This problem set is individual and worth a total of 310 points. You do **not** need to submit anything for this problem set, the material is solely meant to help you better understand the course material and prepare for the exam.

Your Name: Eben Aceto

1. [5 points] Which of the three following options is the **worst** choice for a pivot for your partition function?

   (a) Always choose a random element to be your pivot.

   (b) Always choose element at index 0 to be your pivot.

   (c) Always choose the middle element to be your pivot.

2. [5 points] What makes this a poor choice of partition?

   It is too easy for it to yeild worst case performance given common inputs (reverse-ordered)

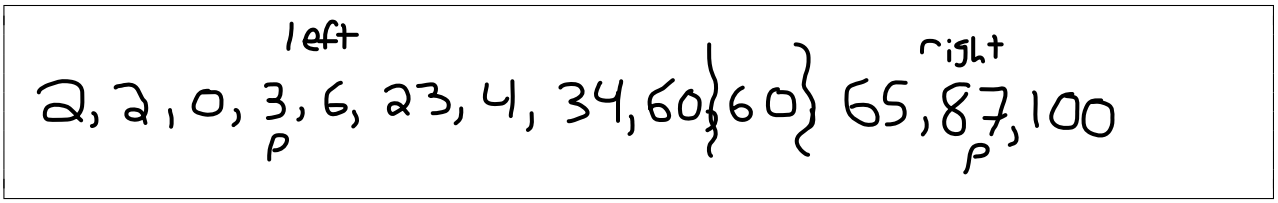
3. [15 points] Choosing the first element to be your partition and using the code in the course slides: Show the state of the following array after the partition function returns. Indicate the pivot points.

   ```
   [60, 100, 4, 2, 6, 23, 65, 34, 87, 3, 2, 0, 60]
   ```

   (a) After the first call to partition:

   3, 60, 4, 2, 6, 23, 0, 34, 2, 60, 87, 65, 100
   p

(b) After the second calls to partition:



4. [5 points] If we were to randomly choose an element from the above array to be the pivot point, which element would provide the most optimal(even) split of the array after the first partition?

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5. [5 points] What is the average big Θ runtime of quick sort?

$\Theta(n \log n)$

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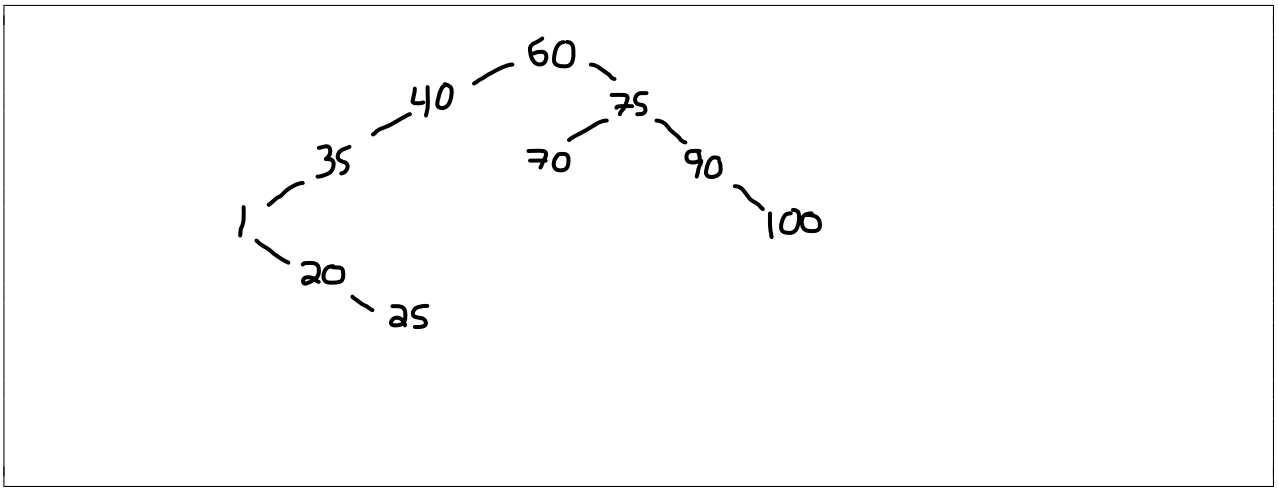
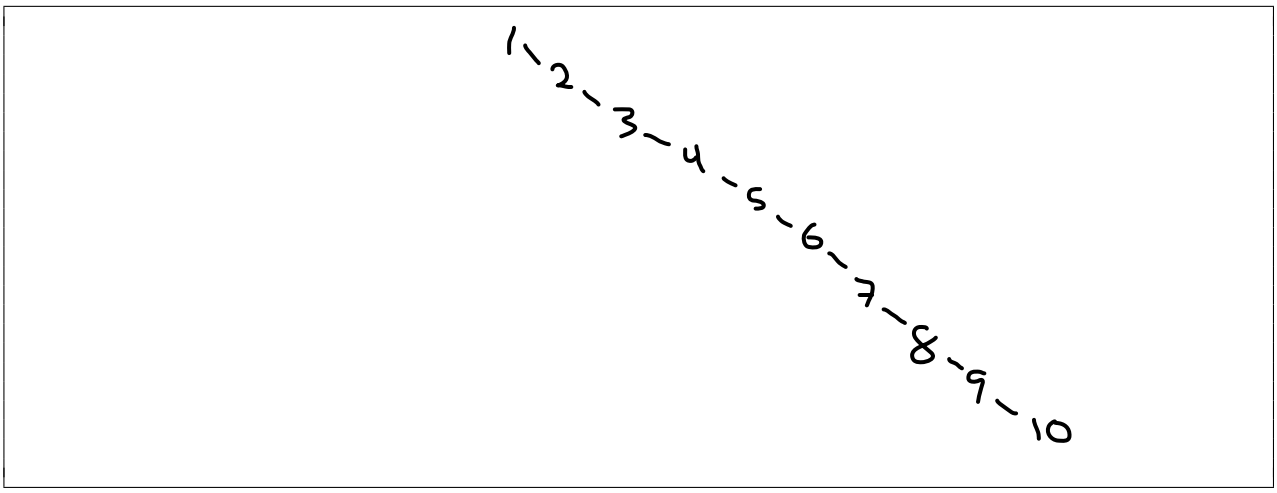
6. [15 points] Draw a binary search tree after the insertion of the following elements in order:

