directions in the low (*μ1*) and high (*μ2*) word imageability conditions. Because the model was fitted to the distribution of report errors, which is centered on zero degrees, the dominant component of the drift rate was expected to be in the *x* direction. The second component of drift was included to allow for the possibility of drift bias. The decision criterion was represented by *a*, which had uniform variability across trials with range *sa*. There were also two standard deviation parameters *η1* and *η2*, which described the standard deviations of the drift rates in the low and high imageability conditions respectively. The standard deviations of the drift rates were assumed to be the same in the *x* and *y* directions. Finally, there was a non-decision time parameter, *Ter­*. Like the standard diffusion model, the circular model assumes that RT is the sum of the decision time and a time for other (encoding and response) processes. These parameters are summarized in Table 2.

The second model variant embodied the thresholded property favored by Harlow and Donaldson (2013), and will be referred to as the *threshold diffusion* model. This was implemented as a mixture of two diffusion processes: one with positive drift and no between-trial drift variability, and a second component that was modelled as a diffusion process with zero drift. The zero-drift process provides a diffusion process implementation of a guessing process, in which the decision process is driven only by noise. Unlike “guessing” in its classical sense, which accounts for accuracy but not RT, the zero-driven process is able to predict both accuracy and RT. Mixing proportions for the two processes were allowed to vary between the word imageability conditions. This model had ten free parameters. Four mean drift rates parameters were shared with the continuous model (*μ1x*, *μ2x*, *μ1y*, *μ2y*), with the same interpretation, as well as *Ter­,* the non-decision time parameter. There were two parameters for the mixing proportions between information-driven and guessing components, one for the low imageability condition (*π1*) and another for the high imageability condition (*π2*). The decision criterion was estimated separately for the information-driven component (*a1*) and the guessing component (*a2*). Both processes shared a parameter for criterion variability (*sa*).

The third model was a synthesis of both the continuous and threshold diffusion models, which was identical to the threshold diffusion model, but also allowed for drift rate variability between trials, and can therefore be regarded as a hybrid model variant which incorporates the continuous and threshold diffusion models. This, the *hybrid diffusion* model, had 11 free parameters, all of which are displayed in Table X.

|  |  |
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| Table X  Symbols and definitions of free parameters estimated in diffusion model variants | |
| Symbol | Parameter |
| *μ1x* | Mean drift, low condition, x direction |
| *μ2x* | Mean drift, high condition, x direction |
| *μ1y* | Mean drift, low condition, y direction |
| *μ2y* | Mean drift, high condition, y direction |
| *η1* | Drift variability, low condition |
| *η2* | Drift variability, high condition |
| *a1* | Decision criteria, information-driven component |
| *a2* | Decision criteria, guessing component |
| *π1* | Mixing proportion, low condition |
| *π2* | Mixing proportion, high condition |
| *Ter* | Non-decision time |
| *sa* | Criterion variability |

*Note.* Not all parameters were estimated for all three models. The continuous diffusion model did not include a mixed guessing process, and therefore lacked *a2*, *π1* and *π2.* The threshold diffusion model did not have drift variability and lacked *η1* and *η2*. The mixture diffusion model included all parameters listed.

The three variants of the circular diffusion model were each fit to data at an individual level. The Bayesian Information Criterion (BIC) and Log Likelihood (LL) for the three models’ fits to each participant are shown in Table X.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Table X | |  | |  | |  | |
| Bayesian Information Criterion (BIC) and Log Likelihoods (LL) for Fits of the Models to Individual Data | | | | | | | |
| Participant | | Continuous | | Threshold | | Mixture | |
| LL | BIC | LL | BIC | LL | BIC |
| High Precision | 2 | 1091.20 | 2244.71 | 960.56 | **1989.65** | 959.85 | 2000.69 |
|  | 3 | 928.05 | **1917.79** | 1053.74 | 2175.35 | 1008.93 | 2098.07 |
|  | 4 | 1838.26 | 3745.60 | 1555.24 | **3186.47** | 1548.51 | 3186.83 |
|  | 5 | 1084.54 | 2231.48 | 967.91 | **2004.47** | 988.83 | 2058.78 |
|  | 6 | 763.05 | **1587.72** | 761.02 | 1589.84 | 762.36 | 1604.85 |
|  | 7 | 880.91 | 1823.91 | 700.20 | **1468.69** | 813.51 | 1707.72 |
|  | 8 | 629.03 | 1320.74 | 338.06 | 745.06 | 314.01 | **709.50** |
|  | 9 | 2372.80 | 4808.12 | 1187.18 | **2443.14** | 1943.53 | 3968.34 |
|  | 10 | 992.03 | 2045.77 | 906.81 | **1881.52** | 1101.11 | 2282.46 |
|  | 11 | 1422.89 | 2908.03 | 1333.17 | 2734.81 | 1119.32 | **2319.56** |
|  | 12 | 854.69 | 1771.85 | 471.71 | **1012.14** | 482.95 | 1047.12 |
|  | 15 | 1185.45 | 2433.16 | 1594.20 | 3256.88 | 1175.89 | **2432.73** |
|  | 16 | 967.46 | 1996.40 | 892.49 | 1852.62 | 876.59 | **1833.12** |
|  | 17 | 992.55 | **2044.90** | 998.73 | 2063.24 | 1082.86 | 2243.44 |
|  | 18 | 1050.21 | 2163.03 | 889.31 | **1847.49** | 1034.86 | 2151.12 |
|  | 19 | 1478.20 | 3017.93 | 878.96 | **1825.60** | 1297.40 | 2674.79 |
|  | 20 | 1839.62 | 3740.91 | 1828.97 | 3725.77 | 1350.13 | **2780.45** |
|  |  |  |  |  |  |  |  |
| Low Precision | 1 | 777.82 | 1609.34 | 663.50 | 1386.08 | 599.46 | **1268.74** |
|  | 13 | 3284.20 | 6629.72 | 1803.14 | **3673.73** | 2782.25 | 5644.21 |

Bold values indicate the lowest BIC for each fits to each participants’ data.

On the basis of the BIC, the threshold and mixture models are favored over the continuous model for most participants (exceptions being participants 3, 6 and 17). Between the threshold and mixture models, the threshold model is generally preferred as the more parsimonious model, which suggests that the inclusion of trial-to-trial drift rate variability does not improve the fit of the diffusion model to data enough to justify the additional complexity it introduces. In contrast, the poor relative performance of the continuous model, which is distinguished by the absence of a zero-drift component,

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Both the threshold and the mixture models consistently outperformed the continuous model without guessing. This strongly suggests that participants sometimes do respond in a no-information guessing state, which is mixed with a distribution of responses driven by information which is centered on the target location. In comparing the two models which utilize a threshold, the mixture model appears to fit the data of most participants better than the pure threshold model, but this advantage is very slight and is outweighed by the penalty for complexity applied by the BIC for the two additional parameters allowing for trial-to-trial variability, as shown by the mixture model having the lowest negative log likelihood (LL) and the threshold model having the lowest BIC for most participants (Table X). This suggests that the addition of drift variability does not improve the fit of the threshold model enough to justify the additional complexity introduced into the model.