Informatics 2D: Solutions for Tutorial 3

Entailment and DPLL*

Week 4

1 The Wumpus World

1.1 Propositional Rules

Translate the following statements into propositional logic formulae. You can use a schematic representation for the location of a square, e.g. use a proposition $W_{i,j}$ to represent that there is a wumpus in the square in the ith row and jth column (don't worry about the edges of the grid when formalising your propositions).

1. A square cannot contain the wumpus and a pit.
2. If a square is breezy then one of the (not diagonally) adjacent squares contains a pit.
3. There is a stench in the square if and only if it contains the wumpus or is (not diagonally) adjacent to the square containing the wumpus.

1.2 Entailment

Using the above rules, and the assumed facts, show the following statements are entailed by the knowledge base (either using a truth table or a diagram showing the possible models):

1. Assuming that there is a pit in square (2, 2) show that the wumpus is not in square (2, 2).
2. Assuming that there is a stench in (1, 1) and that there is not a wumpus in square (1, 1) show that there is either a wumpus in (1, 2) or a wumpus in (2, 1). (Assume the grid begins at (1,1) and ignore the off-grid squares in your rules).
3. Assuming that there is a breeze in square (2, 2) and that there is not a pit in squares (1, 2), (2, 1) or (3, 2), show that there is a pit in square (2, 3).

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Answers

1. Propositional Rules

    1. $\neg(W_{i,j} \wedge P_{i,j})$
    2. $B_{i,j} \Rightarrow P_{i+1,j} \vee P_{i-1,j} \vee P_{i,j+1} \vee P_{i,j-1}$
    3. $S_{i,j} \Leftrightarrow W_{i,j} \vee W_{i+1,j} \vee W_{i-1,j} \vee W_{i,j+1} \vee W_{i,j-1}$

2. Entailment

Either use the diagramatic representation of wumpus world states (as shown in the lecture notes and Russel and Norvig) or construct a truth table for the proposition you are trying to show and demonstrate that each model for the KB is a model of the proposition. Note that if you are using a truth table that you only need to consider the rows in which the propositions in the KB are true.

1. $KB \Leftrightarrow \neg(W_{2,2} \wedge P_{2,2}) \wedge P_{2,2}$
   $\alpha \Leftrightarrow \neg W_{2,2}$
2. $KB \Leftrightarrow S_{1,1} \wedge (S_{1,1} \Leftrightarrow W_{1,1} \vee W_{2,1} \vee W_{1,2}) \wedge \neg W_{1,1}$
   $\alpha \Leftrightarrow W_{2,1} \vee W_{1,2}$
3. $KB \Leftrightarrow (B_{2,2} \Rightarrow P_{1,2} \vee P_{2,1} \vee P_{3,2} \vee P_{2,3}) \wedge B_{2,2} \wedge \neg P_{1,2} \wedge \neg P_{2,1} \wedge \neg P_{3,2}$
   $\alpha \Leftrightarrow \neg P_{2,3}$

2 DPLL algorithm

The DPLL algorithm consists of the following steps:

- Convert proposition to CNF
- Loop through the following steps until a satisfying assignment is found or none is possible:
    - Loop through the following simplifications until the formula can't be simplified anymore:
        * Pure literal heuristic.
        * Unit Clause heuristic.
    - Select a variable and branch the search space into a formula where the variable is true and a formula where the variable is false. (This means that you try the algorithm recursively upon these new formulae, with a satisfying assignment for one of the new formula being a satisfying assignment for the original).

Your lecture notes and R&N chapter 7 section 6 describe the steps in more detail.

