Artificial Intelligence: Optimization in Deep Learning

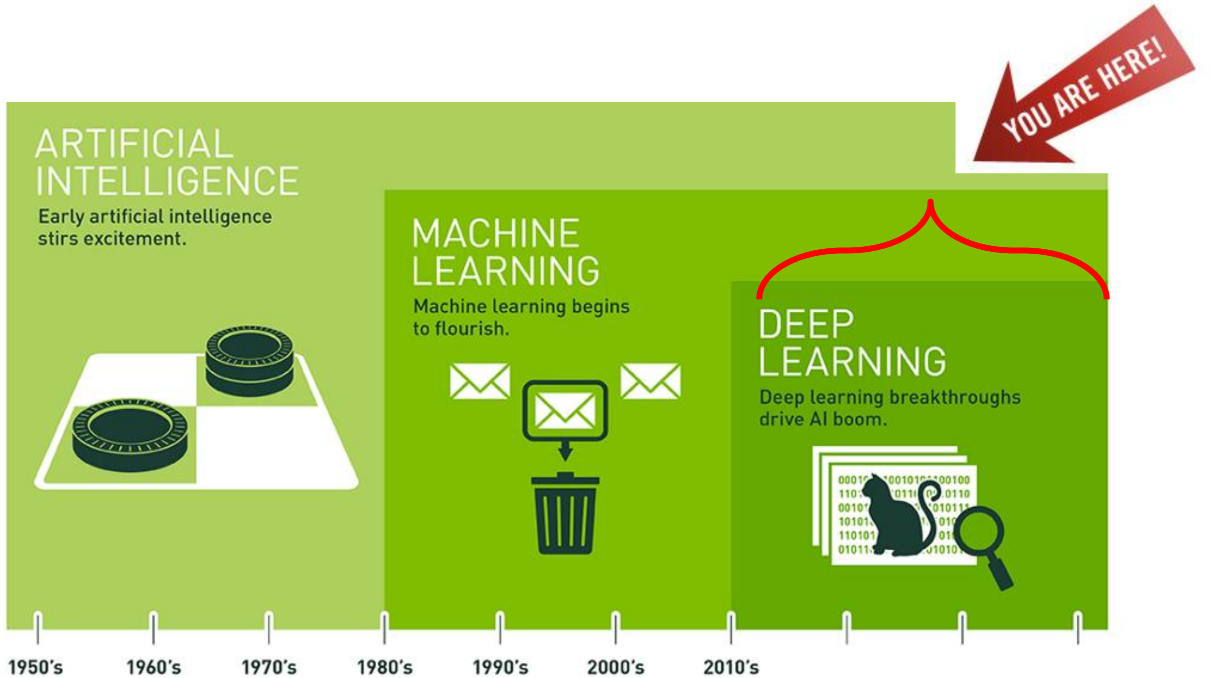
Some Slides from: Goodfellow et al., Deep Learning

Today

1. Feedforward in Deep Learning

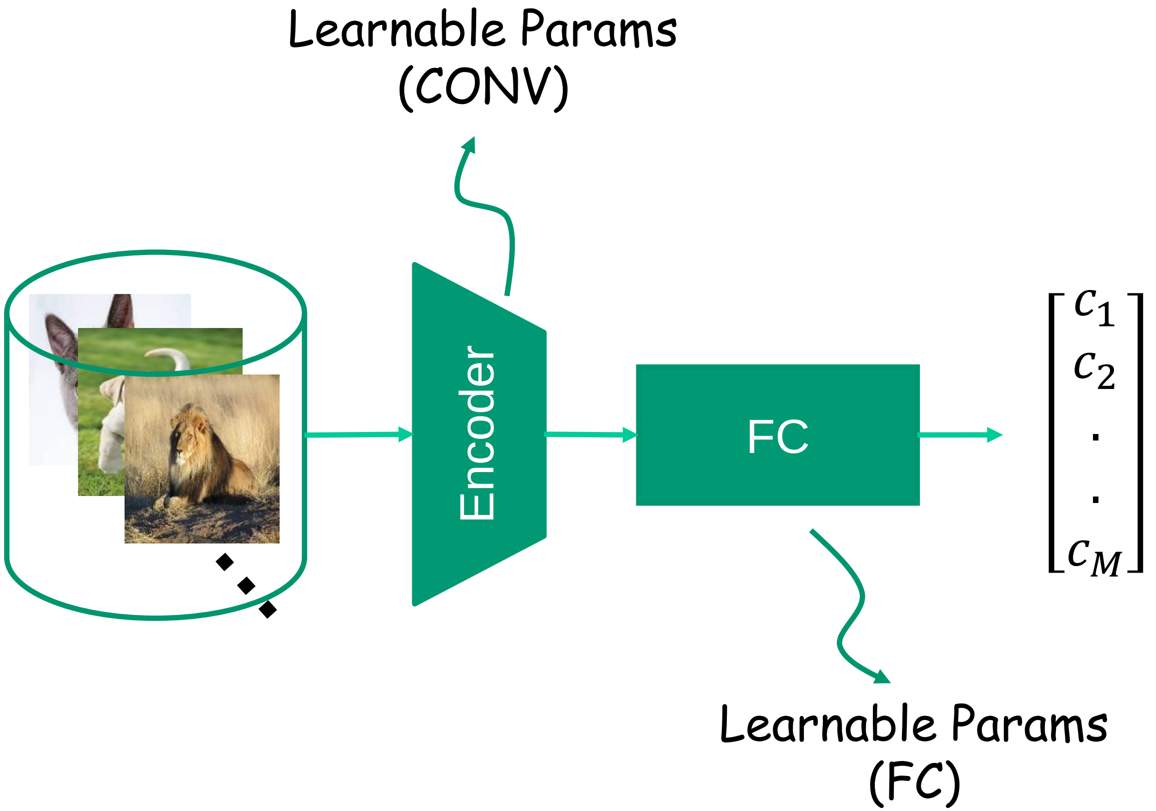
2. Backpropagation in Deep Learning
   - Gradient Descent
   - Stochastic Gradient Decent (SGD)
   - Momentum SGD
   - RMSProp
   - ADAM
3. Scheduled Learning
4. Hyper-Parameter (HP) Tuning