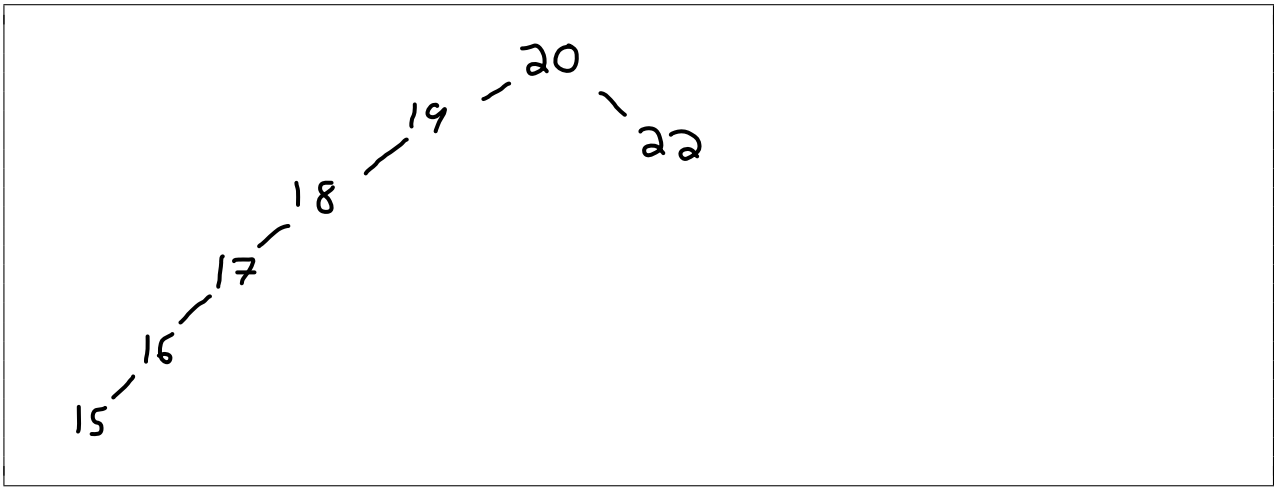
(a) [5 points] [60, 40, 35, 75, 90, 1, 20, 100, 25, 70]



(b) [5 points] [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]



(c) [5 points] [20, 19, 18, 17, 16, 15, 22]



7. [15 points] What do the following terms mean when referring to properties of k-ary trees?

