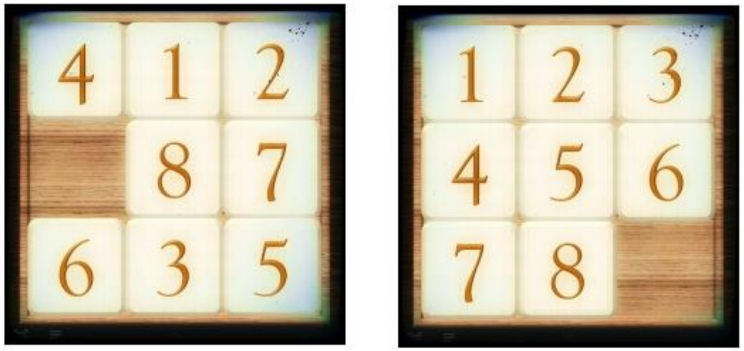
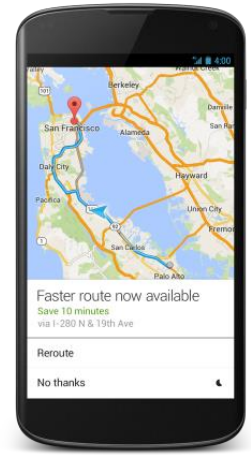
Artificial Intelligence: State Space Search

- Many slides from: robotics.stanford.edu/~latombe/cs121/2003/home.htm

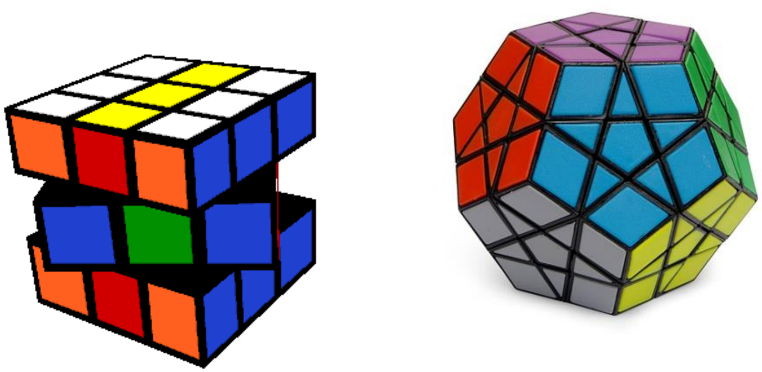
Motivation



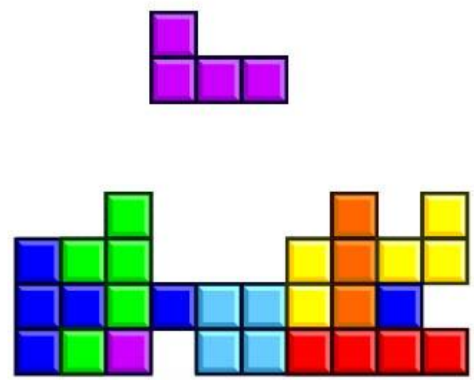
8-puzzle



Google itinerary



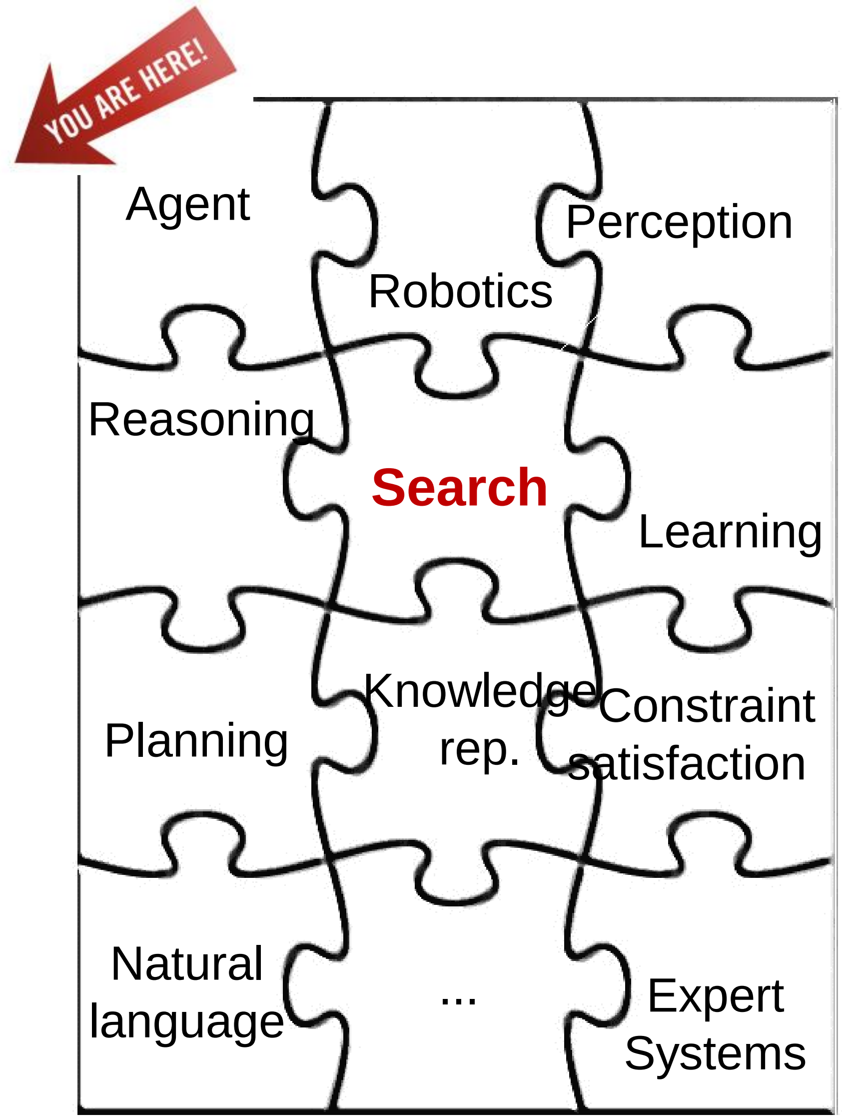
Rubik's cube



Tetris

Today

- State Space Representation
- State Space Search
  - Uninformed search
    - Breadth-first and Depth-first
    - Depth-limited Search
    - Iterative Deepening
    - Uniform Cost
  - Informed search
    - Hill climbing
    - Best-First
    - (Designing Heuristics)
    - A*
- Summary



Example: 8-Puzzle

State: Any arrangement of 8 numbered tiles and an empty tile on a 3x3 board

8	2	
3	4	7
5	1	6

Initial state

1	2	3
4	5	6
7	8	

Goal state

there are several standard goals states for the 8-puzzle

1	2	3
4	5	6
7	8	

1	2	3
8		4
7	6	5

...

(n^2-1)-puzzle

8	2	
3	4	7
5	1	6

8-puzzle

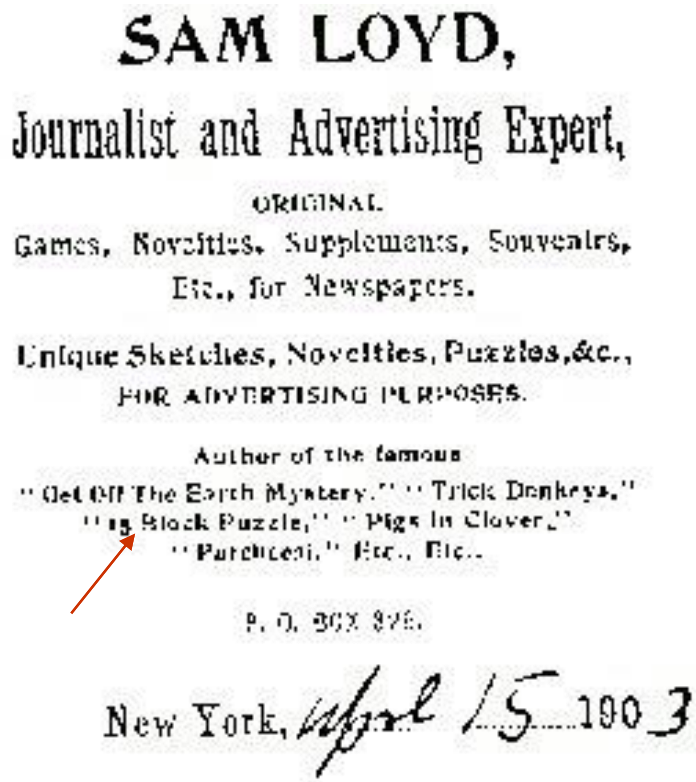
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

15-puzzle

....

15-Puzzle

Invented in 1874 by Noyes Palmer Chapman
… but Sam Loyd claimed he invented it!



SAM LOYD,

Journalist and Advertising Expert,

ORIGINAL
Games, Novelties, Supplements, Souvenirs,
Etc., for Newspapers.

Unique Sketches, Novelties, Puzzles, &c.,
FOR ADVERTISING PURPOSES.

Author of the famous
"Get Off The Earth Mystery," "Trick Donkeys,"
"15 Block Puzzle," "Pigs in Clover,"
"Parcheesi," Etc., Etc.

P. O. BOX [illegible].

New York, April 15 1903

15-Puzzle

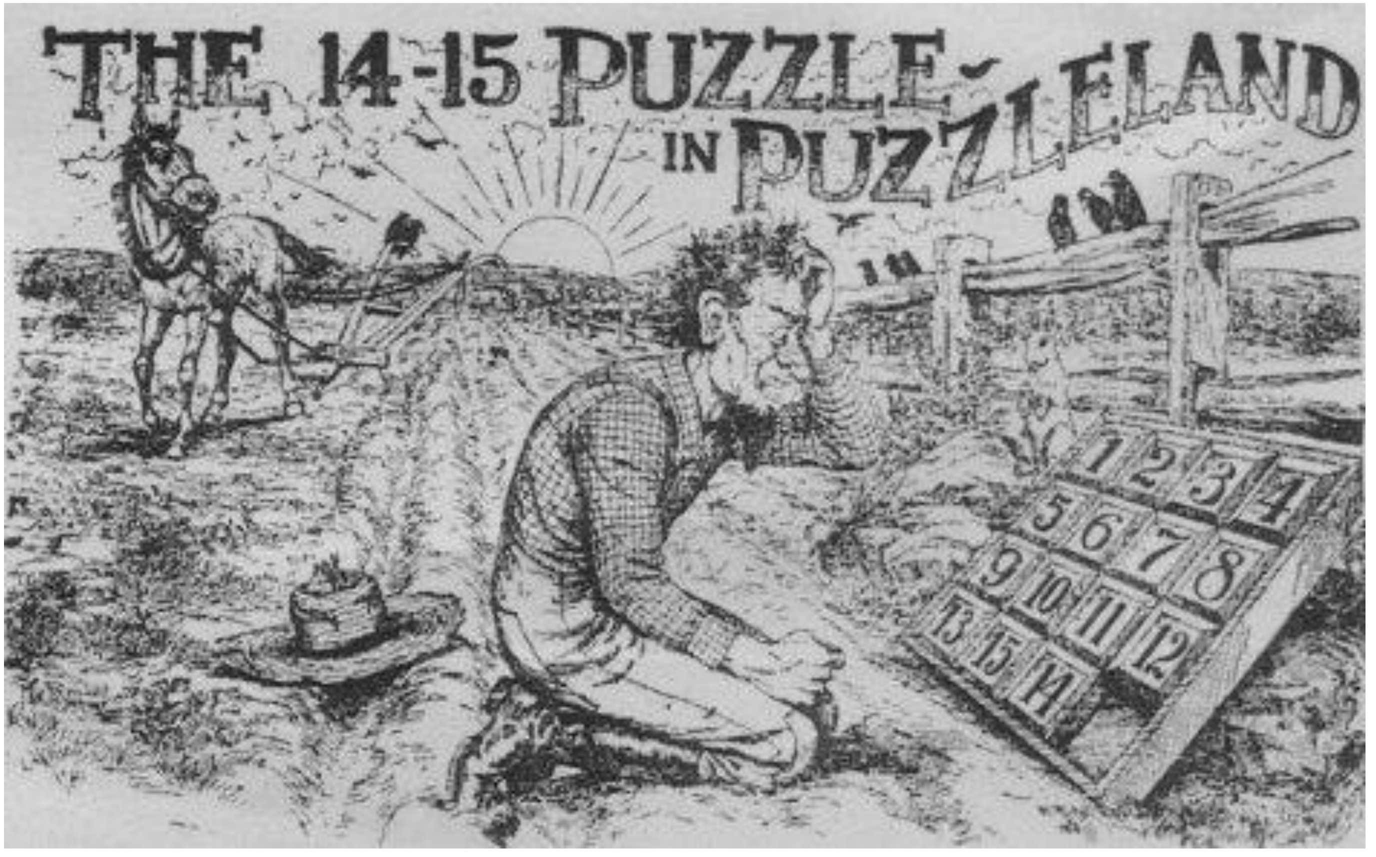
Sam Loyd even offered $1,000 of his own money to the first person who would solve the following problem:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

$\xrightarrow{?}$

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

But no one ever won the prize…



State Space

- Many AI problems, can be expressed in terms of going from an initial state to a goal state
  - *Ex: to solve a puzzle, to drive from home to Concordia…*

- Often, there is no direct way to find a solution to a problem
- but we can list the possibilities and search through them

1. Brute force search:
   - generate and search <u>all</u> possibilities (but inefficient)
2. Heuristic search:
   - only try the possibilities that you *think* (based on your current best guess) are more likely to lead to good solutions

State Space

- Problem is represented by:
  1. Initial State
     - starting state
     - ex. unsolved puzzle, being at home
  2. Set of operators
     - actions responsible for transition between states
  3. Goal test function
     - Applied to a state to determine if it is a goal state
     - ex. solved puzzle, being at Concordia
  4. Path cost function
     - Assigns a cost to a path to tell if a path is preferable to another
- Search space: the set of all states that can be reached from the initial state by any sequence of action
- Search algorithm: how the search space is visited

