

UNIVERSITY OF EDINBURGH  
COLLEGE OF SCIENCE AND ENGINEERING  
SCHOOL OF INFORMATICS

**INFR08010 INFORMATICS 2D: REASONING AND AGENTS**

**Thursday 16<sup>th</sup> May 2013**

**09:30 to 11:30**

**INSTRUCTIONS TO CANDIDATES**

1. Answer Parts A, B and C. The multiple choice questions in Part A are worth 50% in total and are each worth the same amount. Mark one answer only for each question - multiple answers will score 0. Marks will not be deducted for incorrect multiple choice exam answers. Raw multiple choice scores may be rescaled at the discretion of the exam board. Parts B and C are each worth 25%. Answer ONE question from Part B and ONE question from Part C.
2. Use the special mark sheet for Part A. Answer Parts B and C each in a separate script book.

**CALCULATORS ARE PERMITTED.**

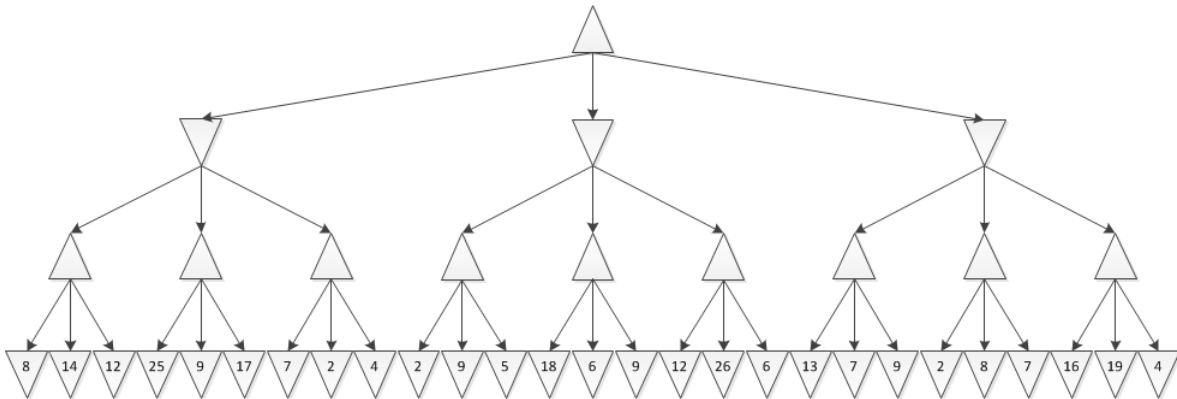
Convener: J Bradfield  
External Examiner: A Preece

**THIS EXAMINATION WILL BE MARKED ANONYMOUSLY**

## Part A

ANSWER ALL QUESTIONS IN PART A. Use the special mark sheet.

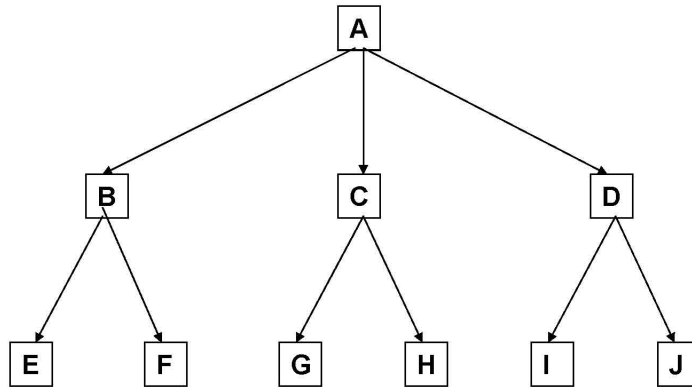
- Consider the following lookahead tree for a 2-person game, which is searched depth-first, left-to-right. Nodes with the symbol  $\Delta$  are where Max is due to play whereas nodes with the symbol  $\nabla$  are where Min is due to play. The number on each leaf is the result of the evaluation function applied to that leaf.



