Assignment 5

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Download latex codes from

https://github.com/jatin−tarachandani/AI1103/blob/main/Assignment_5/main.tex

1 Problem Statement

Let X, Y, have the joint discrete distribution such that $X|Y = y \sim$ Binomial(y, 0.5) and $Y \sim$ Poisson(λ), $\lambda > 0$, where λ is an unknown parameter. Let $T = T(X, Y)$ be any unbiased estimator of λ. Then

1. $Var(T) \leq Var(Y)$ for all λ
2. $Var(T) \geq Var(Y)$ for all λ
3. $Var(T) \geq \lambda$ for all λ
4. $Var(T) = Var(Y)$ for all λ

2 Solution

An unbiased estimator is an estimator on a sample of data whose expected value is exactly equal to the parameter being estimated.
We know that:

$$Var(Y) = \lambda \tag{2.0.1}$$

as Y has a Poisson distribution.
From Bayes Theorem,

$$Pr(X = x, Y = y) = Pr(Y = y) \cdot Pr(X = x|Y = y) \tag{2.0.2}$$

$$= \frac{\lambda^y}{y!} e^{-\lambda} \cdot \binom{y}{x} \frac{1}{2^y} \tag{2.0.3}$$

Let us represent $Pr(X = x, Y = y)$ as a function of x, y, and λ: $f(x, y; \lambda)$.
$T(X, Y)$ is an unbiased estimator of λ. From Cramer-Rao bound,

$$Var(T(X, Y)) \geq \frac{-1}{E(\frac{\partial^2 ln(f(x,y;\lambda))}{\partial\lambda^2})} \tag{2.0.4}$$

$$\geq \frac{-1}{E\left(\frac{-y}{\lambda^2}\right)} \tag{2.0.5}$$

$$\geq \frac{\lambda^2}{E(Y)} \tag{2.0.6}$$

$$\geq \lambda \tag{2.0.7}$$

This is because the expectation value of a Poisson distribution with parameter λ is λ itself.
The correct options are options (2) and (3).