PCA => Principal Component Analysis

Dim of data

USE CASES :-

(i) Reduce Dim of the data.
(ii) Plot our data.

1901 => K. Pearson

?? M2 DL 1953

Statistical models

LR
MLR =>

MLR : y | x_1 x_2 x_3 x_4 ... x_{10} 50

⇓ PCA

Important feature (2)

x_1, x_2, . x_{10} =PCA=> z_1 z_2

When we say Information / Important feature what does that mean??

	Age (x_1)	class	marks (x_2)
1	14	8	95
2	15	8	80
3	19	8	32
4	16	8	12

Age = ✓
class = ✗
marks = ✓

Age ~~clas~~ marks

class => "8" =>

"least" informative ✓

Age ✓ Marks => more variance => very good big ...

Informative => "the feature which has more variation"

what is PCA??

x_1, x_2 => each feature contains how much Info??

case 1

x_1, x_2 (50%, 50%) ✓ PCA => z_1, z_2 (90%, 10%)

z_1 max Info

z_2 min Info

x_1 ✓ ≈ 50%

z_2 ≈ 10%

50% space

case 2

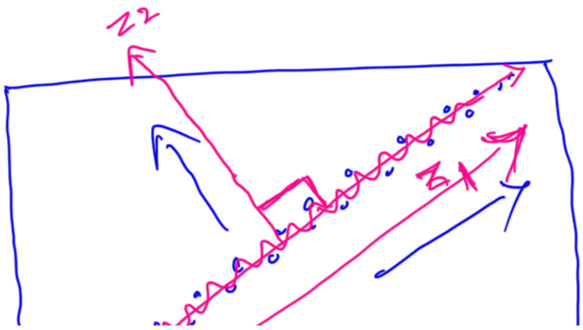
x_1 x_2 x_3 x_4 (≈25%) PCA => z_1, z_2, z_3, z_4 (70%, 25%, 3%, 2%)

50% huge

50%

5%

x_1, x_2,

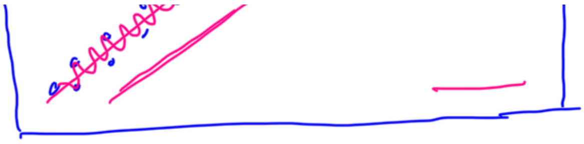


unsharing 50% ↑ x_1

z_1 95%, z_2 5%

z_1 z_2 ✓

no



x_2 50%

x_1	x_2	x_3
1	2	3
1	2	3
1	2	3
⋮	⋮	⋮

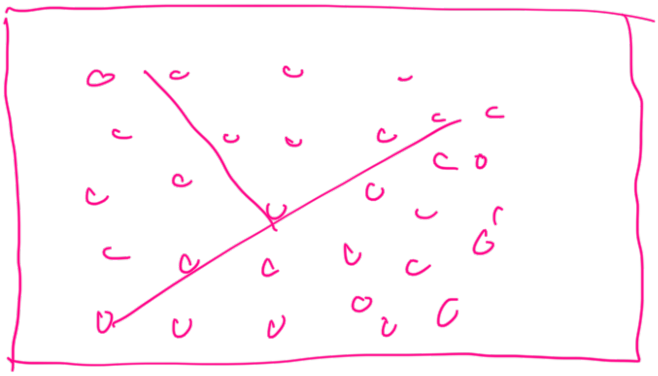
⇒ No

Any ?

100%

x_1 x_2 ⇒ Z_1 Z_2

50% x_1



x_2
50%

x_1 & x_2 Ind

PCA can't help

PCA

x_1, x_2 ⇒ Z_1, Z_2
90% 10%

x_1, x_2 Ind ⇒ Z_1, Z_2
50 50   50 50

x_1, x_2, x_3, x_1 x_2 $x_3 = x_2$

2D

⇒ Z_1 Z_2 Z_3
95% 5% 0

x_1 x_2 x_3 ⇒ Z_1 Z_2 Z_3
33 33 33   70 25 5

KNN,
clustering,
PCA

matrix ✓

eigen vector ✓

PCA math

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} 1 & 7 & 9 \\ 0 & 2 & 1 \end{bmatrix}_{2\times 3}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2}$$

When we can multiply a, b ??

numbers of columns of first

= numbers of rows of second

$$a = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3} \qquad b = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}_{3\times 2}$$

??

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$a_{p\times q}$ $b_{q\times n}$

$$= \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}_{2\times 2}$$

$a_{p\times q} \longleftrightarrow b_{q\times n}$

$c = a_{p\times q}\, b_{q\times n} \rightarrow p\times n$

eigen vectors:

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

A vector with a matrix it will change

(i) direction ✓

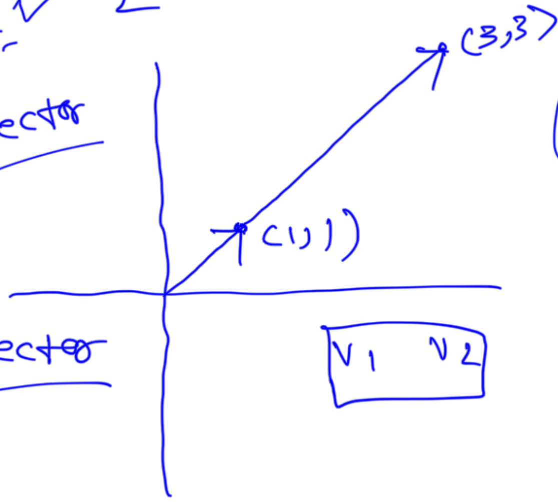
(ii) magnitude. ✓

eigenvectors are those vectors the direction will not be changed

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ → eigen vector

A V



$A\underline{V} = \lambda \underline{V}$ → eigen vector

λ → eigen value

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} = (1) \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \end{bmatrix} \quad \begin{bmatrix} -1 \end{bmatrix}$$ eigen vectors

Data →

$$X = \begin{bmatrix} x_1 & x_2 & \cdots & x_p \\ \vdots & \vdots & & \vdots \end{bmatrix}$$

$n \times p$ → n → rows
p → feature

$X^T_{p \times n} = ???$

$(X^T X)_{p \times p}$

$X^T_{p \times n} \longleftrightarrow X_{n \times p}$

$X^T X \, v_1 = \lambda_1 v_1$

$X^T X \, v_2 = \lambda_2 v_2$

$X^T X \, v_p = \lambda_p v_p$

$X^T X$

$v_1 \quad v_2 \quad v_3 \quad \ldots \quad v_p$

$\lambda_1 > \lambda_2 > \ldots > \lambda_p$

$\lambda_1 \to v_1 \to z_1$

$\lambda_2 \to v_2 \to z_2$

$\lambda_p \to v_p \to z_p$

$z_1 = X v_1 =$

$z_2 = X v_2 =$

$z_p = X v_p =$

$x_1 \quad x_p \; - \; z_1 \quad z_p$

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} \quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

L 3XL