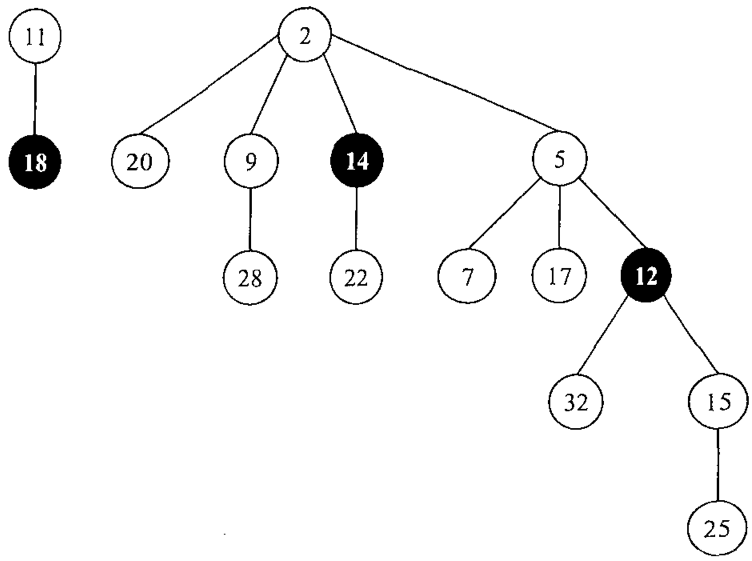
CS31005 Algorithms–II, Autumn 2022–2023

30-Sep-2022, 09:00am–11:00am | **Mid-Semester Test** | Maximum marks: 60

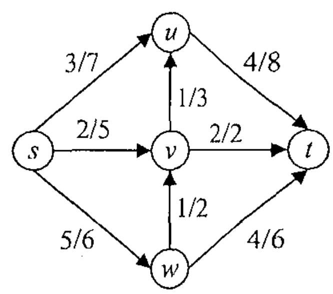
[*Answer all questions.*]

1. **(a)** The adjacent figure shows a Fibonacci heap consisting of two trees. The marked nodes are shown as solid black nodes. Consider the node storing the priority value 15. You need to decrease this priority to 10. Draw the Fibonacci heap just after this decrease-priority operation. Show also all the marked nodes in this modified heap. No explanation is needed.



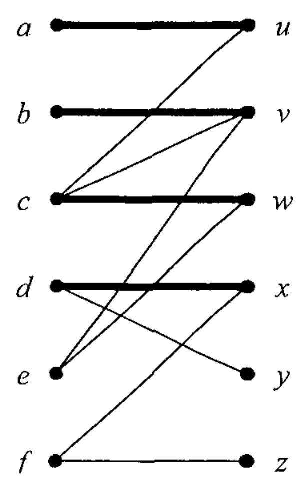
(5)

(b) A flow network with source s and sink t is shown in the adjacent figure. The flow and the capacity of an edge e is shown as $f(e)/c(e)$ beside that edge. Draw the residual network for this flow, and find an augmenting path that the Edmonds–Karp algorithm would select. No explanation is needed.



(5)

(c) The adjacent figure shows the matching $M = \{(a,u),(b,v),(c,w),(d,x)\}$ in a bipartite graph with the bipartitioning $L = \{a,b,c,d,e,f\}$ and $R = \{u,v,w,x,y,z\}$. In order to find an augmenting path, we make a (restricted) DFS traversal from free nodes in the left part L. The free nodes are to be chosen from top to bottom. If one (or more) free nodes fail to identify any augmenting path, we continue the DFS from another free node in L (if any is remaining). The DFS neighbors of nodes in L should be explored in the top-to-bottom order. Draw the alternating forest, identify an alternating path that augments M, and show the augmented matching. No explanation is needed.