(a) [5 points] Full (proper):

Every node has exactly 0 or K children

(b) [5 points] Complete:

Every level is entirely filled except possibly the deepest, where all nodes are as far left as possible

(c) [5 points] Perfect:

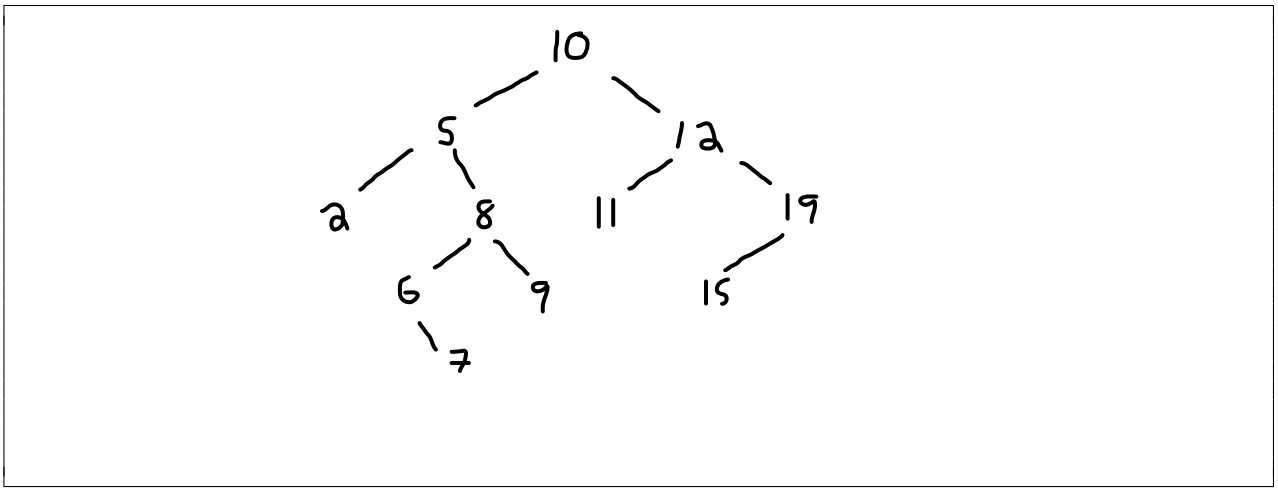
Tree is full and all leaves are the same depth

8. [5 points] If a binary tree is complete, does that necessarily mean it is also full? Justify your answer.

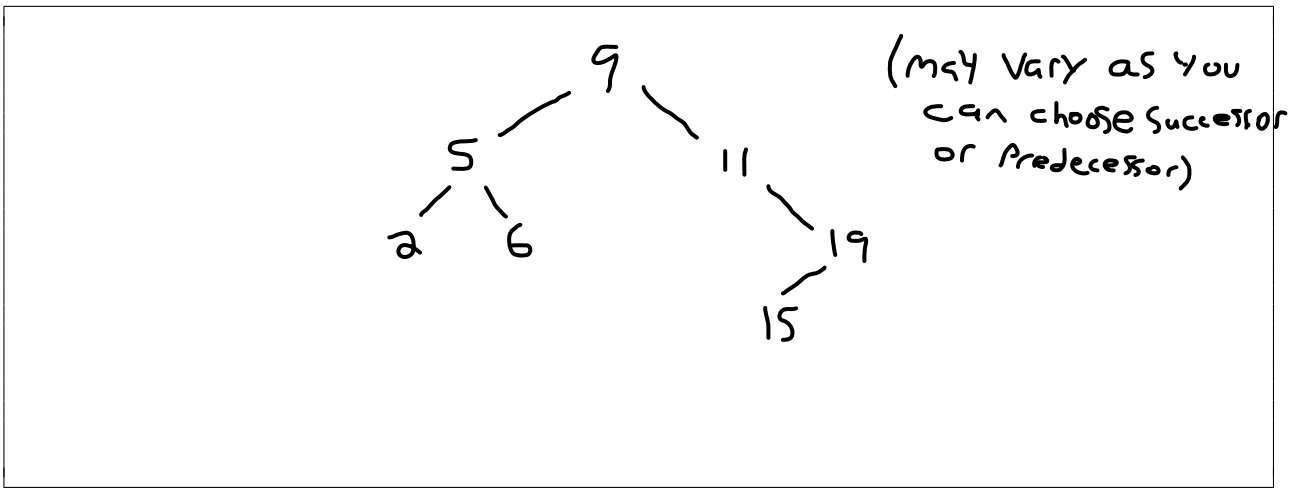
no, on the deepest level of a complete tree, there could be a node which does not have 0 or k children (see 13d)

