CPSC 322 Assignment 4

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We are claiming 2 late days for this assignment

1   Question 1 [20 points] Probabilities

A	B	C	p
T	T	T	0.1
T	T	F	0.1
T	F	T	0.4
T	F	F	0.3
F	T	T	0.1
F	T	F	0.2
F	F	T	0.1
F	F	F	0.1

Consider the above joint probability distribution over the three random variables A, B, C.

(a) **5 points** You are told that it contains only one incorrect value. What is that value? Do you have sufficient information to fix it? If that is the case, how would you fix it? If not, why not?
If we add the probabilities of all the rows, it must sum to 1, but instead it sums to 1.4. Assuming that the table only contains one incorrect value, the incorrect probability should have a value of 0.4, to make the table valid.

Thus we can fix the table by setting the probability of $(A\bar{B}C)$ to 0

(b) **8 points** With the joint resulting from the previous question (after you fixed the error), compute the marginal probability distributions for P(A) and P(B). Are A and B independent? Also, what is the value of P(A=T | B=F)?

$$P(A) = 0.1 + 0.1 + 0 + 0.3 = 0.5$$
$$P(B) = 0.1 + 0.1 + 0.1 + 0.2 = 0.5$$
$$P(A|B) = P(A, B)/P(B) = (0.1+0.1)/0.5 = 0.4$$

From the calculations above, we can notice that P(A) != P(A|B), this means that A and B are not independent.

$$P(A|\bar{B}) = P(A \text{ and } \bar{B})/P(\bar{B}) = (0+0.3)/0.5 = 0.6$$

(c) **7 points Without performing any calculations**, is it true that P(A) * P(B | A) * P(C | A, B) = P(C) * P(B | C) * P(A | B, C)? Why or why not?

$$P(A) = 0.5$$
$$P(C) = 0.1 + 0 + 0.1 + 0.1 = 0.3$$
$$P(B|A) = P(B, A)/P(A) = (0.1+0.1)/0.5 = 0.4$$
$$P(C|AB) = P(A, B, C)/P(A, B) = 0.1/(0.1+0.1) = 0.5$$
$$P(B|C) = P(B, C)/P(C) = (0.1+0.1)/0.3 \approx 0.66667$$
$$P(A|BC) = P(A, B, C)/P(B, C) = 0.1/(0.1+0.1) = 0.5$$

$$P(A) * P(B \mid A) * P(C \mid A, B) = 0.5 * 0.4 * 0.5 = 0.1$$
$$P(C) * P(B \mid C) * P(A \mid B, C) = 0.3 * 0.66667 * 0.5 = 0.1$$

The calculation above proves that the given formula is true, but we can also prove this by simplifying the provided formula.

$$P(A) * P(B \mid A) * P(C \mid A, B) = \cancel{P(A)} * \frac{\cancel{P(A, B)}}{\cancel{P(A)}} * \frac{P(C, A, B)}{\cancel{P(A, B)}} = P(C, A, B)$$
$$P(C) * P(B \mid C) * P(A \mid B, C) = \cancel{P(C)} * \frac{\cancel{P(C, B)}}{\cancel{P(C)}} * \frac{P(A, C, B)}{\cancel{P(C, B)}} = P(A, C, B)$$

So we can reduce everything to the next clause P(C, A, B) = P(A, B, C), we know that must be true because the intersection of the three variables will always be the same.

2   Question 2 [20 points] Bayes' rule

Youve taken your prized sports car into a repair shop for a routine maintenance check. As part of a standard set of diagnostics, the cars onboard computer is checked for logged problems. The mechanic tells you that they have good and bad news. The bad news is that the cars computer has reported that it has detected a failure in the electrical system that will definitely cause an electrical fire in the near future (thus you may treat the failure and the resulting fire as the same thing). The probability of the computer reporting this warning given that there is actually a failure in the electrical system is 0.95, as is the probability that the computer will correctly not report anything if the failure isnt present. The good news is that this type of failure and the consequent electric fire will only occur in one out of every ten thousand cars of this model on the road.

