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PHYSICS 7611981111

Mechanics and Oscillation

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सीमा यादव (मनोहरपुर, उदवाला)

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Unit - I

Chap-I (Physical laws & Frame of Refrence)

Frame of Refrence :-

The system w.r.t. that position of a particle is denoted, is known as Frame of refrence.

There are two types of frame of refrence :-

(a) Inertial frame of refrence

(b) Non-inertial frame of refrence

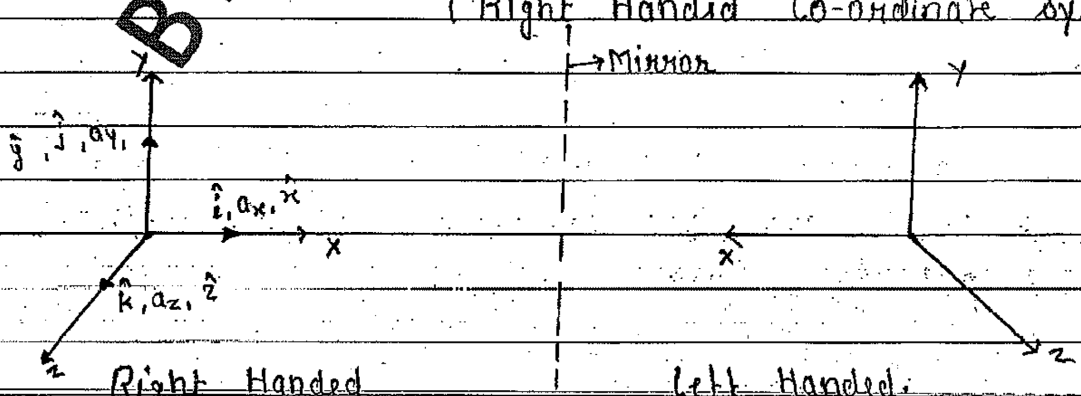
Mathematical tools for Solving mathematics calculation

There are three type of co-ordinate

1. Cartesian Co-ordinate System
2. Spherical Co-ordinate System
3. Cylindrical Co-ordinate System

1 Cartesian Co-ordinate System :-

(Right Handed Co-ordinate system)



Here $\hat{i}, \hat{j}, \hat{k}$ are unit vector in x, y, z dirxn respectively

Limit

$$-\infty < x < +\infty$$
$$-\infty < y < +\infty$$
$$-\infty < z < +\infty$$

Position vector of Stationary point

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

length element

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

In Mathematical calculation Right Handed Cartesian co-ordinate system is used.

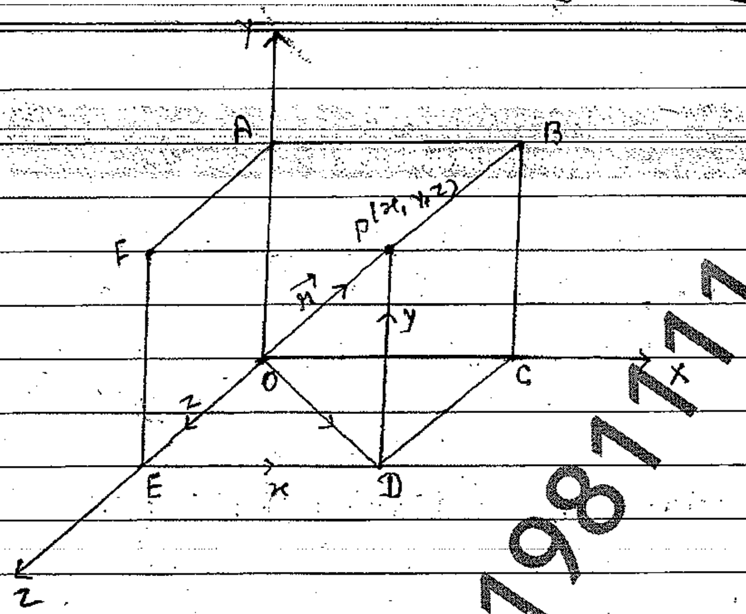
Position Vector :-

It can be described by line vector $\vec{OP}$ joining to observer or origin to observer (position of particle).

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

proof,

let a point 'P' whose co-ordinate are x, y, z and position vector is $\vec{r}$ with respect to origin.



In ΔOED, using vector Δ law of addition

$$\vec{OD} = \vec{OE} + \vec{ED}$$

$$\Rightarrow \quad \vec{OD} = z\hat{k} + x\hat{i} \quad \text{--- (i)}$$

In ΔOPD, using vector Δ law

$$\Rightarrow \quad \vec{OP} = \vec{OD} + \vec{DP}$$

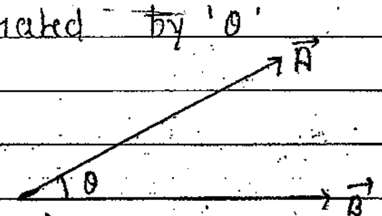
by eq^n (i)

$$\Rightarrow \quad \vec{OP} = \vec{OE} + \vec{ED} + \vec{DP}$$

$$\Rightarrow \quad \boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}} \quad \text{--- (ii)}$$

eq^n (ii) represent position of a particle in stationary condition.

Magnitude of Position Vector :- let two vectors are $\vec{A}$ and $\vec{B}$ seperated by 'θ'



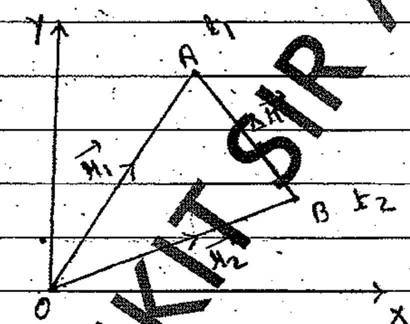
let $\vec{A} = \vec{B} = \vec{r}$,

$\theta = 0°$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$
$$\vec{r}\cdot\vec{r} = |\vec{r}||\vec{r}|\cos 0$$
$$\vec{r}\cdot\vec{r} = r^2$$
$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k}) = r^2$$
$$r^2 = x^2+y^2+z^2$$
$$\boxed{r = \sqrt{x^2+y^2+z^2}}$$

Position Vector of a Moving Particle :-

Let at time 't_1', a particle is at A and its position vector w.r.t. origin is $\vec{r_1}$ and at 't_2' time particle is at B and its position vector is $\vec{r_2}$.