(5)

(d) Show the working of Jarvis march for computing the convex hull of the seven points

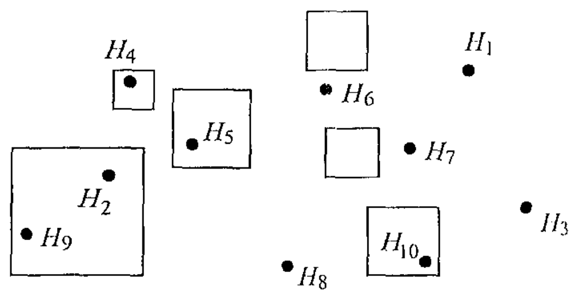
$$P_1 = (8,10),\ P_2 = (12,15),\ P_3 = (2,9),\ P_4 = (9,5),\ P_5 = (5,2),\ P_6 = (16,4),\ \text{and}\ P_7 = (11,7).$$

Briefly explain how each step works. (5)

2. IIT Kharagpur has hired you to write an algorithm to schedule their final exams. Each semester, IIT Kgp offers n different courses. There are r different rooms on campus and m different time slots in which exams can be offered. You are given two arrays $E[1 \ldots n]$ and $S[1 \ldots r]$, where $E[i]$ is the number of students enrolled in the i-th course, and $S[j]$ is the number of seats in the j-th room. The final exam of at most one course can be held in each room during each time slot. Course i can hold its final exam in room j only if $E[i] \leqslant S[j]$. Assume that no two courses share common registrants, so any number of courses can be scheduled in the same time slot. The only restriction is imposed by the availability and suitability of the rooms as mentioned above. Describe and analyze an efficient algorithm to assign a room and a time slot to each course (or report correctly that no such assignment is possible). **(10)**

3. We run the Gale–Shapley algorithm to compute a stable matching between n men and n women. Assume that men propose, and women take decisions (accept, reject, or replace). Prove that no matter how large n is, at most one man can be paired with his last choice. **(8)**

4. A town in Geomerica suffers from a power outage caused by an intense solar storm. The town has m emergency light sources $L_1, L_2, \ldots, L_m$. The i-th light source L_i can illuminate an axis-aligned square region of side length r_i with $L_i = (x_i, y_i)$ at the center. For simplicity, assume that the m illumination zones do not overlap with each other. The town has n houses $H_1, H_2, \ldots, H_n$ considered as points (h_j, k_j) for $j = 1, 2, \ldots, n$. The mayor of the town wants to figure out which houses can be illuminated by the emergency light sources, so he can notify the other house-owners for making their own arrangements. The following figure illustrates the situation for six light sources and ten houses. The squares in the figure show the illumination zones (their centers L_i are not shown). Only the five houses $H_2, H_4, H_5, H_9, H_{10}$ can be illuminated.



The inputs to the problem are x_i, y_i, r_i for $i = 1, 2, \ldots, m$ and h_j, k_j for $j = 1, 2, \ldots, n$. For each j, the mayor can find out whether H_j can be illuminated by any of the light sources L_i. This gives an $O(mn)$-time algorithm. You are required to design an $O((m+n)\log(m+n))$-time algorithm based on the line-sweep paradigm, where one vertical line sweeps from left to right. You may make suitable general-position assumptions if needed. Clearly mention the following in your solution.

(a) What the types of the events are.
(b) How each type of event is handled.
(c) What data structure you use for maintaining the event queue. Justify.
(d) What data structure you use for maintaining the information pertaining to the sweep line. Justify.
(e) A justification that your algorithm runs in $O((m+n)\log(m+n))$ time. **(12)**

5. Let $\Phi(x_1, x_2, \ldots, x_n)$ be a Boolean formula in n variables $x_1, x_2, \ldots, x_n$. By COMPLEMENTSAT, denote the problem of deciding whether there exists a truth assignment $(t_1, t_2, \ldots, t_n)$ of the variables such that both $\Phi(t_1, t_2, \ldots, t_n)$ and $\Phi(\overline{t_1}, \overline{t_2}, \ldots, \overline{t_n})$ are true, where $\overline{t_i}$ denotes the complement of the truth value t_i. Prove that COMPLEMENTSAT is NP-Complete. **(10)**