Example: The 8-puzzle

8	2	
3	4	7
5	1	6

Initial state

1	2	3
4	5	6
7	8	

Goal state

Set of operators:

blank moves up, blank moves down, blank moves left, blank moves right

Goal test function:

state matches the goal state

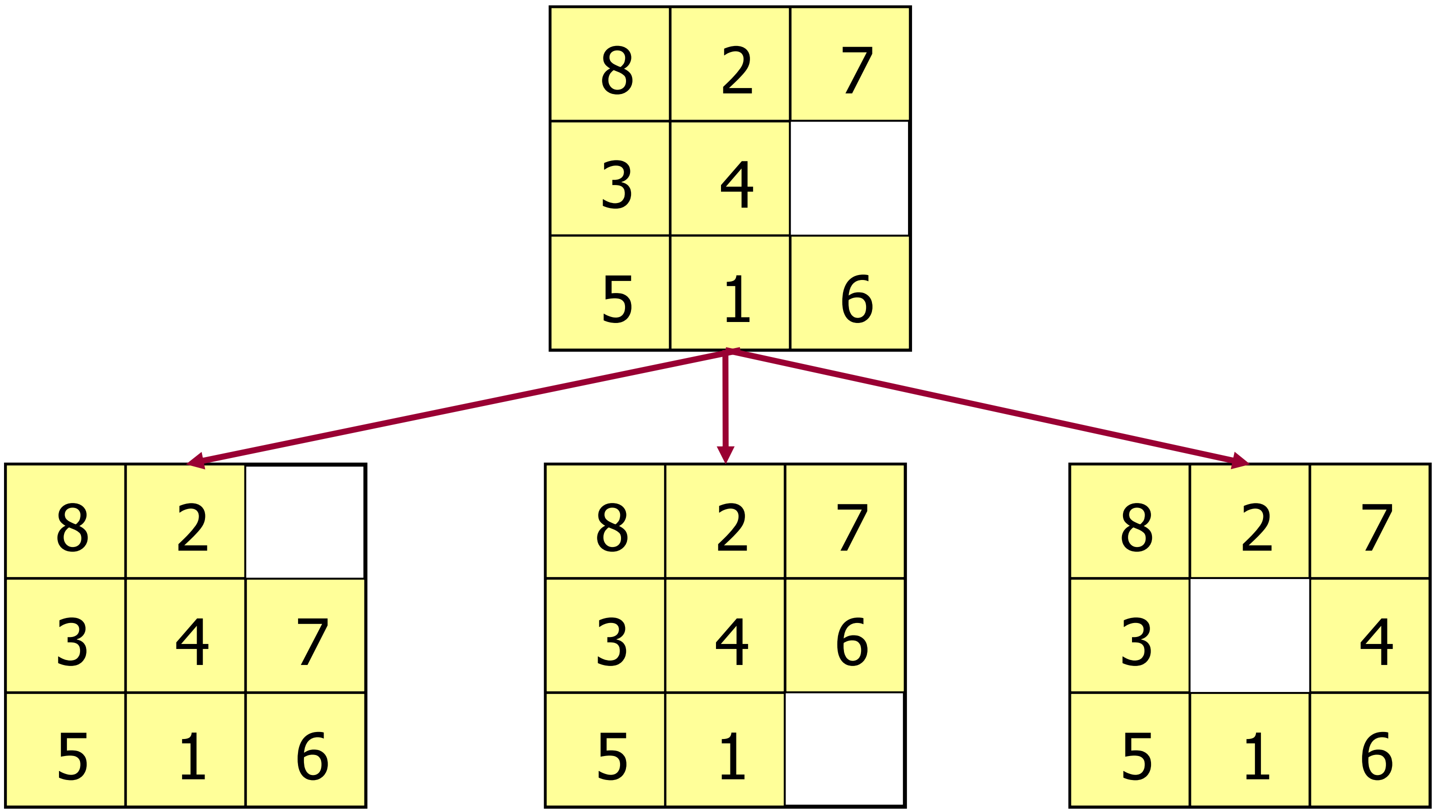
Path cost function:

each movement costs 1

so the path cost is the length of the path (the number of moves)

source: G. Luger (2005)

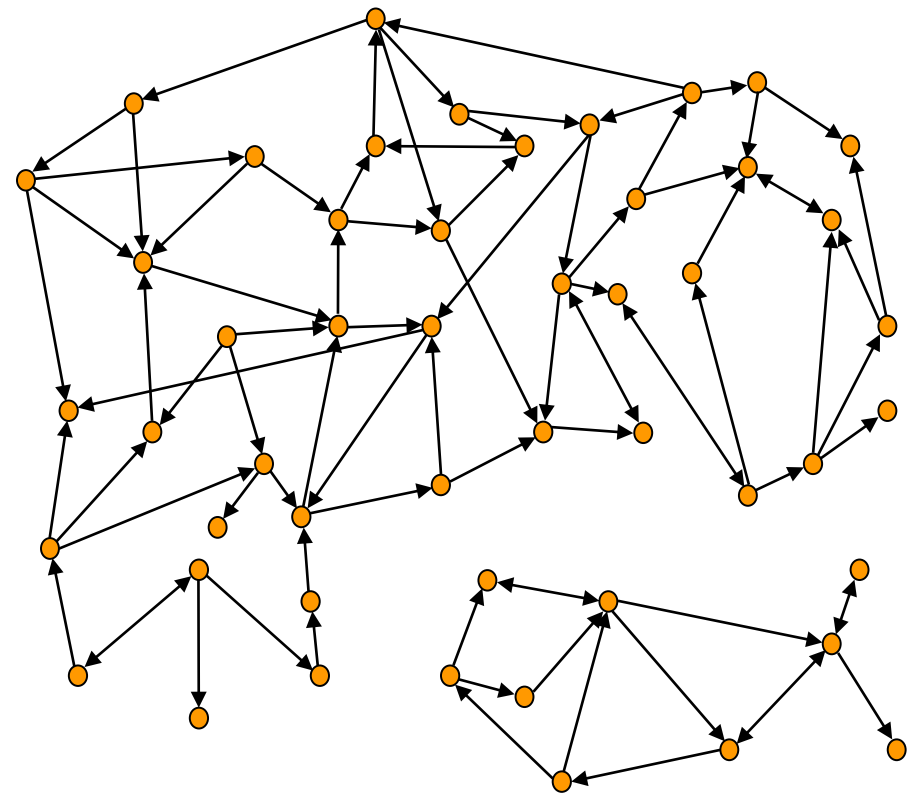
8-Puzzle: Successor Function



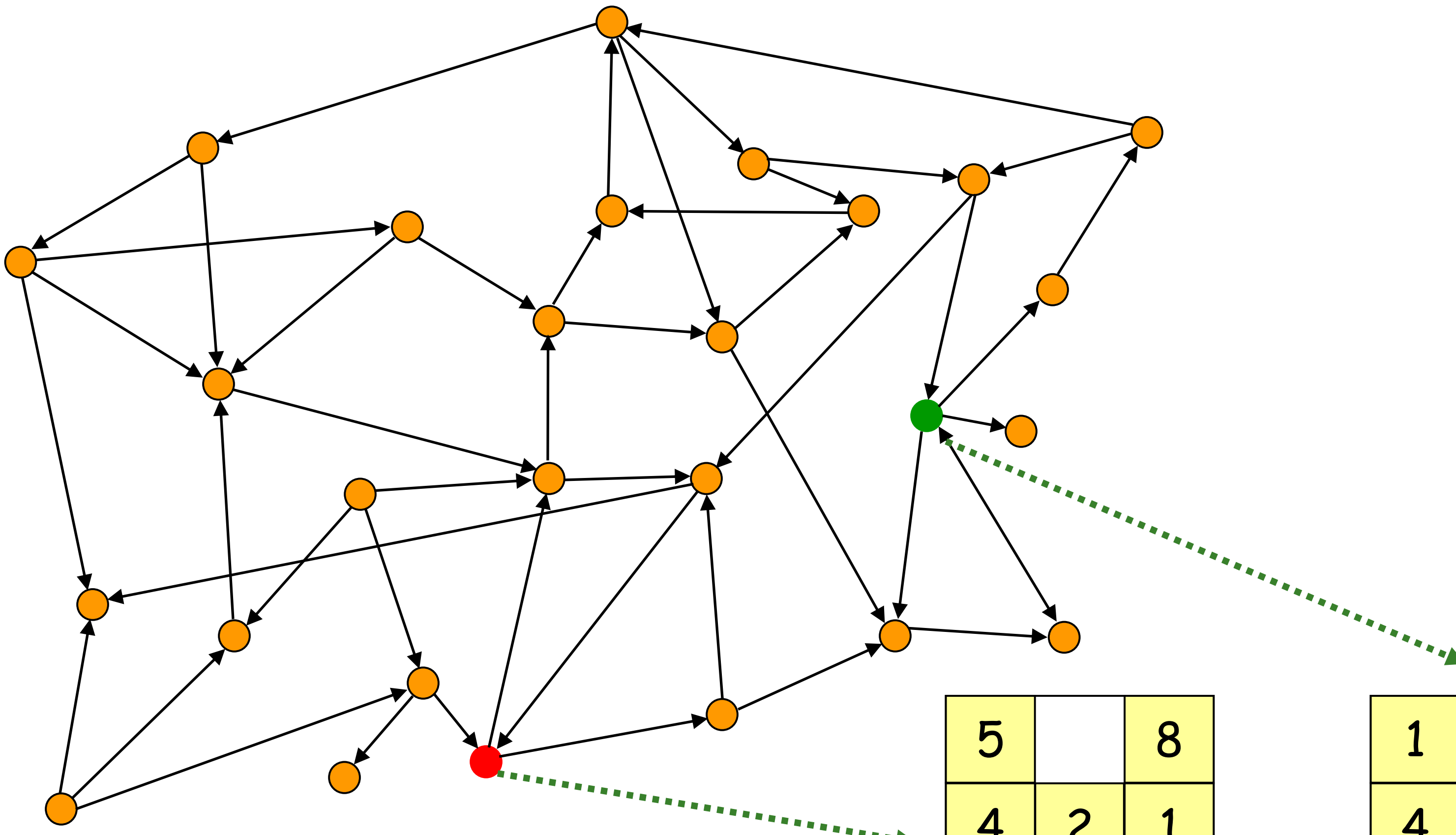
Search is about the exploration of alternatives

State Graph

- Each state is represented by a distinct node
- An arc (or edge) connects a node s to a node s' if s' ∈ SUCCESSOR(s)
- The state graph may contain more than one connected component



Just to make sure we're clear…



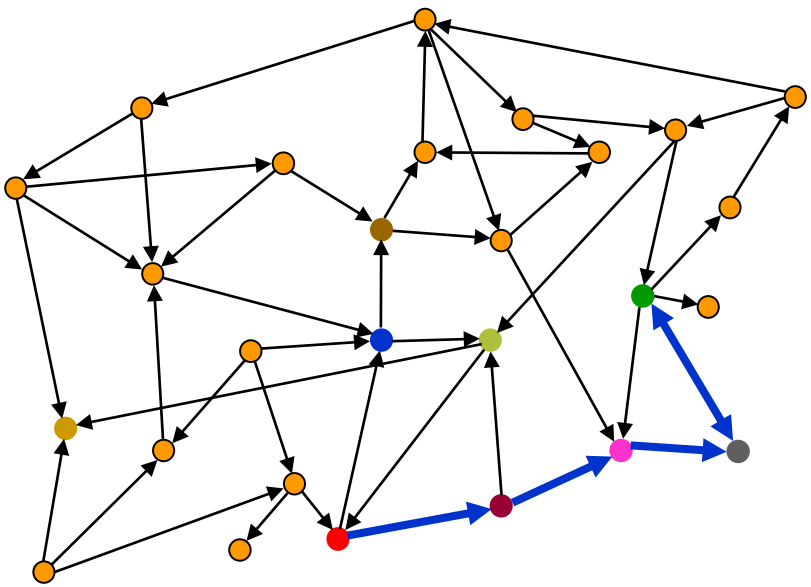
5		8
4	2	1
7	3	6

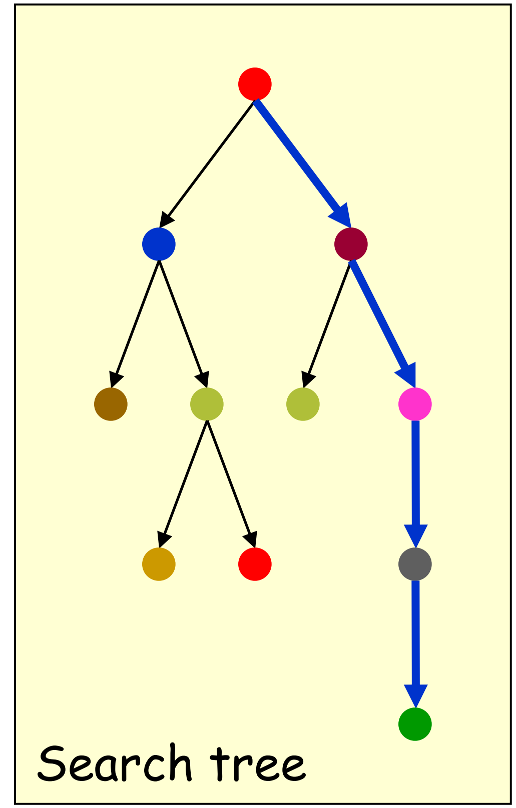
Initial state

1	2	3
4	5	6
7	8	

Goal state

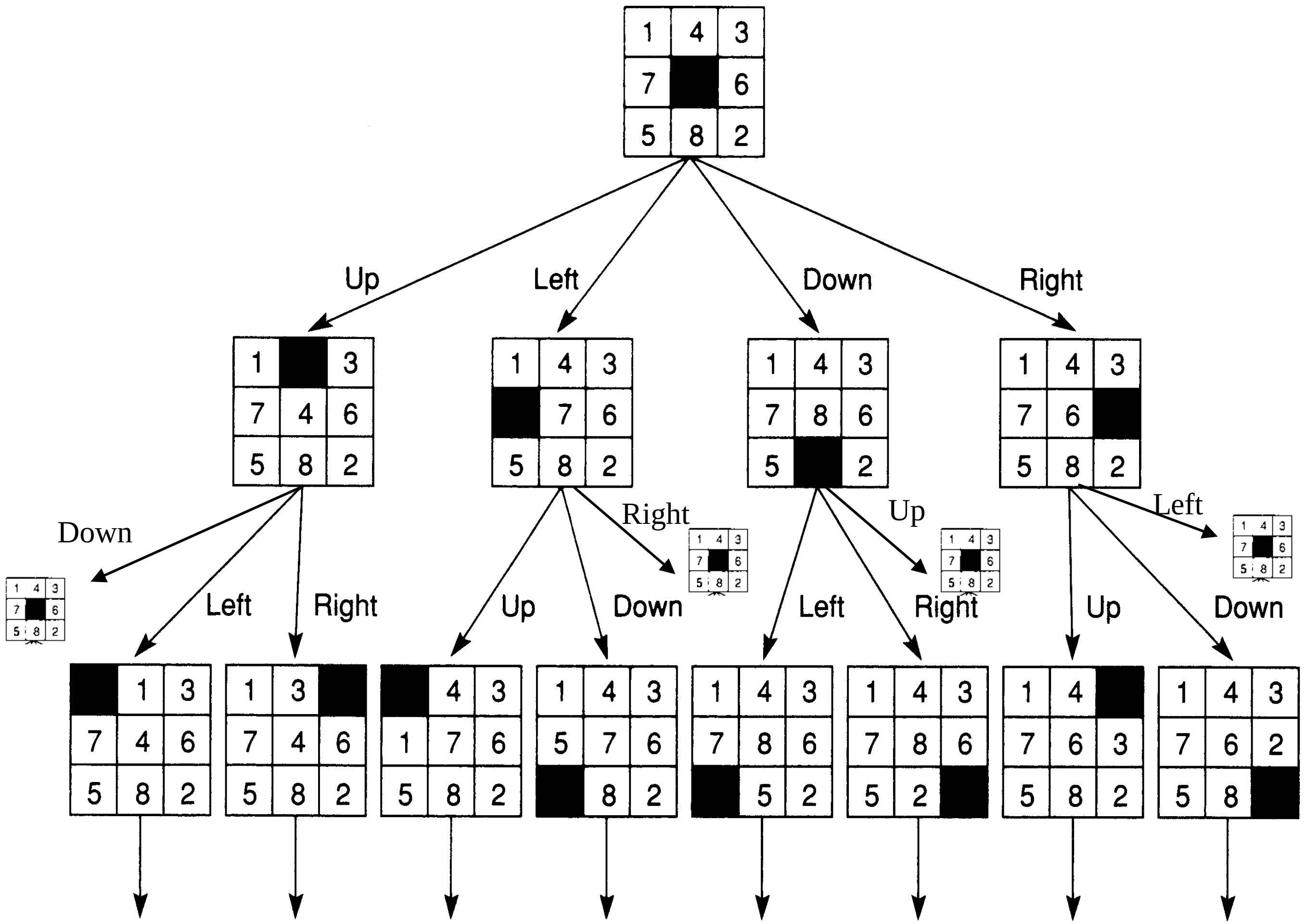
State Space as a Search Tree





- In graph representation, cycles can prevent termination
  - Blind search without cycle check may never terminate
- Use a tree representation, and check for cycles

State Space for the 8-puzzle



source: G. Luger (2005)

How large is the state space of the (n^2-1)-puzzle?