In ΔOAB,
$$\vec{OB} = \vec{OA} + \vec{AB}$$
$$\vec{r_2} = \vec{r_1} + \Delta\vec{r}$$
$$\boxed{\Delta\vec{r} = \vec{r_2} - \vec{r_1}}$$

Average velocity :-
$$\vec{V}_{av.} = \frac{\Delta\vec{r}}{\Delta t}$$
$$\vec{V}_{av.} = \frac{\vec{r_2}-\vec{r_1}}{t_2-t_1}$$

limit,

$$\vec{V}_{av} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

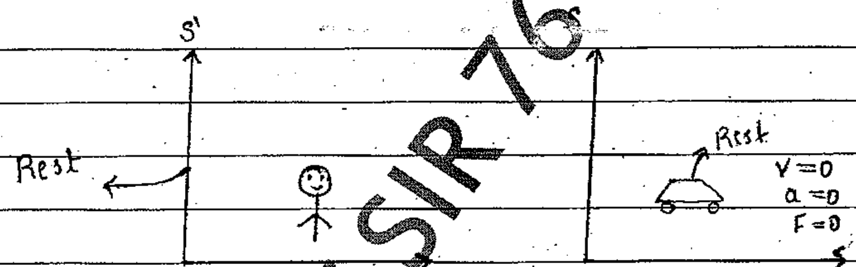
$$\boxed{\vec{V}_{av} = \frac{d\vec{r}}{dt}}$$

There are two types of F.O.R :-

(a) Inertial frame of ref.

(b) Non Inertial frame of ref.

(a) Inertial F.O.R :-

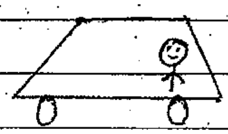


A frame of refrence 'S' having a stationary particle (non accelerated). This particle is seen by an other F.O.R 'S''. We find particle will be stationary. So this type of refrence is known as Inertial frame of refrence.

Here, in Inertial frame, Newton's 1^{st} and 2^{nd} law are valid.

For uniform accelerated particle having inertial frame of refrence.

uniform accelerated



A car accelerated by uniform velocity, then a man sitting in car, this car w.r.t. man work as inertial frame.

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{d^2\vec{x}}{dt^2} \qquad \vec{x}\ \text{constant}$$

$$\vec{F} = m \times 0$$

$$\boxed{\vec{F} = 0}$$

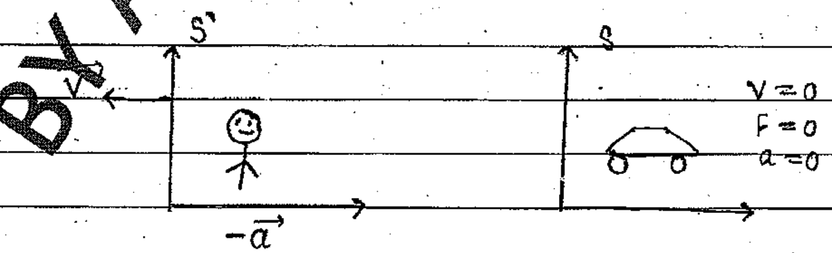
Newton's 2nd law,

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{net} = \frac{dm\vec{v}}{dt}$$

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = 0$$

(b) Non Inertial F.O.R :-



'S' F.O.R. is moving with $-\vec{a}$ w.r.t. 'S''
Hence,

$$\vec{F} = -m\vec{a}$$

In this situation, a pseudo force work on particle and it's dirxn is opposite to motion.

Non Inertial F.O.R. are valid by Rotational frame of refrence.

Newton's law are not valid.

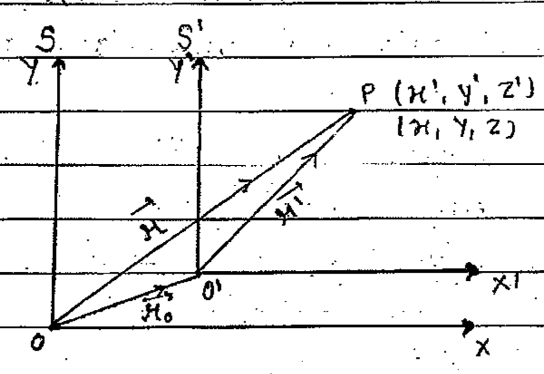
Transformation eqn :-

If the co-ordinate of a particle are given in a frame of ref. and co-ordinate of same particle are given in another frame of refrence then the eqn which define relation b/w these co-ordinate is known as Transformation eqn.

(a) Transformation eqn for Stationary F.O.R :-

let's consider two F.O.R. S and S' in which distance b/w origin is constant ($\vec{r_0}$) this type of frame is Stationary F.O.R.

The position vector of a point 'P' w.r.t. S and S' are $\vec{r}$ and $\vec{r'}$.



In $\Delta OO'P$, using vector Δ law of addition

$$\vec{OP} = \vec{OO'} + \vec{O'P}$$
$$\vec{r} = \vec{r'} + \vec{r_0}$$
$$\boxed{\vec{r'} = \vec{r} - \vec{r_0}} \quad \text{—— (i)}$$

eq^n (i) is known as Transformation eq^n of position.

On differentiating (i) w.r.t 't'

$$\frac{d\vec{r'}}{dt} = \frac{d\vec{r}}{dt} - \frac{d\vec{r_0}}{dt}$$
$$\vec{r_0} = \text{constant}$$
$$\vec{v'} = \vec{v} - 0$$
$$\boxed{\vec{v'} = \vec{v}} \quad \text{—— (ii)}$$

eq^n (ii) is known as Velocity Transformation eq^n
Here, velocity is invariant qty.

On differentiating eq^n (ii) w.r.t. 't'

$$\frac{d\vec{v'}}{dt} = \frac{d\vec{v}}{dt}$$
$$\boxed{\vec{a'} = \vec{a}} \quad \text{—— (iii)}$$

eq^n (iii) is known as Acceleration Transformation eq^n
Here, acceleration is invariant qty.

On multiplying eq^n (ii) and (iii) by 'm'

$$m\vec{v'} = m\vec{v}$$
$$\boxed{\vec{p'} = p} \quad \text{—— (iv)}$$

$$m\vec{a'} = m\vec{a}$$

$$\boxed{\vec{F'} = \vec{F}} \quad \text{---(v)}$$

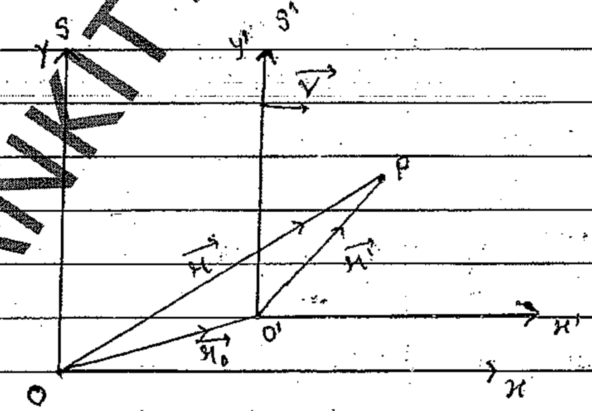
eq^n (IV) is known as momentum Transformation eq^n and eq^n (V) is known as Force Transformation eq^n

Both momentum and force are invariant q.ty.