9. [10 points] Draw an initially empty BST after performing the following operations:

(a) [5 points] Insert the following elements in order: 10, 5, 12, 8, 19, 6, 2, 11, 15, 9, 7



(b) [5 points] Remove the following elements from the tree created by part a: 7, 12, 8, 10



10. [15 points] Briefly explain the the following tree traversal methods.

    (a) [5 points] in-order traversal:

    Visit left, root, right

    (b) [5 points] pre-order traversal:

    root, left, right

    (c) [5 points] post-order traversal:

    left, right, root

11. [20 points] Let T be a full k-ary tree, where k = 2 (a.k.a. binary tree), with n nodes. Let h denote the height of T.

    (a) [5 points] What is the minimum number of leaves for T?

    $h+1$

(b) [5 points] What is the maximum number of leaves for T?

2^h

(c) [5 points] What is the minimum number of internal nodes for T?

h

(d) [5 points] What is the maximum number of internal nodes for T?

$2^h - 1$

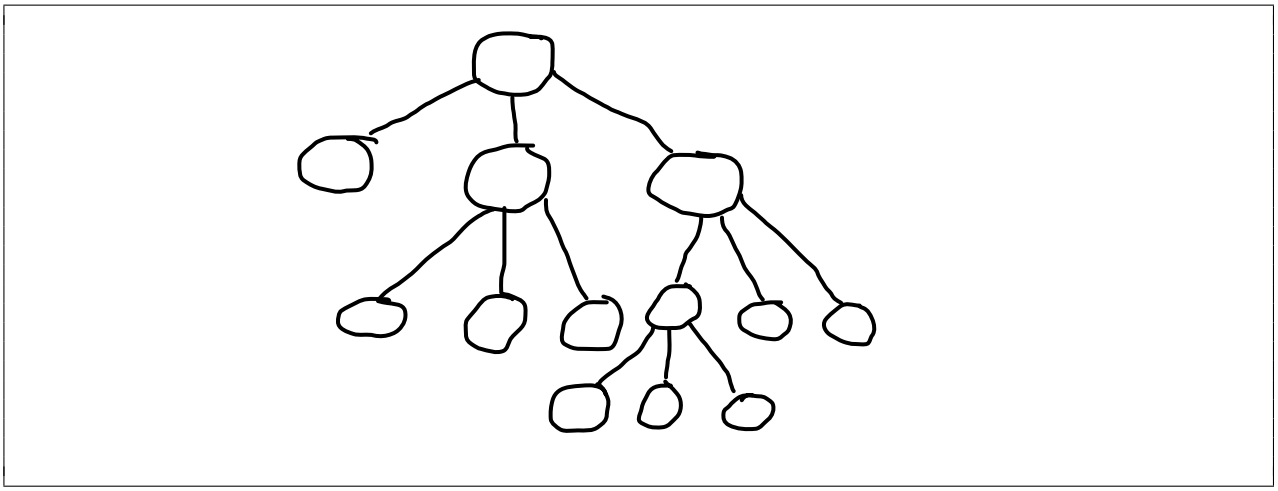
12. [10 points] Show that the maximum number of nodes in a binary tree of height h is $2^{h+1} - 1$.

