Informatics 2D Tutorial 5

Resolution and Situation Calculus Ponens*

Week 6

1 Generalised Modus Ponens

Part 1: Convert the following sentences to first-order logic formulae suitable for use with Generalised Modus Ponens.

1. Horses, cows and pigs are mammals.
2. An offspring of a horse is a horse.
3. Bluebeard is a horse.
4. Bluebeard is Charlie's parent.
5. Offspring and parent are inverse relations.

Part 2: Use the sentences to answer a query using a backward-chaining algorithm.

- Draw the proof tree generated by an exhaustive backward-chaining algorithm for the query $Horse(h)$, where clauses are matched in the order given.
- How many solutions are a logical consequence of your knowledge base?
- How could we solve this problem?

Answer

Part 1:

1. $Horse(x) \Rightarrow Mammal(x)$
   $Cow(x) \Rightarrow Mammal(x)$
   $Pig(x) \Rightarrow Mammal(x)$

*Credits: Kobby Nuamah, Michael Rovatsos

2. $Offspring(y, x) \wedge Horse(x) \Rightarrow Horse(y)$ (y is offspring of x)
3. $Horse(Bluebeard)$
4. $Parent(Bluebeard, Charlie)$ (x is parent of y)
5. $Offspring(x, y) \Leftrightarrow Parent(y, x)$

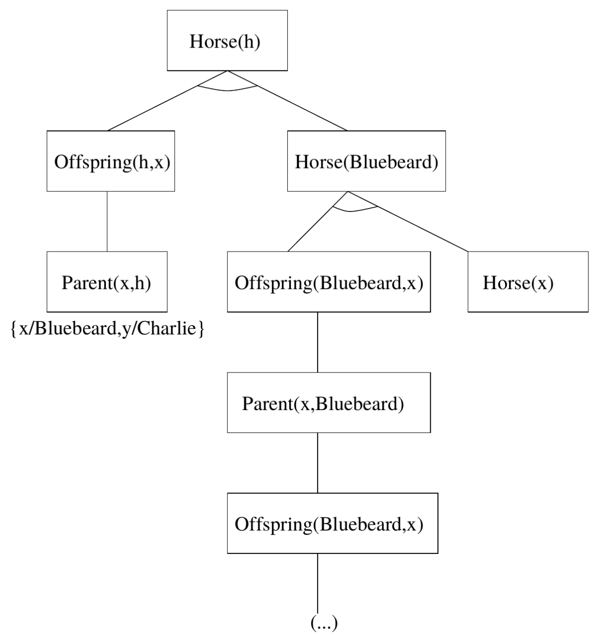

Figure 1: Solution to the Generalised Modus Ponens problem.

This question deals with the problem of looping in backward-chaining proofs. The proof tree is shown in Figure 1.

The branch with $OffSpring(Bluebeard, y)$ and $Parent(y, Bluebeard)$ repeats indefinitely, so the rest of the proof is never reached.

We get an infinite loop because of rule 2, $Offspring(x, y) \wedge Horse(y) \Rightarrow Horse(x)$.

The specific loop appearing in the figure arises because of the ordering of the clauses. We could be order $Horse(Bluebeard)$ before rule 2, which solve the problem of finding $Horse(Bluebeard)$ as a possible answer to the query.

2 Resolution

From "Horses are animals" it follows that "The head of a horse is the head of an animal". Demonstrate that this inference is valid by carrying out the following steps:

1. Translate the premise and the conclusion into the language of First-Order Logic. Use three predicates: $HeadOf(h, x)$ (meaning "h is the head of x"), $Horse(x)$, and $Animal(x)$.

2. Negate the conclusion, and convert the premise and the negated conclusion into Conjunctive Normal Form.

3. Use resolution to show that the conclusion follows from the premises.

Answer

(1):
$\forall x. Horse(x) \Rightarrow Animal(x)$
$\forall x, h. Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y. Animal(y) \wedge HeadOf(h, y)$

(2):
A: $\neg Horse(x) \vee Animal(x)$

B: $Horse(G)$

C: $HeadOf(H, G)$

D: $\neg Animal(y) \vee \neg HeadOf(H, y)$
Here A comes from the first sentence in (1), while the others come from the second. H and G are Skolem constants.

(3):
Resolve D and C to yield $\neg Animal(G)$. Resolve this with A (in (2) above to give $\neg Horse(G)$. Resolve this with B to obtain a contradiction.