(b) Transformation eq^n for constant speed (velocity) moving frame of ref. :-

let two refrence frame S and S'. S' frame of refrence is moving with constant velocity (v) w.r.t. 'S' frame of refrence.

Position vector of 'P' in F.O.R. S and S' are $\vec{r}$ and $\vec{r'}$ respectively. Here the distance b/w origin is not constant.



(I) Position Transformation eq^n :-

In $\Delta OO'P$

$$\vec{OP} = \vec{OO'} + \vec{O'P}$$

$$\vec{r} = \vec{r_0} + \vec{r'}$$

$$\vec{r'} = \vec{r} - \vec{r_0}$$

$\because$ Speed = $\frac{\text{Distance}}{\text{Time}}$

$$\vec{V} = \frac{\vec{r_0}}{t}$$

$$\vec{r_0} = \vec{V}t$$

$$\boxed{\vec{r'} = \vec{r} - \vec{V}t} \quad \text{---(i)}$$

eq^n (i) is position transformation eq^n.

(II) Velocity Transformation eq^n :-

On D. (i) w.r.t. 't'.

$$\frac{d\vec{r'}}{dt} = \frac{d\vec{r}}{dt} - \frac{d}{dt}\vec{V}t$$

$$\boxed{\vec{v'} = \vec{v} - \vec{V}} \quad \text{---(ii)}$$

eq^n (ii) is velocity transformation eq^n.

Here

$\vec{v'}$ = Velocity of 'P' w.r.t. 'S'' frame

$\vec{v}$ = Velocity of 'P' w.r.t. 'S' frame

$\vec{V}$ = Velocity of 'S'' frame

(III) Acceleration Transformation eq^n :-

D. (ii) w.r.t. 't'

$$\frac{d\vec{v'}}{dt} = \frac{d\vec{v}}{dt} - \frac{d}{dt}\vec{V}$$

$$\vec{a'} = \vec{a} - 0$$

$$\boxed{\vec{a'} = \vec{a}} \quad \text{---(iii)}$$

(IV) Momentum & Force Transformation eq^n :-

multiply eq^n (II) and eq^n (III) by m

$$m\vec{v'} = m\vec{v} - m\vec{V}$$

$$\boxed{\vec{p'} = \vec{p} - m\vec{V}} \quad \text{—— (iv)}$$

momentum Transformation eq^n

$$m\vec{a'} = m\vec{a}$$

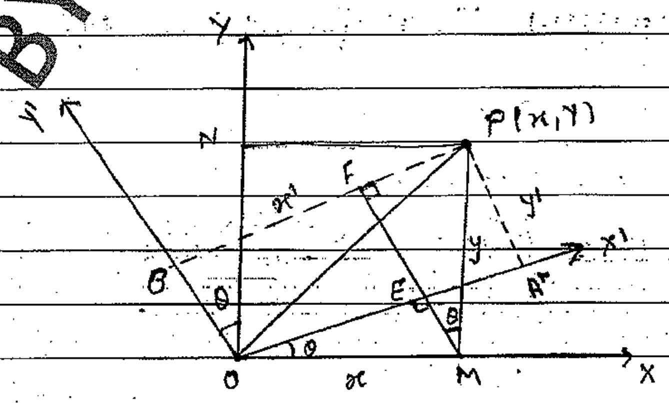
$$\boxed{\vec{F'} = \vec{F}} \quad \text{—— (v)}$$

Force Transformation eq^n

In this type of refrence frame, acceleration, and force are invariant qty while position, velocity and momentum are variant qty.

(C) Transformation eq^n for refrence frame inclined at an angle :-

let a frame of refrence is S in which co-ordinate of P are (x, y, z) and now it is rotated with an angle 'θ' and we get a new frame S' in which co-ordinate of P are (x', y', z')



Here,

$$x = PN = OM$$
$$y = PM = ON$$
$$x' = PB = OA$$
$$y' = PA = OB$$

In ΔOEM

$$EM = x\sin\theta \quad \text{—(i)}$$
$$OE = x\cos\theta \quad \text{—(ii)}$$

In ΔMFP,

$$PF = y\sin\theta \quad \text{—(iii)}$$
$$MF = y\cos\theta \quad \text{—(iv)}$$

Now,

$$x' = BF + PF = PB$$
$$x' = OE + PF$$
$$\boxed{x' = x\cos\theta + y\sin\theta} \quad \text{—(v)}$$

$$y' = MF - EM = PA$$
$$y' = y\cos\theta - x\sin\theta$$
$$\boxed{y' = -x\sin\theta + y\cos\theta} \quad \text{—(vi)}$$

eq^n (v) and (vi) represent Transformation eq^n for frame inclined at angle 'θ'

$$x' = (x, y) \text{ component addition}$$
$$x' = x\cos(x'Ox) + y\sin(90 - x'Oy)$$
$$\boxed{x' = x\cos(x'Ox) + y\cos(x'Oy)} \quad \text{—(vii)}$$

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$$y' = y\cos(y'oy) - x\sin(90 + y'ox)$$

$$y' = y\cos(y'oy) + x\cos(y'ox)$$

$$\boxed{y' = x\cos(y'ox) + y\cos(y'oy)} \quad \text{---}(ix)$$

3-D :-

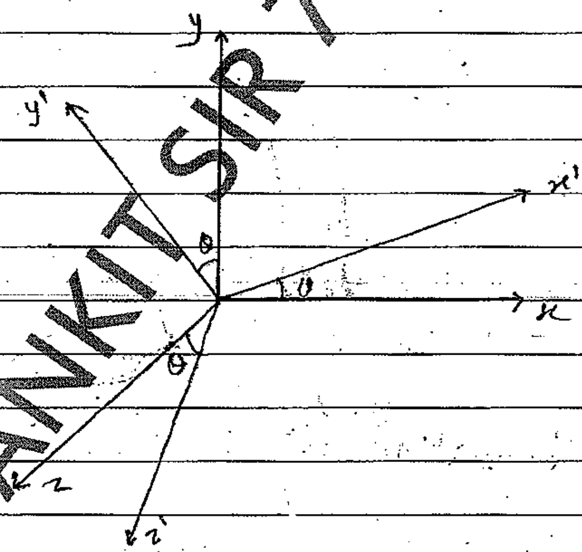
To calculate transformation eq^n in 3-D, obtained with help 2-D transformation eq^n

by eq^n:

$$x' = x\cos(x'ox) + y\cos(x'oy) + z\cos(x'oz)$$

$$y' = x\cos(y'ox) + y\cos(y'oy) + z\cos(y'oz)$$

$$z' = x\cos(z'ox) + y\cos(z'oy) + z\cos(z'oz)$$



let $\cos(x'ox) = a_{11}$; $\cos(x'oy) = a_{12}$; $\cos(x'oz) = a_{13}$

$\cos(y'ox) = a_{21}$; $\cos(y'oy) = a_{22}$; $\cos(y'oz) = a_{23}$

$\cos(z'ox) = a_{31}$; $\cos(z'oy) = a_{32}$; $\cos(z'oz) = a_{33}$

$$x' = a_{11}x + a_{12}y + a_{13}z \quad \text{---}(I)$$

$$y' = a_{21}x + a_{22}y + a_{23}z \quad \text{---}(II)$$

$$z' = a_{31}x + a_{32}y + a_{33}z \quad \text{---}(III)$$

eq^n (I), (II) and (III) represent Transformation eq^n in 3-D.

