UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

# INFORMATICS 2D: REASONING AND AGENTS

**Tuesday 15 May 2012**

**14:30 to 16:30**

Convener: J Bradfield
External Examiner: A Preece

## INSTRUCTIONS TO CANDIDATES

1. **Candidates in the third or later year of study for the degrees of MA(General), BA(Relig Stud), BD, BCom, BSc(Social Science), BSc (Science) and BEng should put a tick ($\surd$) in the box on the front cover of the script book.**

2. **Answer Parts A, B and C. The multiple choice questions in Part A are worth 50% in total and are each worth the same amount. Mark one answer only for each question - multiple answers will score 0. Marks will not be deducted for incorrect multiple choice exam answers. Raw multiple choice scores may be rescaled at the discretion of the exam board. Parts B and C are each worth 25%. Answer ONE question from Part B and ONE question from Part C.**

3. **Use the special mark sheet for Part A. Answer Parts B and C each in a separate script book.**

**Write as legibly as possible.**
**CALCULATORS MAY BE USED IN THIS EXAMINATION.**

**Part A**

**ANSWER ALL QUESTIONS IN PART A. Use the special mark sheet.**

1. Which of the following formulae of propositional logic is valid?

   (a) $\neg P \rightarrow P$

   (b) $P \rightarrow Q$

   (c) $P$

   (d) $(\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P)$

   (e) $P \rightarrow (Q \rightarrow \neg P)$

2. If two heuristics $h_1$ and $h_2$ are admissible, which one of the following is always admissible?

   (a) $h_1 + h_2$

   (b) $h_1 \times h_2$

   (c) $|h_1 + h_2|$

   (d) $h_1^2 + h_2^2$

   (e) $max(h_1, h_2)$

3. Which of the following sets of clauses arises from putting the sentence:

$$\forall xy.\,[P(y) \Longleftrightarrow (\exists y.\, Q(x, f(y)))]$$

   in clausal form (CNF). Note that $c_0$ is a Skolem constant, $g$ is a Skolem function, and that $z$ is a variable.

   (a) $\{\neg P(y) \vee Q(x, f(c_0)), \neg Q(x, f(y)) \vee P(y)\}$

   (b) $\{\neg P(y) \vee Q(x, f(g(x, c_0))), \neg Q(x, f(y)), P(y)\}$

   (c) $\{\neg P(y) \vee Q(x, f(g(x, y))), \neg Q(x, f(z)) \vee P(y)\}$

   (d) $\{\neg P(g(x)) \vee Q(x, f(y)), \neg P(x, f(z)) \vee P(y)\}$

   (e) $\{\neg P(y) \vee Q(x, f(g(x, y))), \neg P(x, f(z)) \vee P(z)\}$

4. Within the context of constraint satisfaction problems (CSPs), which of the following statements is false?

   (a) The most constraining variable heuristic (degree heuristic) chooses the variable with the fewest legal values.

   (b) Backtracking search is commonly used for solving CSPs.

   (c) The most constrained variable (minimum remaining value) heuristic helps prune the search space.

   (d) The forward checking method is weaker than the arc-consistency method.

   (e) The least constraining value heuristic acts as a tie-breaker for the minimum remaining value heuristic.

5. Which of the following is the most general unifier (MGU) of $f(x)+f(x) = x$ and $y + y = 0$, if it exists? Note that $x$ and $y$ are variables.

   (a) $\{x/2f(x), y/0\}$

   (b) $\{x/f(0), y/0\}$

   (c) $\{x/0, y/f(x)\}$

   (d) $\{x/0, y/f(0)\}$

   (e) Unification fails due to an occurs-check violation.

6. What is the result of doing a full resolution step between the clauses

$$\neg P(G(y)) \vee Q(G(y)) \vee Q(z)$$

   and

$$P(x) \vee \neg Q(x)$$

   where $x$, $y$, and $z$ are variables.

   (a) $Q(G(y)) \vee Q(z) \vee \neg Q(G(y))$

   (b) The empty clause

   (c) $\neg P(G(y)) \vee P(x) \vee Q(z)$

   (d) $\neg P(G(y)) \vee Q(G(y)) \vee P(x)$

   (e) $\neg P(G(y)) \vee P(G(y))$

7. Which of the following statements is false of Generalized Modus Ponens (GMP) for first order logic?

   (a) GMP may not terminate on non-theorems.

   (b) GMP is used for forward checking.

   (c) GMP is sound.

   (d) GMP uses unification.

   (e) GMP is a rule of inference.

8. Which of the following statements about WALKSAT (as given in the lectures and Russell & Norvig) is *false*?

   (a) It is an incomplete algorithm.