How many nodes will be pruned by an  $\alpha/\beta$  search?

- 0
  - 6
  - 8
  - 10
  - 15
- Consider the following statement: *A CSP problem is commutative.* This means that:
    - Only a single variable is considered at each node in the search tree.
    - The order in which the heuristics are applied is irrelevant.
    - The order in which a value is chosen for a given variable is irrelevant.
    - The constraints involve only commutative constraints.
    - The constraints involve commutative operators.

3. Consider the following search tree, where each node is labelled with a letter and the arcs are unordered:



Which order does *not* correspond to a depth-first search of the tree?

- (a) A, B, E, F, C, G, H, D, I, J.
  - (b) A, C, H, G, D, I, J, B, E, F.
  - (c) A, D, J, I, B, F, E, C, G, H.
  - (d) A, B, C, G, H, D, I, J, E, F.
  - (e) A, D, I, J, B, E, F, C, G, H.
4. Consider a best-first search with an evaluation function given by  $f(n)$  at node  $n$ , where  $depth(n)$  returns the depth of node  $n$ . What type of search is implemented if  $f(n) = 1/depth(n)$ ?
- (a)  $A^*$  search.
  - (b) Breadth-first.
  - (c) Depth-first.
  - (d) Depth-limited.
  - (e) Iteration-deepening.

5. Consider the following algorithm for **WALKSAT** with a missing part indicated by the black horizontal box:

```

function WALKSAT(clauses, p, max-flips) returns a satisfying model or failure
  inputs: clauses, a set of clauses in propositional logic
         p, the probability of choosing to do a “random walk” move
         max-flips, number of flips allowed before giving up
  model ← a random assignment of true/false to the symbols in clauses
  for i = 1 to max-flips do
    if model satisfies clauses then return model
    clause ← a randomly selected clause from clauses that is false in model
    with probability p flip the value in model of a randomly selected symbol
      from clause
  else
    return failure

```

Which of the following steps is taken by the algorithm in the missing **else** branch?

- (a) Flip a random symbol in *clause*.
- (b) Flip whichever symbol in *clause* minimises the number of satisfied literals.
- (c) Flip whichever symbol in *clause* maximises the number of unsatisfied clauses.
- (d) Flip whichever symbol in *clause* maximizes the number of satisfied clauses.
- (e) Set to false whichever symbol in *clause* minimises the number of satisfied clauses.

6. Consider the following statement:

$$\neg[(\forall x.U(x) \rightarrow R(x)) \rightarrow (\forall x.U(x) \rightarrow M(x))]$$

Which of the following sets of clauses, if any, is its clausal normal form (CNF)? Note that  $a$  and  $b$  are Skolem constants,  $F$  is a Skolem function and any other terms are variables.

- (a)  $\{\neg U(z) \vee R(z), U(a), \neg M(a)\}$ .
- (b)  $\{\neg U(z) \vee R(z), U(F(x)), \neg M(F(x))\}$ .
- (c)  $\{\neg U(a) \vee R(a), U(a), \neg M(a)\}$ .
- (d)  $\{\neg U(a) \vee R(a), U(x), \neg M(x)\}$ .
- (e) None of the above.

7. Which of the following statements is *false* of alpha-beta pruning?

- (a) It computes the same optimal move as minimax.
- (b) It is a simple form of meta-reasoning.
- (c) It can solve a tree roughly twice as deep as minimax in the same amount of time.
- (d) It is independent of the order in which the states are examined.
- (e) It eliminates subtrees that are provably irrelevant.

8. Which of the following substitutions is the most general unifier (MGU) of  $f(x) + g(x) = x$  and  $y + y = 0$ , if it exists. Note that  $x$  and  $y$  are variables.