In matrix form,

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Above matrix represent Transformation matrix in 3-D.

Velocity Transformation eq^n :-

D. eq^n (I), (II) and (III) w.r.t. 't'

$$\frac{dx'}{dt} = a_{11}\frac{dx}{dt} + a_{12}\frac{dy}{dt} + a_{13}\frac{dz}{dt}$$

$$V'_x = a_{11}V_x + a_{12}V_y + a_{13}V_z \quad \text{—(iv)}$$

Similarly,

$$V'_y = a_{21}V_x + a_{22}V_y + a_{23}V_z \quad \text{—(v)}$$

$$V'_z = a_{31}V_x + a_{32}V_y + a_{33}V_z \quad \text{—(vi)}$$

Acceleration Transformation eq^n :-

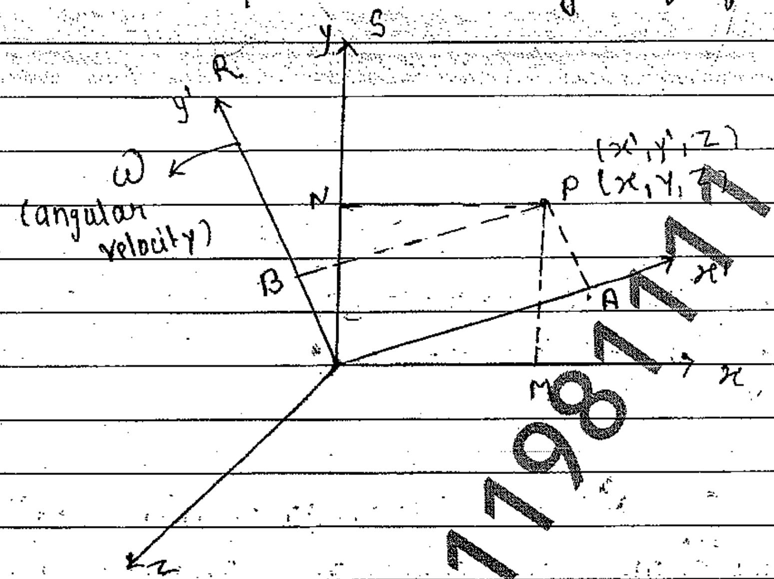
D. eq^n (IV), (V), (VI) w.r.t. 't'

$$a'_x = a_{11}a_x + a_{12}a_y + a_{13}a_z \quad \text{—(VII)}$$

$$a'_y = a_{21}a_x + a_{22}a_y + a_{23}a_z \quad \text{—(VIII)}$$

$$a'_z = a_{31}a_x + a_{32}a_y + a_{33}a_z \quad \text{—(IX)}$$

Transformation eq^n in Rotating ref. frame :-



let us consider two frame of ref. S and R. In initial ($t=0$) coincide at origin both frame of refrence.

let refrence frame (R) is moving with angular velocity (ω) and Z-axis and both refrence frames origin are coincide.

$$\text{angular velocity} = \frac{\text{angular displacement}}{\text{time}}$$

$$\omega = \frac{\theta}{t}$$

$$\boxed{\theta = \omega t}$$

Initially co-ordinate of P are x, y, z and after rotating frame of ref., co-ordinate are x', y', z'.

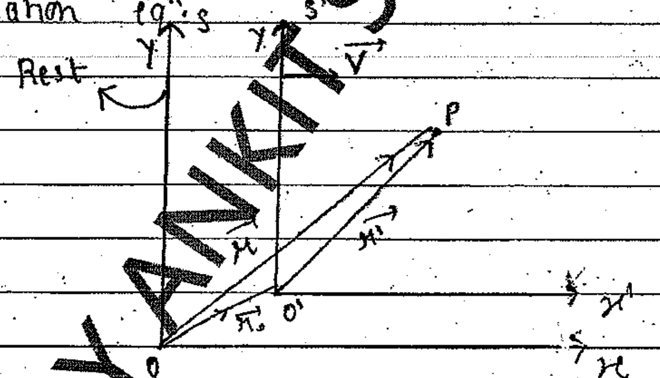
$x' = (x, y)$ component addition

$$x' = x\cos\omega t + y\sin\omega t \qquad \text{—(I)}$$
$$y' = -x\sin\omega t + y\cos\omega t \qquad \text{—(II)}$$
$$z' = z \qquad \text{—(III)}$$

eq^n (I), (II) and (III) is known as transformation eq^n for rotating ref. frame.

Galilean Co-ordinate Transformation eq^n :-
(Relation b/w one inertial and another inertial refrence frame).

Transformation eq^n b/w one inertial frame to another inertial frame is known as galilean transformation eq^{n}s



let us consider a F.O.R. 'S' in which a point 'P' having co-ordinate (x, y, z) and its position vector is $\vec{r}$ w.r.t. origin and another F.O.R. S' in which co-ordinate are (x', y', z') and its position vector is $\vec{r'}$.

By Vector addition

In $\Delta OO'P$

$$\vec{OP} = \vec{OO'} + \vec{O'P}$$

$$\vec{r} = \vec{r_0} + \vec{r'}$$

$$\vec{r'} = \vec{r} - \vec{r_0}$$

$$\because \vec{r_0} = \vec{v}t$$

$$\boxed{\vec{r'} = \vec{r} - \vec{v}t} \quad \text{—— (I)}$$

eq^n (I) represent galilion position transformation eq^n

Position vector w.r.t. origin in 'S'

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad \text{—— (II)}$$

Position vector w.r.t. origin in S'

$$\vec{r'} = x'\hat{i} + y'\hat{j} + z'\hat{k} \quad \text{—— (III)}$$

put value of eq. (II), (III) in (I)

$$x'\hat{i} + y'\hat{j} + z'\hat{k} = (x\hat{i} + y\hat{j} + z\hat{k}) - (v_x\hat{i} + v_y\hat{j} + v_z\hat{k})t$$

$$\because \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$x'\hat{i} + y'\hat{j} + z'\hat{k} = (x - v_x t)\hat{i} + (y - v_y t)\hat{j} + (z - v_z t)\hat{k}$$

On comparing coefficient of unit vector

$$x' = x - v_x t \quad \text{—— (IV)}$$

$$y' = y - v_y t \quad \text{—— (V)}$$

$$z' = z - v_z t \quad \text{—— (VI)}$$

eq^n (IV), (V) and (VI) are known as galilean co-ordinate transformation eq^n.