(a) **12 points** What are the chances that an electrical fire will occur in your car if dont opt to repair it? (Show your calculations as well as giving the final result.)

$$P(\text{Failure}) = 1/10000 = 0.0001$$
$$P(\text{Detection} \mid \text{Failure}) = 0.95$$
$$P(\text{F} \mid \text{D}) = P(\text{FD})/P(\text{D}) = (0.0001\text{*}0.95)/(0.0001\text{*}0.95 + 0.9999\text{*}0.05) = \boxed{0.001896}$$

(b) **4 points** Repeat the calculation of part (a), but for the case where the failure occurs in one out of every hundred cars of this model.

$$P(\text{Failure}) = 1/100 = 0.01$$
$$P(\text{Detection} \mid \text{Failure}) = 0.95$$
$$P(\text{F} \mid \text{D}) = P(\text{FD})/P(\text{D}) = (0.01\text{*}0.95)/(0.01\text{*}0.95 + 0.99\text{*}0.05) = \boxed{0.1610}$$

(c) **4 point** Given the logged report of a problem, why is it good news that the issue is so rare in part (a)?
Because the low probabilities of having a failure, decrease the probabilities of getting a real failure. We can say that it is so rare to get a failure that it is more probable that the computer is detecting a false positive, so the car won't need to be repaired.
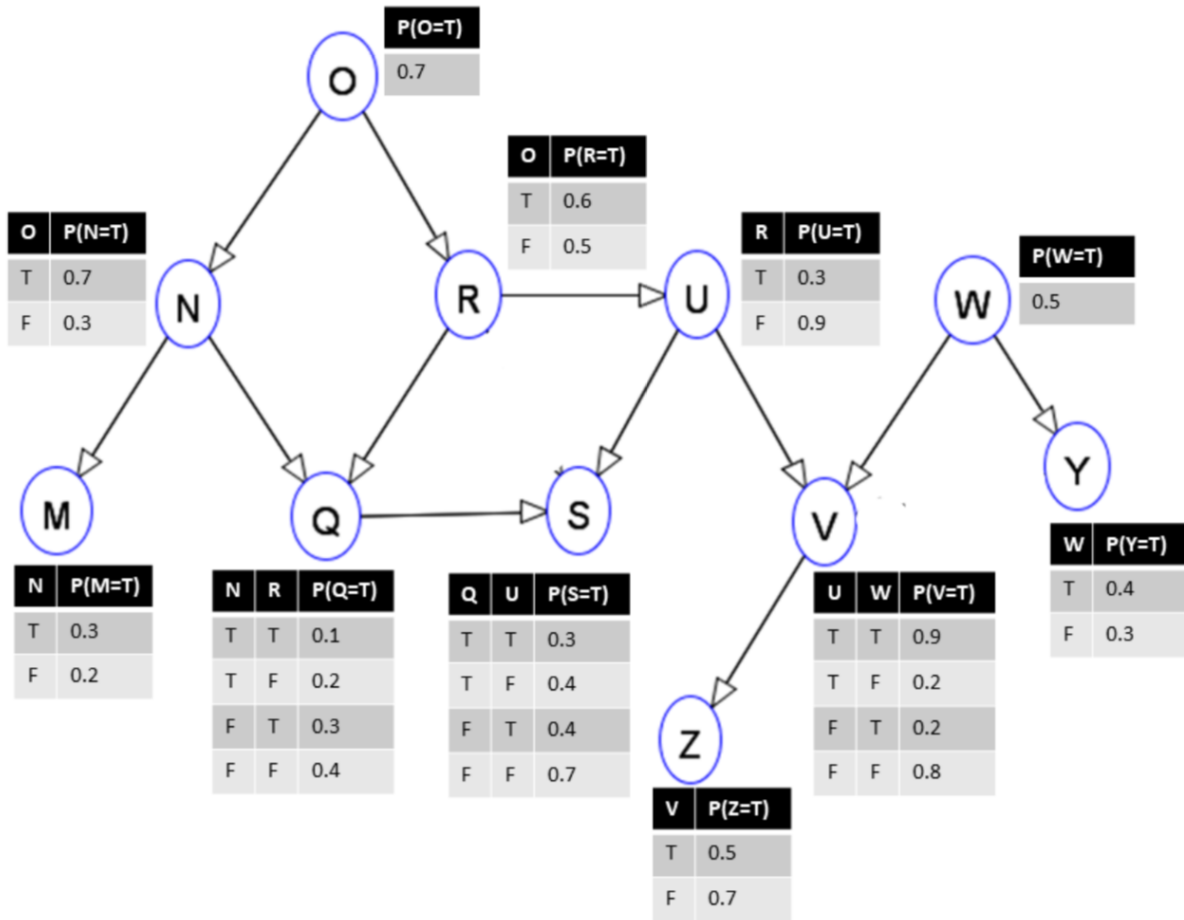
3 Question 3 [15 points] Bayesian/Belief networks