- (a)  $\{y/f(0), y/g(0), x/0\}$ .
- (b)  $\{f/g, x/0, y/g(0)\}$ .
- (c) Unification fails due to a conflict.
- (d)  $\{x/0, y/g(x), f(x)/g(x)\}$ .
- (e) Unification fails due to an occurs-check violation.

9. Which of the following is *false* of Generalized Modus Ponens (GMP)?

- (a) GMP is sound.
- (b) GMP may not terminate on non-entailed queries.
- (c) GMP uses unification.
- (d) GMP can be applied both forwards and backwards.
- (e) GMP is only complete when combined with factoring.

10. In the Situation Calculus, which of the following statements about the successor-state axiom is *true*?
- (a) It enables the inference process to deal with impossible actions.
  - (b) It completely specifies the next state from the current state.
  - (c) It solves the qualification problem.
  - (d) It ensures that for every successor node in the search tree, the evaluation heuristic remains consistent.
  - (e) It ensures that distinct names refer to distinct objects.

11. Which of the following statements is incorrect?
- (a) In contrast to problem-solving agents based on heuristic search, planning agents don't need to define heuristic values for every possible state.
  - (b) Problem-solving agents based on search will often explore irrelevant actions.
  - (c) Search-based agents may consider undoing the effects of actions achieved by previous actions.
  - (d) Partial-order planning ensures that the agent does not commit to overly restrictive courses of execution at the time of planning.
  - (e) State-space search based planning methods are never goal-directed.
12. Assume you have to add an action  $D$  with postconditions  $\neg q$  and  $\neg p$  to a plan with existing causal links  $A \xrightarrow{p} B$  and  $B \xrightarrow{q} C$ . Which of the following total orderings resolves all conflicts that might arise from the addition of  $D$ ?
- (a)  $A \prec B \prec D \prec C$
  - (b)  $A \prec D \prec B \prec C$
  - (c)  $A \prec B \prec C \prec D$
  - (d)  $D \prec A \prec C \prec B$
  - (e)  $A \prec C \prec D \prec B$

13. In how many ways is the action schema

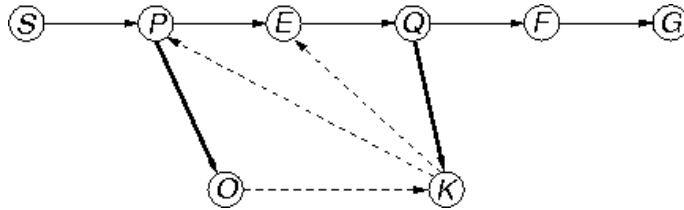
$Action(Fly(p, from, to),$   
 $PRECOND: At(p, from) \wedge Plane(p) \wedge Airport(from) \wedge Airport(to)$   
 $EFFECT: \neg At(p, from) \wedge At(p, to))$

applicable in the following state:

$At(P_1, SFO) \wedge At(P_2, Heathrow) \wedge$   
 $Airport(CDG) \wedge Airport(Heathrow) \wedge Airport(SFO) \wedge$   
 $Plane(P_1) \wedge Plane(P_2)$

- (a) 2
- (b) 4
- (c) 6
- (d) 9

14. The following diagram shows possible reactions to failure using replanning, with the original (linear) plan going through the sequence of states  $\langle S, P, E, Q, F, G \rangle$ :



Assuming that the cost of different sub-plans is proportional to the geometric distances between the states, what is the optimal replanning procedure the agent should execute if it ends up in unexpected states  $O$  and/or  $K$  (dashed lines denote possible intermediate goals during replanning, bold lines denote the only possible failures in this domain)?