If frame is moving along x-axis dirxn

$$v_x \neq 0 \; ; \; v_y = v_z = 0$$

$$x' = x - v_x t \qquad \text{—(vii)}$$

$$y' = y \qquad \text{—(viii)}$$

$$z' = z \qquad \text{—(ix)}$$

* Velocity transformation eqⁿ

D. (I) w.r.t. 't'

$$\frac{d}{dt}\vec{r'} = \frac{d}{dt}\vec{r} - \frac{d}{dt}\vec{v}t$$

$$\boxed{\vec{v'} = \vec{v} - \vec{v}} \qquad \text{—(x)}$$

* Acceleration transformation eqⁿ.

D. (x) w.r.t. 't'

$$\frac{d}{dt}\vec{v'} = \frac{d}{dt}\vec{v} - \frac{d}{dt}\vec{v}$$

$$\boxed{\vec{a'} = \vec{a}} \qquad \text{—(xi)}$$

Que. 1. Prove that in galilean transformation displacement is invariant qty.

proof,

let a frame of ref. 'S' in which two point A and B are situated and their position vector are $\vec{r_1}$ and $\vec{r_2}$.

In another ref. frame S', their position vector are $\vec{r_1'}$ and $\vec{r_2'}$. Here S' moves with constant velocity.

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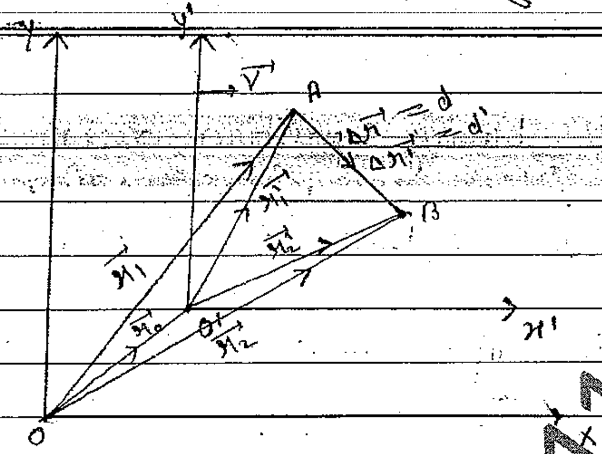


Position vector in F.O.R. 'S''

$$\vec{r'} = \vec{r} - \vec{V}t$$

$$\vec{r_1'} = \vec{r_1} - \vec{V}t \quad \text{—(i)}$$

In 'S''

$$\vec{r_2'} = \vec{r_2} - \vec{V}t \quad \text{—(ii)}$$

eq^n (ii) − eq^n (i)

$$\vec{r_2'} - \vec{r_1'} = \vec{r_2} - \vec{r_1} \quad \text{—(iii)}$$

In 'S' F.O.R. displacement b/w A and B is

$$\vec{d} = \vec{r_2} - \vec{r_1} \quad \text{—(iv)}$$

In S' F.O.R. displacement b/w A and B

$$\vec{d'} = \vec{r_2'} - \vec{r_1'} \quad \text{—(v)}$$

so, $\boxed{\vec{d'} = \vec{d}}$

Hence displacement is invariant qty. in galilean transformation eq^n.

Numerical

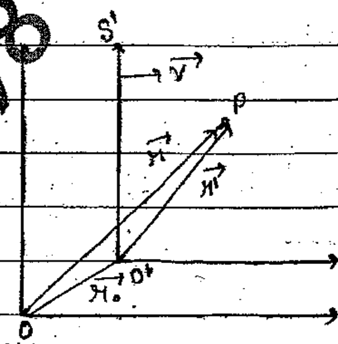
1. A point 'P' having position vector in cartesian co-ordinate is $\vec{r} = \hat{i}(6t^2 - 3t) + \hat{j}(-5t^3) + 7\hat{k}$. If this point is placed in S' then $\vec{r'} = \hat{i}(6t^2 + 3t) + \hat{j}(-5t^3) + 7\hat{k}$

Find,

(i) Velocity of S' w.r.t S

(ii) prove. $\vec{a'} = \vec{a}$

Solⁿ



Given,

$$\vec{r} = \hat{i}(6t^2 - 3t) + \hat{j}(-5t^3) + 7\hat{k}$$
$$\vec{r'} = \hat{i}(6t^2 + 3t) + \hat{j}(-5t^3) + 7\hat{k}$$

Solⁿ

(i) Velocity of S' w.r.t. S

By transformation eqⁿ of position

$$\vec{r'} = \vec{r} - \vec{V}t$$
$$\Rightarrow \quad \vec{V}t = \vec{r} - \vec{r'}$$
$$\Rightarrow \quad \vec{V}t = [\hat{i}(6t^2 - 3t) + \hat{j}(-5t^3) + 7\hat{k}] - [\hat{i}(6t^2 + 3t) + \hat{j}(-5t^3) + 7\hat{k}]$$
$$\Rightarrow \quad \vec{V}t = 6t^2\hat{i} - 3t\hat{i} - 5t^3\hat{j} + 7\hat{k} - 6t^2\hat{i} - 3t\hat{i} + 5t^3\hat{j} - 7\hat{k}$$
$$\Rightarrow \quad \vec{V}t = -6t\hat{i}$$
$$\boxed{\vec{V} = -6\hat{i}}$$

or $\boxed{|\vec{V}| = 6\,m/s}$

(ii) prove $\vec{a'} = \vec{a}$

proof,

$$\vec{r} = (6t^2 - 3t)\hat{i} + \hat{j}(-5t^3) + 7\hat{k}$$
$$\Rightarrow \quad \frac{d\vec{r}}{dt} = (12t - 3)\hat{i} - 15t^2\hat{j}$$
$$\Rightarrow \quad \vec{a} = \frac{d^2\vec{r}}{dt^2} = 12\hat{i} - 30t\hat{j} \quad \text{---(1)}$$

Similarly

$$\vec{r'} = \hat{i}(6t^2+3t) + \hat{j}(-5t^3) + 7\hat{k}$$

$$\Rightarrow \frac{d\vec{r'}}{dt} = (12t+3)\hat{i} - 15\hat{j}t^2 + 0$$

$$\Rightarrow \vec{a'} = \frac{d^2\vec{r'}}{dt^2} = 12\hat{i} - 30t\hat{j} \quad \text{—(ii)}$$

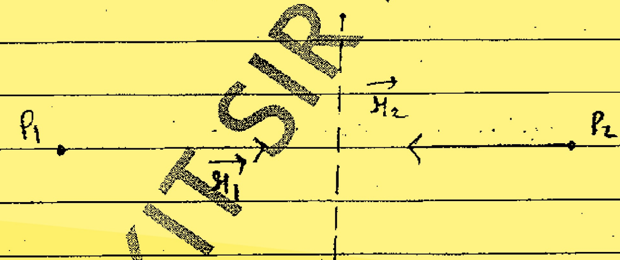
by (i) and (ii)

$$\boxed{\vec{a'} = \vec{a}}$$

Q.2. At any instant of time, two particles having position and velocity vector $\vec{r_1}$, $\vec{r_2}$ and $\vec{V_1}$, $\vec{V_2}$. These particles do Head-on-collision. then prove

$$(\vec{r_2} - \vec{r_1}) \times (\vec{V_2} - \vec{V_1}) = 0$$

Sol^n



let us consider two particles having position vector $\vec{r_1}$ and $\vec{r_2}$. After the collision particles having position vector $\vec{r_1'}$ and $\vec{r_2'}$.

