

\*When relations are otherwise unnamed, I arbitrarily called them all “F” in order to display their schema in addition to the table.

$$1. R \text{ natural join } S = F(R.A, R.B, R.C, S.D) = \\ \pi_{R.A, R.B, R.C, S.D} \\ (\sigma_{R.B = S.B \wedge R.C = S.C} \\ (R(A,B,C,D) \times S(B,C,D)))$$

F

R.A	R.B	R.C	S.D
6	4	3	9
3	5	1	6

$$2. R \text{ theta-join } S = F(R.A, R.B, S.B, S.C, S.D) = \\ \sigma_{R.B = S.B} (R(A,B) \times S(B,C,D))$$

F

R.A	R.B	S.B	S.C	S.D
10	t	t	8	9

3. Given that natural-join for the described relations R (with r unique tuples within R) and S (with s unique tuples within S) can be expanded:

$$R \text{ natural join } S = \\ \pi_{R.A, R.B, S.C} \\ (\sigma_{R.B = S.B} \\ (R(A,B) \times S(B,C)))$$

We will have to consider the resulting relation from the selection statement of the cross product (theta-join) of these sets. Because the tuples within S and R are respectively unique within S and R, each value in the B column can only have exactly one match within  $R \times S$ . Given that a tuple can also be defined as one row (or record) in any given relation, we know that for every matching value between the two different B columns, one tuple will be included in the natural-join of R and S. As stated before, this is equivalent to as many unique tuples there are in each B column of each set, which is r or s in R or S respectively.

Thus, the number of tuples in the final relation is in the range from 0 (where there are no matching values between R.B and S.B) to  $t = r$  or  $t = s$  depending on which value, s or r, is larger. For example, if  $s < r$ , then  $t = s$  because we know there can then only be s possible tuple combination which could overlap in the B columns.

In the case of R natural-join R, however, there would be no overlapping values between R.B and S.B because all values of the set R must be unique within R. Therefore,  $t = 0$  in all instances of R.

$$4. F(R.B, R.A) = \pi B, A ( R )$$

F

R.B	R.A
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2	1
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2	4
---	---

5	4
---	---

5	2
---	---

5.

$$R \cup S = F(R.A, R.B, R.C)$$

R.A	R.B	R.C
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a	b	c
---	---	---

d	b	c
---	---	---

d	e	f
---	---	---

b	e	c
---	---	---

a	b	f
---	---	---

b	e	d
---	---	---

$$R \cap S = F(R.A, R.B, R.C)$$

R.A	R.B	R.C
-----	-----	-----

a	b	c
---	---	---

d	e	f
---	---	---

b	e	c
---	---	---

$$R - S = F(R.A, R.B, R.C)$$

R.A	R.B	R.C
-----	-----	-----

d	b	c
---	---	---

a	b	f
---	---	---

$$S - R = F(R.A, R.B, R.C)$$

S.A	S.B	S.C
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b	e	d
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I, Justin Anthony Timberlake, declare that I have completed this assignment completely and entirely on my own, without any consultation with others. I understand that any breach of the UAB Academic Honor Code may result in severe penalties.