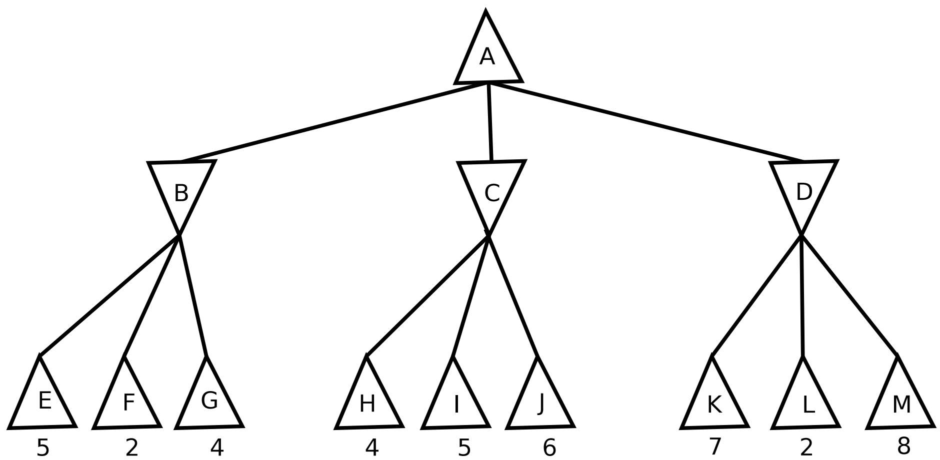
   (b) It uses a heuristic which maximizes the number of satisfied clauses.

   (c) It randomly flips the values of pure literals.

   (d) It is usually much faster than DPLL for satisfiable problems.

   (e) Initially it makes a random assignment of truth values to the symbols in the clauses.

9. Consider the following lookahead tree for a two-person game, which is searched depth-first, **right-to-left**. Each node is named by a letter. Nodes with shape $\Delta$ are where Max is due to play; nodes with shape $\nabla$ are where Min is due to play. The numbers below the leaves of the tree are the results of the evaluation function applied to that leaf.




Which nodes, if any, would an $\alpha/\beta$ search *not* need to visit?

(a) $E$ and $F$.

(b) $E$, $F$, and $G$.

(c) $E$.

(d) $I$, $J$, and $M$.

(e) It has to visit them all.

10. If $b$ is maximum branching factor of a search tree, $d$ is the depth of the least-cost solution and $m$ is the maximum depth of the search space, what is the time complexity of iterative depth-first search?

(a) $O(bm)$

(b) $O(b^m)$

(c) $O(b^{d+1})$

(d) $O(b^d)$

(e) $O(bd)$

11. Assume action $D$ makes postcondition $p$ true and you are given a plan with existing causal links $A \xrightarrow{\neg p} B$ and $B \xrightarrow{\neg p} C$. Which of the following total orderings does *not* resolve all potential conflicts that might arise from the addition of $D$?

    (a) $D \prec A \prec B \prec C$

    (b) $A \prec B \prec C \prec D$

    (c) $C \prec A \prec B \prec D$

    (d) $C \prec A \prec D \prec B$

    (e) $D \prec C \prec A \prec B$

12. In how many ways is the action schema

$$\begin{aligned}&Action(Fly(p, from, to),\\ &\quad \textsc{Precond}{:}At(p, from) \wedge Plane(p) \wedge Airport(from) \wedge Airport(to)\\ &\quad \textsc{Effect}{:}\neg At(p, from) \wedge At(p, to))\end{aligned}$$

    applicable in the following state:

$$\begin{aligned}&At(P_1, SFO) \wedge At(P_2, Heathrow) \wedge\\ &Airport(CDG) \wedge Airport(Heathrow) \wedge Airport(SFO) \wedge\\ &Plane(P_1) \wedge Plane(P_2)\end{aligned}$$

    (a) 2

    (b) 4

    (c) 6

    (d) 9

13. You are given five action descriptions with conditional effects:

$Action(A, \text{PRECOND:}\{X\}, \text{EFFECT:}\{(\textbf{when } P : \neg X), Z\})$
$Action(B, \text{PRECOND:}\{Y\}, \text{EFFECT:}\{(\textbf{when } Z : \neg P), \neg Y, \neg Z, X\})$
$Action(C, \text{PRECOND:}\{\neg Z\}, \text{EFFECT:}\{(\textbf{when } P : \neg X), Y\})$
$Action(D, \text{PRECOND:}\{\neg X\}, \text{EFFECT:}\{(\textbf{when } Q : X)\})$
$Action(E, \text{PRECOND:}\{Z\}, \text{EFFECT:}\{(\textbf{when } Q : \neg Z)\})$

What state would result from executing the action sequence $[E, D, A, B, C]$ in the state $\{P, Q, Y, Z\}$?