Necessary condition of Head on collision.

$$\vec{r_1'} = \vec{r_2'}$$

$$t_1 = t_2$$

by transformation eq^n

$$\vec{r'} = \vec{r} - \vec{V}t$$

For P_1 particle,

$\Rightarrow \quad \vec{r_1'} = \vec{r_1} + \vec{V_1'}t \qquad \because \vec{V'} = -\vec{V_1'} \qquad \text{—(i)}$

and P_2 particle

$\Rightarrow \quad \vec{r_2'} = \vec{r_2} + \vec{V_2'}t \qquad \text{—(ii)}$

$\vec{r_1'} = \vec{r_2'}$ for Head on collision

$\Rightarrow \quad \vec{r_1} + \vec{V_1'}t = \vec{r_2} + \vec{V_2'}t$

$\Rightarrow \quad \vec{r_1} - \vec{r_2} = (\vec{V_2'} - \vec{V_1'})t$

$\Rightarrow \quad (\vec{V_2'} - \vec{V_1'})t + (\vec{r_2} - \vec{r_1}) = 0$

cross multiply of $(\vec{V_2'} - \vec{V_1'})$

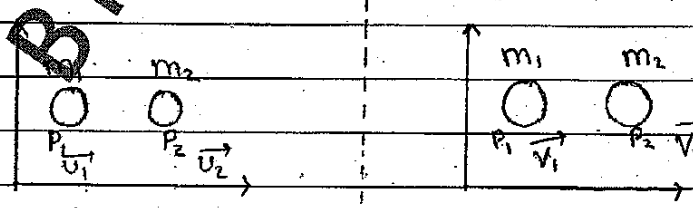
$\Rightarrow \quad (\vec{V_2'} - \vec{V_1'})t \times (\vec{V_2'} - \vec{V_1'}) + (\vec{r_2} - \vec{r_1}) \times (\vec{V_2'} - \vec{V_1'}) = 0$

$$\boxed{(\vec{r_2} - \vec{r_1}) \times (\vec{V_2'} - \vec{V_1'}) = 0}$$ H.P.

Law of momentum conservation :-

Let us consider a reference frame S in which particle P_1 and P_2 having mass m_1 and m_2.

Before collision, velocity of particle are $\vec{U_1}$ and $\vec{U_2}$. and after collision, velocity is $\vec{V_1}$ and $\vec{V_2}$



Bf collision — After collision

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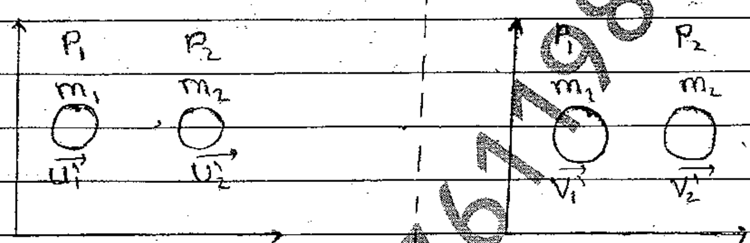
Acc. to law of momentum conservation

$$\vec{P_i} = \vec{P_f}$$

$$\Rightarrow \quad m_1\vec{u_1} + m_2\vec{u_2} = m_1\vec{v_1} + m_2\vec{v_2} \quad \text{---(I)}$$

let another inertial frame S' in which particle P_1 and P_2 having mass m_1 and m_2. before collision velocity of particle is $\vec{u_1'}$ and $\vec{u_2'}$ and after collision velocity are $\vec{v_1'}$ and $\vec{v_2'}$.



By collision   After collision

Acc. to law of momentum conservation

$$p_i = p_f$$

$$\Rightarrow \quad m_1\vec{u_1'} + m_2\vec{u_2'} = m_1\vec{v_1'} + m_2\vec{v_2'} \quad \text{---(II)}$$

Acc. to galilean transformation eq^n

$$\vec{u'} = \vec{u} - \vec{v}$$

so,
$$\left.\begin{array}{ll} \vec{u_1'} = \vec{u_1} - \vec{v} & \vec{v_1'} = \vec{v_1} - \vec{v} \\ \vec{u_2'} = \vec{u_2} - \vec{v} & \vec{v_2'} = \vec{v_2} - \vec{v} \end{array}\right\} \text{(III)}$$

put value of eq^n (III) in eq. (II)

$$\Rightarrow m_1(\vec{u_1} - \vec{v}) + m_2(\vec{u_2} - \vec{v}) = m_1(\vec{v_1} - \vec{v}) + m_2(\vec{v_2} - \vec{v})$$

$$\Rightarrow m_1\vec{u_1} - m_1\vec{v} + m_2\vec{u_2} - m_2\vec{v} = m_1\vec{v_1} - m_1\vec{v} + m_2\vec{v_2} - m_2\vec{v}$$

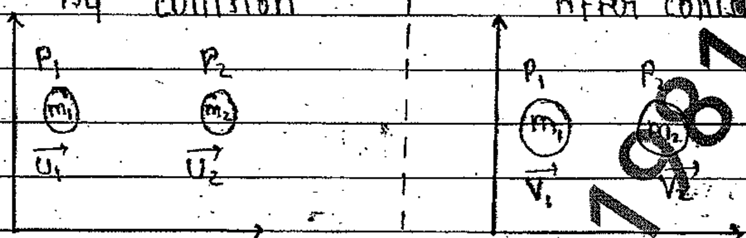
$$\Rightarrow \quad m_1\vec{u_1} + m_2\vec{u_2} = m_1\vec{v_1} + m_2\vec{v_2} \quad \text{---(IV)}$$

by (I) and (IV), we can say that under Galilean transformation, momentum is invariant qty.

Energy Conservation :-

let us consider a ref. frame S in which two particles P_1 and P_2 mass of P_1 and P_2 are m_1 and m_2.
B4 collision their velocity are $\vec{u_1}$ and $\vec{u_2}$. After collision their velocity are $\vec{v_1}$ and $\vec{v_2}$.

