Assignment 5

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Download latex codes from

https://github.com/jatin−tarachandani/AI1103/blob/main/Assignment_5/main.tex

1 Problem Statement

Let X, Y, have the joint discrete distribution such that $X|Y = y \sim$ Binomial(y, 0.5) and $Y \sim$ Poisson(λ), $\lambda > 0$, where λ is an unknown parameter. Let $T = T(X, Y)$ be any unbiased estimator of λ. Then

1. $Var(T) \leq Var(Y)$ for all λ
2. $Var(T) \geq Var(Y)$ for all λ
3. $Var(T) \geq \lambda$ for all λ
4. $Var(T) = Var(Y)$ for all λ

2 Solution

If an estimator of a certain parameter of a sample of data has the expected value exactly equal to the parameter it estimates, then it is said to be an unbiased estimator.
Suppose we had an estimator $T(X_1, X_2, X_3...X_n)$ acting on a set of n RVs $X_1, X_1, X_2, X_3...X_n$ which estimates a parameter θ, then if

$$E\left(T(X_1, X_2, X_3...X_n)\right) = \theta \tag{2.0.1}$$

the estimator is unbiased.
We know that:

$$Var(Y) = \lambda \tag{2.0.2}$$

as Y has a Poisson distribution.
From Bayes Theorem,

$$Pr\left(X = x, Y = y\right) = Pr\left(Y = y\right) \cdot Pr\left(X = x|Y = y\right) \tag{2.0.3}$$

$$= \frac{\lambda^y}{y!}e^{-\lambda} \cdot \binom{y}{x}\frac{1}{2^y} \tag{2.0.4}$$

Let us represent $Pr\left(X = x, Y = y\right)$ as a function of x, y, and λ: $f(x, y; \lambda)$.
$T(X, Y)$ is an unbiased estimator of λ. From Cramer-Rao bound,

$$Var(T(X, Y)) \geq \frac{-1}{E(\frac{\partial^2 ln(f(x,y;\lambda))}{\partial\lambda^2})} \tag{2.0.5}$$

$$\geq \frac{-1}{E\left(\frac{-y}{\lambda^2}\right)} \tag{2.0.6}$$

$$\geq \frac{\lambda^2}{E\left(Y\right)} \tag{2.0.7}$$

$$\geq \lambda \tag{2.0.8}$$

This is because the expectation value of a Poisson distribution with parameter λ is λ itself.
The correct options are options (2) and (3).