- Nb of states:
  - 8-puzzle --> 9! = 362,880 states
  - 15-puzzle --> 16! ~ 2.09×10^{13} states
  - 24-puzzle --> 25! ~ 10^{25} states
- At 100 millions states/sec:
  - 8-puzzle --> 0.036 sec
  - 15-puzzle --> ~ 55 hours
  - 24-puzzle --> > 10^{9} years

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    - Hill climbing
    - Best-First
    - (Designing Heuristics)
    - A*
- Summary

Uninformed VS Informed Search

- Uninformed search
  - We systematically explore the alternatives
  - aka: systematic/exhaustive/blind/brute force search
    - Breadth-first
    - Depth-first
    - Uniform-cost
    - Depth-limited search
    - Iterative deepening search
    - Bidirectional search
    - …
- Informed search (heuristic search)
  - We try to choose smartly
    - Hill climbing
    - Best-First
    - A*
    - …

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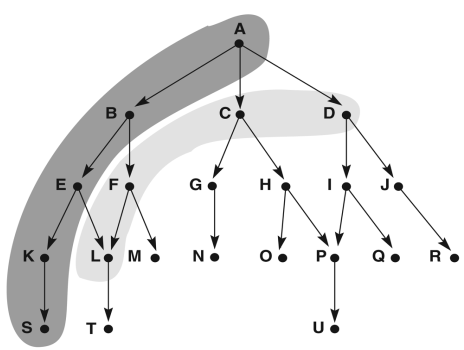
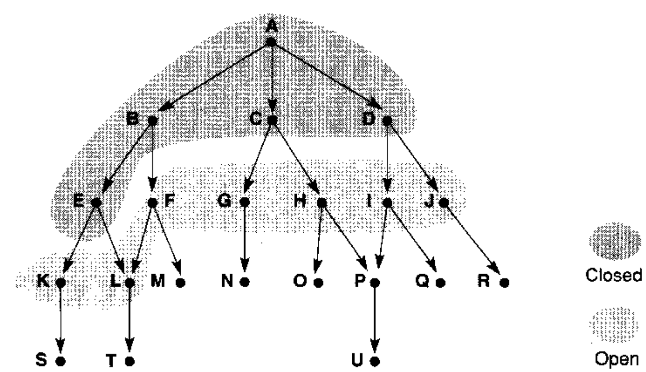
Breadth-first vs Depth-first Search

- Determine order for examining states
  - Depth-first:
    - visit successors before siblings
  - Breadth-first:
    - visit siblings before successors
    - ie. visit level-by-level



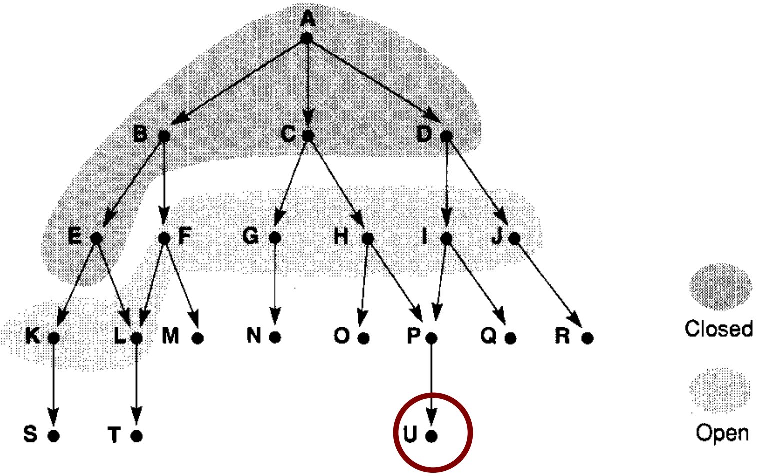


source: G. Luger (2005)

Data Structures

- In all search strategies, you need:
  - ***open list*** (aka the frontier)
    - lists generated nodes not yet expanded
    - order of nodes controls order of search
  - ***closed list*** (aka the explored set)
    - stores all the nodes that have already been visited (to avoid cycles).
- ex:

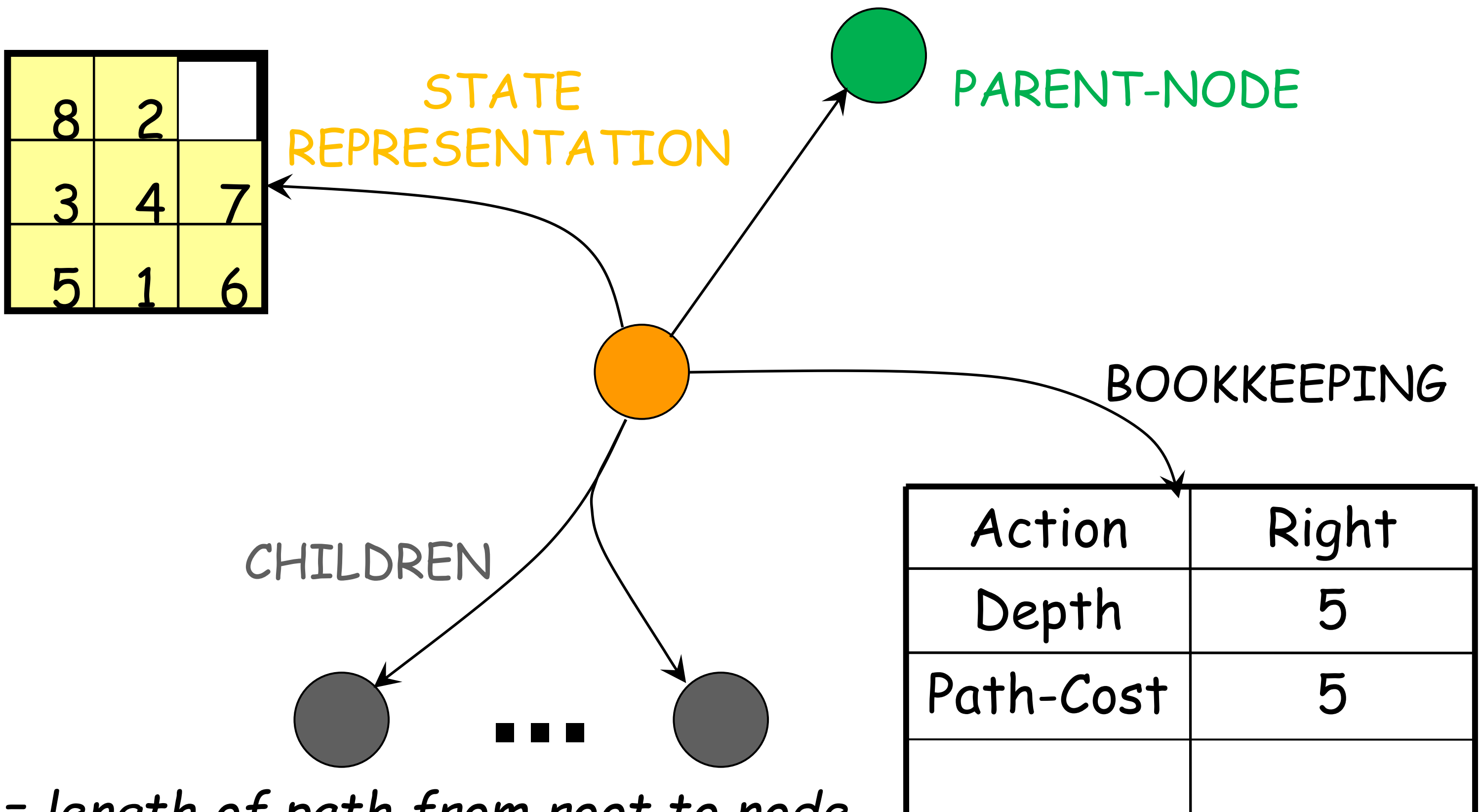
Closed = [A, B, C, D, E]
Open = [F, G, H, I, J, K, L]



source: G. Luger (2005)

Data Structures

- To trace back the entire path of the solution after the search, each node in the lists contain:



Depth = length of path from root to node

Generic Search Algorithm

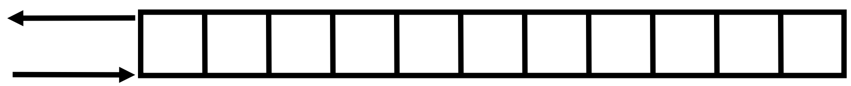
1. Initialize the open list with the initial node s_o (top node)
2. Initialize the closed list to empty
3. Repeat
   a) If the open list is empty, then exit with failure.
   b) Else, take the first node s from the open list.
   c) If s is a goal state, exit with success. Extract the solution path from s to s_o
   d) Else, insert s in the closed list (s has been visited /expanded)
   e) Insert the successors of s in the open list in a certain order if they are not already in the closed and/or open lists (to avoid cycles)

Notes:

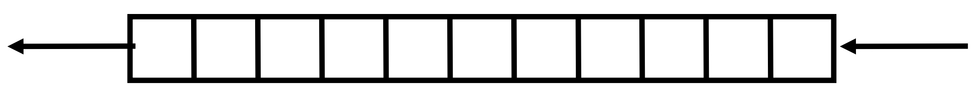
- The order of the nodes in the open list depends on the search strategy

DFS and BFS

- DFS and BFS differ only in the way they order nodes in the open list:
  - DFS uses a stack:
    - nodes are added on the top of the list.



  - BFS uses a queue:
    - nodes are added at the end of the list.



Breadth-First Search

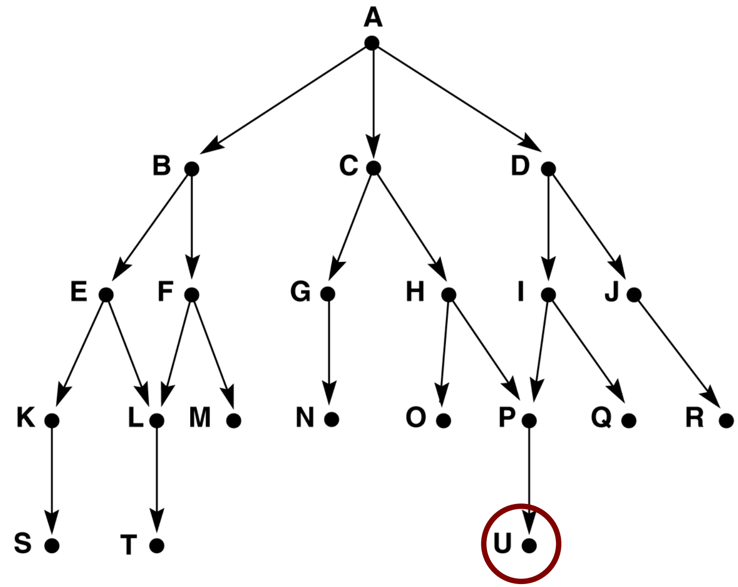
```
begin
   open := [Start];                                                    % initialize
   closed := [ ];
   while open ≠ [ ] do                                                 % states remain
      begin
         remove leftmost state from open, call it X;
            if X is a goal then return SUCCESS                         % goal found
               else begin
                  generate children of X;
                  put X on closed;
                  discard children of X if already on open or closed;  % loop check
                  put remaining children on right end of open          % queue
               end
      end
   return FAIL                                                         % no states left
end.
```

source: G. Luger (2005)

Breadth-First Search Example

- BFS: (open is a queue)

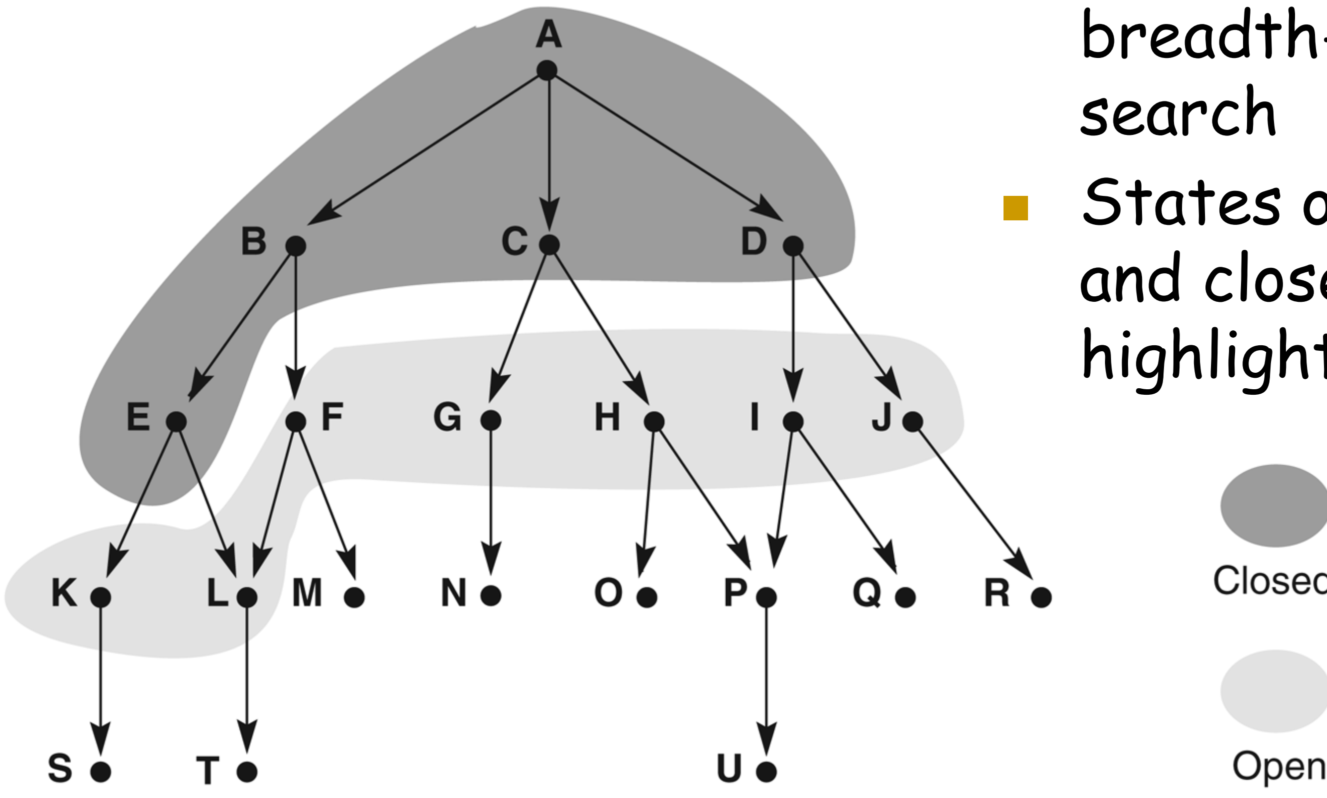
Assume U is goal state



1. open = [A-null] closed = []
2. open = [B-A C-A D-A] closed [A]
3. open = [C-A D-A E-B F-B] closed = [B A]
4. open = [D-A E-B F-B G-C H-C] closed = [C B A]
5. open = [E-B F-B G-C H-C I-D J-D] closed = [D C B A]
6. open = [F-B G-C H-C I-D J-D K-E L-E] closed = [E D C B A]
7. open = [G-C H-C I-D J-D K-E L-E M-F] *as L is already in open* closed = [F E D C B A]
8. and so on until either U is found or open = []

source: G. Luger (2005)

