Assignment 2 - GATE problem 49

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Download the python codes from

https://github.com/jatin−tarachandani/AI1103/blob/main/Assignment%202/codes/Assignment_2.py

and latex codes from

https://github.com/jatin−tarachandani/AI1103/blob/main/Assignment%202/main.tex

1 Problem Statement

A fair coin is tossed 3 times in succession. If the first toss is a head, then the probability of getting exactly two heads in three tosses is?

2 Solution

We can see that if the first toss is guaranteed to be a head, then the problem is reduced to finding the probability of getting one head in 2 coin tosses, since all the 3 trials are independent.

Let $K = \{0, 1, 2\}$ be the random variable denoting the number of heads obtained in 2 tosses of a fair coin. The event consists of multiple Bernoulli trials, therefore it can be represented by a binomial distribution b(n,p). In b(n,p), $\Pr(K = i) = \binom{n}{i} p^i \cdot (1 - p)^{n-i}$. Here $n = 2$, $p = 0.5$.

$$Pr(K = 0) = \binom{2}{0} 0.5^2 = \frac{1}{4} \tag{1}$$

$$Pr(K = 1) = \binom{2}{1} 0.5^2 = \frac{1}{2} \tag{2}$$

$$Pr(K = 2) = \binom{2}{2} 0.5^2 = \frac{1}{4} \tag{3}$$

We can see that the probability of getting 1 head in 2 tosses is

$$Pr(K = 1) = \binom{2}{1} 0.5^2 \qquad = \frac{1}{2}$$

The probability of getting exactly 2 heads in 3 tosses, if the first toss is a head, is 0.5.