- (a) when in  $O$ , plan for  $K$ ; when in  $K$ , plan for  $P$
- (b) when in  $O$ , plan for  $P$ ; when in  $K$ , plan for  $E$
- (c) when in  $O$ , plan for  $P$ ; when in  $K$ , plan for  $P$
- (d) when in  $O$ , plan for  $E$ ; when in  $K$ , plan for  $E$
- (e) when in  $O$ , plan for  $E$ ; when in  $K$ , plan for  $P$



15. You are given the following two PDDL planning operators with conditional effects:

$$\begin{aligned} & \text{Action}(A, \text{PRECOND:}\{P, Q\}, \text{EFFECT:}\{(\mathbf{when} \ R : \neg P), S\}) \\ & \text{Action}(B, \text{PRECOND:}\{P, S\}, \text{EFFECT:}\{(\mathbf{when} \ Q : T), U\}) \end{aligned}$$

Assume you are attempting to achieve goal state  $\{Q, T\}$  from initial state  $\{P, Q, R\}$ . Which of the following statements is correct?

- (a) The action sequence  $[B]$  is a valid solution to this problem.
  - (b) Action  $A$  is not applicable in the initial state.
  - (c) The action sequence  $[A, B]$  is not a valid solution to this problem.
  - (d) Action  $B$  is applicable in the initial state.
  - (e) The successor state that results from applying  $A$  in the initial state is  $\{P, Q, S, R\}$ .
16. The following table specifies a joint probability distribution for three Boolean random variables  $X, Y, Z$ :

	$x$		$\neg x$	
	$y$	$\neg y$	$y$	$\neg y$
$z$	a	b	c	d
$\neg z$	e	f	g	h

Which of the following is the correct value for  $P(x \vee y|z)$ ?

- (a)  $(a+b+c)$
- (b)  $(a+c)/(a+b+c+d)$
- (c)  $(a+b+c+d)/(e+f+g+h)$
- (d)  $(a+b+c)/(e+f+h)$
- (e)  $(a+b+c)/(a+b+c+d)$

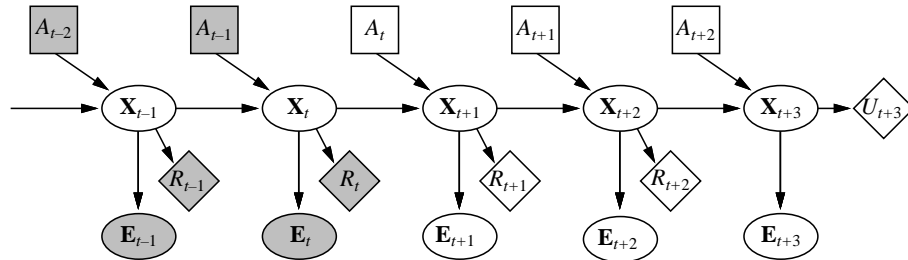
17. Which of the following statements about a Bayesian network with variables labelled  $X_1$  to  $X_n$  is incorrect?
- (a) The network can be viewed as a compact representation of the joint probability distribution  $P(X_1, \dots, X_n)$ .
  - (b) A simple multiplication suffices to calculate any concrete value  $P(x_1, \dots, x_n)$  of the joint distribution.
  - (c) Answering probabilistic queries may take exponentially many calculations in the worst case.
  - (d) If there is no direct link in the network between  $X_i$  and  $X_j$  then  $P(X_i|X_k) = P(X_i|X_j, X_k)$ .
  - (e) Compact representations of conditional probability distributions among continuous variables can be used for canonical families of distributions (such as Gaussians).
18. Assume the following inhibition probabilities between Boolean cause variables  $A$ ,  $B$ ,  $C$  and Boolean effect variable  $X$ :

$$\begin{aligned} P(\neg x|a, \neg b, \neg c) &= p \\ P(\neg x|\neg a, b, \neg c) &= q \\ P(\neg x|\neg a, \neg b, c) &= r \end{aligned}$$

What is the probability  $P(x|a, \neg b, c)$  assuming that the conditional probabilities of  $X$  are computed using a noisy-OR relation?