A Perfect binary tree of height h has $2^{h+1}-1$ nodes,

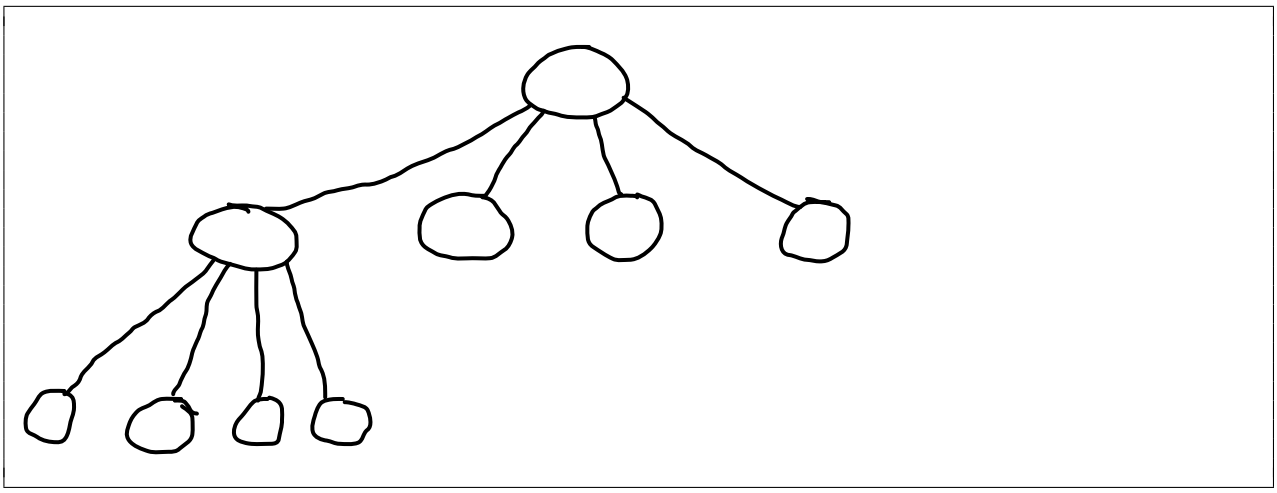
can't do better

13. [20 points] Draw an example of k-ary trees matching the following descriptions

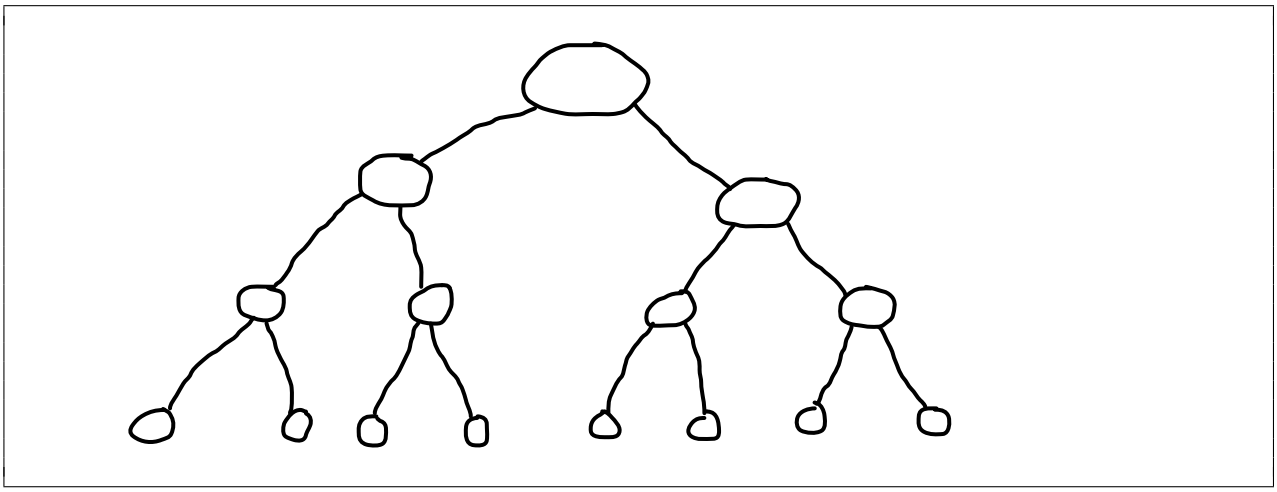
(a) [5 points] Full(proper), k = 3, height = 3