Snapshot of BFS



- Search graph at iteration 6 of breadth-first search
- States on open and closed are highlighted

source: *G. Luger (2005)*

Function Depth-First Search

Function depth_first_search algorithm

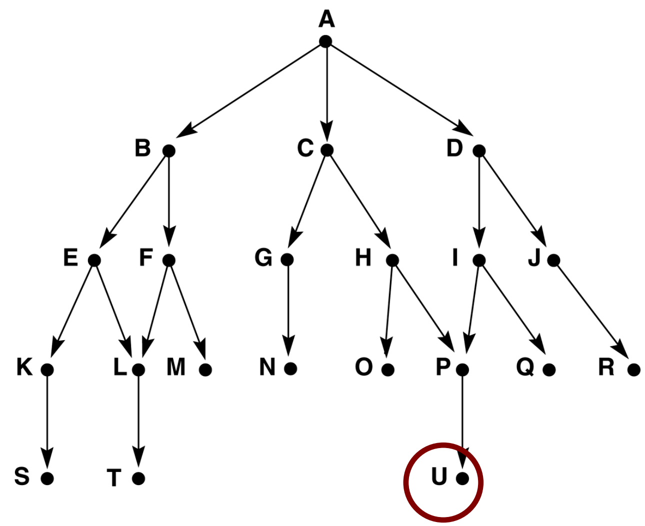
```
begin
  open := [Start];                                                    % initialize
  closed := [ ];
  while open ≠ [ ] do                                                 % states remain
    begin
      remove leftmost state from open, call it X;
      if X is a goal then return SUCCESS                              % goal found
        else begin
          generate children of X;
          put X on closed;
          discard children of X if already on open or closed;         % loop check
          put remaining children on left end of open                  % stack
        end
    end;
  return FAIL                                                         % no states left
end.
```

source: *G. Luger (2005)*

Depth-First Search Example

- DFS: (open is a stack)

Assume U is goal state



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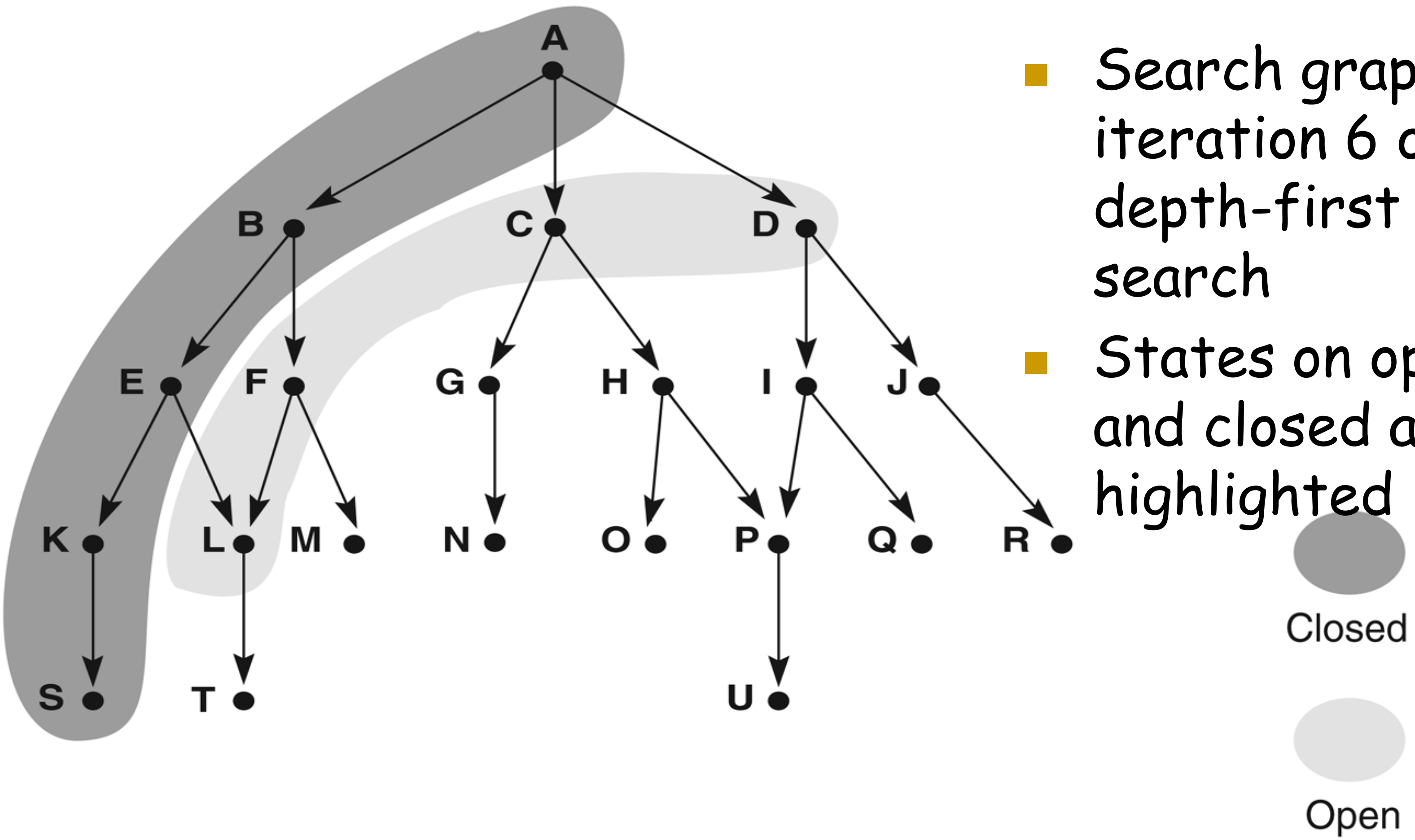
2. open = [B-A C-A D-A] closed [A]
3. open = [E-B F-B C-A D-A] closed = [B A]
4. open = [K-E L-E F-B C-A D-A] closed = [E B A]
5. open = [S-K L-E F-B C-A D-A] closed = [K E B A]
6. open = [L-E F-B C-A D-A] closed = [S K E B A]
7. open = [T-L F-B C-A D-A] closed = [L S K E B A]
8. open = [F-B C-A D-A] closed = [T L S K E B A]
9. open = [M-F C-A D-A] *as L is already on closed* closed = [F T L S K E B A]
10. open = [C-A D-A] closed = [M F T L S K E B A]
11. open = [G-C H-C D-A] closed = [C M F T L S K E B A]

source: *G. Luger (2005)*

Snapshot of DFS



- Search graph at iteration 6 of depth-first search
- States on open and closed are highlighted

source: *G. Luger (2005)*

Depth-first vs. Breadth-first

- Breadth-first:
  - Optimal: will always finds shortest path

  But:
  - inefficient if branching factor **B** is very high
  - memory requirements high -- exponential space for states required: $\mathbf{B}^n$

- Depth-first:
  - Not optimal (no guarantee to find the shortest path)

  But:
  - Requires less memory

- But both search are impractical in real applications because search space is too large!

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Depth-Limited Search

Compromise for DFS :

- Do depth-first but with depth cutoff k (depth at which nodes are not expanded)

- Three possible outcomes:
  - Solution
  - Failure (no solution)
  - Cutoff (no solution within cutoff)

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Iterative Deepening

Compromise between BFS and DFS:

- use depth-first search, but
- with a maximum depth before going to next level

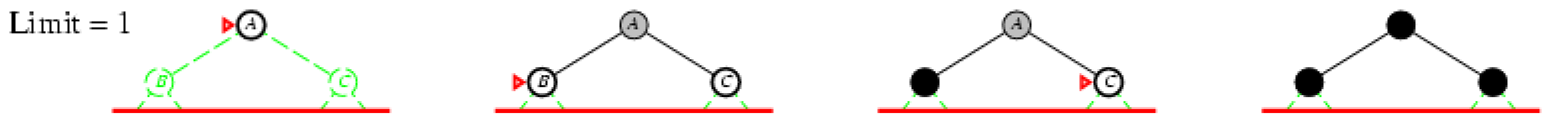
- Repeats depth first search with gradually increasing depth limits
  - Requires little memory (fundamentally, it's a depth first)
  - Finds the shortest path (limited depth)

- Preferred search method when there is a large search space and the depth of the solution is unknown

Iterative Deepening: Example

source: *Russel & Norvig (2003)*

Iterative Deepening: Example



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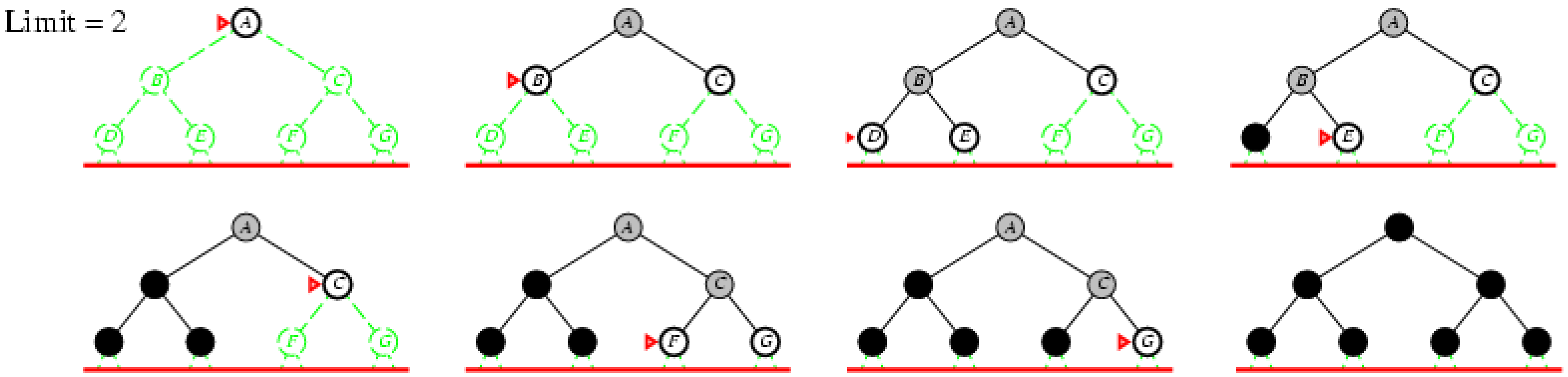

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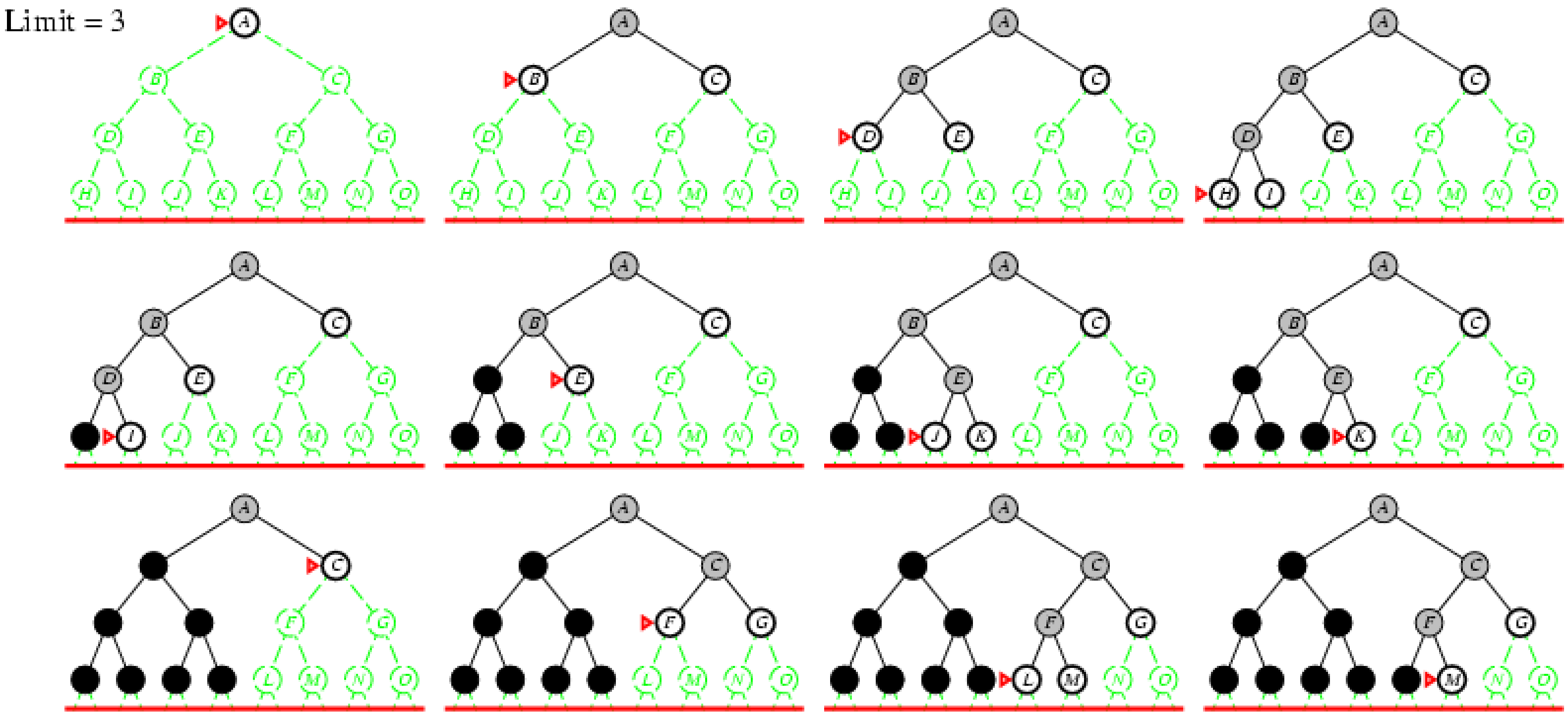
Iterative Deepening: Example



