Assignment 4

Jatin Tarachandani-CS20BTECH11021

March 2021

Download latex codes from

https://github.com/jatin-tarachandani/AI1103/blob/main/Assignment_4/main.tex

1 Problem Statement

CSIR UGC NET Dec 2012 Q60

Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2 . The arrivals of men and women are independent. The probability that the first person to arrive in the queue is a man is:

1.
$$\frac{\lambda_1}{\lambda_1 + \lambda_2}$$

$$2. \ \frac{\lambda_2}{\lambda_1 + \lambda_2}$$

3.
$$\frac{\lambda_1}{\lambda_2}$$

4.
$$\frac{\lambda_2}{\lambda_1}$$

2 Solution

Let *X* and *Y* be Poisson random variables, with the values *X* takes being the number of men joining the queue in an arbitrary time *t*, and the values *Y* takes being the number of women joining the queue in an arbitrary time *t*.

$$Pr(X = i) = \frac{\lambda_1^i \cdot e^{-\lambda_1}}{i!}$$
 (2.0.1)

$$Pr(Y = i) = \frac{\lambda_2^i \cdot e^{-\lambda_2}}{i!}$$
 (2.0.2)

For 2 independent Poisson distributions with means λ_1 and λ_2 , the simultaneous distribution can be represented by:

$$Pr(X + Y = i) = \frac{(\lambda_1 + \lambda_2)^i \cdot e^{-(\lambda_1 + \lambda_2)}}{i!}$$
 (2.0.3)

Now we take conditional probability that if only one person entered the queue within a certain time t, then the probability of them being a man and not a woman is given by:

$$Pr(X = 1 | (X + Y) = 1) = \frac{Pr((X = 1) + (Y = 0))}{Pr(X + Y = 1)}$$
(2.0.4)
(2.0.5)

Since X and Y are independent,

$$Pr(X = 1 | (X + Y) = 1) = \frac{Pr(X = 1) \cdot Pr(Y = 0)}{Pr(X + Y = 1)}$$
(2.0.6)

$$= \frac{\frac{\lambda_1^1 \cdot e^{-\lambda_1}}{1!} \cdot \frac{\lambda_2^0 \cdot e^{-\lambda_2}}{0!}}{\frac{(\lambda_1 + \lambda_2)^1 \cdot e^{-(\lambda_1 + \lambda_2)}}{1!}}$$
(2.0.7)

$$= \frac{\frac{(\lambda_1 + \lambda_2)^{1} \cdot e^{-(\lambda_1 + \lambda_2)}}{1!}}{\lambda_1 + \lambda_2}$$
 (2.0.8)

The probability that the first person to arrive in the queue is a man is option A, i.e $\frac{lambda_1}{\lambda_1 + \lambda_2}$