History of AI



https://www.linkedin.com/pulse/goedels-incompleteness-theorem-emergence-ai-eberhard-schoeneburg/

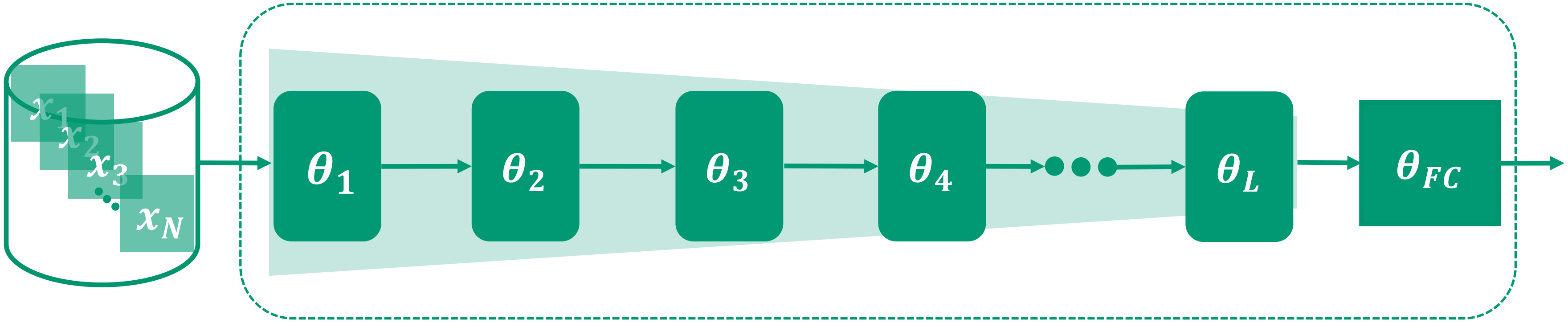
Feed-Forward in Deep Learning



Feed-Forward in Deep Learning

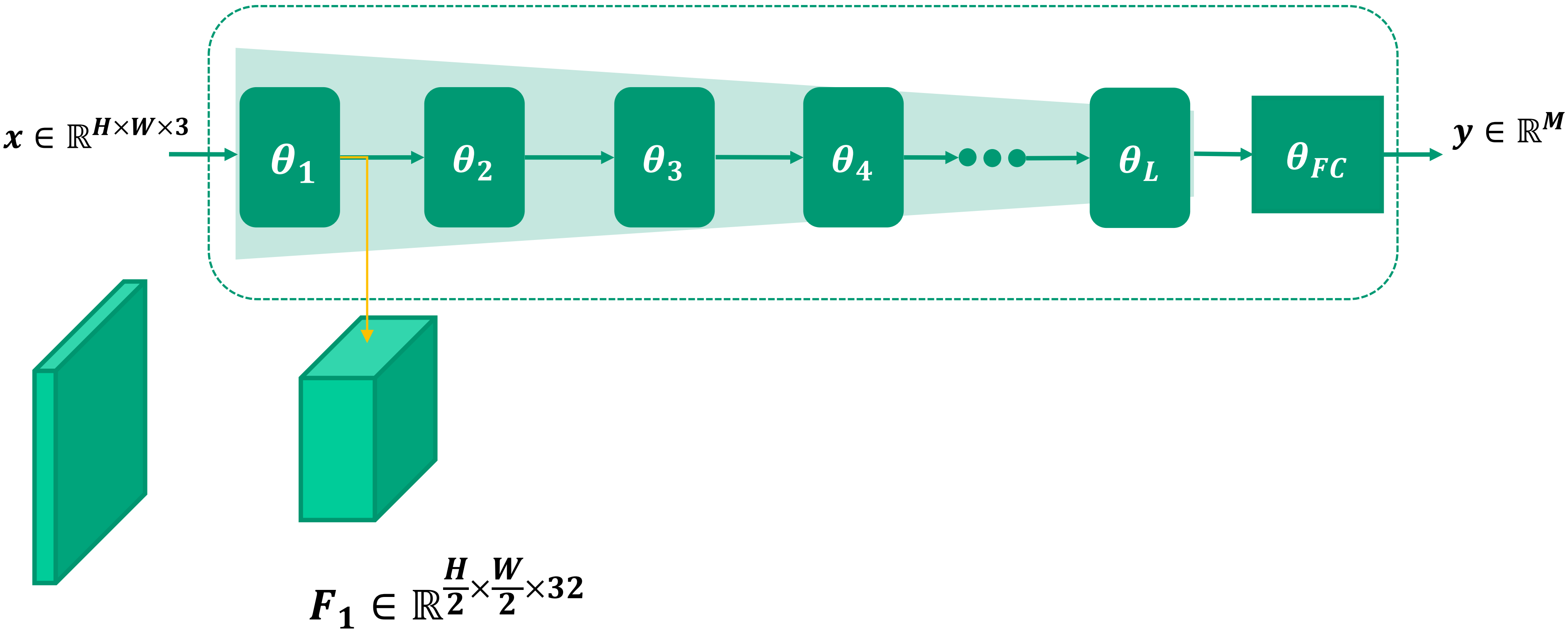
N - Number of images for training
M - Number of Class Images
c - Confidence prediction score
$\boldsymbol{\theta}$ - Learnable parameter
$\boldsymbol{x}$ - Input Image
$\boldsymbol{y}$ - Prediction label
$\boldsymbol{y}^{\boldsymbol{GT}}$ - Ground-truth label

$$y = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ c_M \end{bmatrix}, e.g.\ y_{cat} = \begin{bmatrix} 0.11 \\ 0.78 \\ 0.06 \\ . \\ 0.23 \end{bmatrix}, y_{cat}^{GT} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ . \\ 0 \end{bmatrix}$$