B4 collision | After collision

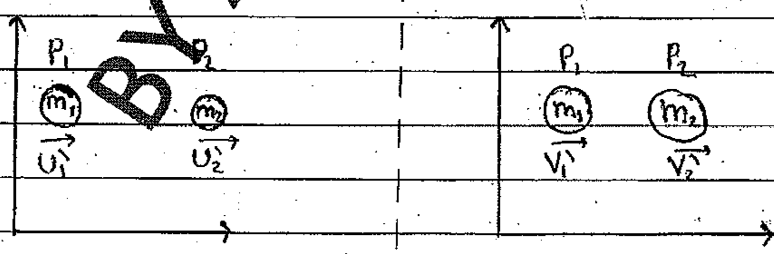


According to law of energy conservation

$$E_i = E_f$$

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + Q \quad \text{---(i)}$$

let's consider a reference frame S' in which two particle P_1 and P_2 having mass and velocity m_1, m_2 and $\vec{u_1'}, \vec{u_2'}$ before collision and $\vec{v_1'}, \vec{v_2'}$ after collision.



B4 collision | After collision

Acc. to law of conservation of energy

$$\Rightarrow \frac{1}{2}m_1u_1'^2 + \frac{1}{2}m_2u_2'^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 + Q \quad \text{---(II)}$$

By galilean transformation eq^n

$$\left.\begin{aligned} \vec{u_1'} &= \vec{u_1} - \vec{v} \; ; \quad \vec{v_1'} = \vec{v_1} - \vec{v} \\ \vec{u_2'} &= \vec{u_2} - \vec{v} \; ; \quad \vec{v_2'} = \vec{v_2} - \vec{v} \end{aligned}\right\} \text{(III)}$$

put value of eq^n (III) in (II)

$$\Rightarrow \frac{1}{2}m_1(\vec{u_1} - \vec{v})^2 + \frac{1}{2}m_2(\vec{u_2} - \vec{v})^2 = \frac{1}{2}m_1(\vec{v_1} - \vec{v})^2 + \frac{1}{2}m_2(\vec{v_2} - \vec{v})^2 + Q$$

$$\Rightarrow \frac{1}{2}m_1(u_1^2 - 2\vec{u_1}\cdot\vec{v} + v^2) + \frac{1}{2}m_2(u_2^2 - 2\vec{u_2}\cdot\vec{v} + v^2) = \frac{1}{2}m_1(v_1^2 - 2\vec{v_1}\cdot\vec{v} + v^2)$$
$$+ \frac{1}{2}m_2(v_2^2 - 2\vec{v_2}\cdot\vec{v} + v^2) + Q$$

$$\Rightarrow \frac{1}{2}m_1u_1^2 - m_1\vec{u_1}\cdot\vec{v} + \frac{1}{2}\cancel{m_1v^2} + \frac{1}{2}m_2u_2^2 - m_2\vec{u_2}\cdot\vec{v} + \frac{1}{2}\cancel{m_2v^2} =$$
$$\frac{1}{2}m_1v_1^2 - m_1\vec{v_1}\cdot\vec{v} + \frac{1}{2}\cancel{m_1v^2} + \frac{1}{2}m_2v_2^2 - m_2\vec{v_2}\cdot\vec{v} + \frac{1}{2}\cancel{m_2v^2} + Q$$

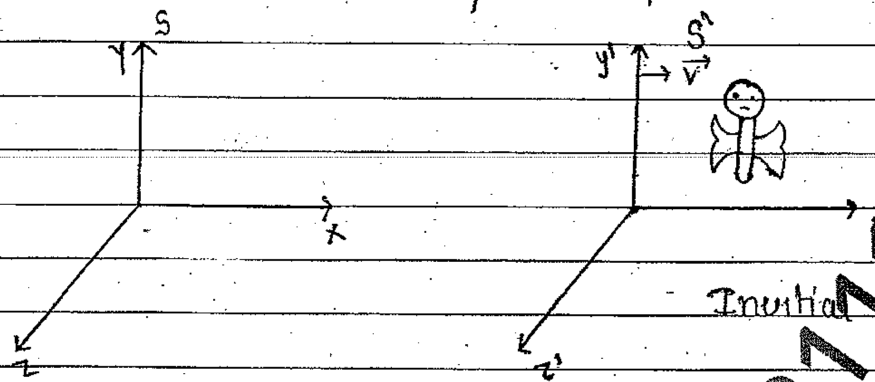
$$\Rightarrow \frac{1}{2}m_1u_1^2 - \vec{P_1}\cdot\cancel{\vec{v}} + \frac{1}{2}m_2u_2^2 - \vec{P_2}\cdot\cancel{\vec{v}} = \frac{1}{2}m_1v_1^2 - P_1\cdot\cancel{\vec{v}} + \frac{1}{2}m_2v_2^2 \mp P_2\cdot\cancel{\vec{v}}$$

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{---(IV)}$$

by (I) and (IV), we can say that under galilean transformation energy conservation is invariant qty.

2021

Newtonian Relativity Principle :-



Inertial frame of ref.

Velocity of Bird in frame 'S'

$$\vec{U} = U_x\hat{i} + U_y\hat{j} + U_z\hat{k}$$

In S' velocity,

$$\vec{U'} = U'_x\hat{i} + U'_y\hat{j} + U'_z\hat{k}$$

let S' is moving along x-axis dirxn
acc. to galilean eqn.

$$\vec{U'} = \vec{U} - \vec{V_x}$$

$$\vec{x'} = \vec{x} - V_x t$$
$$\vec{y'} = \vec{y}$$
$$\vec{z'} = \vec{z}$$
$$t' = t$$

In vector form:

$$x'\hat{i} + y'\hat{j} + z'\hat{k} = x\hat{i} + y\hat{j} + z\hat{k} - V_x t + t$$

D. w.r.t 't'

$$\boxed{\vec{U'} = \vec{U} - \vec{V}}$$

Under galilean transformation velocity is not absolute.

Acceleration

$$\frac{d\vec{v'}}{dt} = \frac{d\vec{v}}{dt} - \frac{d}{dt}\vec{v}$$

$$\boxed{\vec{a'} = \vec{a}}$$

Force,

$$m\vec{a'} = m\vec{a}$$

$$\boxed{\vec{F'} = \vec{F}}$$

Statement of Newtonian Relativity :- All law of mechanics are same for any inertial frame of ref.

(i) Newton 1st law :- Acc. to this law, if a object is in rest, then it would remain in rest or if a object is in uniform motion, it would remain in motion until any external force work on it.

acc. to galilean transformation eq^n (Inertial F.O.R.)

$\vec{a}$ (S frame)

$\vec{a'}$ (S' frame)

$$\vec{a} = \vec{a'}$$

$$\vec{F'} = \vec{F}$$

$$\boxed{m\vec{a'} = m\vec{a}}$$

(ii) Newton 2nd law :- Rate of change of momentum is equal to net force of object.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

$$\vec{P} = m\vec{V}$$

$$\vec{F}_{net} = \frac{d}{dt} m\vec{V}$$

$$\boxed{F_{net} = \vec{V}\frac{dm}{dt} + m\frac{d\vec{V}}{dt}}$$

This formula is valid for all conditions when mass is variable or invariable.

