CPSC 340 Assignment 1 (due 2019-01-11 at 11:55pm)

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Commentary on Assignment 1: CPSC 340 is tough because it combines knowledge and skills across several disciplines. To succeed in the course, you will need to know or very quickly get up to speed on:

- Basic Python programming, including NumPy and plotting with matplotlib.
- Math to the level of the course prerequisites: linear algebra, multivariable calculus, some probability.
- Statistics, algorithms and data structures to the level of the course prerequisites.
- Some basic LaTeX and git skills so that you can typeset equations and submit your assignments.

This assignment will help you assess whether you are prepared for this course. We anticipate that each of you will have different strengths and weaknesses, so don't be worried if you struggle with *some* aspects of the assignment. But if you find this assignment to be very difficult overall, that is a warning sign that you may not be prepared to take CPSC 340 at this time. Future assignments will be more difficult than this one (and probably around the same length).

Questions 1-4 are on review material, that we expect you to know coming into the course. The rest is new CPSC 340 material from the first few lectures.

A note on the provided code: in the `code` directory we provide you with a file called `main.py`. This file, when run with different arguments, runs the code for different parts of the assignment. For example,

```
python main.py -q 6.2
```

runs the code for Question 6.2. At present, this should do nothing (throws a `NotImplementedError`), because the code for Question 6.2 still needs to be written (by you). But we do provide some of the bits and pieces to save you time, so that you can focus on the machine learning aspects. For example, you'll see that the provided code already loads the datasets for you. The file `utils.py` contains some helper functions. You don't need to read or modify the code in there. To complete your assignment, you will need to modify `grads.py`, `main.py`, `decision_stump.py` and `simple_decision.py` (which you'll need to create).

Instructions

Rubric: {mechanics:5}

IMPORTANT!!! Before proceeding, please carefully read the general homework instructions at `https://www.cs.ubc.ca/~fwood/CS340/homework/`. The above 5 points are for following the submission instructions. You can ignore the words "mechanics", "reasoning", etc.

We use blue to highlight the deliverables that you must answer/do/submit with the assignment.

1 Linear Algebra Review

For these questions you may find it helpful to review these notes on linear algebra:
`http://www.cs.ubc.ca/~schmidtm/Documents/2009_Notes_LinearAlgebra.pdf`

1.1 Basic Operations

Rubric: {reasoning:7}

Use the definitions below,

$$\alpha = 2, \quad x = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad y = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \quad z = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix},$$

and use x_i to denote element i of vector x. Evaluate the following expressions (you do not need to show your work).

1. $\sum_{i=1}^n x_i y_i$ (inner product).

$$\sum_{i=1}^n x_i y_i = \boxed{14}$$

2. $\sum_{i=1}^n x_i z_i$ (inner product between orthogonal vectors).

$$\sum_{i=1}^n x_i z_i = \boxed{0}$$

3. $\alpha(x+z)$ (vector addition and scalar multiplication)

$$\alpha(x+z) = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$$

4. $x^T z + \|x\|$ (inner product in matrix notation and Euclidean norm of x).

$$x^T z + \|x\| = 0 + \sqrt{0+1+4} = \sqrt{5} \approx \boxed{2.23606}$$

5. Ax (matrix-vector multiplication).

$$Ax = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

6. $x^T Ax$ (quadratic form).

$$x^T Ax = \boxed{19}$$

7. $A^T A$ (matrix tranpose and matrix multiplication).

$$A^T A = \begin{bmatrix} 11 & 10 & 10 \\ 10 & 14 & 10 \\ 10 & 10 & 14 \end{bmatrix}$$

1.2 Matrix Algebra Rules

Rubric: {reasoning:10}

Assume that $\{x, y, z\}$ are $n \times 1$ column vectors, $\{A, B, C\}$ are $n \times n$ real-valued matrices, 0 is the zero matrix of appropriate size, and I is the identity matrix of appropriate size. State whether each of the below is true in general (you do not need to show your work).

1. $x^T y = \sum_{i=1}^n x_i y_i$. $\boxed{\text{true}}$
2. $x^T x = \|x\|^2$. $\boxed{\text{false}}$
3. $x^T x = x x^T$. $\boxed{\text{true}}$
4. $(x - y)^T (x - y) = \|x\|^2 - 2x^T y + \|y\|^2$. $\boxed{\text{false}}$
5. $AB = BA$. $\boxed{\text{false}}$
6. $A^T (B + IC) = A^T B + A^T C$. $\boxed{\text{true}}$
7. $(A + BC)^T = A^T + B^T C^T$. $\boxed{\text{true}}$
8. $x^T A y = y^T A^T x$. $\boxed{\text{false}}$
9. $A^T A = AA^T$ if A is a symmetric matrix. $\boxed{\text{false}}$
10. $A^T A = 0$ if the columns of A are orthonormal. $\boxed{\text{false}}$

2 Probability Review

For these questions you may find it helpful to review these notes on probability:
`http://www.cs.ubc.ca/~schmidtm/Courses/Notes/probability.pdf`
And here are some slides giving visual representations of the ideas as well as some simple examples:
`http://www.cs.ubc.ca/~schmidtm/Courses/Notes/probabilitySlides.pdf`

2.1 Rules of probability

Rubric: {reasoning:6}

Answer the following questions. You do not need to show your work.

1. You are offered the opportunity to play the following game: your opponent rolls 2 regular 6-sided dice. If the difference between the two rolls is at least 3, you win \$15. Otherwise, you get nothing. What is a fair price for a ticket to play this game once? In other words, what is the expected value of playing the game? There are 12 ways to win, so the probability to win is $12/36 \approx 0.33$, so the fair price would be $\frac{12*15}{36} = \boxed{5}$
2. Consider two events A and B such that $\Pr(A, B) = 0$ (they are mutually exclusive). If $\Pr(A) = 0.4$ and $\Pr(A \cup B) = 0.95$, what is $\Pr(B)$? Note: $p(A, B)$ means "probability of A and B" while $p(A \cup B)$ means "probability of A or B". It may be helpful to draw a Venn diagram. Pr(B)= 0.95 - 0.4 = $\boxed{0.55}$
3. Instead of assuming that A and B are mutually exclusive ($\Pr(A, B) = 0$), what is the answer to the previous question if we assume that A and B are independent? Pr(B)= $\frac{Pr(A\cup B)-Pr(A)}{1-Pr(A)} = \frac{0.95-0.4}{1-0.4} = \frac{0.55}{0.6} \approx \boxed{0.916}$

2.2 Bayes Rule and Conditional Probability

Rubric: {reasoning:10}

Answer the following questions. You do not need to show your work.