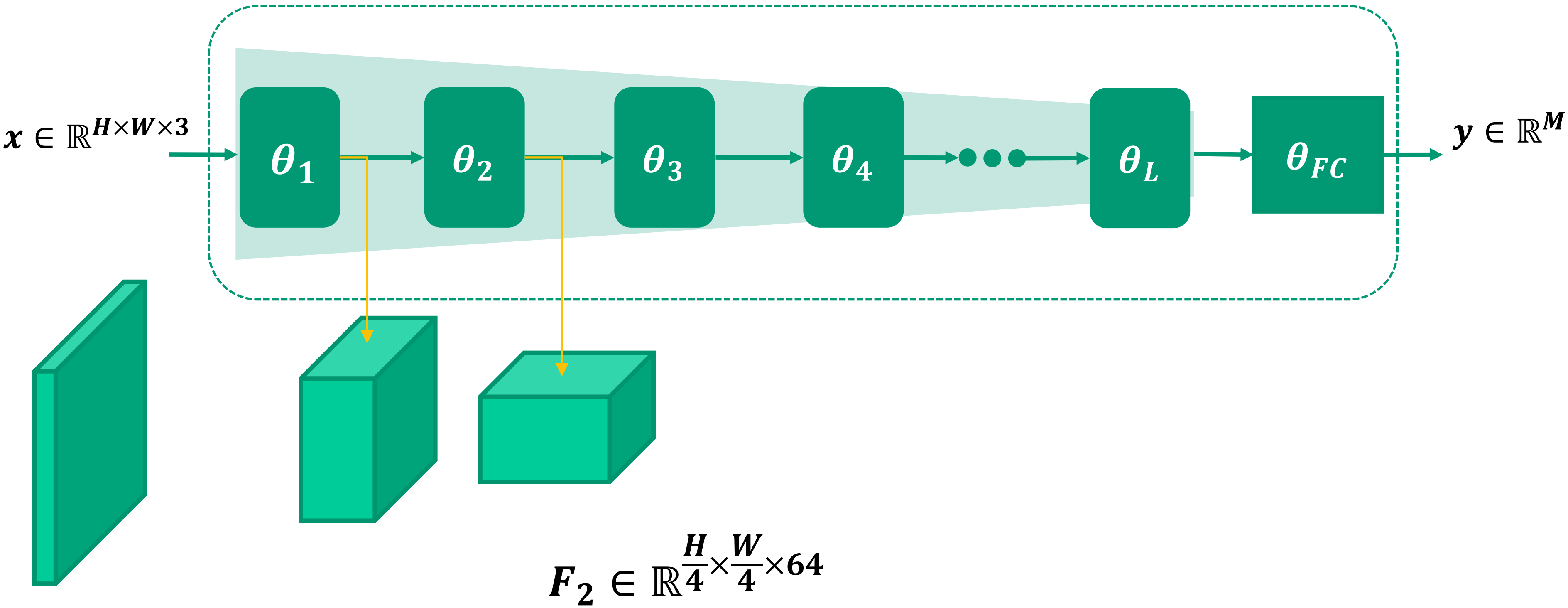
Feed-Forward in Deep Learning

N - N



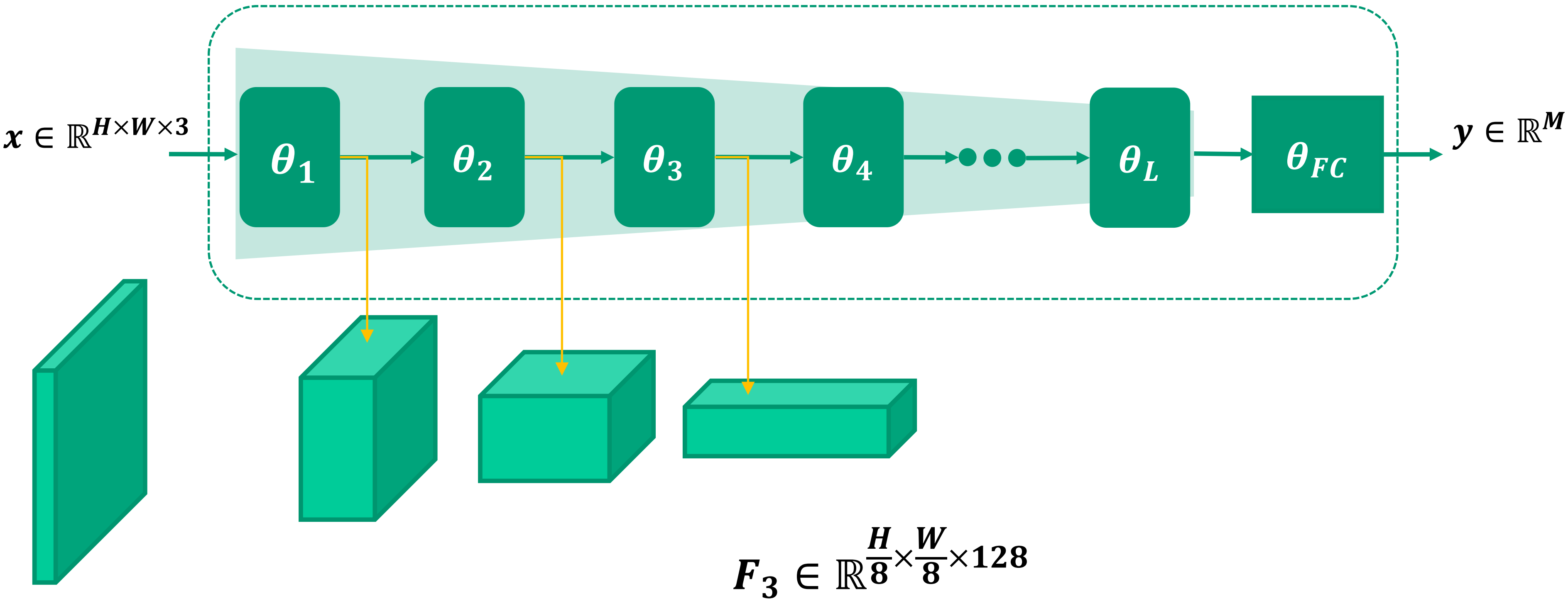
Feed-Forward in Deep Learning

N - N