(a) $\{P, Q, Y\}$

(b) $\{Q, X, Y\}$

(c) $\{P, Q, Y, Z\}$

(d) The plan isn't executable because the preconditions for action $C$ aren't met.

(e) The plan isn't executable because the preconditions for action $D$ aren't met.

14. A house has 5 rooms, and the robot moves from one room to another, cleaning each room. It can sense its location and whether the room it is in is currently clean or dirty. Sometimes when it cleans, its filter deposits dirt onto the floor. Moreover, rooms sometimes get dusty from the airconditioning system that runs throughout the house.

Which of the following statements is *incorrect*?

(a) The robot is working in a partially observable environment.

(b) The robot could use contingent planning to clean the house.

(c) A plan to clean a particular room in the house would include a loop.

(d) It is impossible for the robot to guarantee that it has achieved the goal of making each room in the house clean.

(e) The robot must monitor and replan if it aims to clean each room in the house.

15. Suppose that an agent has a goal $(On(A,B) \vee On(B,C)) \wedge \neg Clear(A)$. Which of the following states satisfies this goal?

   (a) $On(B,C) \wedge On(C,A)$

   (b) $On(A,B) \wedge On(B,C) \wedge Clear(A)$

   (c) $On(B,A)$

   (d) $Clear(C)$

   (e) $On(C,B) \wedge Clear(B)$

16. Which of the following statements is correct for arbitrary Boolean random variables $X$ and $Y$?

   (a) $P(y \wedge x) = P(y|x)P(x|y)$

   (b) $P(x|y) = \frac{P(y|x)P(y)}{P(x)}$

   (c) $P(x|y) = \frac{P(y \wedge x)}{P(x)}$

   (d) $P(y \wedge x) = P(y)P(x)$

   (e) $P(y|x) = \frac{P(x|y)P(y)}{P(x)}$

17. The following table specifies a joint probability distribution for three Boolean random variables $X$, $Y$, $Z$.

| | $x$ | | $\neg x$ | |
|---|---|---|---|---|
| | $y$ | $\neg y$ | $y$ | $\neg y$ |
| $z$ | a | b | c | d |
| $\neg z$ | e | f | g | h |

   Which of the following statements is incorrect?

   (a) $P(x \wedge y \wedge \neg z) = e$

   (b) $P(z|y) = (a+c)/(a+c+e+g)$

   (c) If $Z$ and $Y$ are conditionally independent, then $(a+b+c+d) = (a+c)/(a+e+c+g)$ holds

   (d) $P(\neg x|y) = c+g$

   (e) $\mathbf{P}(X) = \langle a+b+e+f, c+d+g+h \rangle$

18. Assume the following inhibition probabilities between Boolean cause variables $A$, $B$, $C$ and $D$ and Boolean effect variable $X$:

$$P(\neg x|a, \neg b, \neg c, \neg d) = 0.7$$
$$P(\neg x|\neg a, b, \neg c, \neg d) = 0.9$$
$$P(\neg x|\neg a, \neg b, c, \neg d) = 0.3$$
$$P(\neg x|\neg a, \neg b, \neg c, d) = 0.2$$

What is the probability $P(x|a, \neg b, c, d)$ assuming the conditional probabilities of $X$ are computed using a noisy-OR relation?

(a) 0.042
(b) 0.168
(c) 0.958
(d) 0.058
(e) 0.0378

19. You are given $P(o_1|a_1)$, $P(o_1|a_2)$, $P(o_2|a_1)$, $P(o_2|a_2)$ and utility function $u$ for a single-shot decision problem with actions $\{a_1, a_2\}$ and outcomes $\{o_1, o_2\}$. Which of the following is the correct formula for computing the expected utility of $a_1$?

(a) $P(o_1|a_1)u(o_1) + P(o_2|a_1)u(o_2)$
(b) $P(o_1|a_1)u(a_1) + P(o_2|a_1)u(a_2)$
(c) $P(a_1|o_1)u(o_1) + P(a_2|o_1)u(o_1) + P(a_1|o_2)u(o_2) + P(a_2|o_2)u(o_2)$
(d) $P(o_1|a_1)u(o_1) + P(o_2|a_1)u(o_2) + P(o_1|a_2)u(o_1) + P(o_2|a_2)u(o_2)$
(e) $P(o_1|a_1)u(o_1) + P(o_2|a_1)u(o_2) - P(o_1|a_2)u(o_1) + P(o_2|a_2)u(o_2)$

20. Imagine the UK is preparing for the outbreak of an unusual disease. With no treatment, it is expected to kill 600 people. If they adopt Programme A for combating the disease, then 400 people will die. If they adopt Programme B, then there is a $\frac{1}{3}$ chance that no one will die and a $\frac{2}{3}$ chance that 600 people will die.