- (a)  $1 - (pr)$
  - (b)  $(1 - p)(1 - r)$
  - (c)  $(1 - r)pq$
  - (d)  $r(1 - p)$
  - (e)  $(1 - r)(1 - p)q$
19. The process of computing  $\vec{P}(\vec{X}_{t+k}|\vec{e}_{1:t})$  for  $k > 0$  in a temporal probabilistic model with state variables  $\vec{X}$  and evidence variables  $\vec{e}$  is called
- (a) smoothing
  - (b) abstraction
  - (c) monitoring
  - (d) prediction
  - (e) filtering

20. A general-purpose model of probabilistic decision-theoretic agents can be designed by using dynamic decision networks:



Which of the following techniques is *not* absolutely necessary for implementing such agents?

- (a) Expected utility maximisation
- (b) The variable elimination technique
- (c) Inference algorithms for dynamic bayesian networks
- (d) Probability theory and Bayes' rule
- (e) Filtering

**Part B**

**ANSWER ONE QUESTION FROM PART B**

1. (a) Translate the following argument by Lewis Carroll (the author of Alice in Wonderland) into sentences of first order logic. Define all your symbols.

*Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised. Therefore babies cannot manage crocodiles.*

[5%]

- (b) State the binary resolution rule.

[3%]

- (c) Show that the conclusion “babies cannot manage crocodiles” follows from the assumptions using resolution. For full marks, you should show i) how the clauses needed for the proof are obtained and ii) give all the steps of your proof.

[9%]

- (d) Give an algorithm for backward chaining using Generalized Modus Ponens.

[8%]

2. Consider the following problem:

Two agents Romeo and Juliet are lost in a  $N \times N$  grid and are attempting to meet in *any* square as quickly as possible. At each time step, *both* Romeo and Juliet move *simultaneously* and can do one of the following actions: Move {UP, DOWN, LEFT, RIGHT} or {STOP}. Romeo and Juliet do **not** alternate turns.

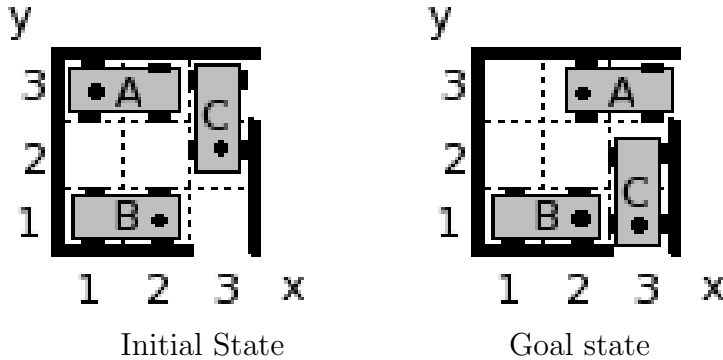
Parts (2a)-(2e) refer to the above problem.

- (a) Assuming a *single agent problem*, give a definition for the states. [2%]
- (b) What is the maximum size of the state space? [2%]
- (c) What is the maximum branching factor? [2%]
- (d) Based on your answer to (2a) above, define the goal test. [2%]
- (e)
  - i. Give a mathematical definition for the property satisfied by an *admissible* heuristic. [2%]
  - ii. Give a *non-trivial* admissible heuristic for the Romeo and Juliet problem. Explain your answer. [4%]
- (f) In the context of  $A^*$  search, consider the following statement about two *consistent* heuristics  $h_1$  and  $h_2$ :  $h_2$  *dominates*  $h_1$ .
  - i. What is meant by the above statement? [2%]
  - ii. Explain why  $h_2$  is better for search than  $h_1$ . [4%]
- (g) *Prove* that every consistent heuristic is also admissible. Hint: Consider giving a proof by induction. [5%]

## Part C

### ANSWER ONE QUESTION FROM PART C

#### 1. Planning



Consider the initial state of a rush hour puzzle shown in the left figure above. There are three cars,  $A$ ,  $B$  and  $C$  that can move on a board with nine positions. The front of these vehicles is shown with a dot. Cars  $A$  and  $B$  can only move left and right, while car  $C$  can only move up and down. The goal is to move cars  $A$  and  $C$  to the two exits as shown in the right hand figure above.