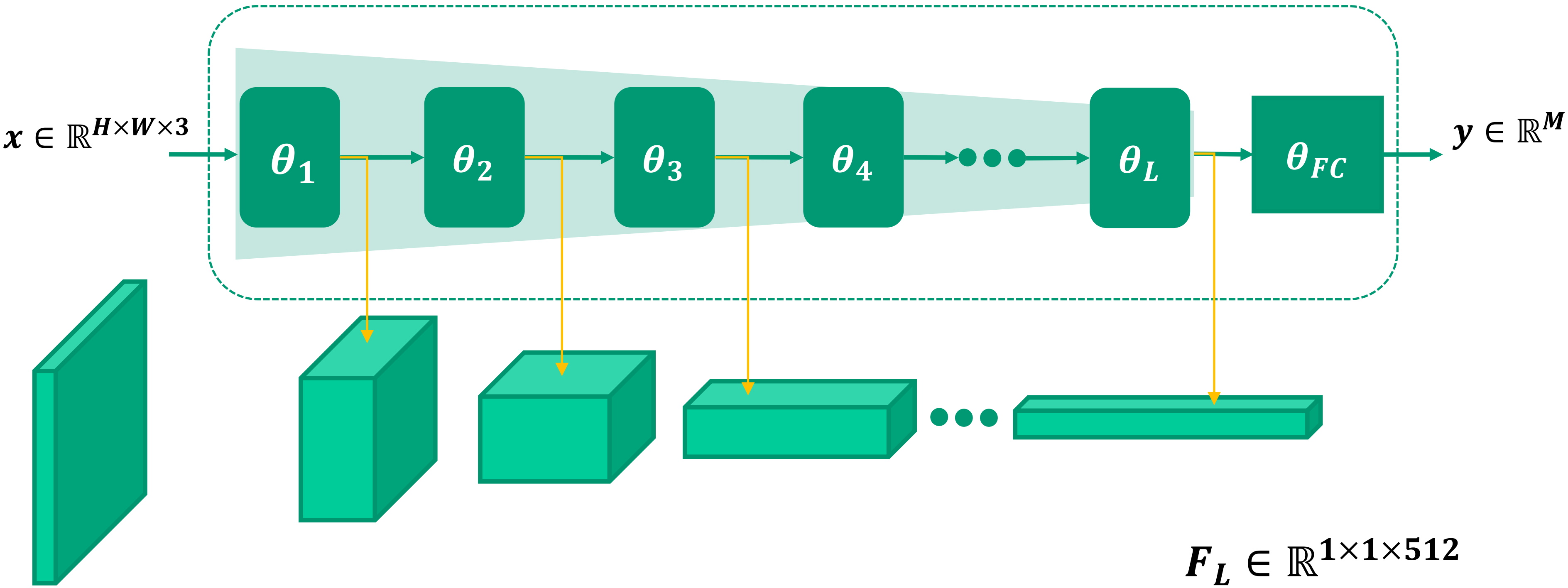
Feed-Forward in Deep Learning

N - N

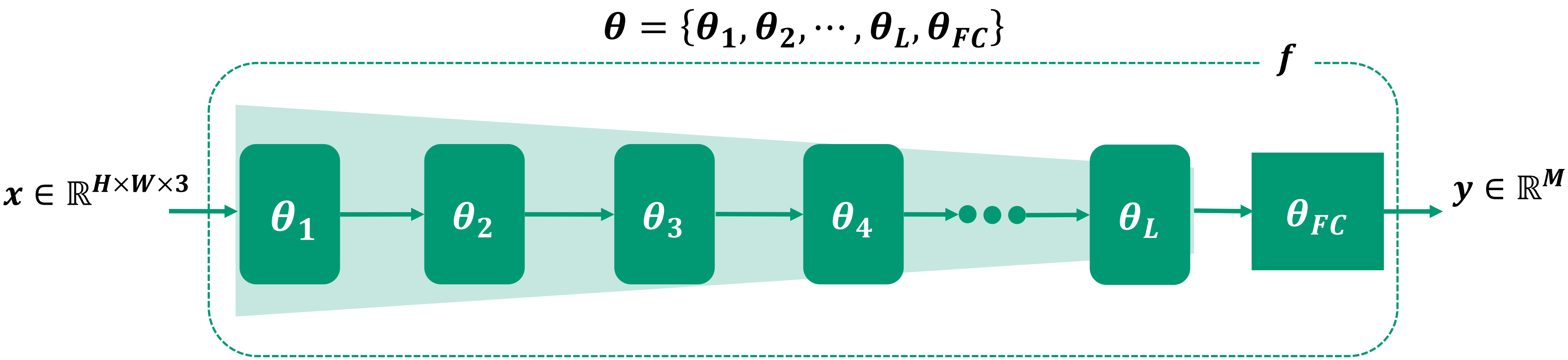


Feed-Forward in Deep Learning

N - N



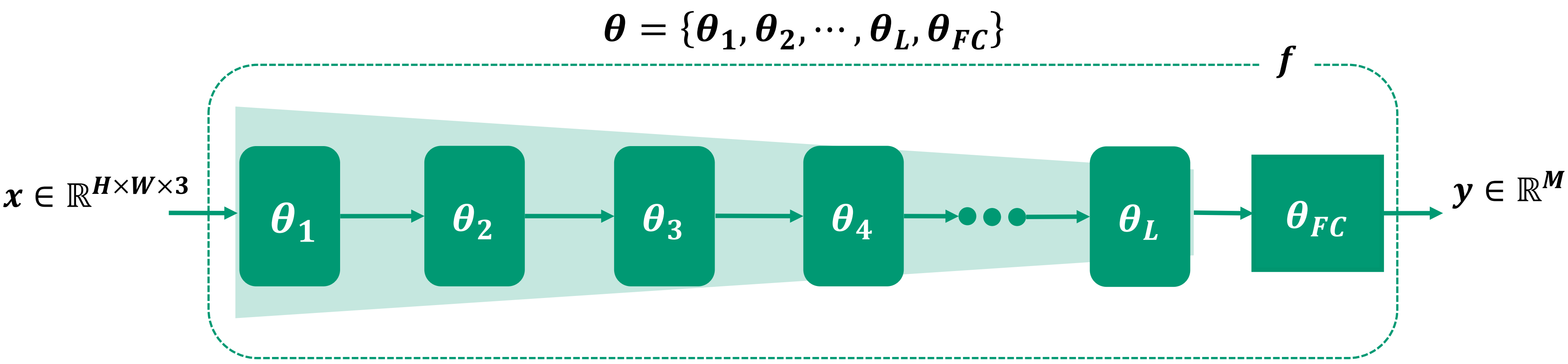
Feed-Forward in Deep Learning



The output prediction label is generated by a function 'f' applied on input image 'x' processed by learnable parameters

