

Lazy caterer's sequence



Pancake cut into seven pieces with three straight cuts.

The **lazy caterer's sequence**, more formally known as the **central polygonal numbers**, describes the maximum number of pieces of a **circle** (a **pancake** or **pizza** is usually used to describe the situation) that can be made with a given number of straight cuts. For example, three cuts across a pancake will produce six pieces if the cuts all meet at a common point inside the circle, but up to seven if they do not. This problem can be formalized mathematically as one of counting the cells in an **arrangement of lines**; for generalizations to higher dimensions, see **arrangement of hyperplanes**.

The analogue of this sequence in 3 dimensions is the **cake number**.

1 Formula and sequence

The maximum number p of pieces that can be created with a given number of cuts n , where $n \geq 0$, is given by the formula

$$p = \frac{n^2 + n + 2}{2}.$$

Using **binomial coefficients**, the formula can be expressed as

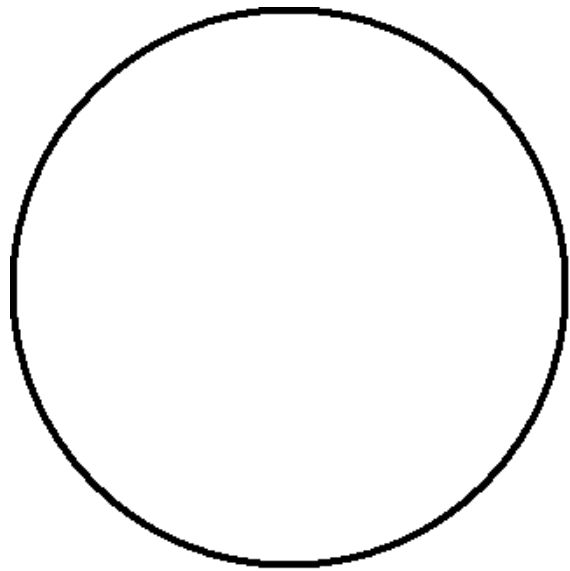
$$p = 1 + \binom{n+1}{2} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2}.$$

This sequence (sequence A000124 in the OEIS), starting with $n = 0$, results in

1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, 121, 137, 154, 172, 191, 211, ...

Each number equals 1 plus a **triangular number**.

2 Proof



$n = 0$

$n = 0$

$f(n) = 1$

The maximum number of pieces from consecutive cuts are the numbers in the Lazy Caterer's Sequence.

When a circle is cut n times to produce the maximum number of pieces, represented as $p = f(n)$, the n th cut must be considered; the number of pieces before the last cut is $f(n - 1)$, while the number of pieces added by the last cut is n .

To obtain the maximum number of pieces, the n th cut line should cross all the other previous cut lines inside the circle, but not cross any intersection of previous cut lines. Thus, the n th line itself is cut in $n - 1$ places, and into n line segments. Each segment divides one piece of the $(n - 1)$ -cut pancake into 2 parts, adding exactly n to the number of pieces. The new line can't have any more segments since it can only cross each previous line once.

A cut line can always cross over all previous cut lines, as rotating the knife at a small angle around a point that is not an existing intersection will, if the angle is small enough, intersect all the previous lines including the last one added.

Thus, the total number of pieces after n cuts is

$$f(n) = n + f(n - 1).$$

This **recurrence relation** can be solved. If $f(n - 1)$ is expanded one term the relation becomes

$$f(n) = n + (n - 1) + f(n - 2).$$

Expansion of the term $f(n - 2)$ can continue until the last term is reduced to $f(0)$, thus,

$$f(n) = n + (n - 1) + (n - 2) + \cdots + 1 + f(0).$$

Since $f(0) = 1$, because there is one piece before any cuts are made, this can be rewritten as

$$f(n) = 1 + (1 + 2 + 3 + \cdots + n).$$

This can be simplified, using the formula for the sum of an **arithmetic progression**:

$$f(n) = 1 + \frac{n(n + 1)}{2} = \frac{n^2 + n + 2}{2}.$$

3 References

- Moore, T. L. (1991), "Using Euler's formula to solve plane separation problems", *The College Mathematics Journal*, Mathematical Association of America, **22** (2): 125–130, doi:10.2307/2686448, JSTOR 2686448.
- Steiner, J. (1826), "Einige Gesetze über die Theilung der Ebene und des Raumes ("A Few Statements about the Division of the Plane and of Space")", *J. Reine Angew. Math.*, **1**: 349–364.
- Wetzel, J. E. (1978), "On the division of the plane by lines" (PDF), *American Mathematical Monthly*, Mathematical Association of America, **85** (8): 647–656, doi:10.2307/2320333, JSTOR 2320333.

4 External links

- Weisstein, Eric W. "Circle Division by Lines". *MathWorld*.

5 Text and image sources, contributors, and licenses

5.1 Text

- **Lazy caterer's sequence** *Source:* https://en.wikipedia.org/wiki/Lazy_caterer%27s_sequence?oldid=720885825 *Contributors:* Michael Hardy, Isopropyl, Giftlite, Anton Mravcek, ArnoldReinhold, Zaslav, Shanes, Oleg Alexandrov, Miss Madeline, Tardis, Ethan, Cydebot, David Eppstein, TXiKiBoT, Dmcq, HansHermans, SieBot, Rumping, Addbot, Yobot, Citation bot 1, Double sharp, EmausBot, ZéroBot, CitationCleanerBot, יהודה שמחה ולדמן, Dane Bouchie, Julietdeltalima and Anonymous: 17

5.2 Images

- **File:Lazy_Caterer%27s_Sequence_(Cuts).gif** *Source:* https://upload.wikimedia.org/wikipedia/commons/d/d6/Lazy_Caterer%27s_Sequence_%28Cuts%29.gif *License:* CC BY-SA 3.0 *Contributors:* Own work *Original artist:* Dane Bouchie
- **File:PancakeCutThrice.agr.jpg** *Source:* <https://upload.wikimedia.org/wikipedia/commons/4/45/PancakeCutThrice.agr.jpg> *License:* CC BY-SA 3.0 *Contributors:* Own work *Original artist:* ArnoldReinhold

5.3 Content license

- Creative Commons Attribution-Share Alike 3.0