    What should a rational agent do?

    (a) It is impossible to say without knowing more about the utility function for people dying (or surviving).

    (b) Take no action.

    (c) Adopt Programme A.

    (d) Adopt Programme B.

    (e) Adopt either Programme A or B; their expected utility is the same.

**Part B**
**ANSWER ONE QUESTION FROM PART B**

1. (a) Briefly explain what is meant by the following statement: *Entailment for first order logic is semi-decidable.*

   [3%]

   (b) Assuming that Modus Ponens is sound, prove that Generalized Modus Ponens (GMP) is also a sound rule of inference for first order logic.

   [4%]

   (c) State Herbrand's theorem.

   [2%]

   (d) Briefly describe a procedure for showing how a first order logic statement is entailed by a first order logic knowledge base through propositionalization.

   [4%]

   (e) Consider the following statements:

$$\forall x.\exists y.\ Q(x) \;\Rightarrow\; P(y,x) \tag{1}$$
$$\exists z.\ Q(z) \;\Rightarrow\; P(z,a) \tag{2}$$

   where $a$ is a constant. Convert statement (1) and the *negation* of statement (2) into clausal form. For full marks, you should show all the transformation steps.

   [4%]

   (f) Using the clauses from part (1e), show that your procedure from part (1d) can be used with *ground* resolution to yield a refutation. For full marks, you should show all your work.

   [8%]

2. Consider a search problem $S$ with a path $P$ consisting of nodes $n_0, n_1, \ldots, n_k$ where $n_0$ is the start node, $n_k$ is the goal node, and the cost of the unique action from a node $n_i$ to its successor $n_{i+1}$ is $+1$.

   (a) Briefly explain what is meant by an admissible heuristic.

   [2%]

   (b) Briefly explain, with the help of the appropriate mathematical formula, what is meant by a consistent heuristic. What property does the *evaluation* function have when the heuristic is consistent?

   [4%]

   (c) For the search problem S given above, what is the true cost $h^*(n_i)$ from a node $n_i$ to the goal node $n_k$.

   [2%]

   (d) Assume that for the search problem S that a heuristic function is defined by

   $$h(n_i) = k - 2\lceil i/2 \rceil$$

   where $\lceil x \rceil$ is the ceiling function that returns the smallest integer greater or equal to $x$.

   i. Is $h$ admissible? Explain your answer.

   [4%]

   ii. Is $h$ consistent? Explain your answer.

   [4%]

   (e) Consider a best-first search with an evaluation function given by $f(n)$ at node $n$, and a function $depth(n)$ that returns the depth of node $n$. What type of search is implemented by defining

   i. $f(n) = 1/depth(n)$?

   [2%]

   ii. $f(n) = depth(n)$?

   [2%]

   (f) *Prove* that $A^*$ using TREE-SEARCH is optimal if its heuristic function $h$ is admissible.

   [5%]

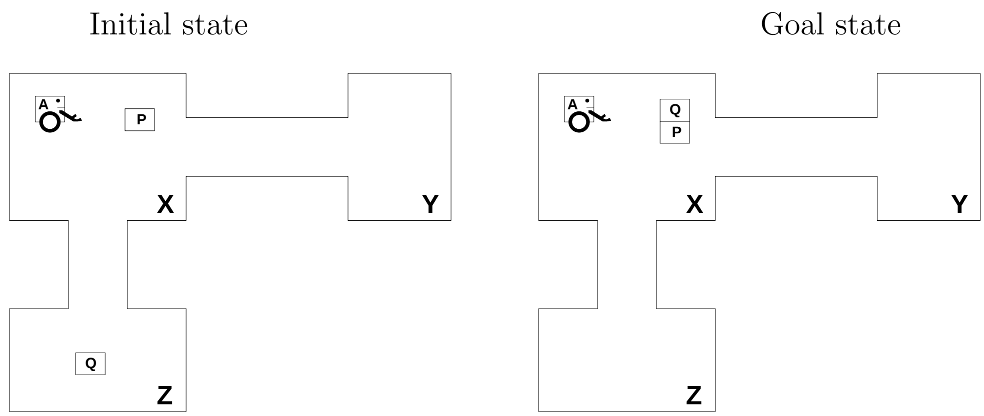



Figure 1: Initial and goal states for an office delivery system.

**Part C**
**ANSWER ONE QUESTION FROM PART C**

1. **Planning**

   What follows is a description of a robotic office delivery system.