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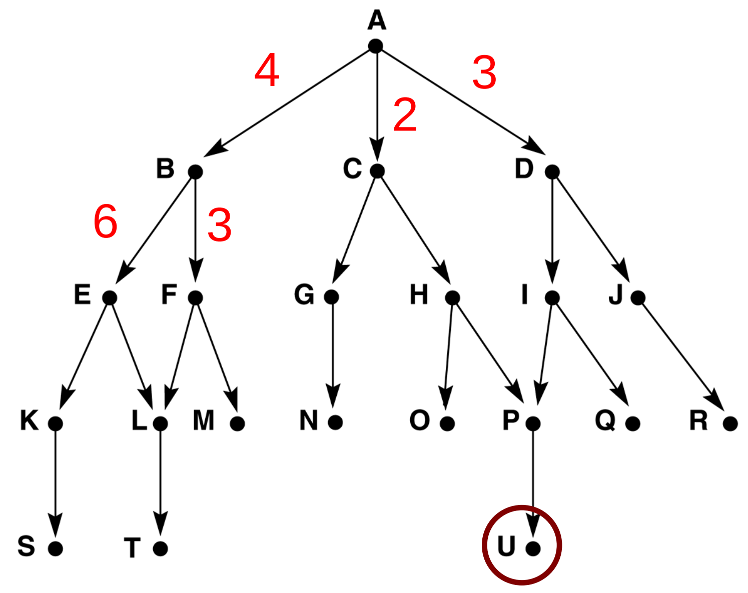
Uniform Cost Search

- Breadth First Search
  - Open is a priority queue sorted using the <u>depth</u> of the nodes from the root
  - guarantees to find the <u>shortest</u> solution path
- But what if all edges/moves do not have the same cost?
- Uniform Cost Search
  - uses a priority queue sorted using the <u>cost</u> from the root to node n – later called g(n)
  - guarantees to find the <u>lowest cost</u> solution path



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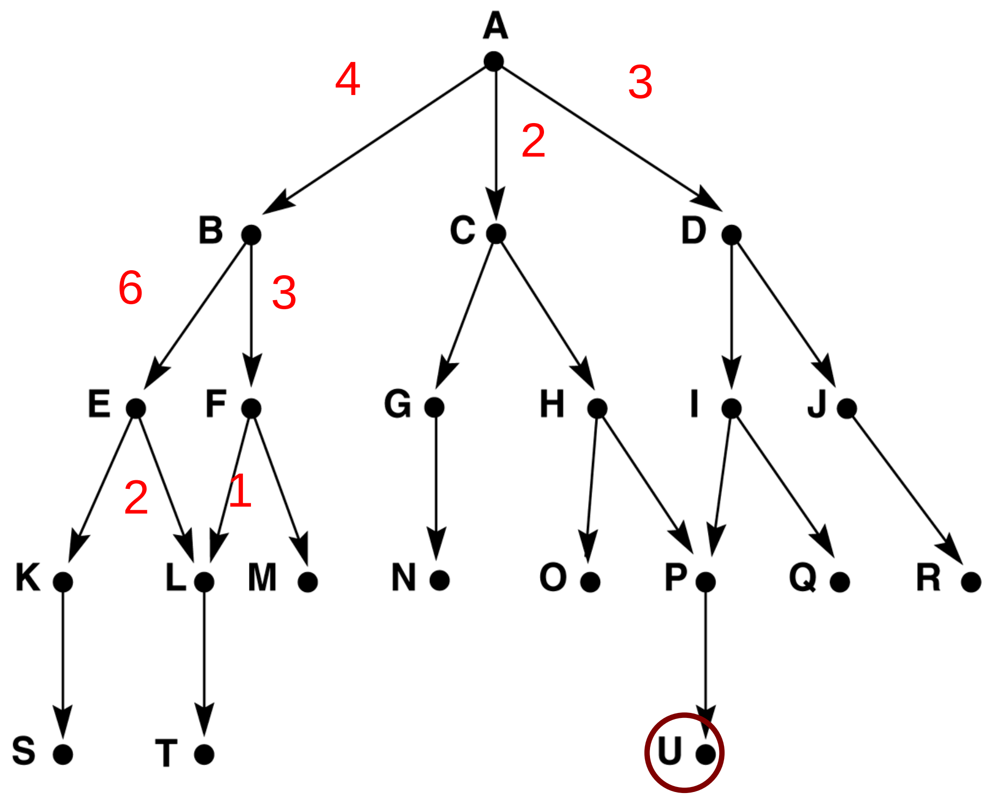
Informed Search (aka heuristic search)

- Most of the time, it is not feasible to do an exhaustive search, search space is too large
  - e.g. state space of all possible moves in chess = 10^{120}
    - 10^{75} = nb of molecules in the universe
    - 10^{26} = nb of nanoseconds since the "big bang"
- so far, all search algorithms have been uninformed (general search)
- so need an informed/heuristic search
- Idea:
  - choose "best" next node to expand
  - according to a selection function (i.e. a heuristic function h(n))
- But: heuristic might fail

Heuristic - *Heureka!*

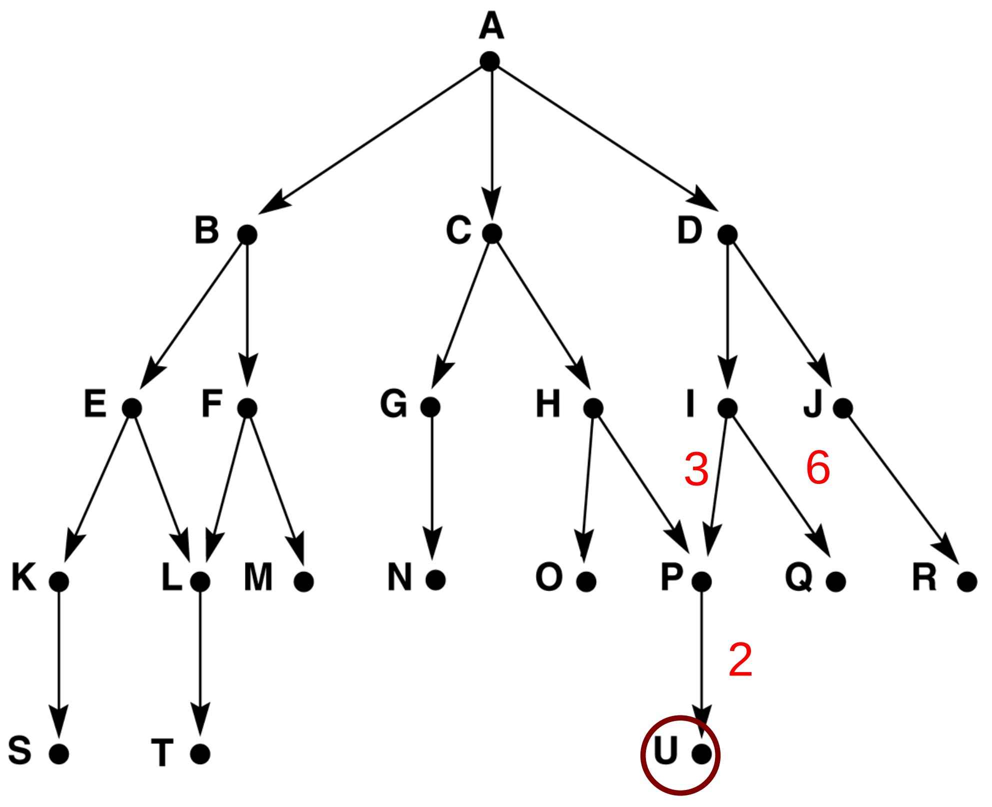
- Heuristic:
  - a rule of thumb, a good bet
  - but has no guarantee to be correct whatsoever!
- Heuristic search:
  - A technique that improves the efficiency of search, possibly sacrificing on completeness
  - Focus on paths that seem most promising according to some function
  - Need an evaluation function (heuristic function) to score a node in the search tree
- Heuristic function h(n) = an approximation of the lowest cost from node n to the goal

g(n)



- **g(n)** = cost of current path from start to node *n*

h(n)



- **h(n)** = estimate of the lowest cost from *n* to *goal*

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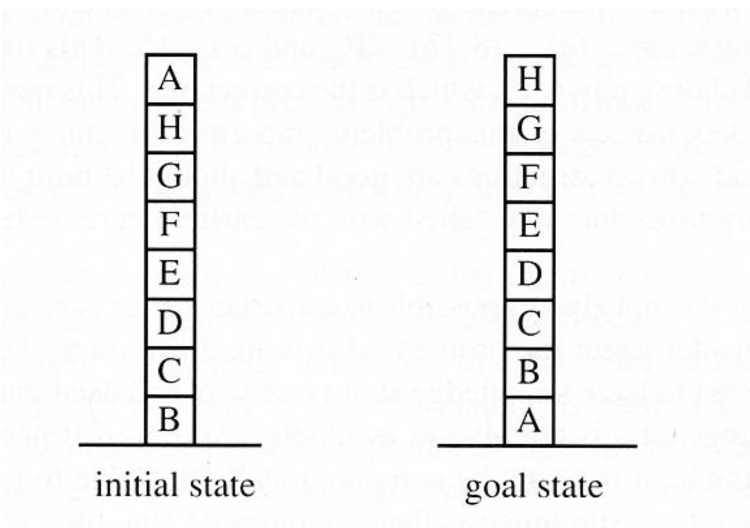
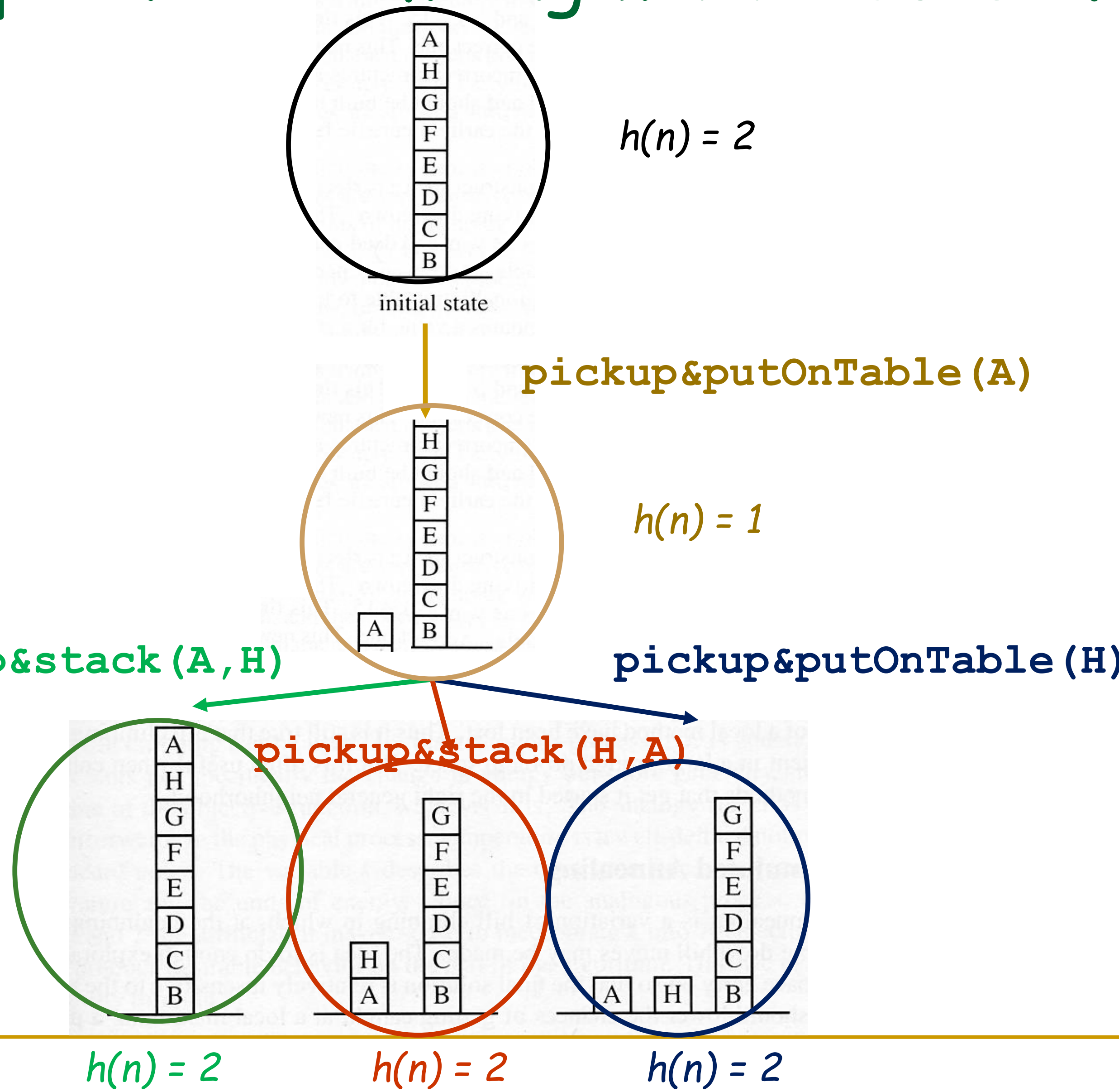
Example: Hill Climbing with Blocks World



- Operators:
  - **`pickup&putOnTable(Block)`**
  - **`pickup&stack(Block1,Block2)`**
- Heuristic:
  - 0pt if a block is sitting where it is supposed to sit
  - +1pt if a block is NOT sitting where it is supposed to sit
  - so lower h(n) is better
    - h(initial) = 2
    - h(goal) = 0

source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991.

