

This problem set is individual and worth a total of 105 points. You do **not** need to submit anything for this problem set, the material is solely meant to help you better understand the course material and prepare for the exam.

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1. Given the following recurrences and their base cases, find the closed-form equivalent (use recurrence unrolling):

(a) [10 points]

$$T(1) = 1; T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$T(n) = n \log_2 n + n$$

(b) [10 points]

$$T(0) = 1; T(n) = T(n-1) + 2^n$$

(n^{th} partial sum of a geometric sequence" formula required)

$$T(n) = 2^{n+1} - 1$$

(c) [10 points]

$$T(1) = 1; T(n) = T\left(\frac{n}{3}\right) + 1$$

$$T(n) = \log_3 n + 1$$

2. Determine the Big Θ runtime of Mergesort given the following input

(a) [5 points] Input in non-decreasing order (already sorted)

$$\Theta(n \log_2 n) \text{ or } \Theta(n) \text{ (depends if you stop when already sorted)}$$

(b) [5 points] Input in non-increasing order (reverse sorted)

$$\Theta(n \log_2 n)$$

(c) [5 points] Identical input (all the same number)

$$\Theta(n \log_2 n)$$

3. [5 points] What is one major disadvantage of Mergesort?

It is not in-place

4. [5 points] What does it mean for a sorting algorithm to be stable?

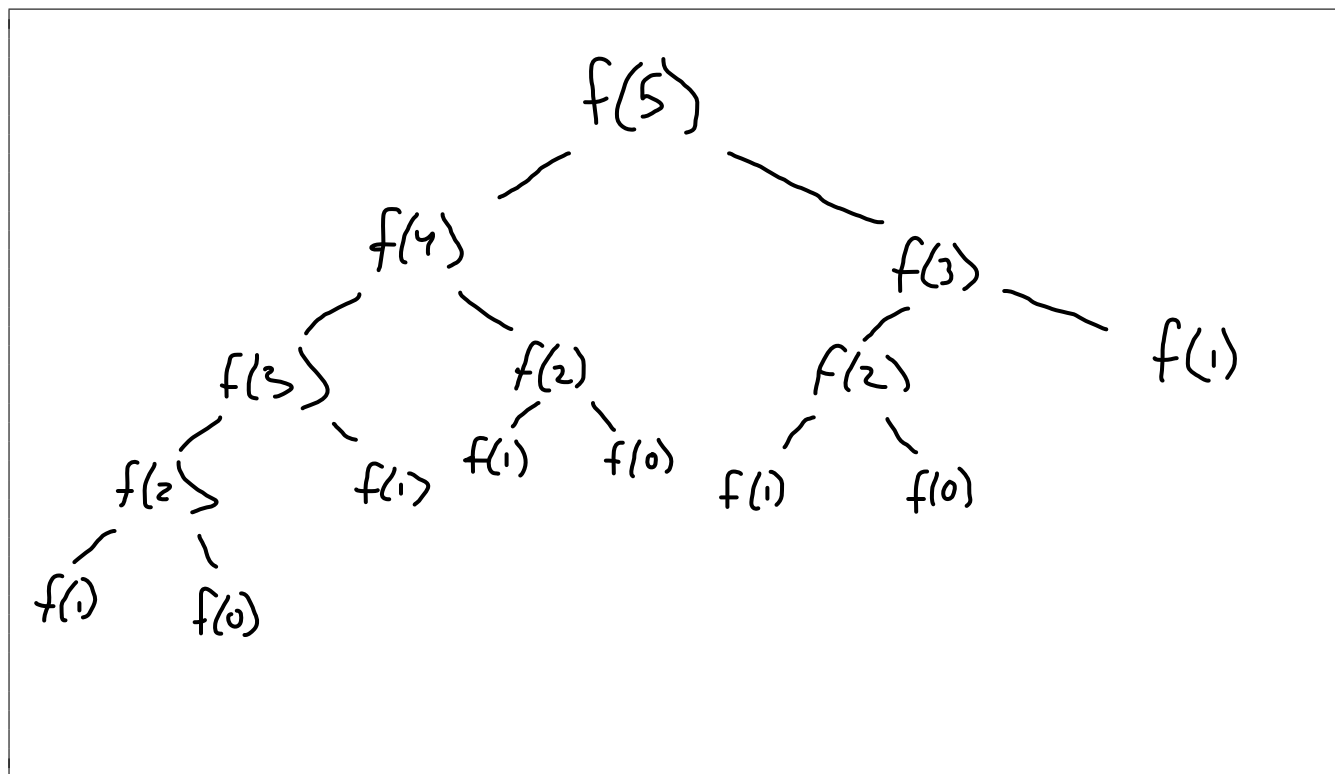
It Preserves the order of equal elements (Equal elements never pass each other)

5. [5 points] Is Mergesort a stable algorithm? Which part(s) of the algorithm can you reference to prove this?

Yes. Nothing is moved by the recursive calls and inside the merge

function if $\text{left}[i] \leq \text{right}[i]$ we choose left for equal elements.

6. [10 points] Draw the recursion call tree for the Recursive implementation of Fibonacci Sequence. Draw the tree for $n = 5$. If you need a refresher on this function, refer to lab-01 readme.



7. [5 points] For the same function given $n = 7$, give the exact number of function calls.

$$(2 * f(n+1)) - 1$$

8. [5 points] What does FIFO stand for? Which data structure supports FIFO operations?

"First in, first out". Queues support FIFO operations.

9. [5 points] What does LIFO stand for? Which data structure supports LIFO operations?

"Last in, first out". Stacks support LIFO operations.

10. [20 points] Given a pointer q to a stack of double values, write a function `double average(Stack *q)` which will return the average of the elements in q . You are provided with the public methods from the Stack class: `void push(double)`, `double top()`, `int size()`, and `void pop()`. q must remain the same after returning from the function.

```
double average(Stack *q){
    double avg=0.0;
    Stack<double> temp;
    int n = q->size();
    for(int i=0; i<n; i++){
        temp.push(q->top());
        avg += q->top();
        q->pop();
    }
    for(int j=0; j<n; j++){
        q->push(temp.top());
        temp.pop();
    }
    return avg/n;
}
```