Suppose a drug test produces a positive result with probability 0.97 for drug users, $P(T = 1 \mid D = 1) = 0.97$. It also produces a negative result with probability 0.99 for non-drug users, $P(T = 0 \mid D = 0) = 0.99$. The probability that a random person uses the drug is 0.0001, so $P(D = 1) = 0.0001$.

1. What is the probability that a random person would test positive, $P(T = 1)$? $= \boxed{0.010096}$
2. In the above, do most of these positive tests come from true positives or from false positives? From false positives, because the probability of a false positive is 0.009999 larger than 0.000097 which is the probability of a true positive
3. What is the probability that a random person who tests positive is a user, $P(D = 1 \mid T = 1)$? $= \boxed{0.009607}$
4. Suppose you have given this test to a random person and it came back positive, are they likely to be a drug user? It is not very likely that this person is a drug user.
5. What is one factor you could change to make this a more useful test? If the probability that a random person uses the drug was higher, the test would have a better performance

3 Calculus Review

3.1 One-variable derivatives

Rubric: {reasoning:8}

Answer the following questions. You do not need to show your work.

1. Find the derivative of the function $f(x) = 3x^2 - 2x + 5$. $\boxed{f'(x) = 6x - 2}$
2. Find the derivative of the function $f(x) = x(1 - x)$. $\boxed{f'(x) = 1 - 2x}$
3. Let $p(x) = \frac{1}{1+\exp(-x)}$ for $x \in \mathbb{R}$. Compute the derivative of the function $f(x) = x - \log(p(x))$ and simplify it by using the function $p(x)$. $\boxed{p(x) = -\exp(-x)}$

Remember that in this course we will $\log(x)$ to mean the "natural" logarithm of x, so that $\log(\exp(1)) = 1$. Also, obseve that $p(x) = 1 - p(-x)$ for the final part.

3.2 Multi-variable derivatives

Rubric: {reasoning:5}

Compute the gradient vector $\nabla f(x)$ of each of the following functions. You do not need to show your work.

1. $f(x) = x_1^2 + \exp(x_1 + 2x_2)$ where $x \in \mathbb{R}^2$. $\boxed{\nabla f(x) = (2x_1 + \exp(x_1 + 2x_2), 2\exp(x_1 + 2x_2))}$
2. $f(x) = \log\left(\sum_{i=1}^3 \exp(x_i)\right)$ where $x \in \mathbb{R}^3$ (simplify the gradient by defining $Z = \sum_{i=1}^3 \exp(x_i)$). $\boxed{\nabla f(x) = (\frac{\exp(x_1)}{Z}, \frac{\exp(x_2)}{Z}, \frac{\exp(x_3)}{Z})}$

3. $f(x) = a^T x + b$ where $x \in \mathbb{R}^3$ and $a \in \mathbb{R}^3$ and $b \in \mathbb{R}$. $\boxed{\nabla f(x) = a}$
4. $f(x) = \frac{1}{2}x^\top Ax$ where $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $x \in \mathbb{R}^2$. $\boxed{\nabla f(x) = Ax}$
5. $f(x) = \frac{1}{2}\|x\|^2$ where $x \in \mathbb{R}^d$. $\boxed{\nabla f(x) = \|x\| + d}$

Hint: it is helpful to write out the linear algebra expressions in terms of summations.

3.3 Optimization

Find the following quantities. You do not need to show your work. You can/should use your results from parts 3.1 and 3.2 as part of the process.

1. $\min 3x^2 - 2x + 5$, or, in words, the minimum value of the function $f(x) = 3x^2 - 2x + 5$ for $x \in \mathbb{R}$. $\boxed{\text{min} = f(\frac{1}{3}) = \frac{14}{3}}$
2. $\max x(1-x)$ for $x \in [0,1]$. $\boxed{\text{max} = f(\frac{1}{2}) = \frac{1}{4}}$
3. $\min x(1-x)$ for $x \in [0,1]$. $\boxed{\text{min}_1 = 0, \text{ min}_2 = 0}$
4. $\arg\max x(1-x)$ for $x \in [0,1]$. $\boxed{\text{argmax}_1 = 0, \text{ arg max}_2 = 1}$
5. $\min x_1^2 + \exp(x_2)$ where $x \in [0,1]^2$, or in other words $x_1 \in [0,1]$ and $x_2 \in [0,1]$. $\boxed{\text{min} = 1}$
6. $\arg\min x_1^2 + \exp(x_2)$ where $x \in [0,1]^2$. $\boxed{\text{arg min} = 0}$

Note: the notation $x \in [0,1]$ means "x is in the interval $[0,1]$", or, also equivalently, $0 \leq x \leq 1$.

Note: the notation "$\max f(x)$" means "the value of $f(x)$ where $f(x)$ is maximized", whereas "$\arg\max f(x)$" means "the value of x such that $f(x)$ is maximized". Likewise for min and arg min. For example, the min of the function $f(x) = (x-1)^2$ is 0 because the smallest possible value is $f(x) = 0$, whereas the arg min is 1 because this smallest value occurs at $x = 1$. The min is always a scalar but the arg min is a value of x, so it's a vector if x is vector-valued.

3.4 Derivatives of code

Rubric: {code:4}

Note: for info on installing and using Python, see
`https://www.cs.ubc.ca/~fwood/CS340/python/`.

The homework zip file contains a file named `grads.py` which defines several Python functions that take in an input variable x, which we assume to be a 1-d array (in math terms, a vector). It also includes (blank) functions that return the corresponding gradients. For each function, write code that computes the gradient of the function in Python. You should do this directly in `grads.py`; no need to make a fresh copy of the file. However, per the homework instructions, you should include it in the latex report as described so that the TA can assess it easily. When finished, you can run `python main.py -q 3.4` to test out your code.

Hint: it's probably easiest to first understand on paper what the code is doing, then compute the gradient, and then translate this gradient back into code.

Note: do not worry about the distinction between row vectors and column vectors here. For example, if the correct answer is a vector of length 5, we'll accept numpy arrays of shape `(5,)` (a 1-d array) or `(5,1)` (a column vector) or `(1,5)` (a row vector). In future assignments we will start to be more careful about this.