Example: Hill Climbing with Blocks World



Hill Climbing

- General hill climbing strategy:
  - as soon as you find a position that is better than the current one, select it.
  - <u>Does not</u> maintain a list of next nodes to visit (an *open* list)
  - Similar to climbing a mountain in the fog with amnesia … always go higher than where you are now,
    but never go back…

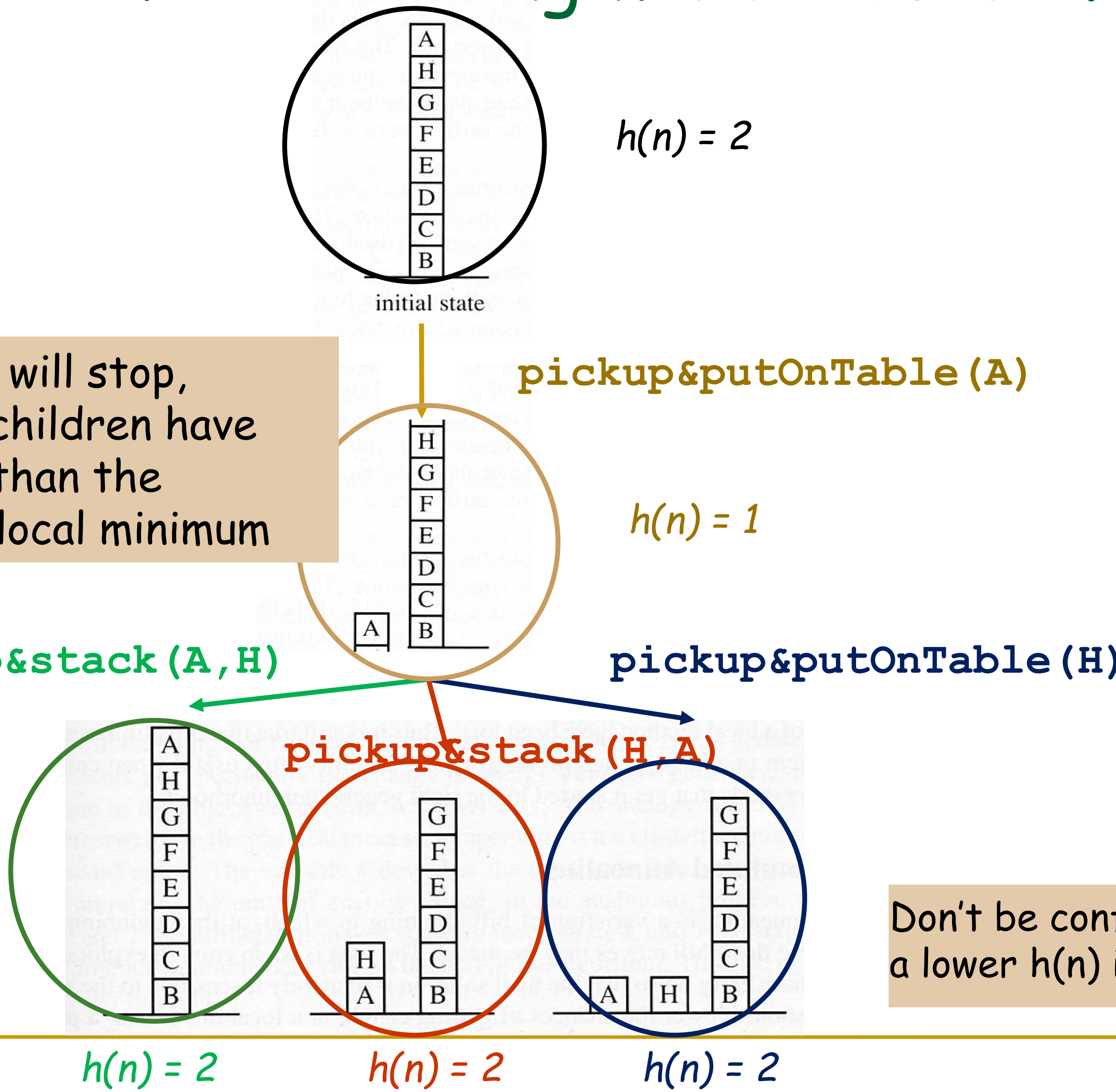
- Steepest ascent hill climbing:
  - instead of moving to the first position that is better than the current one
  - pick the best position out of all the next possible moves

Steepest Ascent Hill Climbing

```
currentNode = startNode;
    loop do
        L = CHILDREN(currentNode);
        nextEval = INFINITY;
        nextNode = NULL;

        for all c in L
            if (HEURISTIC-VALUE(c) < nextEval) // lower h is better
                nextNode = c;
                nextEval = HEURISTIC-VALUE(c);

        if nextEval >= HEURISTIC-VALUE(currentNode)
            // Return current node since no better child state exists
            return currentNode;

        currentNode = nextNode;
```

Adapted from https://en.wikipedia.org/wiki/Hill_climbing

Example: Hill Climbing with Blocks World

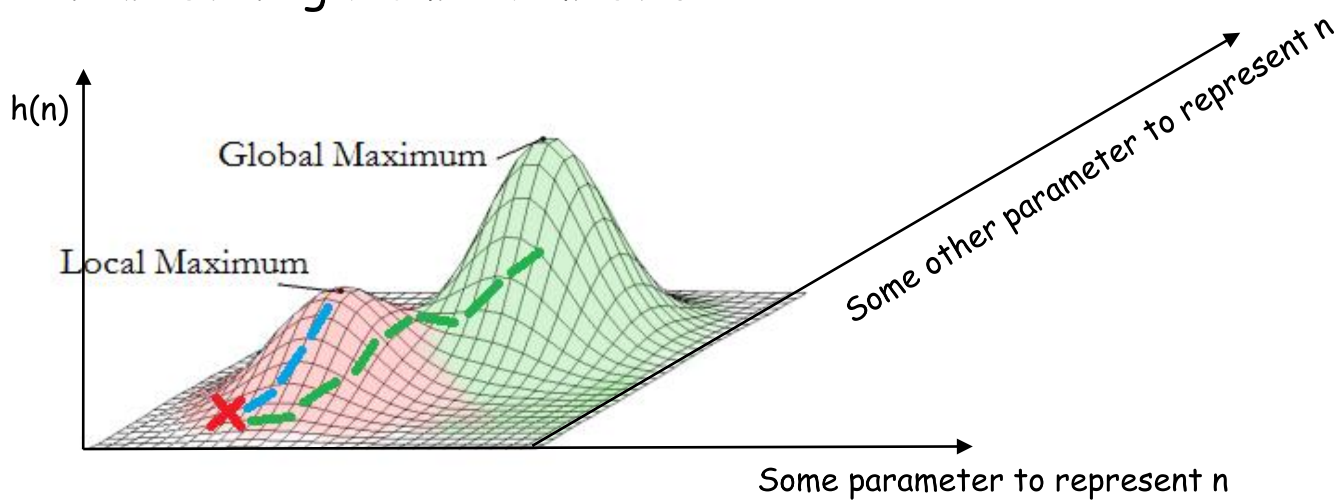


hill-climbing will stop, because all children have higher h(n) than the parent... --> local minimum

Don't be confused... a lower h(n) is better...

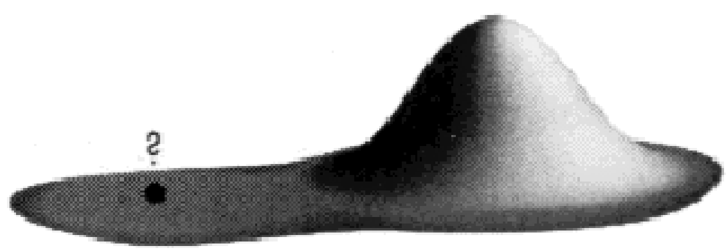
Problems with Hill Climbing

- Foothills (or local maxima)
  - reached a local maximum, not the global maximum
  - a state that is better than all its neighbors but is not better than some other states farther away.
  - at a local maximum, all moves appear to make things worse.
  - ex: 8-puzzle: we may need to move tiles temporarily out of goal position in order to place another tile in goal position
  - ex: TSP: "nearest neighbour" heuristic



Problems with Hill Climbing

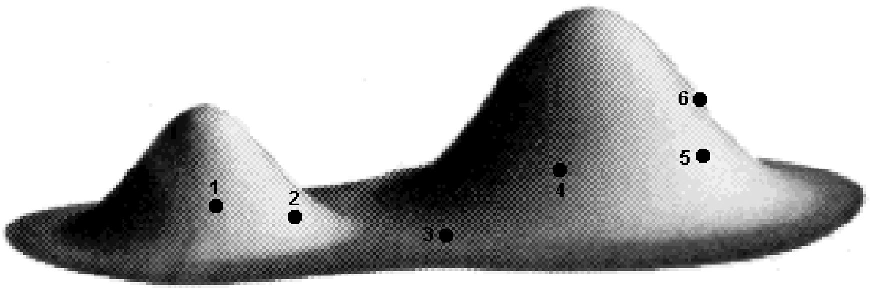
- Plateau
  - a flat area of the search space in which the next states have the same value.
  - it is not possible to determine the best direction in which to move by making local comparisons.



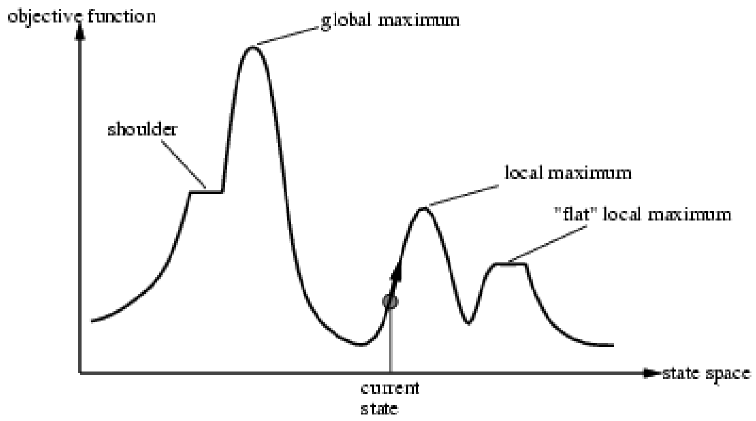
source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991.

Some Solutions to Hill-Climbing

- Random-restart hill-climbing
  - random initial states are generated
  - run each until it halts or makes no significant progress.
  - the best result is then chosen.



- keep going even if the best successor has the same value as current node
  - works well on a "shoulder"
  - but could lead to infinite loop on a plateau



source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991. & Russel & Norvig (2003)

Today

- State Space Representation
- State Space Search
  - Uninformed search
    - Breadth-first and Depth-first
    - Depth-limited Search
    - Iterative Deepening
  - Informed

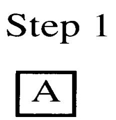
    - Hill Climbin
    - Best-First
    - (Designing Heuristics)
    - A*
- Summary

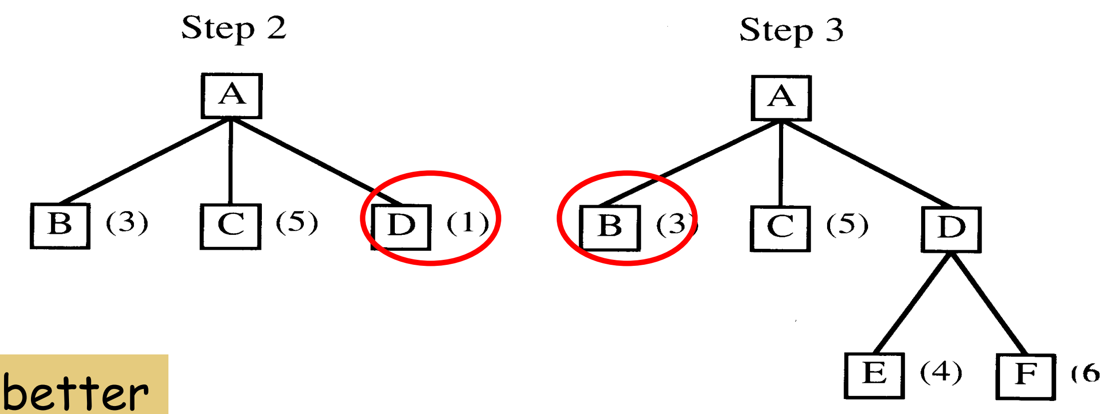
Best-First Search

- problem with hill-climbing:
  - one move is selected and all others are forgotten.
- solution to hill-climbing:
  - use "open" as a priority queue
  - this is called best-first search

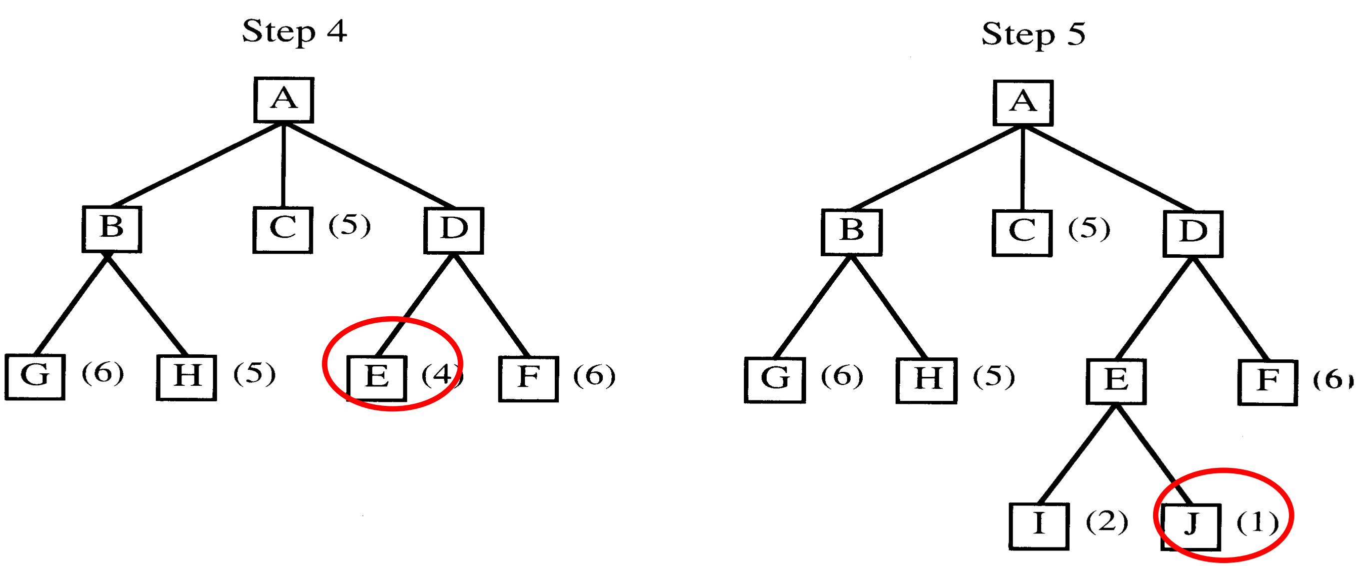
- Best-first search:
  - Insert nodes in *open* list so that the nodes are sorted in ascending **h(n)**
  - Always choose the next node to visit to be the one with the best h(n) -- regardless of where it is in the search space

Best-First: Example





Lower h(n) is better



source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991.

