CS648 Project

Jatin Jindal,
Yash Mahajan

Deterministic Algorithm for unweighted graphs

Algorithm for 3-Spanner sub-graph

Time Complexity

Expected number of edges

Correctness

Algorithm for 5-Spanner sub-graph

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Expected number of

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Jatin Jindal   Yash Mahajan

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Initial Ideas and Failed attempts

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- For the case of unweighted graphs we thought of an deterministic algorithm which would try to find all edges in a cycle and try to remove them in a single BFS/DFS traversal
- However that wouldn't work because we aren't able to find all cycles of a particular length, also we don't get any bound on the number of edges that remain.

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- For every vertex u, add the smallest edge in (E-E')

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- Hence, total time of the algorithm is O(n+m) = O(m) time.

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- Now, we will find the bound in each type of edge.

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$$\frac{\partial v}{\partial d} = (1-p)^{d+1} + d(1-p)^{d+1} Ln(1-p)$$

$$\frac{\partial v}{\partial d} \geq 0 \Rightarrow d \leq \frac{-1}{Ln(1-p)} \leq \frac{1}{p} = \sqrt{n}$$

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- **Inter Cluster Edges with higher weights than center:** Note that each vertex has at most one each edge to each cluster. Total number of such clusters are np thus total number of edges per vertex $= O(\sqrt{n})$ and total such edges $= O(n\sqrt{n})$.

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$$\sum_{k=0}^{d} k(1-p)^k p \leq \frac{1-p}{p} \leq \frac{1}{p} = \sqrt{n}$$

Hence, expected number of edges $= O(n\sqrt{n})$.

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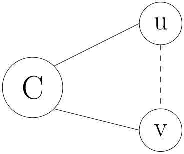

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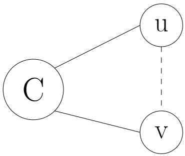

3-stretch holds because the weight of w(C-u) and w(C-v) is less than w(u-v) because of the way we picked the edges.

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  An inter-cluster edge is not present iff the weight of this edge is greater than their centers and also there is another edge from one of the vertex to other cluster with lower weight.

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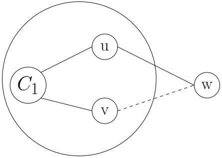

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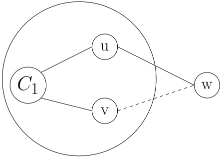
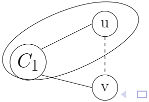


  Case 2: $w(C_1 - v) < w(u - v)$ and $w(u - C_1) < w(u - v)$



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Hence, expected number of edges = $O(n * n^{1/3}) = O(n^{4/3})$.

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- Now in this case our algorithm for getting intra-cluster edges is similar to that of what we used for 3 spanner. So the proof of correctness will be similar in this case.

Correctness
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- Also, if the path is (u,C1,x1,x2,C2,v) where u and x1 are in cluster-1 and v and x2 are in cluster-2; w(u,C1) and w(v,C2) $<$ w(u,v). Also,w(C1,x1) and w(C2,x2) $<$w(x1,x2) $<$w(u,v). Thus all edges on the path are less than w(u,v) and stretch property is maintained for 5 spanner.

Rough Idea for 7 spanner

- First we will repeat all steps done for 3-spanner except the step of creating inter-cluster edges, with probability p= $\frac{1}{n^{1/4}}$

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- Now we will repeat step-1 but instead of picking vertices, we will pick clusters created by previous step and treat them as a single vertex now.
- Now the newly formed clusters will have depth atmost 2. Now we will include the minimum edge between the clusters from step-1 and step-2.

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- First we will repeat all steps done for 3-spanner except the step of creating inter-cluster edges, with probability p$=\frac{1}{n^{1/4}}$
- Now we will repeat step-1 but instead of picking vertices, we will pick clusters created by previous step and treat them as a single vertex now.
- Now the newly formed clusters will have depth atmost 2. Now we will include the minimum edge between the clusters from step-1 and step-2.
- We will see that the clusters of step-1 and 2 have depths 1 and 2 respectively. And for every pair of vertices there will exist a path with the number of edges atmost 7(2+4+1(edge joining the clusters))