Warning: Python uses whitespace instead of curly braces to delimit blocks of code. Some people use tabs and other people use spaces. My text editor (Atom) inserts 4 spaces (rather than tabs) when I press the Tab key, so the file `grads.py` is indented in this manner. If your text editor inserts tabs, Python will complain and you might get mysterious errors... this is one of the most annoying aspects of Python, especially when starting out. So, please be aware of this issue! And if in doubt you can just manually indent with 4 spaces, or convert everything to tabs. For more information see `https://www.youtube.com/watch?v=SsoOG6ZeyUI`.

4 Algorithms and Data Structures Review

4.1 Trees

Rubric: {reasoning:2}

Answer the following questions. You do not need to show your work.

1. What is the minimum depth of a binary tree with 64 leaf nodes? Minimum depth: $\boxed{6}$
2. What is the minimum depth of binary tree with 64 nodes (includes leaves and all other nodes)? Minimum depth: $\boxed{6}$

Note: we'll use the standard convention that the leaves are not included in the depth, so a tree with depth 1 has 3 nodes with 2 leaves.

4.2 Common Runtimes

Rubric: {reasoning:4}

Answer the following questions using big-O notation You do not need to show your work.

1. What is the cost of finding the largest number in an unsorted list of n numbers? $\boxed{\mathrm{O}(n)}$
2. What is the cost of finding the smallest element ansater than 0 in a *sorted* list with n numbers. $\boxed{\mathrm{O}(\log n)}$
3. What is the cost of finding the value associated with a key in a hash table with n numbers? (Assume the values and keys are both scalars.) $\boxed{\mathrm{O}(n)}$
4. What is the cost of computing the inner product $a^T x$, where a is $d \times 1$ and x is $d \times 1$? $\boxed{\mathrm{O}(d)}$
5. What is the cost of computing the quadratic form $x^T Ax$ when A is $d \times d$ and x is $d \times 1$. $\boxed{\mathrm{O}(d^3)}$

4.3 Running times of code

Rubric: {reasoning:4}

Your repository contains a file named `bigO.py`, which defines several functions that take an integer argument N. For each function, state the running time as a function of N, using big-O notation. Please include your answers in your report. Do not write your answers inside `bigO.py`.

1. Func 1 $\rightarrow$ The function goes from 1 all the way to N, so the complexity is $\boxed{\mathrm{O}(N)}$
2. Func 2 $\rightarrow$ The function generates an array with N zeros and later it adds 1000 to each element, so the function runtime goes as far as 2 times N, so the complexity is $\boxed{\mathrm{O}(N)}$
3. Func 3 $\rightarrow$ The function generates an array with 1000 zeros and later it multiplies N to each element, so the function runtime goes as far as 2 times 1000, the parameter N doesn't affects the algorithm in complexity, so the complexity is $\boxed{\mathrm{O}(1)}$
4. Func 4 $\rightarrow$ The function loops through N and by each iteration it will loop through $N - i$, being i the current iteration of the first loop. So the function will loops through $N(N - i)$ wich still gives a complexity of $\boxed{\mathrm{O}(N^2)}$

5 Data Exploration

Your repository contains the file `fluTrends.csv`, which contains estimates of the influenza-like illness percentage over 52 weeks on 2005-06 by Google Flu Trends. Your `main.py` loads this data for you and stores it in a pandas DataFrame `X`, where each row corresponds to a week and each column corresponds to a different region. If desired, you can convert from a DataFrame to a raw numpy array with `X.values()`.

5.1 Summary Statistics

Rubric: {reasoning:2}

Report the following statistics:

1. The minimum, maximum, mean, median, and mode of all values across the dataset.

Stats	NE	MidAtl	ENCentral	WNCentral	SAt1	ESCentral	WSCentral	Mtn	Pac	WtdILI
minimum	0.428	0.483	0.452	0.464	0.468	0.554	0.456	0.352	0.377	0.606
maximum	2.310	2.205	2.515	3.115	2.714	3.859	3.219	4.862	2.660	3.260
mean	1.223	1.233	1.275	1.460	1.298	1.562	1.292	1.270	1.063	1.566
median	1.129	1.116	1.265	1.277	1.102	1.416	1.107	0.978	0.957	1.303
mode	0.428	0.490	0.452	0.505	0.542	0.554	0.499	0.352	0.377	0.715

Global Stats	Value	Region
minimum	0.352	Mtn
maximum	4.862	Mtn
mean	1.324 63	NA
median	1.159	NA
mode	0.77	NA

2. The 5%, 25%, 50%, 75%, and 95% quantiles of all values across the dataset.

Stats	NE	MidAtl	ENCentral	WNCentral	SAt1	ESCentral	WSCentral	Mtn	Pac	WtdILI
5% quantile	0.435	0.492	0.469	0.495	0.517	0.577	0.485	0.398	0.399	0.615
25% quantile	0.728	0.714	0.711	0.701	0.737	0.922	0.708	0.624	0.556	0.795
50% quantile	1.129	1.116	1.265	1.277	1.102	1.416	1.107	0.978	0.957	1.303
75% quantile	1.663	1.710	1.656	1.993	1.756	2.084	1.757	1.695	1.458	2.343
95% quantile	2.240	2.175	2.249	2.934	2.491	3.157	2.537	2.808	2.133	3.127

3. The names of the regions with the highest and lowest means, and the highest and lowest variances.

Performance	Mean	Variance
Highest	WtdILI	Mtn
Lowest	Pac	Pac

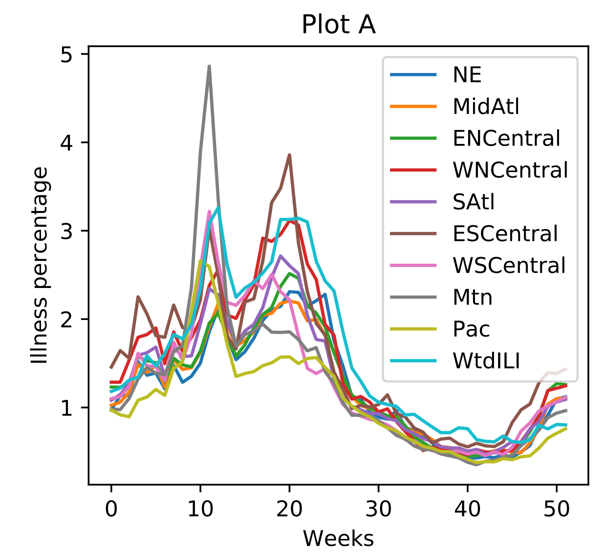
In light of the above, is the mode a reliable estimate of the most “common” value? Describe another way we could give a meaningful “mode” measurement for this (continuous) data. Note: the function `utils.mode()` will compute the mode value of an array for you.