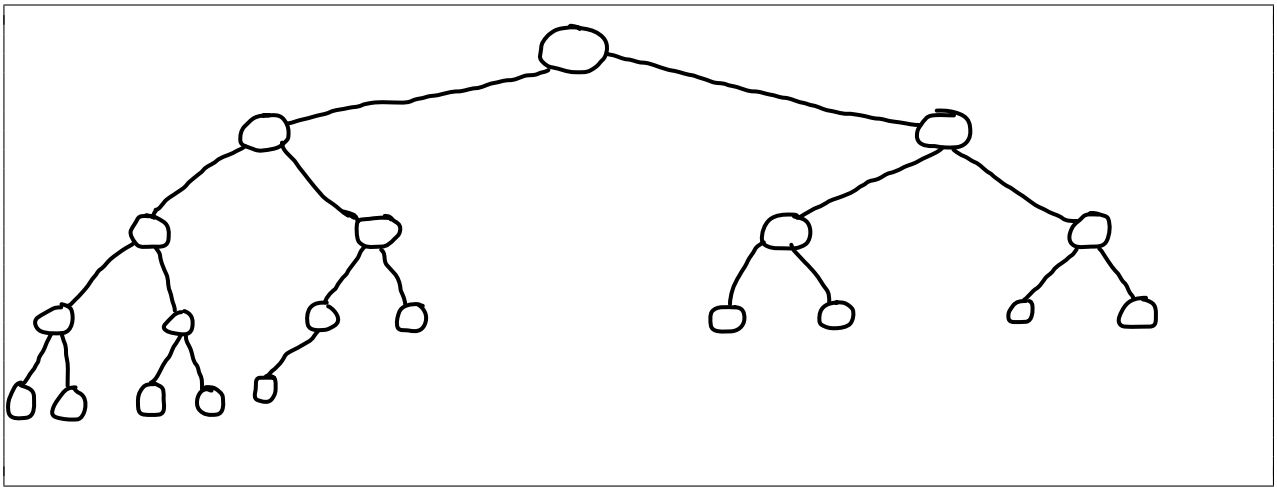
(b) [5 points] Complete, k = 4, height = 2