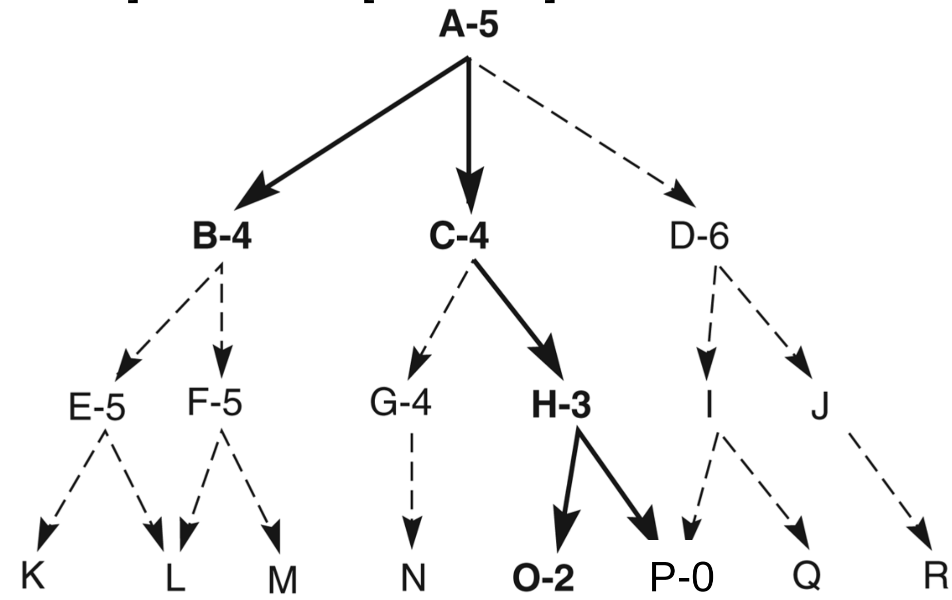
Notes on Best-first

- If you have a good h(n), best-first can find the solution very quickly
- The first solution may not be the best,
  but there is a good chance of finding it quickly
- It is an exhaustive search …
  - will eventually try all possible paths

Best-First Search: Example

1. open = [A-null-5] closed = []
2. open = [B-A-4 C-A-4 D-A-6] *(arbitrary choice)* closed [A]
3. open = [C-A-4 E-B-5 F-B-5 D-A-6] closed = [B A]
4. open = [H-C-3 G-C-4 E-B-5 F-B-5 D-A-6] closed = [C B A]
5. open = [P-H-0 0-H-2 G-C-4 E-B-5 F-B-5 D-A-6] closed = [H C B A]
6. goal P found

solution path: A C H P



Lower h(n) is better

source: adapted from G. Luger (2005)

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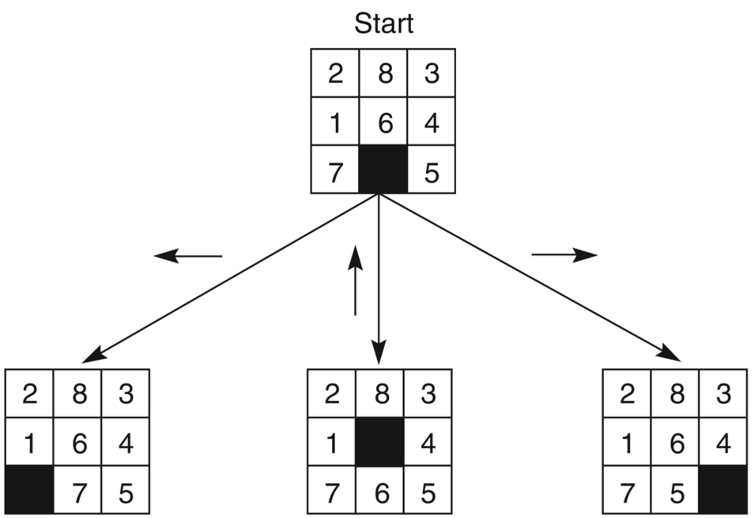
Designing Heuristics

- Heuristic evaluation functions are highly dependent on the search domain
- In general: the more informed a heuristic is, the better the search performance
- Bad heuristics lead to frequent backtracking
- So how do we design a "good" heuristic?

Example: 8-Puzzle - Heuristic 1



- h_1: Simplest heuristic
  - Hamming distance : count number of tiles out of place when compared with goal

5		8
4	2	1
7	3	6

STATE n

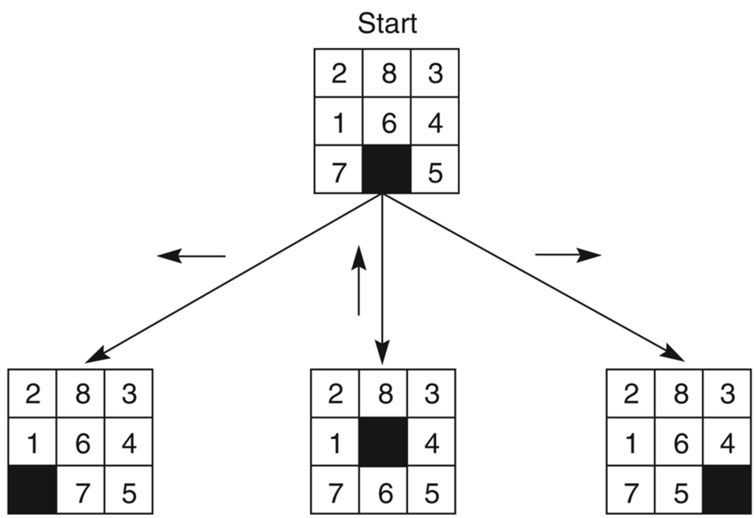
1	2	3
4	5	6
7	8	

Goal state

- $h_1(n) = 6$
  - does not consider the distance tiles have to be moved

source: *G. Luger (2005)*

Example: 8-Puzzle - Heuristic 2



- h_2: Better heuristic
  - Manhattan distance: sum up all the distances by which tiles are out of place

5		8
4	2	1
7	3	6

STATE n

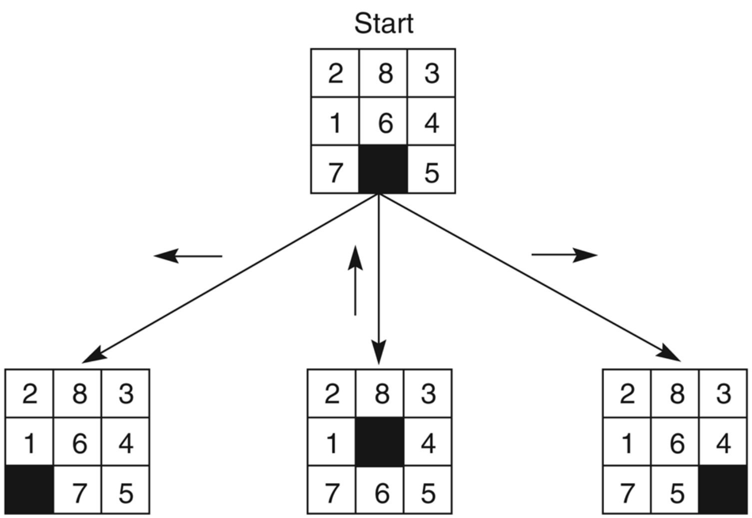
1	2	3
4	5	6
7	8	

Goal state

- $h_2(n) = 2+3+0+1+3+0+3+1$
  $= 13$

source: *G. Luger (2005)*

Example: 8-Puzzle – Heuristic 3



- h_3: Even Better
  - sum of permutation inversions
  - See next slide…

source: G. Luger (2005)

$h_3(N)$ = sum of permutation inversions

5		8
4	2	1
7	3	6

STATE n

→

5		8	4	2	1	7	3	6

- For each numbered tile, count how many tiles on its right should be on its left in the goal state.

- $$\begin{aligned} h_3(n) &= n_5 + n_8 + n_4 + n_2 + n_1 + n_7 + n_3 + n_6 \\ &= 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0 \\ &= 16 \end{aligned}$$

1	2	3
4	5	6
7	8	

Goal state

Heuristics for the 8-Puzzle

5		8
4	2	1
7	3	6

STATE n

1	2	3
4	5	6
7	8	

Goal state

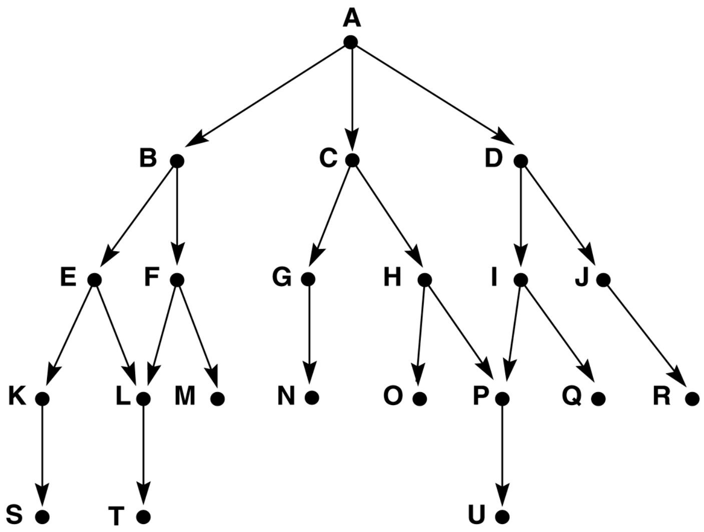
- $h_1(n)$ = misplaced numbered tiles
  = 6

- $h_2(n)$ = Manhattan distance
  = 2 + 3 + 0 + 1 + 3 + 0 + 3 + 1 = 13

- $h_3(n)$ = sum of permutation inversions
  $= n_5 + n_8 + n_4 + n_2 + n_1 + n_7 + n_3 + n_6$
  $= 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0 = 16$

g(n), h(n) and f(n)

- Evaluation function **f(n)** = **g(n)** + **h(n)** for node **n**:
  - **g(n)** current cost from *start* to node *n*
  - **h(n)** estimate of the lowest cost from *n* to *goal*
  - **f(n)** estimate of the lowest cost of the solution path (from *start* to *goal* passing through *n*)
- Now consider **f*(n)** = **g*(n)** + **h*(n)**:
  - **g*(n)** cost of lowest cost path from *start* to node *n*
  - **h*(n)** actual lowest cost from *n* to *goal*
  - **f*(n)** actual cost of lowest cost of the solution path (from *start* to *goal* passing through *n*)



Evaluating Heuristics

1. Admissibility:
   - "optimistic"
   - never overestimates the actual cost of reaching the goal
   - guarantees to find the lowest cost solution path to the goal (if it exists)

1. Monotonicity:
   - "local admissibility"
   - guarantees to find the lowest cost path to each state n encountered in the search

1. Informedness:
   - measure for the "quality" of a heuristic
   - the more informed, the better

Admissibility

- A heuristic is admissible if it never overestimates the cost of reaching the goal
- i.e.:
  - $h(n) \leq h^*(n)$ for all n

- guarantees to find the lowest cost <u>solution path</u> to the goal (if it exists)
- Note: does <u>not</u> guarantee to find the lowest cost <u>search path</u>.
- e.g.: breadth-first is admissible -- it uses $f(n) = g(n) + 0$

Example: 8-Puzzle

5		8
4	2	1
7	3	6

n

1	2	3
4	5	6
7	8	

goal

- h1(n) = Hamming distance = number of misplaced tiles = 6
--> admissible

- h2(n) = Manhattan distance = 13
--> admissible

Monotonicity (aka consistent)

- An admissible heuristics may temporarily reach non-goal states along a suboptimal path
- A heuristic is monotonic if it always finds the optimal path to each state the 1st time it is encountered !
- guarantees to find the lowest cost path to each state n encountered in the search

- h is monotonic if for every node n and every successor n′ of n:
  - $h(n) \leq c(n,n') + h(n')$
- i.e. f(n) is non-decreasing along any path

Informedness

- Intuition: number of misplaced tiles is less informed than Manhattan distance
- For two admissible heuristics h_1 and h_2
  - if $h_1(n) \leq h_2(n)$, for all states n
  - then h_2 is more informed than h_1
  - $h_1(n) \leq h_2(n) \leq h^*(n)$

More informed heuristics search smaller space to find the solution path

- However, you need to consider the computational cost of evaluating the heuristic…
- The time spent computing heuristics must be recovered by a better search

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    - Be
    - (D euristics)
    - A*

- Summary

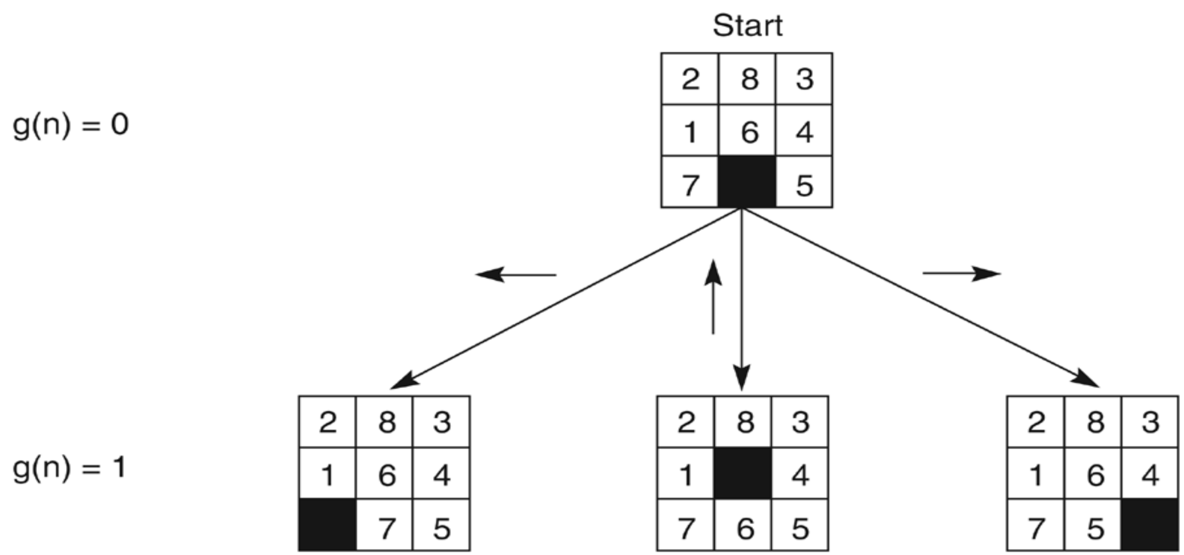
Algorithm A

- Heuristics might be wrong:
  - so search could continue down a wrong path
- Solution:
  - Maintain depth/cost count, i.e., give preference to shorter/least expensive paths
- Modified evaluation function **f**:

  $f(n) = g(n) + h(n)$

  - $f(n)$ estimate of total cost along path through n
  - $g(n)$ actual cost of path from start to node n
  - $h(n)$ estimate of cost to reach goal from node n