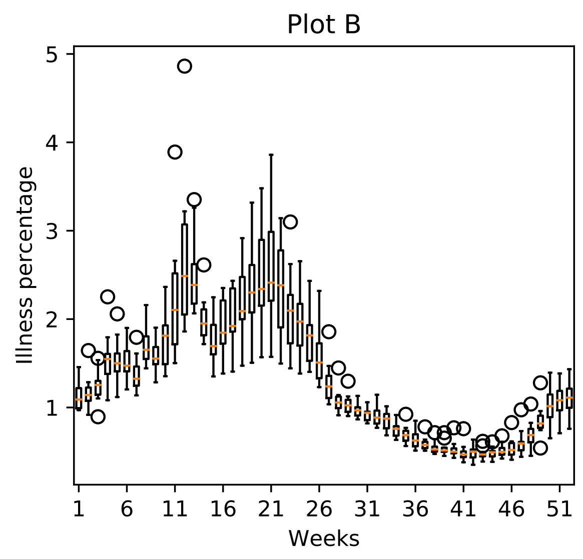
In this particular dataset the mode is not a reliable estimate of the most ”common” value in the dataset, because the mean is very different from the mode.

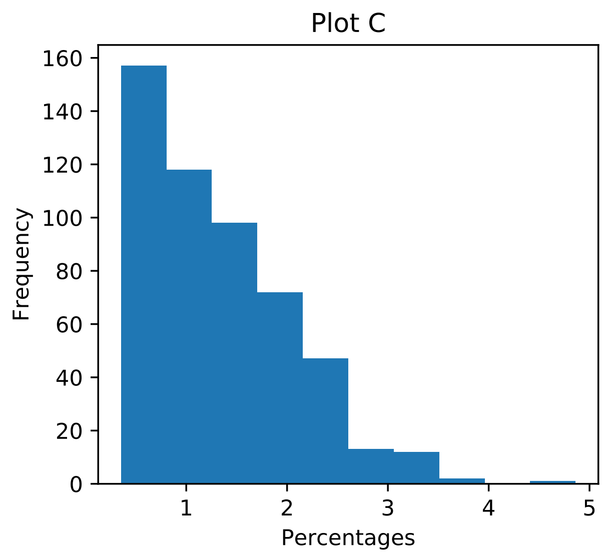
5.2 Data Visualization

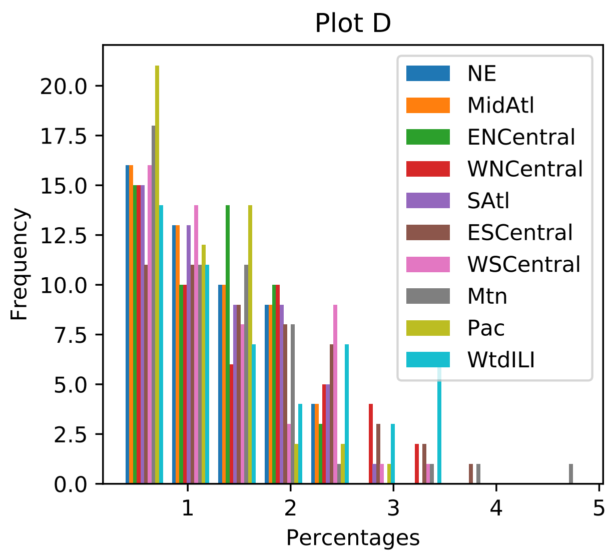
Rubric: {reasoning:3}

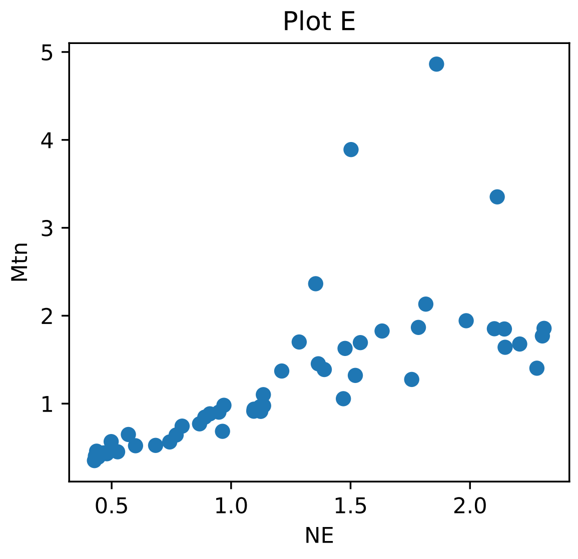
Consider the figure below.

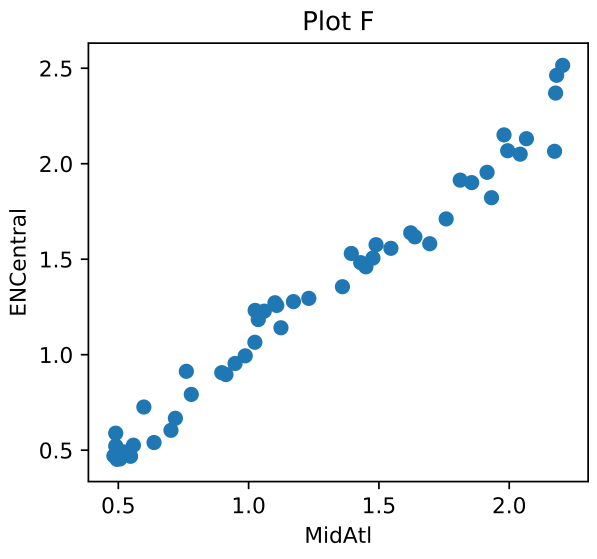

Plot A
Illness percentage
Weeks
NE
MidAtl
ENCentral
WNCentral
SAtl
ESCentral
WSCentral
Mtn
Pac
WtdILI