Question: Use the DPLL algorithm to show whether the following propositional formulae is satisfiable:

$S_{1,1} \land (S_{1,1} \Leftrightarrow W_{1,2} \lor W_{1,1} \lor W_{2,1}) \land \neg\left((W_{1,2} \land P_{1,2}) \lor (W_{2,1} \land P_{2,1})\right) \land \neg P_{1,1} \land$

$\neg\left((W_{1,1} \land W_{2,1}) \lor (W_{1,1} \land W_{1,2})\right)$

Answer:

The proposition consists of the following conjuncts:

- $S_{1,1}$
- $\neg P_{1,1}$
- $S_{1,1} \Leftrightarrow W_{1,2} \lor W_{1,1} \lor W_{2,1}$
- $\neg\left((W_{1,2} \land P_{1,2}) \lor (W_{2,1} \land P_{2,1})\right)$
- $\neg\left((W_{1,1} \land W_{2,1}) \lor (W_{1,1} \land W_{1,2})\right)$

Converting each into CNF:

$S_{1,1} \Leftrightarrow W_{1,2} \lor W_{1,1} \lor W_{2,1}$

$$(S_{1,1} \Rightarrow W_{1,2} \lor W_{1,1} \lor W_{2,1}) \land (W_{1,2} \lor W_{1,1} \lor W_{2,1} \Rightarrow S_{1,1})$$

$$(\neg S_{1,1} \lor W_{1,2} \lor W_{1,1} \lor W_{2,1}) \land ((\neg W_{1,2} \land \neg W_{1,1} \land \neg W_{2,1}) \lor S_{1,1})$$

$$(\neg S_{1,1} \lor W_{1,2} \lor W_{1,1} \lor W_{2,1}) \land (\neg W_{1,2} \lor S_{1,1}) \land (\neg W_{1,1} \lor S_{1,1}) \land (\neg W_{2,1} \lor S_{1,1})$$

$\neg\left((W_{1,2} \land P_{1,2}) \lor (W_{2,1} \land P_{2,1})\right)$

$$\neg(W_{1,2} \land P_{1,2}) \land \neg(W_{2,1} \land P_{2,1})$$

$$(\neg W_{1,2} \lor \neg P_{1,2}) \land (\neg W_{2,1} \lor \neg P_{2,1})$$

$\neg\left((W_{1,1} \land W_{2,1}) \lor (W_{1,1} \land W_{1,2})\right)$

$$(\neg W_{1,1} \lor \neg W_{2,1}) \land (\neg W_{1,1} \lor \neg W_{1,2})$$

cntd. overleaf

So the CNF formula has the following conjuncts:

- $S_{1,1}$
- $\neg P_{1,1}$
- $\neg S_{1,1} \vee W_{1,2} \vee W_{1,1} \vee W_{2,1}$
- $\neg W_{1,2} \vee S_{1,1}$
- $\neg W_{1,1} \vee S_{1,1}$
- $\neg W_{2,1} \vee S_{1,1}$
- $\neg W_{1,2} \vee \neg P_{1,2}$
- $\neg W_{2,1} \vee \neg P_{2,1}$
- $\neg W_{1,1} \vee \neg W_{2,1}$
- $\neg W_{1,1} \vee \neg W_{1,2}$

which has pure literals $\neg P_{1,1}$, $\neg P_{1,2}$ and $\neg P_{2,1}$ and the unit clauses with literals $S_{1,1}$ and $\neg P_{1,1}$.

So we assign $S_{1,1}$ to `true` and $P_{1,1}$, $P_{1,2}$ and $P_{2,1}$ to `false` which, using early termination, leaves us with:

- $W_{1,2} \vee W_{1,1} \vee W_{2,1}$
- $\neg W_{1,1} \vee \neg W_{2,1}$
- $\neg W_{1,1} \vee \neg W_{1,2}$

There are no more simplifications to make so we must pick a literal and branch on it, choosing $W_{1,2}$ and assigning it to true leaves:

- $\neg W_{1,1} \vee \neg W_{2,1}$
- $\neg W_{1,1}$

Assigning $W_{1,1}$ to false, since $\neg W_{1,1}$ is a unit clause literal, satisfies the remaining clauses. This gives us an assignment of $S_{1,1}$ and $W_{1,2}$ to `true` and $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $W_{1,1}$ to `false`. Note that we have not assigned $W_{2,1}$ to either `true` or `false`, since in either case we have a satisfying assignment.