Assignment 4

Jatin Tarachandani-CS20BTECH11021

March 2021

Download latex codes from

https://github.com/jatin−tarachandani/AI1103/blob/main/Assignment_4/main.tex

1 Problem Statement

CSIR UGC NET Dec 2012 Q60
Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of men and women are independent. The probability that the first person to arrive in the queue is a man is:

1. $\frac{\lambda_1}{\lambda_1 + \lambda_2}$
2. $\frac{\lambda_2}{\lambda_1 + \lambda_2}$
3. $\frac{\lambda_1}{\lambda_2}$
4. $\frac{\lambda_2}{\lambda_1}$

2 Solution

Let X and Y be Poisson random variables, with the values X takes being the number of men joining the queue in an arbitrary time t, and the values Y takes being the number of women joining the queue in an arbitrary time t.

$$Pr\left(X = i\right) = \frac{\lambda_1^i \cdot e^{-\lambda_1}}{i!} \tag{2.0.1}$$

$$Pr\left(Y = i\right) = \frac{\lambda_2^i \cdot e^{-\lambda_2}}{i!} \tag{2.0.2}$$

For 2 independent Poisson distributions with means λ_1 and λ_2, the simultaneous distribution can be represented by:

$$Pr\left(X + Y = i\right) = \frac{(\lambda_1 + \lambda_2)^i \cdot e^{-(\lambda_1 + \lambda_2)}}{i!} \tag{2.0.3}$$

Now we take conditional probability that if only one person entered the queue within a certain time t, then the probability of them being a man and not a woman is given by:

$$Pr\left(X = 1 | (X + Y) = 1\right) = \frac{Pr\left((X = 1) + (Y = 0)\right)}{Pr\left(X + Y = 1\right)} \tag{2.0.4}$$

$$\tag{2.0.5}$$

Since X and Y are independent,

$$Pr\left(X = 1 | (X + Y) = 1\right) = \frac{Pr\left(X = 1\right) \cdot Pr\left(Y = 0\right)}{Pr\left(X + Y = 1\right)} \tag{2.0.6}$$

$$= \frac{\frac{\lambda_1^1 \cdot e^{-\lambda_1}}{1!} \cdot \frac{\lambda_2^0 \cdot e^{-\lambda_2}}{0!}}{\frac{(\lambda_1 + \lambda_2)^1 \cdot e^{-(\lambda_1 + \lambda_2)}}{1!}} \tag{2.0.7}$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2} \tag{2.0.8}$$

The probability that the first person to arrive in the queue is a man is option A, i.e $\frac{\lambda_1}{\lambda_1 + \lambda_2}$