Tutorial 8

Part 1: Basic Probability

1) $P(\text{battery}) = 0.7 + 0.085 + 0.05 + 0.03 = 0.865$
$P(\text{Radio} = \text{true}) = 0.7 + 0.05 + 0.002 + 0.03 = 0.782$
$P(\text{Radio} = \text{false}) = 1 - P(\text{Radio} = \text{true}) = 1 - 0.782 = 0.218$
$P(\text{radio} \wedge \neg\text{ignition}) = 0.05 + 0.03 = 0.8$
$P(\neg\text{radio} \vee \neg\text{ignition}) = 0.085 + 0.03 + 0.003 + 0.1 + 0.05 + 0.03 = 0.298$

2) From the product rule we can derive Bayes rule:

$$P(a|b)P(b) = P(b|a)P(a) => P(a|b) = \frac{P(b|a)P(a)}{P(b)}.$$

3) In order to compute $P(\text{Ignition}|\neg\text{radio})$ we need to first compute $P(\text{ignition}|\neg\text{radio})$ and $P(\neg\text{ignition}|\neg\text{radio})$ separately, so:

$$P(\text{ignition}|\neg\text{radio}) = \frac{P(ignition \wedge \neg\text{radio})}{P(\neg\text{radio})} = \frac{0.085+0.003}{0.085+0.03+0.003+0.1} = 0.4037$$

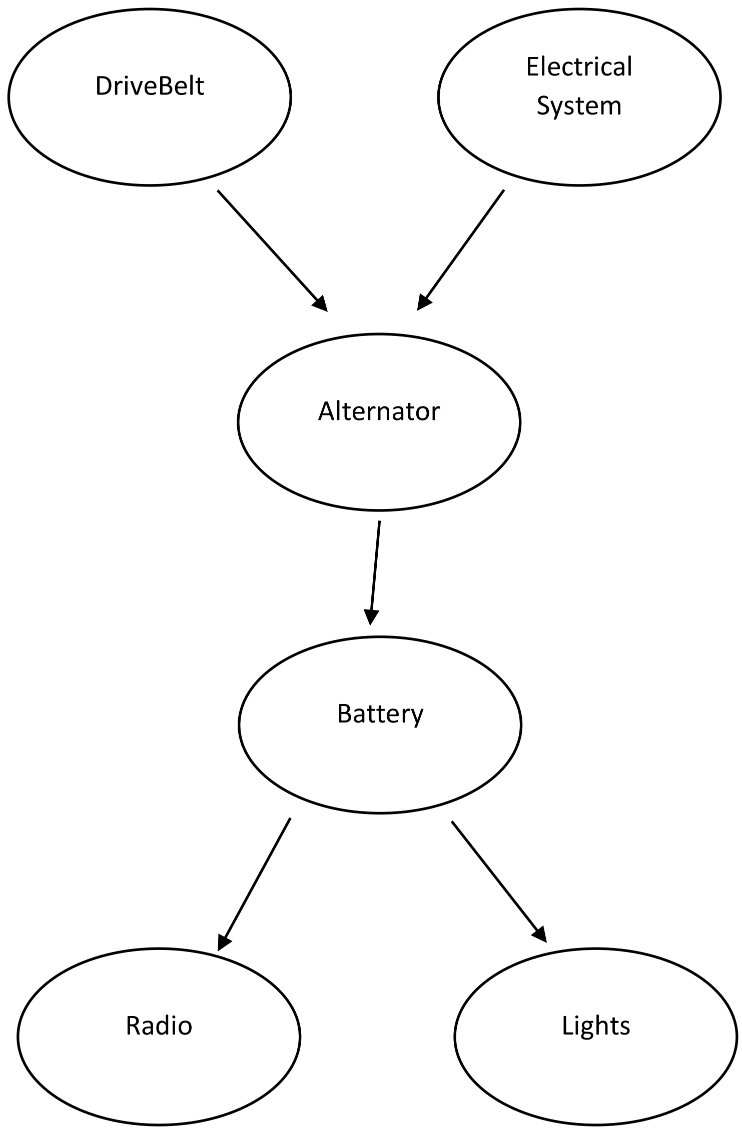
$$P(\text{ignition}|\neg\text{radio}) = \frac{P(\neg ignition \wedge \neg\text{radio})}{P(\neg\text{radio})} = \frac{0.03+0.01}{0.085+0.03+0.003+0.1} = 0.5963.$$

4) The easiest way would be:

$$P(\text{Battery}, \text{Ignition}, \text{Radio}) = P(\text{Battery}) * P(\text{Ignition}|\text{Battery}) * P(\text{Radio}|\text{Battery})$$

Part 2: Bayesian Networks

1) The diagram would be:



2) The probability would be:

$\mathrm{P}(\mathrm{d}, \mathrm{e}, \mathrm{a}, \mathrm{b}, \neg\mathrm{r}, \neg\mathrm{l}) = 0.75 * 0.95 * 0.9 * 0.8 * (1 - 0.8) * (1 - 0.9) = 0.1026.$
$\mathrm{P}(\neg\mathrm{d}, \mathrm{e}, \neg\mathrm{a}, \mathrm{b}, \mathrm{r}, \mathrm{l}) = (1 - 0.75) * 0.95 * (1 - 0.1) * 0.2 * 0.8 * 0.9 =$
$0.03078.$

Part 3: Extract Inference in Bayesian Networks

We will have to find out $\mathrm{P(D = true|a), P(D = false|a)}$ separately:

$$\mathrm{P(D = true|a) = P(d|a) = \alpha \sum_E P(d, a, E)}$$

$$\mathrm{= \alpha \sum_E P(d)P(a|d, E)P(E)}$$

$$\mathrm{= \ \alpha P(d) \sum_E P(a|d, E)P(E)}$$

$$\mathrm{= \alpha P(d)\big[(P(a|d, e)P(e)) + \big(P(a|d, \neg \mathit{e})P(\neg e)\big)\big]}$$

$$= \alpha * 0.75[(0.9 * 0.95) + (0.3 * 0.05)] = \ \alpha 0.6525.$$

$$\mathrm{P(D = false|a) = P(\neg d|a) = \alpha \sum_E P(\neg d, a, E)}$$

$$\mathrm{= \alpha \sum_E P(\neg d)P(a|\neg d, E)P(E)}$$

$$\mathrm{= \ \alpha P(\neg d) \sum_E P(a|\neg d, E)P(E)}$$

$$\mathrm{= \alpha P(\neg d)\big[(P(a|\neg d, e)P(e)) + \big(P(a|\neg d, \neg \mathit{e})P(\neg e)\big)\big]}$$

$$= \alpha * 0.25[(0.1 * 0.95) + (0.1 * 0.05)] = \ \alpha 0.025.$$

Where α the normalisation factor is computed from:

$$\mathrm{P(d|a)} = 0.6525 * \frac{1}{0.6525 + 0.025} = 0.963.$$

$$\mathrm{P(\neg d|a)} = 0.025 * \frac{1}{0.6525+0.025} = 0.036.$$