   - The robot $A$ transports parcels $P$ and $Q$ between different rooms, $X$, $Y$ and $Z$. The initial and goal states are shown in Figure 1.
   - The robot can pick up and drop parcels, put one parcel on top of another and move between rooms.
   - The robot can sense its own location and those of the parcels. It can also sense when its battery is low. If the robot is moving and its battery is low, it will go to room $Y$ (regardless of where it initially intended to move) to recharge.

   The following predicates from the language PDDL can describe the domain:

   - $In(o, r)$—robot or parcel $o$ is in room $r$.
   - $Holds(r, p)$—robot $r$ is carrying parcel $p$.
   - $OnFloor(p)$—parcel $p$ is on the floor.
   - $Clear(p)$—the top of parcel $p$ is clear.
   - $Robot(r)$—$r$ is a robot.
   - $Parcel(p)$—$p$ is a parcel.
   - $Room(rm)$—$rm$ is a room.
   - $On(p_1, p_2)$—parcel $p_1$ is on parcel $p_2$.
   - $LowBattery(r)$—robot $r$'s battery is low.

(a) Using the above predicates, describe the initial and goal states illustrated in Figure 1. Assume that in the initial state the robot's battery level is not low.
*Note: You need only stipulate the values of fluents in the goal states.*
[5%]

(b) Using PDDL, define the following action schemata so as to reflect the preconditions and consequences of actions as described, taking into account also the above knowledge about the domain. You can use disjunctions and conditional effects as required:

i. $Go(r, rm_1, rm_2)$: robot $r$ moves from room $rm_1$ to room $rm_2$, moving anything it is potentially carrying to $rm_2$ with it.
*Hint: Recall what happens when a robot is moving and its battery is low.*
[5%]

ii. $Drop(r, p)$: robot $r$ drops parcel $p$, resulting in parcel $p$ being on the floor.
[2%]

iii. $Pickup(r, p)$: robot $r$ picks up parcel $p$, which requires $r$ and $p$ to be in the same room and as a result of this action, $r$ holds $p$.
[2%]

iv. $Put(r, p_1, p_2)$: robot $r$ puts parcel $p_1$ onto parcel $p_2$. This requires $r$ to be holding $p_1$, the top of parcel $p_2$ to be clear and the robot $r$ to be in the same room as $p_2$. Its effect is that $p_1$ is on $p_2$, $r$ no longer holds $p_1$, and parcel $p_2$ is no longer clear.
[2%]

(c) Assuming that each robot is planning his own actions but has no control over the actions of the other robot, what sort of planner is most suitable for this domain? Explain your answer.
[3%]

(d) Using the action schemata, write a plan for robot $A$ to put parcel $Q$ onto parcel $P$ in room $X$, as shown in the goal state.
*Hint: To handle steps that contain loops, you can use the syntax* **while** *$S$* **do** *$A$ (where $S$ is a description of a state and $A$ is an action). You may use [] to stipulate the robot to do no action.*
[6%]

| $P(S)$ |
|---|
| 0.9 |

| $P(F)$ |
|---|
| 0.7 |

| $S$ | $F$ | $P(H)$ |
|---|---|---|
| T | T | 0.95 |
| F | T | 0.7 |
| T | F | 0.8 |
| F | F | 0.1 |

| $F$ | $H$ | $P(G)$ |
|---|---|---|
| T | T | 0.95 |
| F | T | 0.6 |
| T | F | 0.55 |
| F | F | 0.2 |

Figure 2: Prior and Conditional Probabilities for Farming

2. **Bayesian Networks**

A farmer wants to work out the likelihood that he needs to increase the price of his grain for the month of August 2011. He represents a grain price increase with the Boolean variable $G$. The price of his grain is dependent on whether the cost of harvesting the grain increases (represented with the Boolean variable $H$) and on whether there is a fuel price increase (represented with the Boolean variable $F$), since fuel is needed to transport the grain to buyers. Fuel increases can also cause an increase in the cost of harvesting. Finally, labour relations with his farm workers have been poor lately, and if there is a strike (Boolean variable $S$), then this also increases the cost of harvesting.

The prior and conditional probabilities are given in Figure 2.

(a) Draw a Bayesian network that captures the causal relationships among the various variables described above.

[4%]

(b) Given that a price increase to both the cost of harvesting and to the grain has occurred, calculate the probability that there was a fuel price increase using inference by enumeration.
*Hint: Do all your calculations to 2 decimal places.*

[15%]

(c) Assume you were given the following information:

i. Given that a grain price increase ($G$) has occurred, the probability that there was a fuel increase ($F$) is 0.7.

ii. Given that an grain price increase ($G$) has occurred, the probability that the cost of harvesting increased ($H$) is 0.65.

iii. Given that a grain price increase ($G$) has occurred and the price of fuel ($F$) has increased, the probability that the cost of harvesting incrased ($H$) is 0.85.