(c) [5 points] Perfect, k = 2, height = 3

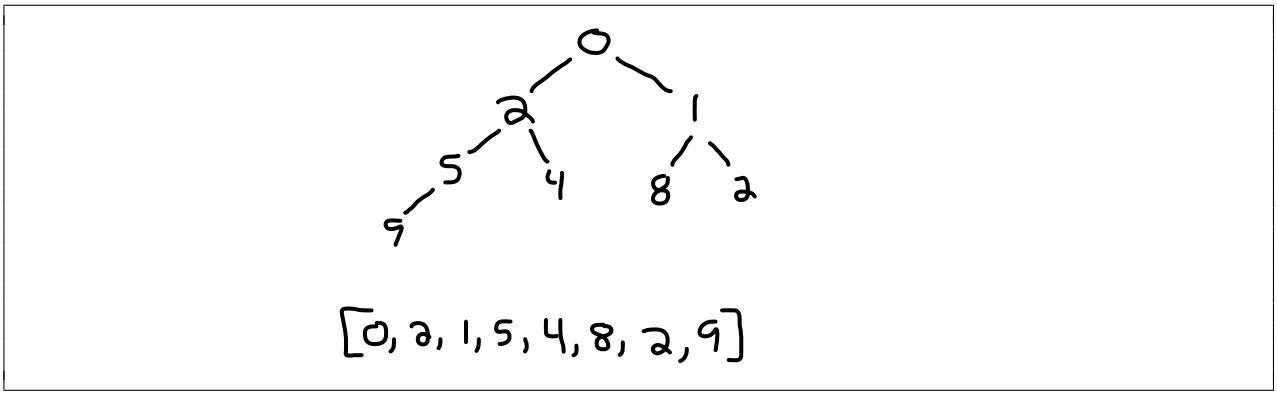


(d) [5 points] Complete, k = 2, height = 4

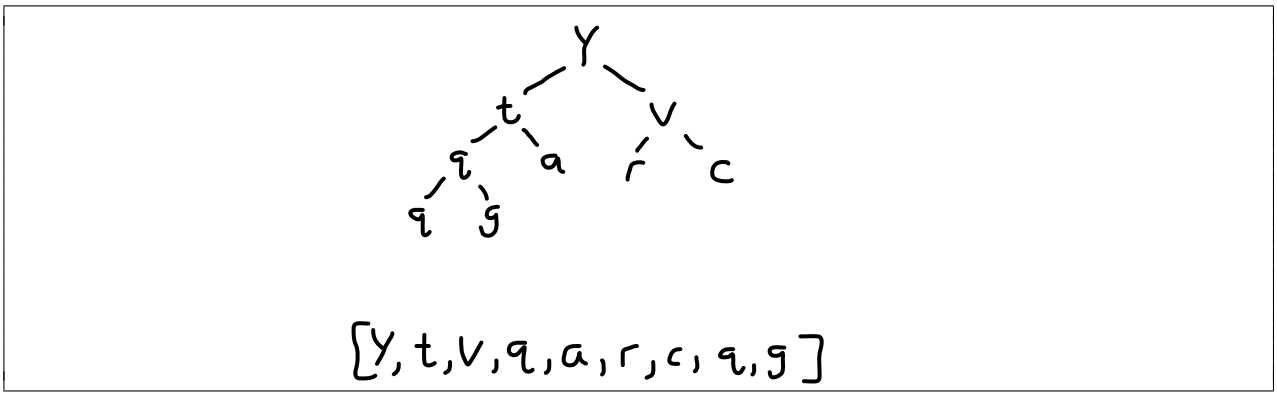


14. [15 points] Take the following sequences and apply the buildHeap algorithm, draw the tree *and* the array which represent the final state of the heap after the algorithm finishes. Strings and characters are in lexicographic order.

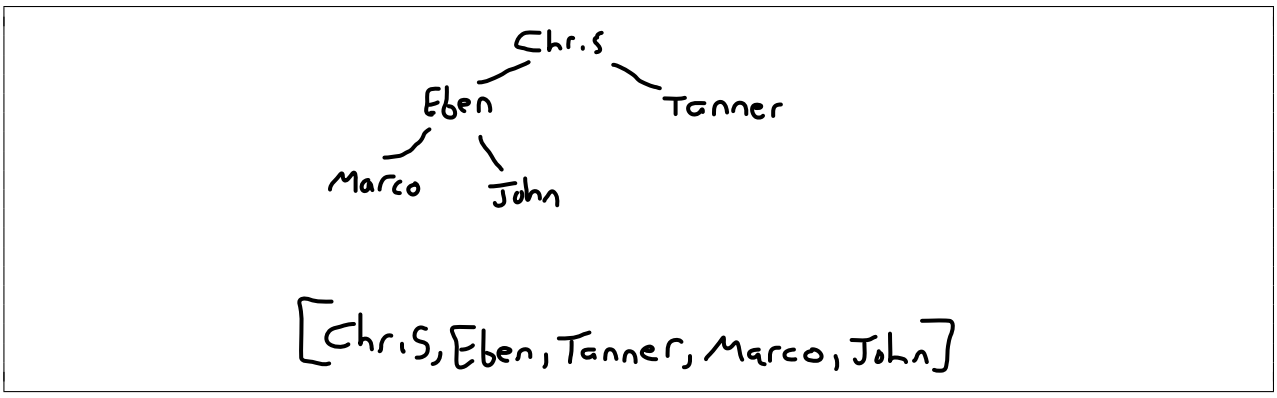
    (a) [5 points] Min Heap: [0, 4, 2, 5, 2, 8, 1, 9]



    (b) [5 points] Max Heap: [a, g, r, q, t, v, c, y, q]

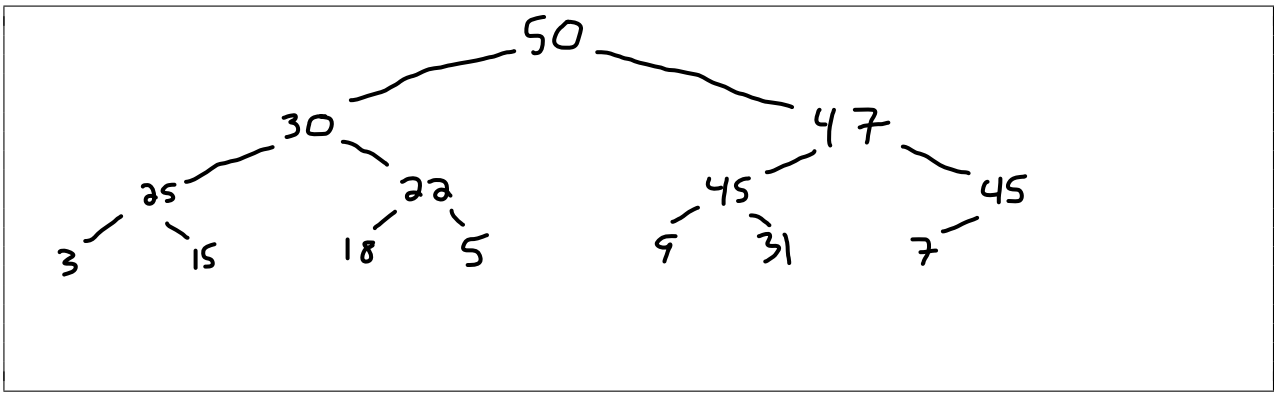


    (c) [5 points] Min Heap: ["Marco", "John", "Tanner", "Eben", "Chris"]