The following predicates from the language PDDL can describe the domain:

- $at(v, x, y)$ : the front of vehicle  $v$  is located at the square denoted by the xy-coordinates  $(x, y)$ . The front of the vehicle is shown in the above diagrams with a dot ( $\bullet$ ).
  - $clear(x, y)$  the square denoted by the xy-coordinates  $(x, y)$  is clear (or not occupied) by any vehicle  $v$ .
  - $horizontal(v)$ : vehicle  $v$  is in a horizontal position.
  - $vertical(v)$ : vehicle  $v$  is in a vertical position.
  - $m < n$ :  $m$  is (strictly) less than  $n$ . Assume that you can also write  $m \leq n$ ,  $m \geq n$ ,  $m > n$ . Furthermore, assume that you can refer to coordinates using expressions such as  $x + 1$  or  $x - 2$  (similarly for  $y$ ). So, for instance  $x + 1 < 3$  means that the x-coordinate that is 1 to the right of  $x$  is less than 3, and  $y - 2 \geq 1$  means that the y-coordinate that is 2 down from  $y$  is greater than or equal to 1.
- (a) Using the above predicates, describe the initial and goal states illustrated above.

**Note:** You don't have to describe the shape or size of the grid.

[5%]

(b) Using PDDL, define the following action schemata. Note that you must define these actions in a sufficiently general manner that they can handle states where the vehicles face the opposite directions from the ones shown in the above initial state and goal.

- i. *MoveRight*( $v, x, y$ ): move vehicle  $v$  so that its front, which is at  $xy$ -coordinates  $(x, y)$ , moves one square to the right. This action can be executed only if  $v$  is in a horizontal position, its front *and* back will remain within the grid after the move is done, and the square it is to move into is clear.
- ii. *MoveLeft*( $v, x, y$ ): move vehicle  $v$  so that its front, which is at  $xy$ -coordinates  $(x, y)$ , moves one square to the left. This action can be executed only if  $v$  is in a horizontal position, both its front and back will remain within the grid at the end of the move, and the square it is to move into is clear.
- iii. *MoveUp*( $v, x, y$ ): move vehicle  $v$  so that its front, which is located at  $xy$ -coordinates  $(x, y)$ , moves one square up. This action can be executed only if  $v$  is in a vertical position, both its front and back will remain within the grid at the end of the move, and the square it is to move into is clear.
- iv. *MoveDown*( $v, x, y$ ): move vehicle  $v$  so that its front, which is located at  $xy$ -coordinates  $(x, y)$ , moves one square down. This action can be executed only if  $v$  is in a vertical position, both its front and back will remain within the grid at the end of the move, and the square it is to move into is clear.

**Hint:** *You will need to use conditional effects and disjunction.*

[8%]

Suppose we want to find an optimal plan—that is, a plan that gets us from the initial to the goal state with the minimum number of actions. We assume that doing each action costs 1. For the purposes of finding an optimal plan, we define a heuristic function  $h(s)$  that given a state  $s$  returns the cost of an optimal solution to a relaxed version of the planning problem. The relaxation we deploy is to ignore collisions by allowing a car to move to a position that's occupied by another car.

(c) What is the value of  $h$  in the initial state? What is the value of the cost  $g$  of the initial state, and why?

[2%]

(d) Is  $h$  an admissible and consistent heuristic? Explain your answer.

[2%]

- (e) Draw the search tree generated by A\* search for the above planning problem, with the initial state and goal states given above. Draw each search node as a board state (i.e., as shown above for the initial and goal states, draw each state as a grid that shows where the vehicles  $A$ ,  $B$  and  $C$  are), and note at each state  $s$  the value of  $h(s)$ , the cost  $g(s)$  of getting to that state and the total estimated cost  $f(s) = h(s) + g(s)$ . Indicate the path in the search tree corresponding to the optimal plan found. Include unexpanded nodes in the search tree.