$$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\theta})$$

Back-Propagation in Deep Learning



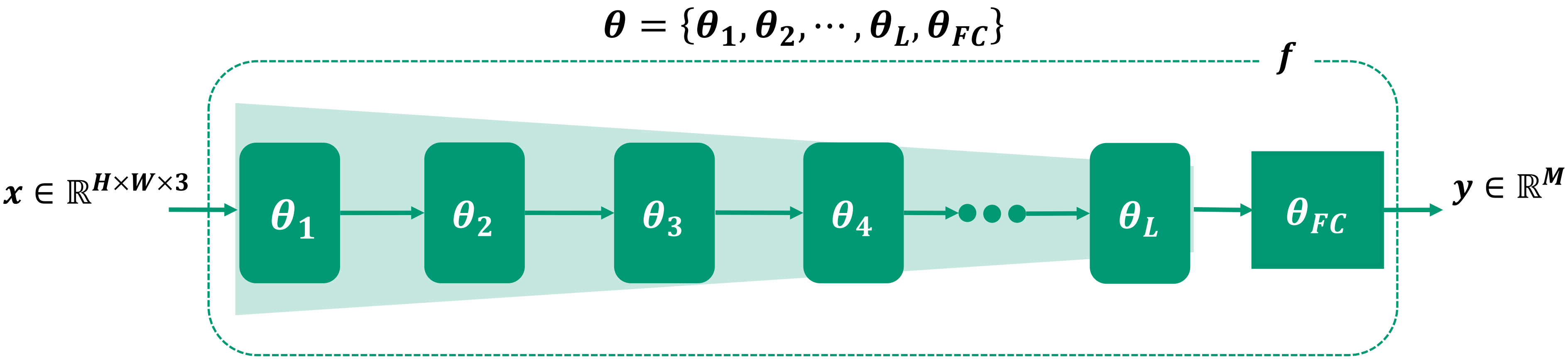
- For each input image $\boldsymbol{x}_i$ there is a corresponding ground-truth label $\boldsymbol{y}^{GT}$ which should be matched with output prediction label $\boldsymbol{y}$

$$\boldsymbol{\epsilon} = \boldsymbol{L}(\boldsymbol{y}, \boldsymbol{y}^{GT})$$

L: Loss-Function

Ideally Speaking: $\boldsymbol{\epsilon} \rightarrow \mathbf{0}$

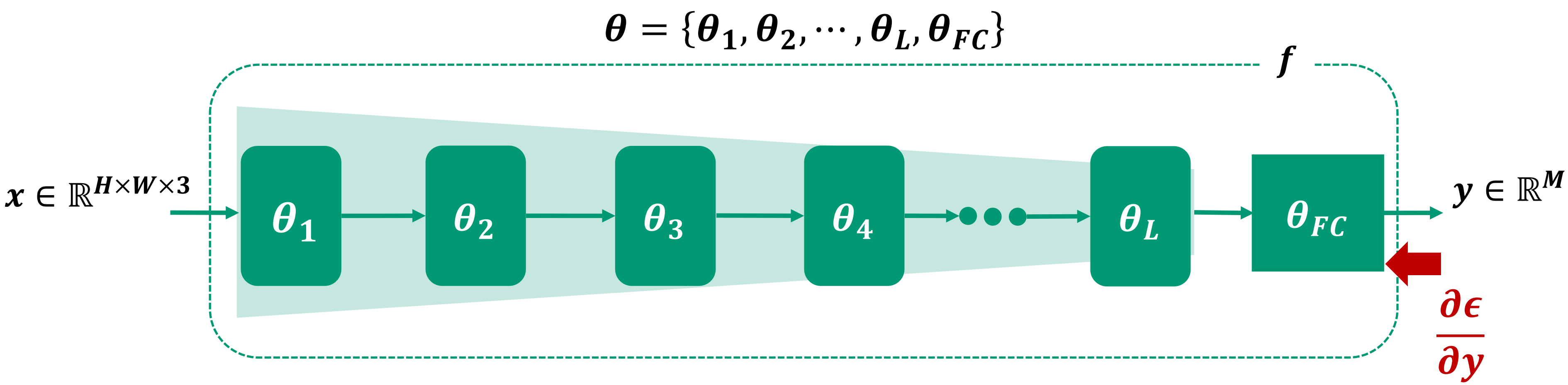
Back-Propagation in Deep Learning



- Calculate the gradient of loss prediction in terms of prediction label

$$\frac{\partial \epsilon}{\partial y} = \frac{\partial L\left(y, y^{GT}\right)}{\partial y}$$