Show how the probability of a fuel increase given the observation that the price of grain and the cost of harvesting both increased (i.e., the same query as in part (a)) can be alternatively computed using this information.

*Note: the probability you get by calucating the query using the information in part (c) is different from that for part (a). Again, do the calculations to 2 decimal places.*

[6%]

# Specimen Answers

### Part A

1. d
2. e
3. c
4. a or e
5. d
6. b
7. b
8. c
9. a
10. d
11. d
12. c
13. b
14. e
15. a
16. e
17. d
18. c
19. a
20. a

**Part B**

1. (a) (Bookwork) This means that a method (algorithm) exists that says yes to every entailed sentence for first order logic (FOL), but no method exists that also says no to every non-entailed sentence.

   Marking guide: Any answer (including more formal ones) along these lines is acceptable.

   (b) (Bookwork) We need to show that $p'_1, \ldots, p'_n, (p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models q\theta$ provided $p'_i\theta = p_i\theta$ for all $i$ and some most general unifier $\theta$.

   *Proof*: For any sentence $p$, we have $p \models p\theta$ by the Universal Instantiation rule. Using this (and a few basic rules), we thus have:

   i. $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models (p_1 \wedge \ldots \wedge p_n \Rightarrow q)\theta = (p_1\theta \wedge \ldots \wedge p_n\theta \Rightarrow q\theta)$
   ii. $p'_1, \ldots, p'_n \models p'_1 \wedge \ldots \wedge p'_n \models (p'_1 \wedge \ldots \wedge p'_n)\theta = p'_1\theta \wedge \ldots \wedge p'_n\theta = p_1\theta \wedge \ldots \wedge p_n\theta$ since by definition of GMP, we have $p'_i\theta = p_i\theta$ for all $i$.
   iii. From 1(b)i and 1(b)ii, $q\theta$ follows by Modus Ponens.

   Marking guide: 1 mark for stating what is being proved, 1 mark for each of the steps 1(b)i to 1(b)iii in the proof.

   (c) (Bookwork) Herbrand's theorem (as given in the lectures): If a sentence $\alpha$ is entailed by a first order logic knowledge base, it is entailed by a finite subset of the propositionalized knowledge base.

   Marking guide: Alternative statements of the theorem are also acceptable.

   (d) (Bookwork) Following from Herbrand's theorem, we can have an algorithm that basically does the following when trying to show that some FOL statement $\alpha$ is entailed by some knowledge base $KB$:

   For $n = 0$ to $\infty$ do

   - Create a propositionalized version of $KB$ and $\alpha$ by instantiating with depth $n$ terms;
   - Check whether this ground $\alpha$ is entailed by the ground $KB$ using any complete propositional algorithm (e.g. DPLL or ground resolution).

   Marking guide: 2 marks for mentioning propositionalization and 2 marks for stating that entailment can then be checked using a complete propositional algorithm.

(e) The transformation to CNF converts the first statement to

$$\neg Q(x) \lor P(f(x), x) \tag{3}$$

where $f$ is a Skolem constant. The negated second statement is converted to

$$Q(z) \tag{4}$$
$$\neg P(z, a) \tag{5}$$

Marking guide: 2 marks for clause (3) and 1 mark for each of the remaining ones. Deduct 1 mark if the Skolem function is not explicitly stated.

(f) The procedure involves a propositionalization of the clauses with ground terms of increasing depth, starting with constants, until a proof by ground resolution can be found:

At depth 0, the ground terms are $\{a\}$, so instantiating the clauses yields $\neg Q(a) \lor P(f(a), a)$, $Q(a)$, and $\neg P(a, a)$, from which one resolution step is possible between the first two clauses yielding $P(f(a), a)$. No more resolution proof step is possible after this since there are no other complementary literals.

At depth 1, the ground terms are $\{a, f(a)\}$, so instantiating the clauses with the depth-1 term yields the additional instances: $\neg Q(f(a)) \lor P(f(f(a)), f(a))$, $Q(f(a))$, and $\neg P(f(a), a)$.

At this depth, a refutation proof using resolution *is* possible by first resolving $Q(f(a))$ with $\neg Q(f(a)) \lor P(f(a), a)$ to give $P(f(a), a)$, which can then resolved with $\neg P(f(a), a)$ to give the empty clause.

Marking guide: 2 marks the propositionalization to depth 0. 1 mark for stating that one resolution step but no full proof i.e. no refutation is possible at this depth. 2 marks for the propositionalization to depth 1. 3 marks for the resolution proof yielding the empty clause.

2. (a) (Bookwork) A heuristic $h(n)$ is admissible if for every node $n$, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach goal state $n$. An admissible heuristic never over-estimates the cost to reach the goal i.e. it is optimistic.