Alternative:

The above is an abbreviated version of the proof. The full proof would use the (negated) conclusion (which might have been a bit too cumbersome for the tutorial)

$$\neg (\forall x, h. Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y. Animal(y) \wedge HeadOf(h, y))$$

$$\Leftrightarrow \neg (\forall h. (\forall x. Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y. Animal(y) \wedge HeadOf(h, y)))$$

$$\Leftrightarrow \exists h. \neg (\forall x. Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y. Animal(y) \wedge HeadOf(h, y))$$

$$\Leftrightarrow \exists h. \neg (\neg (\forall x. Horse(x) \wedge HeadOf(h, x)) \vee (\exists y. Animal(y) \wedge HeadOf(h, y)))$$

$$\Leftrightarrow \exists h. \neg (\exists x. \neg (Horse(x) \wedge HeadOf(h, x)) \vee (\exists y. Animal(y) \wedge HeadOf(h, y)))$$

$$\Leftrightarrow \exists h. (\neg (\exists x. \neg (Horse(x) \wedge HeadOf(h, x)))) \wedge \neg (\exists y. Animal(y) \wedge HeadOf(h, y))$$

$$\Leftrightarrow \exists h. (\forall x. (Horse(x) \wedge HeadOf(h, x))) \wedge \forall y. (\neg Animal(y) \vee \neg HeadOf(h, y))$$

Skolemisation:

$$\Rightarrow Horse(x) \wedge HeadOf(H, x) \wedge (\neg Animal(y) \vee \neg HeadOf(H, y))$$

In order to show that this contradicts the facts we need to have one of the conjuncts to be inconsistent with the facts. We should first substitute $x \backslash G$. Next, we observe that the third clause is the (negated) conclusion in D above, which we have already disproved in part (3), such that we reach an contradiction as well for the full conclusion.

3 Situation Calculus

Before the break you learnt about the frame problem and you were shown how it can be fixed by adding frame axioms.

Consider the following predicates and functions:

1. $At(sq, s)$ means that the agent is at square sq in situation s.
2. $Heading(dir, s)$ means that the agent is facing in direction dir in situation s.
3. $Next(sq1, dir, sq_2)$ means that square sq_2 is adjacent to square sq_1 in direction dir.
4. $Result(act, s)$ is the situation resulting from executing the action act in situation s.
5. $Turn(x)$ is the action of turning x where $x \in \{left, right\}$.
6. $Shoot$ is the action of shooting once forward.
7. $Newdir(dir1, x, dir_2)$ means that dir_2 is the new direction the agent will face if it is facing in direction dir_1 and turns $x \in \{left, right\}$
8. $Wumpus(sq, s)$ means that that the Wumpus is in square sq in situation s.

In the following we assume that the action $Shoot$ only has an effect in directly adjacent squares.

a. Formalise a precondition and an effect axiom for the Wumpus World that best describes the action $Turn(x)$.

b. Formalise a precondition and an effect axiom that best describes the $Shoot$ action in the Wumpus World.

c. Formalise a frame axiom that best describes the $Shoot$ action in the Wumpus World. You only need to do this for the $Wumpus$ fluent.

Answer

Note that, the formalisation of actions using precondition and effect axioms differs from the lecture notes, where only one axiom is used. Alternative effect axioms, in the form used in the lecture notes, are given below.

- Preconditions describe the fluents that must hold for an action to be possible.
- Effects describe the fluents that will hold as a result of taking the action.
- Frame axioms state what doesn't change as a result of taking an action

In the following axioms universal quantifiers (whose scope is the entire sentence) are omitted.

a) If the agent is heading in direction dir_1 and the result of turning x is dir_2 then the agent is heading in direction dir_2 in the situation following after turning x.

- Precondition: $Heading(dir_1, s) \wedge Newdir(dir_1, x, dir_2) \Rightarrow Poss(Turn(x), s)$
- Effect: $Poss(Turn(x), s) \Rightarrow Heading(dir_2, Result(Turn(x), s))$

Alternatively: $Heading(dir_1, s) \wedge Newdir(dir_1, x, dir_2) \Rightarrow Heading(dir_2, Result(Turn(x), s))$

b) If the agent is at square sq_1 and heading in direction dir and the next square in direction dir is sq_2 then the result of shooting will be that the wumpus is not in square sq_2 (if it was there then it's dead).

- Precondition: $At(sq_1, s) \wedge Heading(dir, s) \wedge Next(sq_1, dir, sq_2) \Rightarrow Poss(Shoot, s)$
- Effect: $Poss(Shoot, s) \Rightarrow \neg Wumpus(sq_2, Result(Shoot, s))$

Alternatively:

$At(sq), s) \wedge Heading(dir, s) \wedge Next(sq_1, dir, sq_2) \Rightarrow \neg Wumpus(sq_2, Result(Shoot, s))$

c) If the agent is at square sq_1 and heading in direction dir and the next square in direction dir is sq_2 and the wumpus is in square sq_3 and square sq_2 doesn't equal sq_3 then the Wumpus is still in square sq_3 after shooting.

$At(sq_1, s) \wedge Heading(dir, s) \wedge Next(sq_1, dir, sq_2) \wedge Wumpus(sq_3, s) \wedge sq_2 \neq sq_3$

$\Rightarrow Wumpus(sq_3, result(Shoot, s))$