Plot B
Illness percentage
Weeks


Plot C
Frequency
Percentages


Plot D
Frequency
Percentages
NE
MidAtl
ENCentral
WNCentral
SAtl
ESCentral
WSCentral
Mtn
Pac
WtdILI


Plot E
Mtn
NE


Plot F
ENCentral
MidAtl

The figure contains the following plots, in a shuffled order:

1. A single histogram showing the distribution of *each* column in X. $\boxed{\text{Plot D}}$ → shows the distribution of illness percentage of each region over the weeks, and in X each column is a region
2. A histogram showing the distribution of each the values in the matrix X. $\boxed{\text{Plot C}}$ → shows the distribution of illness percentage of all the values in X, no taking in count the region
3. A boxplot grouping data by weeks, showing the distribution across regions for each week. $\boxed{\text{Plot B}}$ → because this is a boxplot showing the information of each value in X by weeks
4. A plot showing the illness percentages over time. $\boxed{\text{Plot A}}$ → because the illness percentage of the plot is in represented over time
5. A scatterplot between the two regions with highest correlation. $\boxed{\text{Plot F}}$ → the values on this plot show a high correlation between each other because they all seem to follow the same pattern
6. A scatterplot between the two regions with lowest correlation. $\boxed{\text{Plot E}}$ → as opposed to the Plot F this values do not follow a pattern

Match the plots (labeled A-F) with the descriptions above (labeled 1-6), with an extremely brief (a few words is fine) explanation for each decision.

6 Decision Trees

If you run `python main.py -q 6`, it will load a dataset containing longitude and latitude data for 400 cities in the US, along with a class label indicating whether they were a "red" state or a "blue" state in the 2012 election.[1] Specifically, the first column of the variable X contains the longitude and the second variable contains the latitude, while the variable y is set to 0 for blue states and 1 for red states. After it loads the data, it plots the data and then fits two simple classifiers: a classifier that always predicts the most common label (0 in this case) and a decision stump that discretizes the features (by rounding to the nearest integer) and then finds the best equality-based rule (i.e., check if a feature is equal to some value). It reports the training error with these two classifiers, then plots the decision areas made by the decision stump. The plot is shown below:

[1]The cities data was sampled from `http://simplemaps.com/static/demos/resources/us-cities/cities.csv`. The election information was collected from Wikipedia.

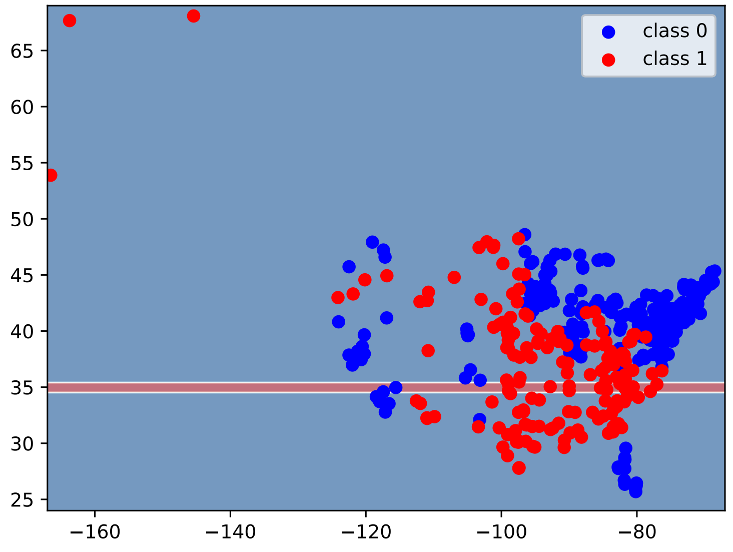


***Fig. 1:* Boundary regions of `DecisionStumpEquality`**

As you can see, it is just checking whether the latitude equals 35 and, if so, predicting red (Republican). This is not a very good classifier.

6.1 Splitting rule

Rubric: {reasoning:1}

Is there a particular type of features for which it makes sense to use an equality-based splitting rule rather than the threshold-based splits we discussed in class?

If the domain of a feature is a finite number (i.e., The feature has labels representing the age of people as 'Child','Adult' and 'Old'), in those scenarios it makes sense to use an equality-based splitting rule rather than a threshold-based splitting.

6.2 Decision Stump Implementation

Rubric: {code:3}

The file `decision_stump.py` contains the class `DecisionStumpEquality` which finds the best decision stump using the equality rule and then makes predictions using that rule. Instead of discretizing the data and using a rule based on testing an equality for a single feature, we want to check whether a feature is above or below a threshold and split the data accordingly (this is a more sane approach, which we discussed in class). Create a `DecisionStumpErrorRate` class to do this, and report the updated error you obtain by using inequalities instead of discretizing and testing equality. Also submit the generated figure of the classification boundary.

Hint: you may want to start by copy/pasting the contents `DecisionStumpEquality` and then make modifications from there.

The `DecisionStumpErrorRate` predict y with an error of $\boxed{0.253}$. The following figure represents the boundary regions from the model.

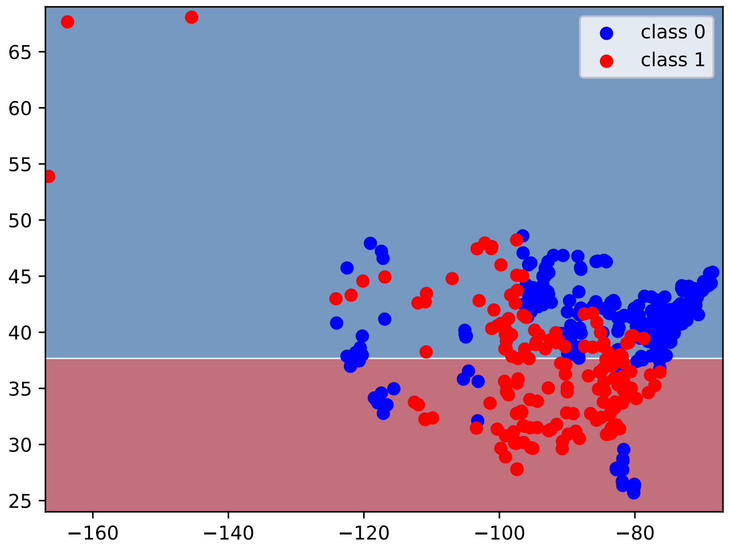


Fig. 2: **Boundary regions of** `DecisionStumpErrorRate`

See **Appendix A** for the implementation of `DecisionStumpErrorRate`.