(b) (Bookwork) A heuristic $h(n)$ is consistent if, for every node $n$ and every successor $n'$ of $n$ generated by an action $a$, the estimated cost of reaching the goal from $n$ is no greater than the step cost $c(n, a, n')$ of getting to $n'$ from $n$ plus the estimated cost of getting from $n'$:

$$h(n) \leq c(n, a, n') + h(n') \tag{6}$$

Such a heuristic ensures that the values of the evaluation function are non-decreasing along any path.

Marking guide: 3 marks for explanation of consistency. 1 mark for inequality.

(c) The definition of the cost function is straightforward, given that each action costs 1: $h^*(n_i) = k - i$.

(d) For the given problem S:

   i. The heuristic is admissible since $h(n_i) = k - 2\lceil i/2 \rceil \leq k - i = h^*(n_i)$ for all $i$ i.e. for all nodes.

   ii. The heuristic is not consistent because for even $i$, we have that $h(n_i) > 1 + h(n_{i+1})$, which falsifies condition (6) in (2b) above.

(e) With $depth(n)$ as defined, we have

   i. Depth-first search when $f(n) = 1/depth(n)$.

   ii. Breadth-first search when $f(n) = depth(n)$.

Marking guide: Full marks can be given in each case for just stating the type of search achieved:

(f) (Bookwork)

Let $f(n) = g(n) + h(n)$ where $g(n)$ be the cost so far to reach node $n$, $h(n)$ be the heuristic (i.e. estimated) cost from $n$ to the goal. Thus, $f(n)$ is the total estimated cost of the path through $n$ to the goal. Moreover, let $h^*(n)$ be the true cost to reach the goal from $n$.

Now, suppose some suboptimal goal $G_2$ has been generated and is in the fringe. Let $n$ be an unexpanded node in the fringe such that $n$ is on a shortest path to an optimal goal $G$.

We then have the following chain of reasoning:

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since $G_2$ is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above
- $h(n) \leq h^*(n)$, since $h$ is admissible
- $g(n) + h(n) \leq g(n) + h^*(n)$
- and so $f(n) \leq f(G)$
- Hence, we have $f(G_2) > f(n)$ and $A^*$ will never select $G_2$ for expansion.

Marking guide: The proof is given in the lecture and in Russell & Norvig. For full marks, all the terms ($f(n)$, $g(n)$, etc.) must be explained and a mathematical proof must be given.

**Part C**

1. The Planning Question

   (a) The initial state is:

$$\begin{aligned}&In(A,X) \wedge In(P,X) \wedge In(Q,Z)\wedge\\&OnFloor(P) \wedge OnFloor(Q)\wedge\\&Robot(A) \wedge Parcel(P) \wedge Parcel(Q)\wedge\\&Room(X) \wedge Room(Y) \wedge Room(Z)\wedge\\&\neg LowBattery(A)\end{aligned}$$

   The goal state is:

$$\begin{aligned}&In(A,X) \wedge In(P,X) \wedge In(Q,X)\wedge\\&On(Q,P) \wedge OnFloor(P)\end{aligned}$$

   *Deduct 1 point for each missing literal; deduct 1 point for each literal that is present in the description but shouldn't be.*

   (b) *For the following, deduct 1 point for each incorrect literal.*

$$\begin{aligned}&Action(Go(r, rm_1, rm_2),\\&\quad \text{PRECOND:} Robot(r) \wedge Room(rm_1) \wedge Room(rm_2) \wedge In(r, rm_1)\\&\quad \text{EFFECT:} (In(r, Y) \wedge (\textbf{when } Holds(r,p); In(p,Y)))\vee\\&\qquad\qquad (In(r, rm_2) \wedge (\textbf{when } Holds(p,r); In(p, rm_2))))\end{aligned}$$

   ii.

$$\begin{aligned}&Action(Drop(r,p),\\&\quad \text{PRECOND:} Holds(r,p) \wedge Robot(r) \wedge Parcel(p)\\&\quad \text{EFFECT:} \neg Holds(r,p) \wedge OnFloor(p))\end{aligned}$$

   iii.

$$\begin{aligned}&Action(Pickup(r,p),\\&\quad \text{PRECOND:} OnFloor(p) \wedge In(r, rm) \wedge In(p, rm)\\&\quad \text{EFFECT:} Holds(r,p) \wedge \neg OnFloor(p))\end{aligned}$$

   iv.

$$\begin{aligned}&Action(Put(r, p_1, p_2),\\&\quad \text{PRECOND:} Robot(r) \wedge Parcel(p_1) \wedge Parcel(p_2)\wedge\\&\qquad OnFloor(p_2) \wedge Holds(r, p_1)\wedge\\&\qquad In(r, rm) \wedge In(p_2, rm)\wedge\\&\qquad Clear(p_2)\\&\quad \text{EFFECT:} On(pi_1, p_2) \wedge \neg Holds(r, p_1) \wedge \neg \mathrm{Clear}(p_2))\end{aligned}$$

(c) The most suitable planner is a contingency planner, because this is a domain where the robot can't sense all of the current state but it can sense all that it needs to in order to know the optimal action (making sensorless planning unsuitable), and the outcomes of actions may not be as intended (due to low battery) but are bounded (so contingency planning is necessary, but online planning or replanning is not).

