Contents

Tutorial for the Sliding-Tile Puzzle

In the N^2 sliding-tile puzzle, there are N^2 squares, and N^2-1 tiles, numbered from 1 to N^2-1. These tiles are placed on the board leaving one square blank. Given some start configuration of numbered tiles on the squares, the aim is to move tiles, one at a time, until some given goal configuration is reached. A move involves placing one of the tiles that neighbours the blank square into that space, excluding the tile that was previously on the square. (You can only move in the horizontal and vertical directions of course.) The aim is to reach the goal configuration with a minimum number of tile moves.

The puzzle was first created in the late 19th century. It became a craze in the USA in 1880, and 'everybody' played it. Here's a cartoon about the difficult in choosing the republican candidate at the 1880 U.S. presidential elections.

First set of exercises

1. Consider a 2^2 puzzle.
   a. How many different possible configurations of tiles are there?
   b. Generate all possible configurations given:
      i. start configuration

1	2
3	

      ii. start configuration

2	1
3	

      iii. A configuration is called reachable if it can be reached from a given start configuration. How many reachable configurations were there from each of the two start configurations, and how does that related to the total number in the puzzle?
   c. Given the start configuration:

1	2
3	

and goal configuration

1	3
2	

can you solve the puzzle?

2. How many different possible configurations of tiles are there in the:
   a. 3^2 puzzle?
   b. 4^2 puzzle?
   c. n^2 puzzle?
3. Given the 3^2 puzzle goal configuration:

1	2	3
8		4
7	6	5

and start configuration:

8	3	5
4	1	6
2	7	

draw the **solution tree** that represents every configuration by a vertex and every move by an edge. Do not repeat configurations, and go no deeper than level 3 (where level 0 is the root vertex).

Solvability

It is not always possible to reach a given goal state from some given start state. This was realised very soon after the game was 'invented' (about 1879).

There is an algorithm to determine whether two states are reachable. This algorithm involves differentiating between even-sized boards, 2^2, 4^2, 6^2 etc, and odd-sized boards, 3^2, 5^2 etc.

We also need to define the **disorder** of a tile, and of a board:

- the *disorder* of a tile is the number of lower-numbered tiles that appear after it in left-right top-down order
- the *disorder* of a board is:
  - *odd-sized board*: the sum of the tile disorders
  - *even-sized board*: the sum of the tile disorders plus the row number of the blank (row numbers start with 1)

Finally we can define the **parity** of a board:

- the parity is **even** if the board disorder is even
- the parity is **odd** if the board disorder is odd

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A goal board is reachable from a start board if and only if both boards
have the same parity.
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For example, if you have a goal board:

1	2
3	

we note that:

- tile '1': disorder = 0 (in its correct position)
- tile '2': disorder = 0 (correct position)
- tile '3': disorder = 0 (correct position)
- blank is in row 2

so the disorder of this board is 0+0+0+2, hence the board has even parity.

If you have the start board:

3	2
	1

the disorders are:

- tile '3': disorder = 2 (tiles '2' and '1' appear after it)
- tile '2': disorder = 1 (tile '1' appears after it)
- blank is in row 2
- tile '1': disorder = 0

so the start board has disorder 2+1+2+0 = 5. The parity of this board is hence odd, which is different to the parity of the goal board, hence you cannot reach the goal board from this start board. Hence the game cannot be solved.

A different start board:

	1
3	2

yields 1+0+1+0 = 2, where the 1's come from the blank and tile 3. The disorder is hence even, which is the same parity as the goal board, hence you can reach that goal board from this board, and the game is solvable.

Another example, this time a 3x3 goal board:

1	2	3
4	5	6
7	8	

This board is odd-sized, and we can see all tiles are in the correct position, hence the parity is even.

Computing the disorder of the following start board:

8	7	6
5	4	3
2	1	

yields disorder = 7+6+5+4+3+2+1+0 = 28. The parity of the start and goal boards are both even, hence the game is solvable.

Sometimes board configurations are written sequentially: e.g. the previous start board can be written *8 7 6 5 4 3 2 1 b*, where *b* is the blank square and the rows are written sequentially from top to bottom.

Second set of exercises

1. a. Show that the disorders of <u>all</u> the boards reachable from the start board *1 2 3 b* are even.
   b. Show that the disorders of <u>all</u> the boards reachable from the start board *2 1 3 b* are odd.
2. a. What is the parity of the following goal board?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

   b. Is that goal board reachable from the following start board?

1	2	3	4
5	6	7	8
9	10	11	
12	13	14	15

   If the goal is reachable, how many moves are required?

   c. Is that goal board reachable from the following start board?

1	2	3	4
5	6	7	8
9	10	11	12
13	14		15

   If the goal is reachable, how many moves are required?
3. Is the puzzle with start board *1 6 7 2 5 8 3 4 b* and goal board *1 2 3 8 b 4 7 6 5* solvable?

TuteSTPuzzle (last edited 2019-06-07 16:47:33 by AlbertNymeyer)