6.3 Decision Stump Info Gain Implementation

Rubric: {code:3}

In `decision_stump.py`, create a `DecisionStumpInfoGain` class that fits using the information gain criterion discussed in lecture. Report the updated error you obtain, and submit the classification boundary figure.

Notice how the error rate changed. Are you surprised? If so, hang on until the end of this question!

Note: even though this data set only has 2 classes (red and blue), your implementation should work for any number of classes, just like `DecisionStumpEquality` and `DecisionStumpErrorRate`.

Hint: take a look at the documentation for `np.bincount`, at `https://docs.scipy.org/doc/numpy/reference/generated/numpy.bincount.html`. The `minlength` argument comes in handy here to deal with a tricky corner case: when you consider a split, you might not have any examples of a certain class, like class 1, going to one side of the split. Thus, when you call `np.bincount`, you'll get a shorter array by default, which is not what you want. Setting `minlength` to the number of classes solves this problem.

The `DecisionStumpInfoGain` predict y with an error of $\boxed{0.325}$. The following figure represents the boundary regions from the model.

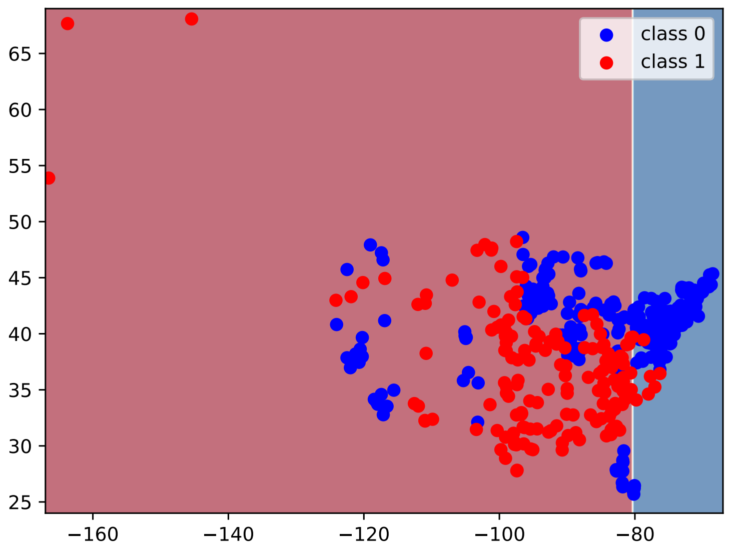


***Fig. 3:* Boundary regions of `DecisionStumpInfoGain`**

From the figure we can observe that this decision stump considered as split variable another feature of the data X, this is why the regios are divided by a vertical line.

See **Appendix B** for the implementation of `DecisionStumpInfoGain`.

6.4 Constructing Decision Trees

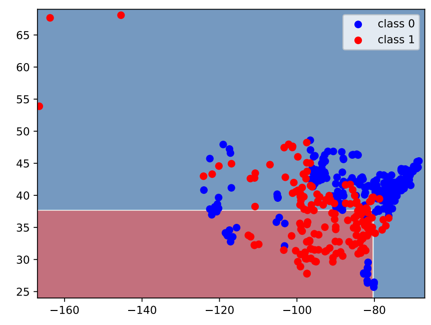
Rubric: {code:2}

Once your `DecisionStumpInfoGain` class is finished, running `python main.py -q 6.4` will fit a decision tree of depth 2 to the same dataset (which results in a lower training error). Look at how the decision tree is stored and how the (recursive) `predict` function works. Using the splits from the fitted depth-2 decision tree, write a hard-coded version of the `predict` function that classifies one example using simple if/else statements (see the Decision Trees lecture). Save your code in a new file called `simple_decision.py` (in the `code` directory) and make sure you include the text of this file in your LaTeX report.

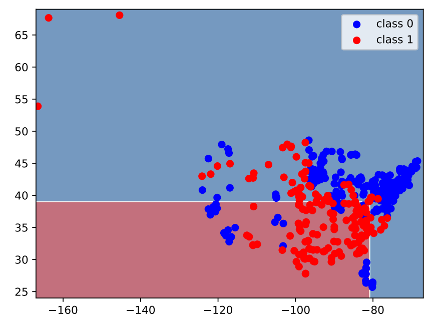
Note: this code should implement the specific, fixed decision tree which was learned after calling `fit` on this particular data set. It does not need to be a learnable model. You should just hard-code the split values directly into the code. Only the `predict` function is needed.

Hint: if you plot the decision boundary you can do a visual sanity check to see if your code is consistent with the plot.

Here are the two figures of the plot boundary regions of both DecisionTrees, the InfoGain and the hard coded one.



***(a)* DecisionStumpInfoGain of depth 2**



***(b)* DecisionStumpHardCoded**

Here is the code from `simple_decision.py`. It contains the definition offered the class DecisionStumpHard-Coded, which just implements the function *predict*, by using the split values and split variables acquired from running the file `main.py -q 6.4`

See **Apendix C**, for de implementation of the `DecisionStumpHardCoded` classifier

6.5 Decision Tree Training Error

Rubric: {reasoning:2}

Running `python main.py -q 6.5` fits decision trees of different depths using the following different implementations:

1. A decision tree using `DecisionStump`
2. A decision tree using `DecisionStumpInfoGain`
3. The `DecisionTreeClassifier` from the popular Python ML library *scikit-learn*

Run the code and look at the figure. Describe what you observe. Can you explain the results? Why is approach (1) so disappointing? Also, submit a classification boundary plot of the model with the lowest training error.

Note: we set the `random_state` because sklearn's `DecisionTreeClassifier` is non-deterministic. This is probably because it breaks ties randomly.

Note: the code also prints out the amount of time spent. You'll notice that sklearn's implementation is substantially faster. This is because our implementation is based on the $O(n^2d)$ decision stump learning algorithm and sklearn's implementation presumably uses the faster $O(nd\log n)$ decision stump learning algorithm that we discussed in lecture.

The approach(1) or DecisionStumpErrorRate has the best performance at the initial depth, but it has also the worst performance on the next depths. This is due because of the way the DecisionStump calculates the thresholds, it is just taking in count the error of the prediction, whereas the DecisionStumpInfoGain reduces the entropy of the predictions allowing this classifier to get a higher precision than the DecisionStumpErrorRate.