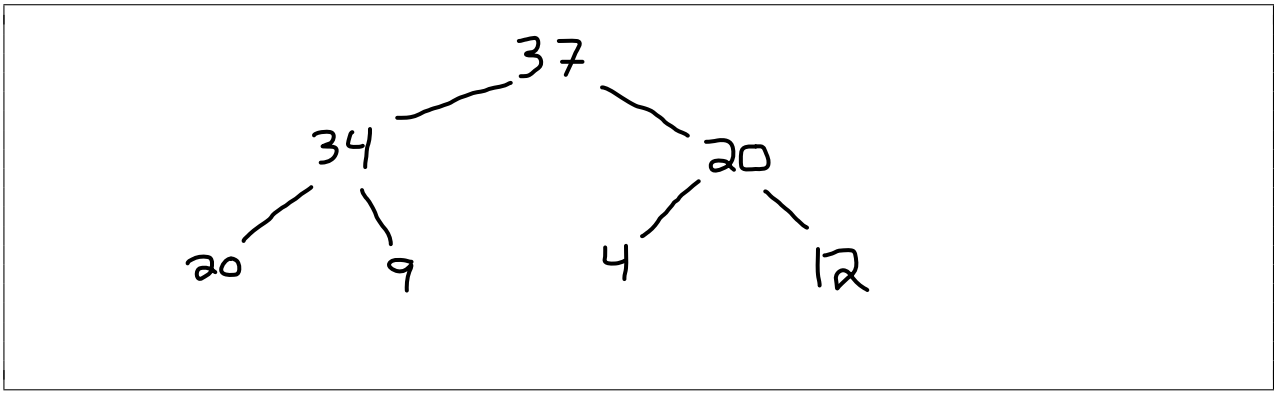


15. [20 points] Take the following sequences representing min and max heaps and draw the resulting heap after the operation.

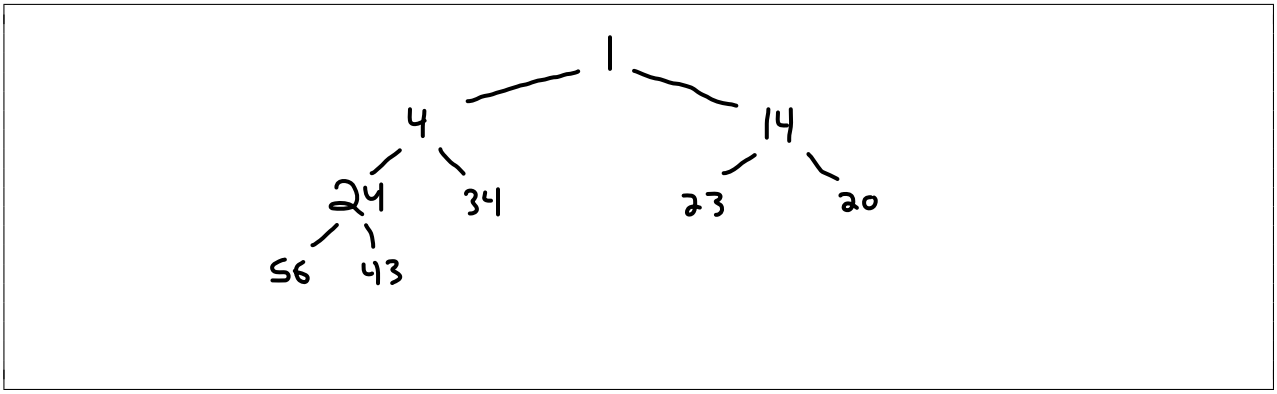
(a) [5 points] Insert 47 into max heap [50, 30, 45, 25, 22, 45, 7, 3, 15, 18, 5, 9, 31]



(b) [5 points] Remove max value from max heap [43, 37, 20, 20, 34, 4, 12, 9]



(c) [5 points] Insert 4 into min heap [1, 24, 14, 43, 34, 23, 20, 56]



(d) [5 points] Remove min value from min heap [4, 23, 12, 23, 23, 76, 23, 79, 95, 79, 45, 80]