

# Assignment 2 - GATE problem 49

Jatin Tarachandani-CS20BTECH11021

March 2021

## 1 Problem Statement

A fair coin is tossed 3 times in succession. If the first toss is a head, then the probability of getting exactly two heads in three tosses is?

The probability of getting exactly 2 heads in 3 tosses, if the first toss is a head, is 0.5.

## 2 Solution

Let  $K \in \{0, 1, 2, 3\}$  be the random variable denoting the possible numbers of heads we obtain in three consecutive tosses of the coin. Let  $L \in \{0, 1\}$  be the random variable denoting the result of the first flip, with 0 representing a result of tails. We know that,

$$Pr(K = 0) = 0.125 \quad (1)$$

$$Pr(K = 1) = 0.375 \quad (2)$$

$$Pr(K = 2) = 0.375 \quad (3)$$

$$Pr(K = 3) = 0.125 \quad (4)$$

$$Pr(L = 1|K = 0) = 0 \quad (5)$$

$$Pr(L = 1|K = 1) = \frac{1}{3} \equiv 0.33 \quad (6)$$

$$Pr(L = 1|K = 2) = \frac{2}{3} \equiv 0.67 \quad (7)$$

$$Pr(L = 1|K = 3) = 1 \quad (8)$$

Using Bayes theorem, we get:

$$\begin{aligned} Pr(K = 2|L = 1) &= \frac{Pr(L = 1|K = 2) \cdot Pr(K = 2)}{\sum_{i=0}^3 Pr(L = 1|K = i) \cdot Pr(K = i)} \\ &= \frac{0.67 \cdot 0.375}{0 \cdot 0.125 + 0.33 \cdot 0.375 + 0.67 \cdot 0.375 + 1 \cdot 0.125} \\ &= \frac{0.25}{0.5} \\ &= 0.5. \end{aligned}$$