The model with the lowest training error would be the DecisionStumpInfoGain and the DecisionTreeClassifier from *scikit-learn*. The following figure is the classification boundary plot of that model.

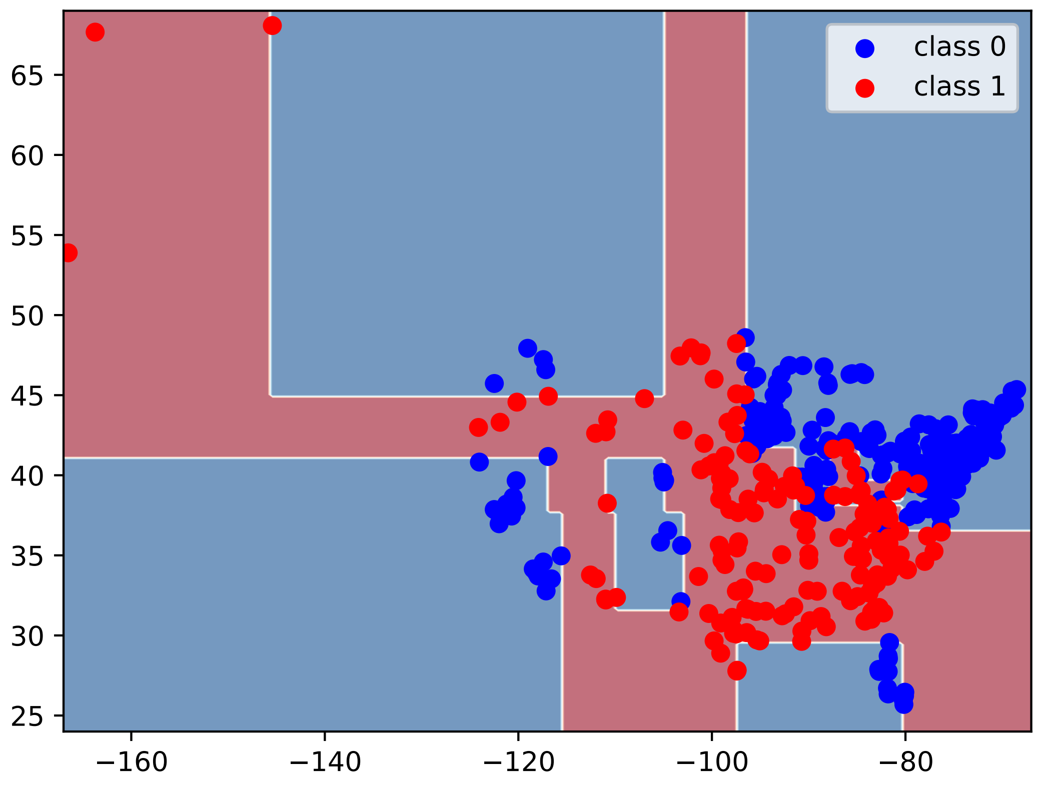


Fig. 5: **Boundary regions of DecisionStumpInfoGain**

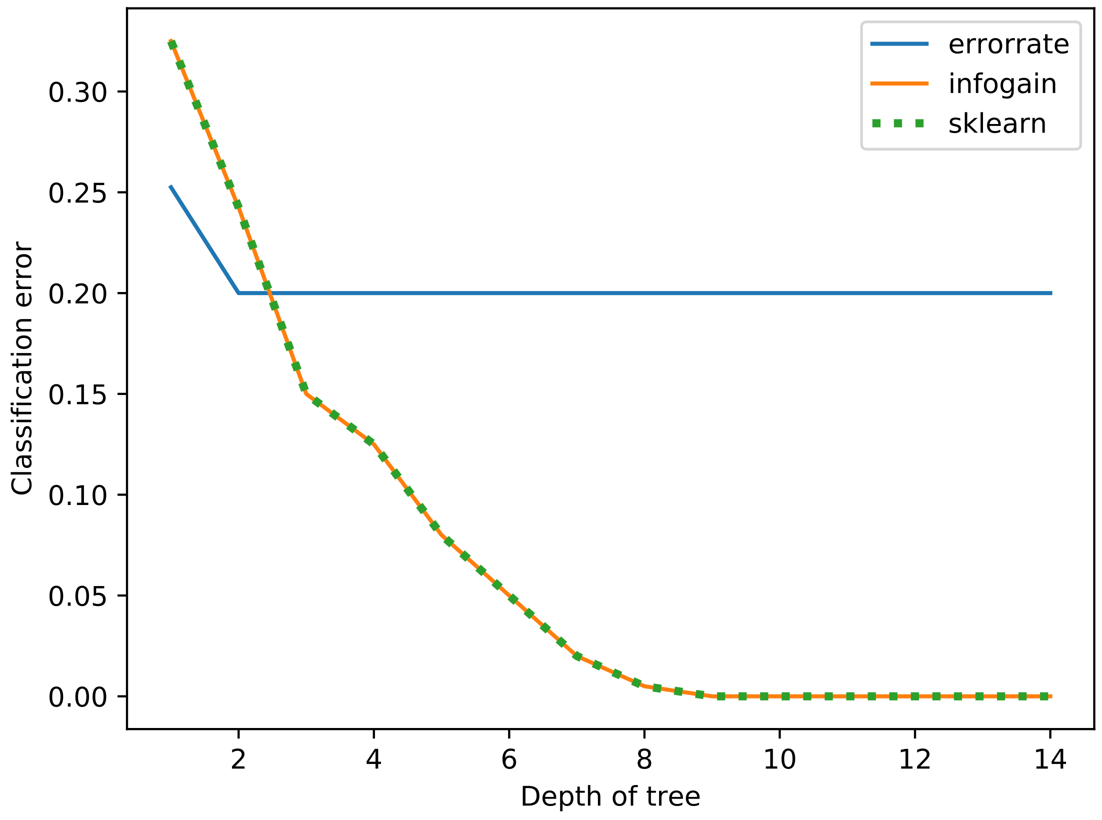


Fig. 6: **Error over Depth of each classifier**

6.6 Comparing implementations

Rubric: {reasoning:2}

In the previous section you compared different implementations of a machine learning algorithm. Let's say that two approaches produce the exact same curve of classification error rate vs. tree depth. Does this conclusively demonstrate that the two implementations are the same? If so, why? If not, what other experiment might you perform to build confidence that the implementations are probably equivalent?

