

*When relations are otherwise unnamed, I arbitrarily called them all “F” in order to display their schema in addition to the table when asked.

1.

$$F(A, B) = \text{PROJ}_{A,B}(R) \text{ INTERSECT } \text{RENAME}_{S(A,B)}(\text{PROJ}_{B,C}(R))$$

A	B
3	1
2	3
2	3
1	2

2. a.) For sets R and S, $(R \text{ INTERSECT } S) = (R \text{ JOIN } S)$ always holds true. This is because for set operations, all duplicates in any given resultant set will be eliminated. This is not the case for bag operations, however, as the JOIN operation (theta-join in relational algebra) (for example JOIN in SQL still presents the duplicate tuples, whereas INTERSECT does not and instead only displays the minimum number of occurrences of any give tuple.

b.) In either set operations or bag operations, $((R - S) \text{ UNION } (S - R)) \text{ INTERSECT } (R \text{ INTERSECT } S) = \text{EMPTYSET}$ is always necessarily true. Despite differing numbers of duplicates being displayed in intermediate relations (those in parentheses), a hierarchical view of this problem shows very apparently that $((R - S) \text{ UNION } (S - R))$ and $(R \text{ INTERSECT } S)$ will never intersect, because they are the antitheses of one another. When computing $R - S$ and $S - R$, we are looking for elements which are in one relation, but not the other. Once we form the union of these two resultant relations, we effectively have found all the values except those which *intersect* between the two, or are commonalities in both. Therefore, when computing $((R - S) \text{ UNION } (S - R)) \text{ INTERSECT } (R \text{ INTERSECT } S)$, the answer will always be the empty set, for all relations R and S.

c.) In terms of sets and set operations, $\text{DELTA}(R) = \text{GAMMA}_{\{A,B,C\}}(R)$ will always hold true because the purpose of DELTA is to eliminate all duplicates presented in the set. Gamma, takes the attributes A, B, and C to delineate grouping together common values of each of those columns. Since there are no other attributes present, this means that a new relation is made with the distinct tuples of the set as well. In other words, each distinct value appears only once with no duplicates. This will be the same with bags as well as distinct values will never have duplicates, even when taken from a bag.

d.) Given that both relations R and S, interpreted as sets, using set-operations, are both filled with non-null values, the statement $(R \text{ UNION } S) = R$ can only be true when $R = S$ or S only contains the empty set, as $R \text{ UNION } S$ will simply concatenate all values of R and S and then delete duplicates in set operations. As for bag operations, $(R \text{ UNION } S) = R$ can never be true. Even if the two bags are exactly the same, all duplicates will be displayed and all tuples will be represented from both bags.

3.

a.) $\text{PI}_{\text{item}} (\text{SIGMA}_{\text{age} > 18} (\text{Shop_list NATURAL JOIN Person}))$

b.) $\text{PI}_{\text{name}} (\text{SIGMA}_{\text{supermarket} = \text{'Walmart'}} ((\text{SIGMA}_{\text{gender} = \text{'M'}} (\text{Shop_list NATURAL JOIN Person})) \text{ NATURAL JOIN Sell}))$

c.) $\text{PI}_{\text{name, item, lowest_price}} (\text{Person NATURAL JOIN Shop_list NATURAL JOIN } (\text{GAMMA}_{\text{item; min(price)} \rightarrow \text{lowest_price}} (\text{Sell})))$

I, Justin Anthony Timberlake, declare that I have completed this assignment completely and entirely on my own, without any consultation with others. I understand that any breach of the UAB Academic Honor Code may result in severe penalties.