Algorithm A on the 8-puzzle

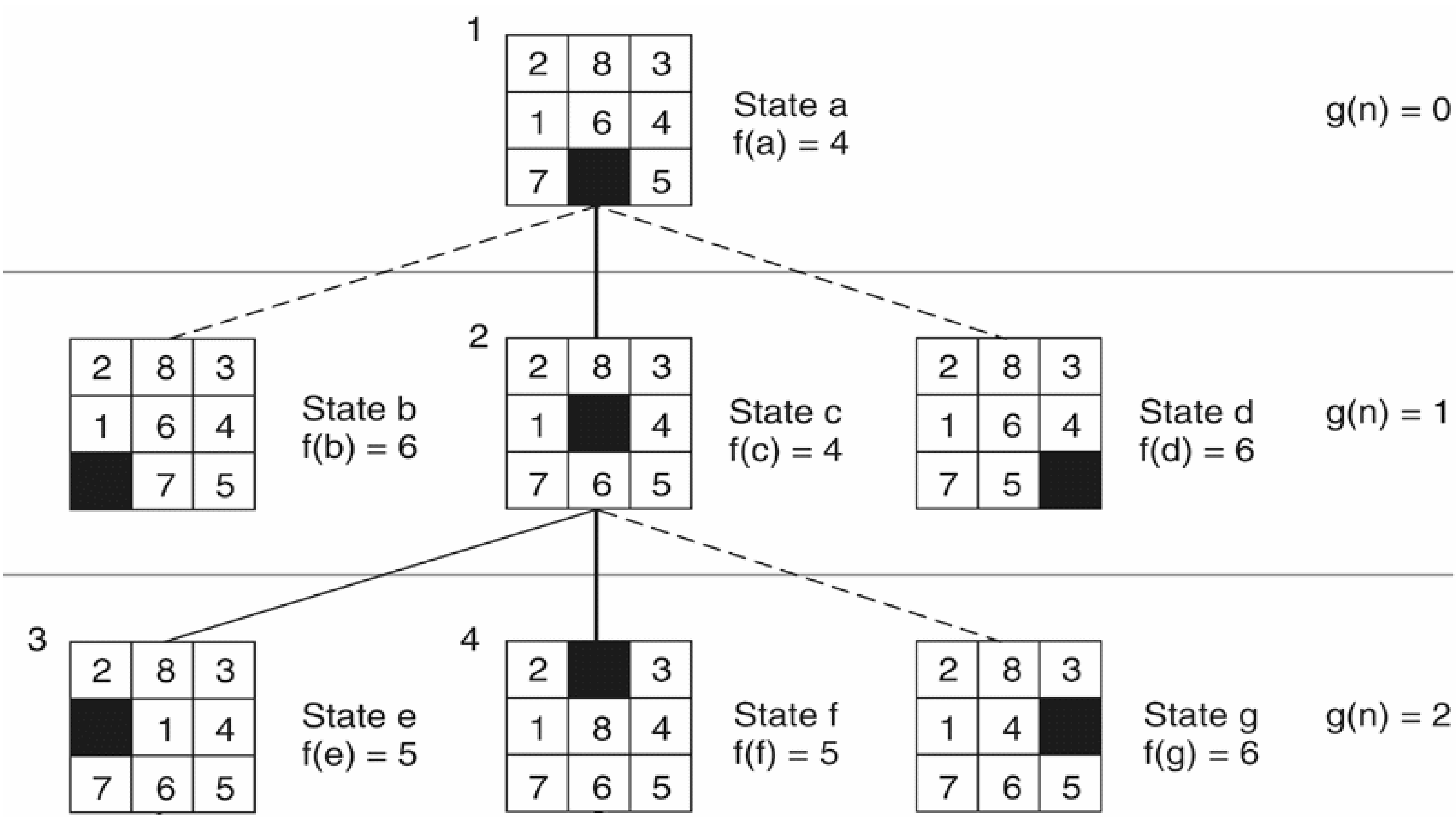


Values of f(n) for each state, **6** **4** **6**

where:
$f(n) = g(n) + h(n)$,
$g(n) =$ actual distance from n to the start state, and
$h(n) =$ number of tiles out of place.

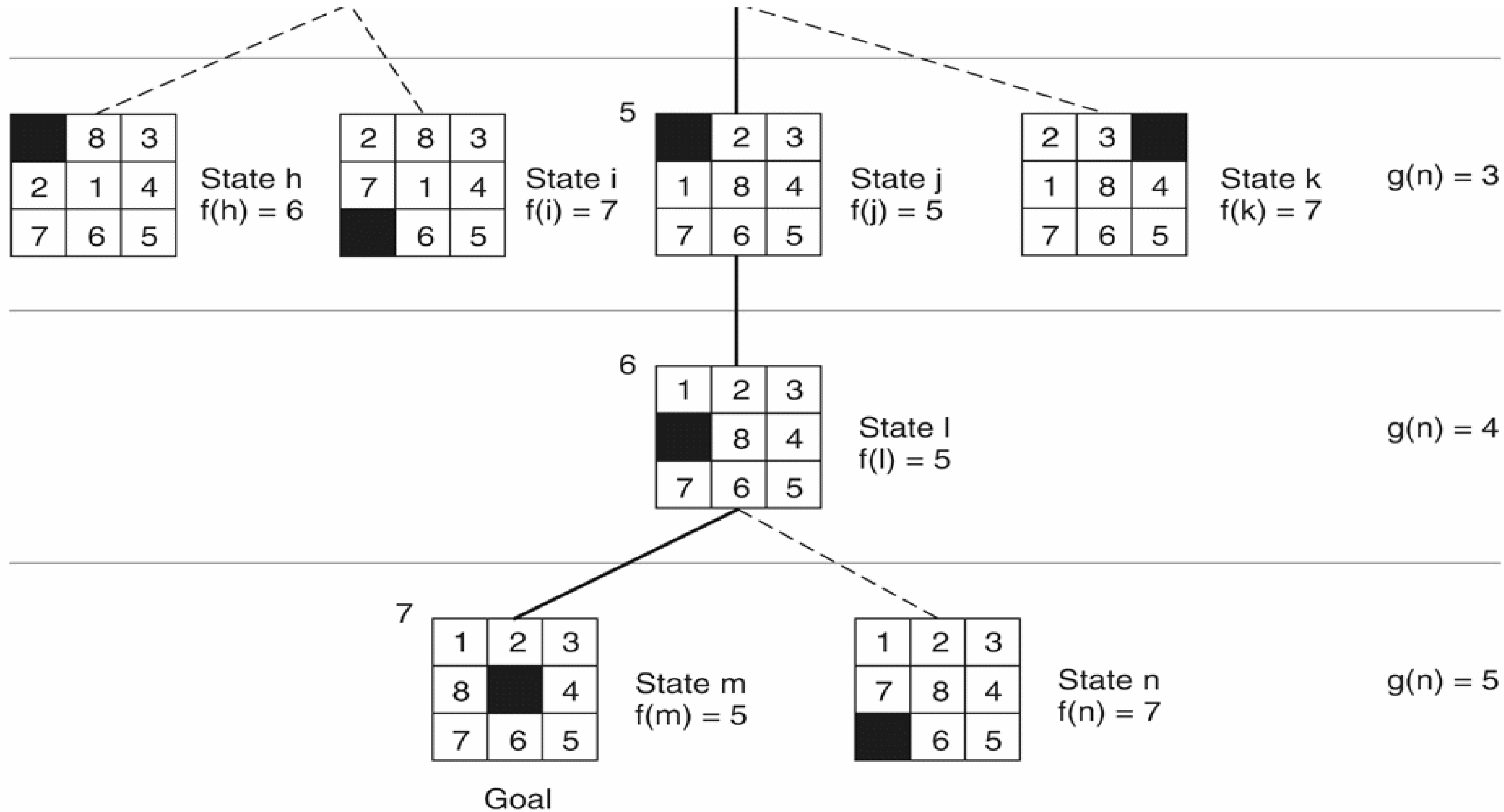
source: *G. Luger (2005)*

Algorithm A on the 8-puzzle



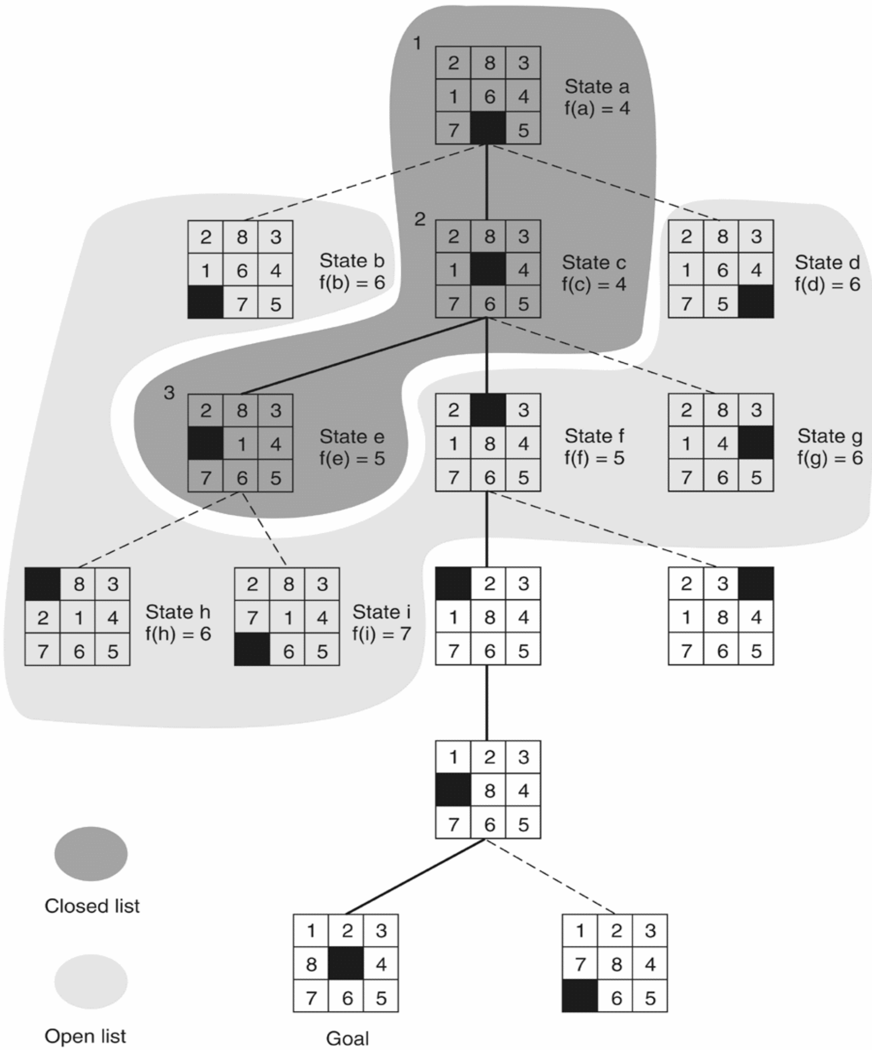
source: *G. Luger (2005)*

Algorithm A on the 8-puzzle

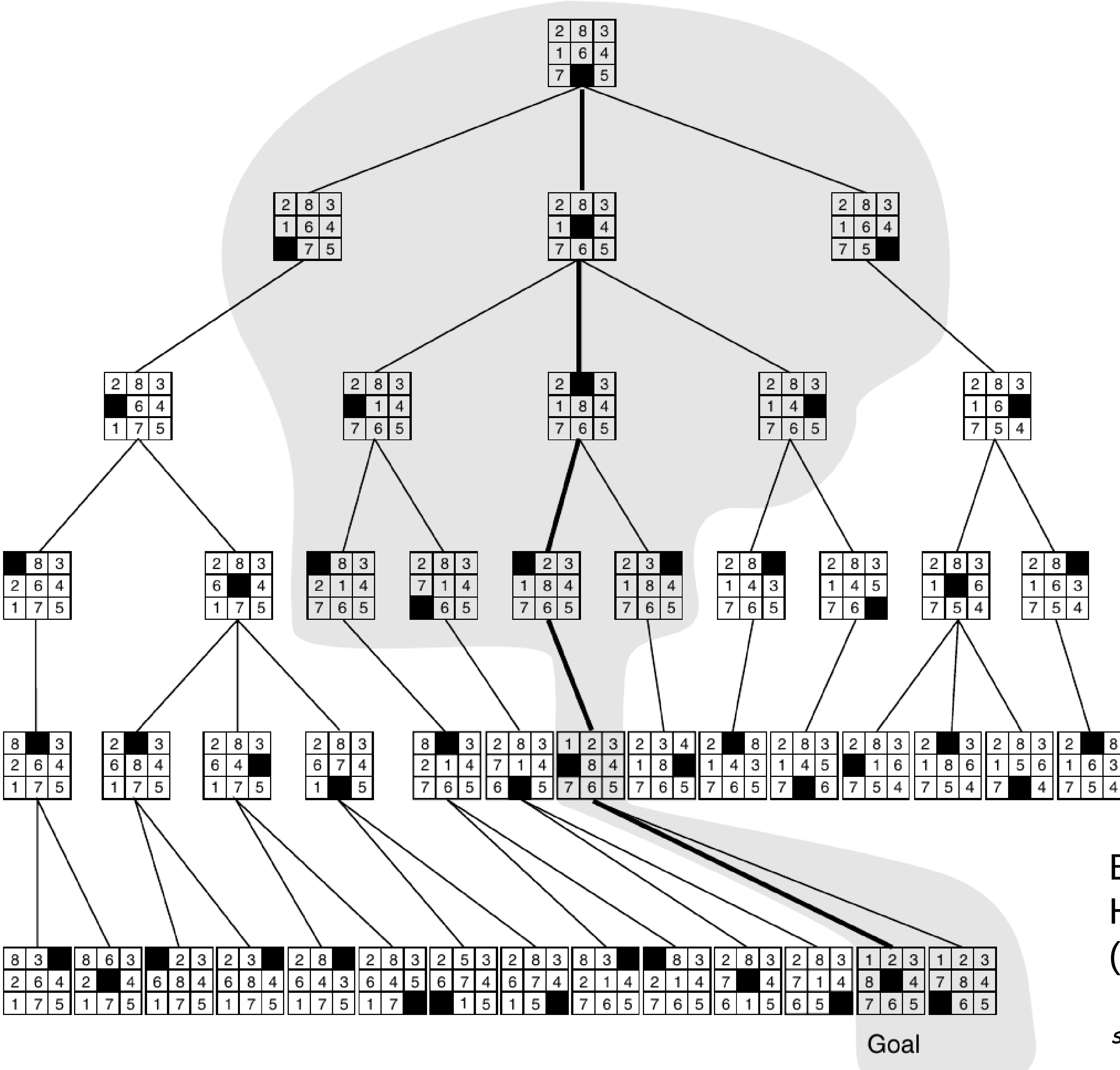


source: G. Luger (2005)

Algorithm A on the 8-puzzle



source: *G. Luger (2005)*



BFS vs. Heuristic search (tiles out of place)

source: G. Luger (2005)

Algorithm A vs Algorithm A*

- if **$g(n) \geq g^*(n)$** for all n
  - best-first search with $f(n) = g(n) + h(n)$ is called "algorithm A"
- if **$h(n) \leq h^*(n)$** for all n
  - i.e. h(n) never overestimates the true cost from n to a goal
  - algorithm **A** used with such an h(n) is called "algorithm A*"
  - → an A* algorithm is admissible
  - → i.e. it guarantees to find the lowest cost solution path from the initial state to the goal

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    - Hill climbing
    - Best-First
    - (Designi ics)
    - A*

- Summary

Summary

Search	Uses h(n)?	Open is a…
Breadth-first	No	Queue
Depth-first	No	Stack
Depth-limited	No	Stack
Iterative Deepening	No	Stack
Uniform Cost	No	Priority queue sorted by g(n)
Hill Climbing	Yes	none
Best-First	Yes	Priority queue sorted by h(n)
Algorithm A - no constraints on h(n)	Yes	Priority queue sorted by f(n) f(n) = g(n) + h(n)
Algorithm A* - same as A, but h(n) must be admissible - guarantees to find the lowest cost solution path	Yes	Priority queue sorted by f(n) f(n) = g(n) + h(n)

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