Consider a probabilistic problem that can be represented by the simple Bayesian Network given below. Let L1 and L2 be Boolean variables, and let C1, C2, and N have 51 possible values each. Compute the representational saving (i.e. how many fewer values you need to store) between this belief network and the joint distribution for this problem. Show your work.
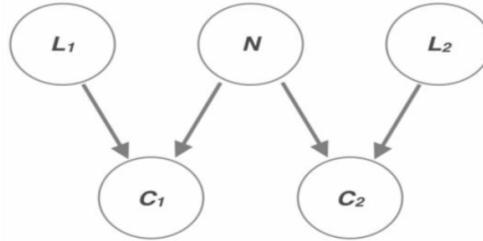
The joint probability table would need to store 2*2*51*51*51-1 = 530603 values, which can be reduced to 10252. Nodes L1 and L2 only need to store one value on their probability table given they are Boolean variables with no parents. N only needs to store 50 values on its table given it has 51 variables and no parents. C1 and C2 need to store 51* 50 * 2 each. This calculations can be obtained using the formula $n2^k$ for Boolean variables and nd^{k+1} for non Boolean variables. Adding the values stored for all probability tables gives us $2 + 50 + 10200 = 10252$. That is a reduction of 520351.

4   Question 4 [40 points] Variable Elimination

O	P(N=T)
T	0.7
F	0.3

P(O=T) = 0.7

O	P(R=T)
T	0.6
F	0.5

R	P(U=T)
T	0.3
F	0.9

P(W=T) = 0.5

N	P(M=T)
T	0.3
F	0.2

N	R	P(Q=T)
T	T	0.1
T	F	0.2
F	T	0.3
F	F	0.4

Q	U	P(S=T)
T	T	0.3
T	F	0.4
F	T	0.4
F	F	0.7

U	W	P(V=T)
T	T	0.9
T	F	0.2
F	T	0.2
F	F	0.8

W	P(Y=T)
T	0.4
F	0.3

V	P(Z=T)
T	0.5
F	0.7

Carry out variable elimination (VE) on this network to compute P(S—Q=F).

1. **10 marks** indicate which nodes can be pruned (justifying each pruning step), and list the factors (with their values) that VE needs to initially create.