**Bookwork**

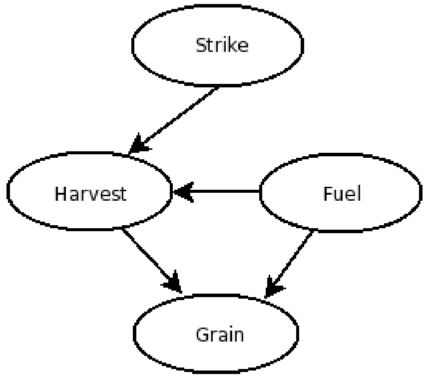
*Remove 1 point for having no explanation as to why contingency planning is optimal. Remove both points if they don't say contingency planning.*

(d)

$$
\begin{array}{ll}
[ & \mathit{Go}(A, X, Z), \\
 & \textbf{if } \mathit{In}(A, Y) \textbf{ then while } \neg \mathit{In}(A, Z) \textbf{ do } \mathit{Go}(A, Y, Z) \\
 & \qquad\qquad\quad \textbf{else } [], \\
 & \mathit{Pickup}(A, Q), \mathit{Go}(A, Z, X) \\
 & \textbf{if } \mathit{In}(A, Y) \textbf{ then while } \neg \mathit{In}(A, X) \textbf{ do } \mathit{Go}(A, Y, X) \textbf{ else } [], \\
 & \mathit{Put}(A, Q, P)]
\end{array}
$$

*Remove 1 point for each flaw in the plan, including flaws arising from not taking account of all contingencies and from not including the loops or imposing the wrong scope on the loops.*

2. The Dynamic Bayesian Network Question

(a) 


*Deduct 1 point for each incorrect link.*

(b) The quantity we are looking for us $P(f|h,g)$ (*2 marks*). So:

$$\begin{aligned}
P(f|h,g) = &\ \alpha\mathbf{P}(f,h,g,S) \\
= &\ \alpha P(g|h,f)P(f)P(S)P(h|f,S) \\
= &\ \alpha P(g|h,f)P(f)\textstyle\sum_s P(s)P(h|f,s)
\end{aligned}$$

*6 marks should be given for developing this formula (2 for each line), and 3 marks for the following calculation.*

$$\begin{aligned}
P(f|h,g) = &\ \alpha \times 0.95 \times 0.7 \times ((0.9 \times 0.95) + (0.1 \times 0.3)) \\
= &\ \alpha \times 0.59
\end{aligned}$$

For normalisation purposes we also need to calculate:

$$\begin{aligned}
P(\neg f|h,g) = &\ \alpha P(g|h,\neg f)P(\neg f)\textstyle\sum_s P(s)P(h|\neg f,s) \\
= &\ \alpha \times 0.6 \times 0.3 \times ((0.9 \times 0.8) + (0.1 \times 0.1)) \\
= &\ \alpha \times 0.13
\end{aligned}$$

*Another 3 marks can be obtained for the above calculation.*

So after normalisation:

$$\begin{aligned}
P(f|h,g) = &\ \tfrac{0.59}{0.59+0.13} \\
= &\ 0.82
\end{aligned}$$

*1 mark for this final calculation.*

(c) Because of Bayes' Rule:

$$\begin{aligned}
P(f|h,g) = &\ \tfrac{P(f,h,g)}{P(h,g)} \\
= &\ \tfrac{P(h|f,g)P(f,g)}{P(h,g)} \\
= &\ \tfrac{P(h|f,g)P(f|g)P(g)}{P(h|g)P(g)} \\
= &\ \tfrac{P(h|f,g)P(f|g)}{P(h|g)}
\end{aligned}$$

*3 marks for getting this far.*

$P(h|f,g)$ is given by statement (iii). $P(f|g)$ is given by statement (i). and $P(h|g)$ is given by statement (ii).

*2 marks for getting this far.*

So:

$$\begin{aligned} P(f|h,g) &= \tfrac{0.7 \times 0.85}{0.65} \\ &= 0.92 \end{aligned}$$

*1 mark for final calcuation.*