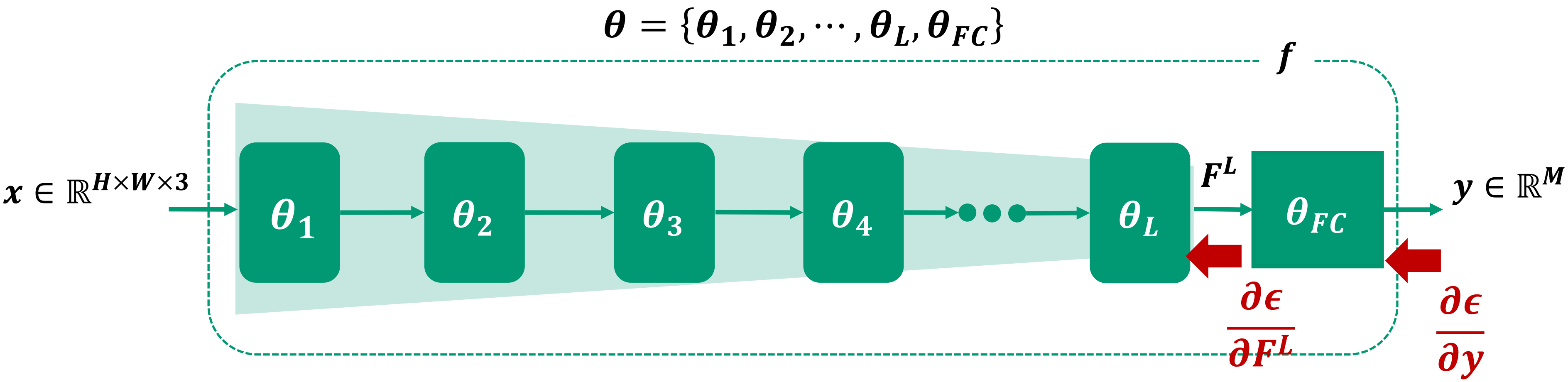
Back-Propagation in Deep Learning



- Calculate the gradient of loss prediction in terms of prediction label

$$\frac{\partial \epsilon}{\partial \boldsymbol{y}} = \frac{\partial L\left(\boldsymbol{y}, \boldsymbol{y}^{GT}\right)}{\partial \boldsymbol{y}}$$

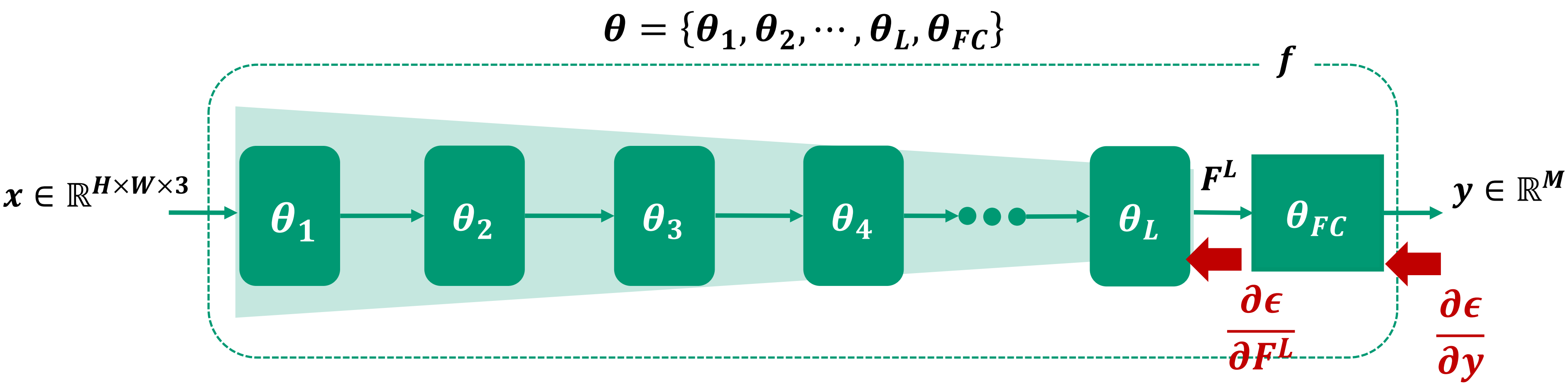
Back-Propagation in Deep Learning



- Back-propagate the gradient of loss-function into inner layers to calculate the gradient of loss-function with respect to the learnable parameter of that particular layer

$$\frac{\partial \epsilon}{\partial F^L} = \frac{\partial \epsilon}{\partial y} \cdot \frac{\partial y}{\partial F^L}$$

Back-Propagation in Deep Learning



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$$\frac{\partial \epsilon}{\partial F^L} = \frac{\partial \epsilon}{\partial y} \cdot \frac{\partial y}{\partial F^L}$$