    1. M can be pruned because it is an unobserved leaf

    2. X can be pruned because it is an unobserved leaf

    3. Z can be pruned because it is an unobserved leaf

    4. W can be pruned because it is conditionally independent from the query

    5. V can now be pruned because it is an unobserved leaf

$$f_0(O) = \begin{bmatrix} O & Val \\ T & 0.7 \\ F & 0.3 \end{bmatrix}$$

$$f_1(O, N) = \begin{bmatrix} O & N & \text{Val} \\ T & T & 0.7 \\ T & F & 0.3 \\ F & T & 0.3 \\ F & F & 0.7 \end{bmatrix}$$

$$f_2(O, R) = \begin{bmatrix} O & R & \text{Val} \\ T & T & 0.6 \\ T & F & 0.4 \\ F & T & 0.5 \\ F & F & 0.5 \end{bmatrix}$$

$$f_3(N, R, Q) = \begin{bmatrix} N & R & Q & \text{Val} \\ T & T & T & 0.1 \\ T & T & F & 0.9 \\ T & F & T & 0.2 \\ T & F & F & 0.8 \\ F & T & T & 0.3 \\ F & T & F & 0.7 \\ F & F & T & 0.4 \\ F & F & F & 0.6 \end{bmatrix}$$

$$f_4(R, U) = \begin{bmatrix} R & U & \text{Val} \\ T & T & 0.3 \\ T & F & 0.7 \\ F & T & 0.9 \\ F & F & 0.1 \end{bmatrix}$$

$$f_5(Q, U, S) = \begin{bmatrix} Q & U & S & \text{Val} \\ T & T & T & 0.3 \\ T & T & F & 0.7 \\ T & F & T & 0.4 \\ T & F & F & 0.6 \\ F & T & T & 0.4 \\ F & T & F & 0.6 \\ F & F & T & 0.7 \\ F & F & F & 0.3 \end{bmatrix}$$

$$f_6(N, R) = \begin{bmatrix} N & R & \text{Val} \\ T & T & 0.9 \\ T & F & 0.8 \\ F & T & 0.7 \\ F & F & 0.6 \end{bmatrix}$$

$$f_7(U, S) = \begin{bmatrix} R & U & Val \\ T & T & 0.4 \\ T & F & 0.6 \\ F & T & 0.7 \\ F & F & 0.3 \end{bmatrix}$$

2. **5 marks** Assuming that the elimination ordering is alphabetical, show how the factors and the summations should be ordered.

$$\sum_U f_7(U, S) \sum_R f_4(R, U) \sum_O f_0(O) f_2(O, R) \sum_N f_1(O, N) f_6(N, R)$$

3. **15 marks** step through VE, showing what operations are performed, the resulting intermediate factors (with their values) and how P(S—Q=F) is finally computed (hint: see the example we traced in class and the factor operations we covered in the slides).

   1. Compute intermediate result for N, $f_8'(O, N, R)$

   2. Eliminate N, result in: $\sum_U f_7(U, S) \sum_R f_4(R, U) \sum_O f_0(O) f_2(O, R) f_8(O, R)$

$$f_8(O, R) = \begin{bmatrix} O & R & Val \\ T & T & 0.87 \\ T & F & 0.74 \\ F & T & 0.76 \\ F & F & 0.66 \end{bmatrix}$$

   3. Compute intermediate result for O, $f_9'(O, R)$

   4. Eliminate O, result in: $\sum_U f_7(U, S) \sum_R f_4(R, U) f_9(R)$

$$f_9(R) = \begin{bmatrix} R & Val \\ T & 0.4794 \\ F & 0.3062 \end{bmatrix}$$

   5. Compute intermediate result for R, $f_{10}'(R, U)$

   6. Eliminate R, result in: $\sum_U f_7(U, S) f_{10}(U)$

$$f_{10}(U) = \begin{bmatrix} U & Val \\ T & 0.4194 \\ F & 0.3661 \end{bmatrix}$$

   7. Compute intermediate result for U, $f_{11}'(U, S)$

   8. Eliminate U, result in: $f_{11}(S)$

$$f_{11}(S) = \begin{bmatrix} S & Val \\ T & 0.4241 \\ F & 0.3614 \end{bmatrix}$$

   9. Normalize $f_{11}(S)$

$$f_{11}(S) = \begin{bmatrix} S & Val \\ T & 0.54 \\ F & 0.46 \end{bmatrix}$$