Having two implementations with the same curve of classification error rate vs. tree depth does not necessary demonstrate that both implementations are the same. This is because this two implementations may have the exact error at certain depth but this do not mean that they use the same feature and threshold to classify. It may just be that the error at that depth is the same, but with different split variables and values.

Another method that we can use in order to clarify if both implementations are equivalent is by plotting the classification boundaries of both models. With this we can visualize the thresholds used by each model and if this looks similar they may be the equivalent implementations.

6.7 Cost of Fitting Decision Trees

Rubric: {reasoning:3}

In class, we discussed how in general the decision stump minimizing the classification error can be found in $O(nd \log n)$ time. Using the greedy recursive splitting procedure, what is the total cost of fitting a decision tree of depth m in terms of n, d, and m?

Hint: even thought there could be $(2^m - 1)$ decision stumps, keep in mind not every stump will need to go through every example. Note also that we stop growing the decision tree if a node has no examples, so we may not even need to do anything for many of the $(2^m - 1)$ decision stumps.

The procedure of calculating one rule of the decision tree has the complexity $O(n)$, but this process needs to be repeated nd times, so this gives a complexity of $O(n^2d)$ the complexity of calculating one decision stump. But we have to this recursively until the depth m, or if there is no more data to split into another decision stump, which might occur before than getting to depth level m. The process of recursively split the data in 2 on each depth has complexity of $O(\text{log}n)$, multiplied by the complexity of calculating a decision stump, it is $O(nd\text{log}n)$

A Appendix: Decision Tree Error Rate

```
class DecisionStumpErrorRate:

    def __init__(self):
        pass

    def fit(self, X, y):
        N, D = X.shape

        # Get an array with the number of 0's, number of 1's, etc.
        count = np.bincount(y, minlength=2)

        # Get the index of the largest value in count.
        # Thus, y_mode is the mode (most popular value) of y
        y_mode = np.argmax(count)

        self.splitSat = y_mode
        self.splitNot = None
        self.splitVariable = None
        self.splitValue = None

        # If all the labels are the same, no need to split further
        if np.unique(y).size <= 1:
            return

        minError = np.sum(y > y_mode)

        # Loop over features looking for the best split

        for d in range(D):
            for n in range(N):
                # Choose value to equate to
                value = X[n, d]
                # Find most likely class for each split
                y_sat = utils.mode(y[X[:,d] > value])
                y_not = utils.mode(y[X[:,d] <= value])

                # Make predictions
                y_pred = y_sat * np.ones(N)
                y_pred[X[:, d] <= value] = y_not

                # Compute error
                errors = np.sum(y_pred != y)

                # Compare to minimum error so far
                if errors < minError:
                    # This is the lowest error, store this value
                    minError = errors
                    self.splitVariable = d
                    self.splitValue = value
                    self.splitSat = y_sat
                    self.splitNot = y_not

    def predict(self, X):

        M, D = X.shape

        if self.splitVariable is None:
            return self.splitSat * np.ones(M)

        yhat = np.zeros(M)

        for m in range(M):
            if X[m, self.splitVariable] > self.splitValue:
                yhat[m] = self.splitSat
            else:
                yhat[m] = self.splitNot

        return yhat
```

B Appendix: Decision Tree Info Gain

```
class DecisionStumpInfoGain(DecisionStumpErrorRate):
    def fit(self, X, y):
        N, D = X.shape

        # Get an array with the number of 0's, number of 1's, etc.
        count = np.bincount(y, minlength=2)

        # Get the index of the largest value in count.
        # Thus, y_mode is the mode (most popular value) of y
        y_mode = np.argmax(count)

        self.splitSat = y_mode
        self.splitNot = None
        self.splitVariable = None
        self.splitValue = None

        # If all the labels are the same, no need to split further
        if np.unique(y).size <= 1:
            return

        maxInfoGain = None

        # Loop over features looking for the best split
        for d in range(D):
            for n in range(N):
                # Choose value to equate to
                value = X[n, d]

                # Find most likely class for each split
                y_sat = utils.mode(y[X[:,d] > value])
                y_not = utils.mode(y[X[:,d] <= value])

                # Make predictions
                y_pred = y_sat * np.ones(N)
                y_pred[X[:, d] <= value] = y_not

                yNorm = np.bincount(y,minlength=2) / y.shape

                yyes = y[X[:, d] > value]
                Nyyes = yyes.shape[0]
                if Nyyes == 0:
                    continue
                yyesNorm = np.bincount(yyes,minlength=2) / Nyyes

                yno = y[X[:, d] <= value]
                Nyno = yno.shape[0]
                if Nyno == 0:
                    continue
                ynoNorm = np.bincount(yno,minlength=2) / Nyno

                # Compute infogain
                infogain = entropy(yNorm) - (Nyyes/N)*entropy(yyesNorm) - (Nyno/N)*entropy(ynoNo

                isBestDecision = False
                if maxInfoGain == None:
                    isBestDecision = True
                # Compare to maximum infogain so far
                elif infogain > maxInfoGain:
                    isBestDecision = True

                if isBestDecision:
                    # This is the lowest error, store this value
                    maxInfoGain = infogain
                    self.splitVariable = d
                    self.splitValue = value
                    self.splitSat = y_sat
                    self.splitNot = y_not
```

C Appendix: Simple Decision Tree

```
class DecisionStumpHardCoded:

    def __init__(self):
        pass

    def fit(self, X, y):
        pass

    def predict(self, X):
        # Info learned from the depth 2 decision three using DecisionStumpInfoGain
        splitVariables = [0,1,1]
        splitVals = [-81.0,40.0,39.0]
        splitSats = [0,0,0]
        splitNots = [1,0,1]

        M, D = X.shape

        yhat = np.zeros(M)

        for m in range(M):
            # We check for the split values in a hierarchical order
            if X[m, splitVariables[0]] > splitVals[0]:
                if X[m, splitVariables[1]] > splitVals[1]:
                    yhat[m] = splitSats[1]
                else:
                    yhat[m] = splitNots[1]
            else:
                if X[m, splitVariables[2]] > splitVals[2]:
                    yhat[m] = splitSats[2]
                else:
                    yhat[m] = splitNots[2]

        return yhat
```