Assignment 3

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Download the latex codes from

https://github.com/jatin−tarachandani/AI1103/blob/main/Assignment_3/main.tex

1 Problem Statement

Let $A_1, A_2, ...A_n$ be n independent events which the probability of occurrence of the event A_i is given by $Pr(A_i) = 1 - \frac{1}{\alpha^i}, \alpha > 1, i = 1, 2, ...n.$. Then the probability that at least one of the events occurs is:

1. 1- $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$
2. $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$
3. $\frac{1}{\alpha^n}$
4. 1- $\frac{1}{\alpha^n}$

2 Solution

$$Pr(A_i') = 1 - Pr(A_i) \tag{2.0.1}$$

$$= \frac{1}{\alpha^i} \tag{2.0.2}$$

for all i in $\{1, 2, 3...n\}$.
The probability that at least one event occurs is $Pr(A_1 \cup A_2 \cup ...A_n)$.

$$Pr(A_1 \cup A_2 \cup ...A_n) = 1 - Pr((A_1 \cup A_2 \cup ...A_n)') \tag{2.0.3}$$

Using de Morgans laws,

$$Pr(A_1 \cup A_2 \cup ...A_n) = 1 - Pr(A_1' \cap A_2' \cap ...A_n') \tag{2.0.4}$$

Because all the n events are independent,

$$Pr(A_1 \cup A_2 \cup ...A_n) = 1 - Pr(A_1') \cdot Pr(A_2') .... \cdot Pr(A_n') \tag{2.0.5}$$

$$= 1 - \frac{1}{\alpha^1} \cdot \frac{1}{\alpha^2} .... \cdot \frac{1}{\alpha^n} \tag{2.0.6}$$

$$= 1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}} \tag{2.0.7}$$

The probability that at least one of the specified events occurs is $1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}}$.