[8%]



## 2. Bayesian Decision Network

The Decision Network in Figure 1 consists of four boolean chance nodes  $A$ ,  $B$ ,  $C$  and  $D$ , with dependencies among them as shown.

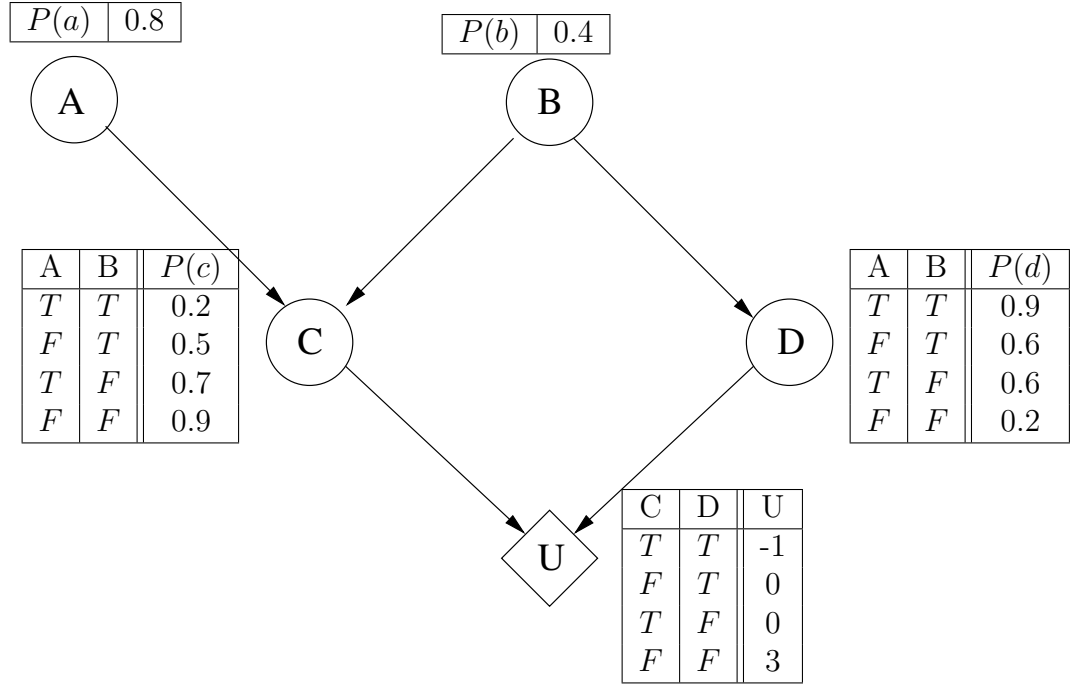


Figure 1: A Decision Network Consisting of Boolean Chance Nodes

- (a) Calculate the joint probability  $P(\neg a, b, \neg c, d)$  and the utility of this particular outcome.

[3%]

- (b) Assume that we generate 100 samples using the above network to estimate the distribution of  $\mathbf{P}(B|a, \neg d)$  and obtain the following results:

	$b$		$\neg b$	
	$a$	$\neg a$	$a$	$\neg a$
$d$	30	3	25	0
$\neg d$	2	12	20	8

What is the estimated distribution of  $\langle P(b|a, \neg d), P(\neg b|a, \neg d) \rangle$  that we would obtain by applying the rejection sampling method to this problem?

**Note:** Give your answer to 2 decimal places.

[3%]

- (c) What is the actual probability distribution  $\mathbf{P}(B|a, \neg d)$  given the conditional probability tables in Figure 1?

[8%]

- (d) What is the expected utility that can be derived from the network?

**Note:** *Be sure to show each step of this calculation.*

[11%]