- We can now update parameter weights using gradient-descent method

$$\underbrace{\boldsymbol{\theta}_L^{k+1}}_{\text{Updated Parameter}} \leftarrow \underbrace{\boldsymbol{\theta}_L^{k}}_{\text{Previous Parameter}} - \underbrace{\alpha}_{\text{Learning Rate}} \cdot \underbrace{\frac{\partial \epsilon}{\partial \theta^L}}_{\text{Error-Gradient}}$$

Back-Propagation in Deep Learning

- Updating on a single image sample introduces noisy gradient direction and we can easily get stuck at local minima
- Select a mini-batch samples (from randomly shuffled data) and average the gradients for updating
- aka we update not for every image but batch-of-images

$$\{\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_B}\} \Longleftrightarrow \{\frac{\partial \epsilon}{\partial F_1^l}, \frac{\partial \epsilon}{\partial F_2^l}, \cdots, \frac{\partial \epsilon}{\partial F_B^l}\}$$

- Superimpose all batch gradients to step into average direction

$$\boldsymbol{\theta_l^{k+1}} \leftarrow \boldsymbol{\theta_l^{k+1}} - \boldsymbol{\rho} \cdot \frac{\mathbf{1}}{\boldsymbol{B}} \sum_{i=1}^{B} \frac{\partial \epsilon}{\partial F_i^l}$$

Updated Parameter

Previous Parameter

Learning Rate

Stochastic Error-Gradient

Stochastic Gradient Descent (SGD)

Algorithm 8.1 Stochastic gradient descent (SGD) update

Require: Learning rate schedule $\boldsymbol{\rho}_1$ $\boldsymbol{\rho}_2, \ldots$

Require: Initial parameter $\boldsymbol{\theta}$

$k \leftarrow 1$

while stopping criterion not met **do**

Sample a minibatch of $\boldsymbol{B}$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(\boldsymbol{B})}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.

Compute gradient estimate: $\hat{\boldsymbol{g}} \leftarrow \frac{1}{\boldsymbol{B}} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$

Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \boldsymbol{\rho}_k \hat{\boldsymbol{g}}$

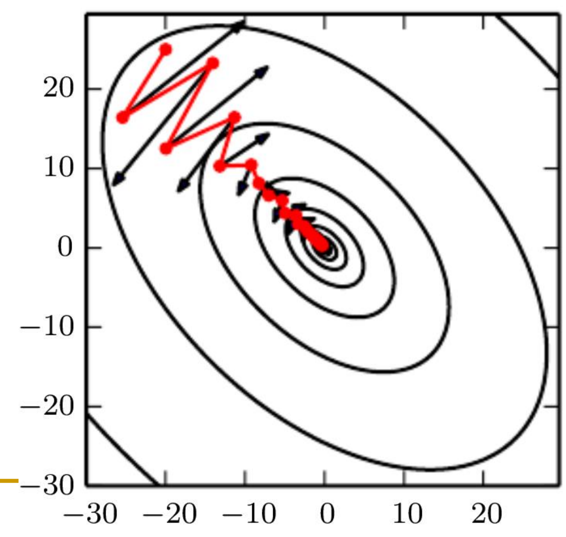
$k \leftarrow k + 1$

end while

SGD with Momentum

- Learning with SGD can be sometimes slow
- Momentum approach we can be used to accelerate learning in the face of exploring local minima of loss function
  - High curvature
  - Small but consistent gradient
  - Noisy gradient
- Momentum approach accumulates an exponentially decaying moving average of past gradients and continues to move in their direction

The contour lines depicts a quadratic loss function with poor Hessian Matrix. The red path cutting across the contour indicates the path followed by momentum learning rule to minimize the loss function



SGD with Momentum

How to formulate it?

- Introduce a hyper-parameter (i.e. momentum) $\alpha \in [0,1)$
- α determines how quickly the contributions of previous gradients exponentially decay
- The update rule is given by

$$\boldsymbol{v} \leftarrow \alpha \boldsymbol{v} - \boldsymbol{\rho} \nabla_{\boldsymbol{\theta}} \left(\frac{1}{\boldsymbol{B}} \sum_{i=1}^{\boldsymbol{B}} L(\boldsymbol{f}(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)}) \right)$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}.$$

Momentum-SGD (MSGD)

Algorithm 8.2 Stochastic gradient descent (SGD) with momentum

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Require: Learning rate $\boldsymbol{\rho}$, momentum parameter α
Require: Initial parameter $\boldsymbol{\theta}$, initial velocity $\boldsymbol{v}$
while stopping criterion not met **do**
  Sample a minibatch of $\boldsymbol{B}$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(\boldsymbol{B})}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.
  Compute gradient estimate: $\boldsymbol{g} \leftarrow \frac{1}{\boldsymbol{B}} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$.
  Compute velocity update: $\boldsymbol{v} \leftarrow \alpha \boldsymbol{v} - \boldsymbol{\rho} \boldsymbol{g}$.
  Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}$.
end while

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Where to read from rest of topics?

Please refer to Reading Material as well as Class Discussions for the rest of topics.