Case I. If $m =$ constant

$$\frac{dm}{dt} = 0$$

$$\vec{F}_{net} = m\frac{d\vec{V}}{dt}$$

$$\boxed{\vec{F}_{net} = m\vec{a}}$$

3. Newton's 3rd law / Action- Reaction law :-

Acc. to this, whenever one object acts a force on 2nd object, 2nd object act equal but opposite force on 1st object so this law is also called Action - Reaction law.

Unit I

ch-2

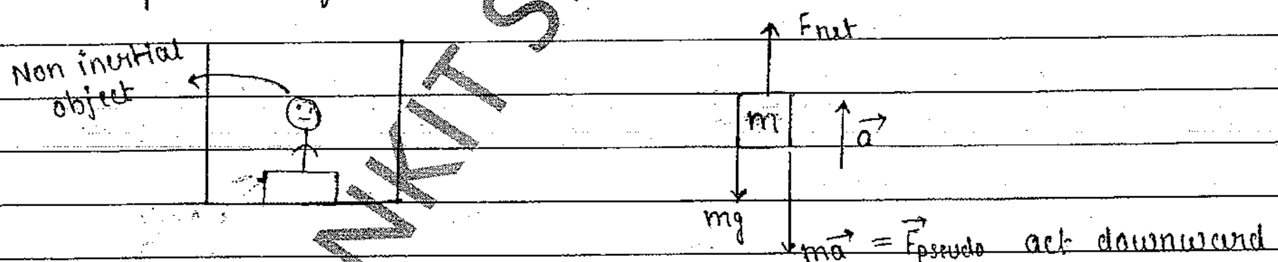
"Force acting on Rotating F.O.R"

Pseudo Force :-
A force which is not applied in real but seems to be exerted on particle is called pseudo force.

$$\vec{F}_{pseudo} = -m\vec{a}$$

Here
(−) = force act on particle is in opposite dirxn
m = object mass
a = acceleration of object

Ex. 1
let a lift is going upward with $\vec{a}$ acceleration then pseudo force will act downward.

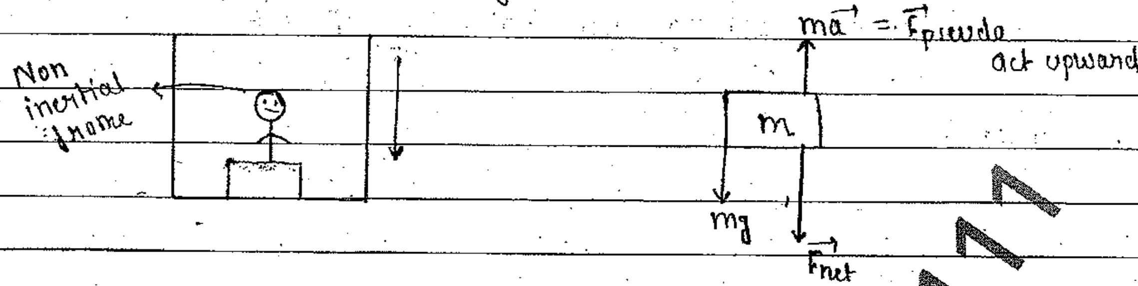


$$\vec{F}_{net} - mg - m\vec{a} = 0$$

$$\boxed{F_{net} = m(\vec{g} + \vec{a})}$$

that's why in this condn, weight of particle seems high.

Ex-2. If a lift goes downward with $\vec{a}$ acceleration



$$m\vec{a} = mg + F_{net}$$

$$\boxed{F_{net} = m(\vec{a} - g)}$$

that's why weight of object ~~same~~ less

Q. Explain pseudo force with example.

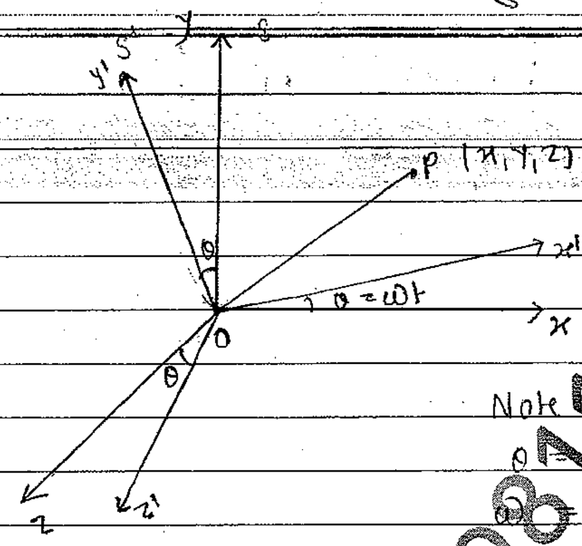
Rotating Frame of ref. & Coriolis force :-

A frame which is rotating about any point or axis is called rotatory or rotating frame of ref.

Earth is a rotating frame of ref.

Earth rotates along axis joining N to S-pole from west to east

let a frame of ref. 'S' in which a point 'P' having co-ordinate (x, y, z) and position vector w.r.t. origin $\vec{OP} = \vec{r}$



Note
θ = angular displacement
$\omega = \frac{\theta}{t}$

$$\boxed{\theta = \omega t}$$

S' F.O.R. moves with angular velocity (ω) w.r.t. 'S' frame of refrence.
In initial state (t = 0) both frame will be coincided and at 't' time, x-axis move with θ = ωt angle.
Here both frame moves w.r.t. origin
In S refrence frame, position vector of P w.r.t. origin

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k} \quad \text{---(I)}$$

Here position vector of P w.r.t. origin in frame S'

$$\vec{r'} = x'\hat{i'} + y'\hat{j'} + z'\hat{k'} \quad \text{---(II)}$$

$$\vec{r} = \vec{r'} = \vec{OP}$$

by eq. (I) and (II)

$$\boxed{\vec{r} = x'\hat{i'} + y'\hat{j'} + z'\hat{k'}} \quad \text{---(III)}$$

eq^n (III) represent position vector in rotating frame of refrence.

* Velocity Transformation eq^n :-

Differentiate eq. (III) w.r.t 't'

$$\vec{r} = x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$$

$$\frac{d\vec{r}}{dt} = \hat{i}'\frac{dx'}{dt} + x'\frac{d\hat{i}'}{dt} + y'\frac{d\hat{j}'}{dt} + \hat{j}'\frac{dy'}{dt} + \hat{k}'\frac{dz'}{dt} + z'\frac{d\hat{k}'}{dt}$$

$$\frac{d\vec{r}}{dt} = \left(\hat{i}'\frac{dx'}{dt} + \hat{j}'\frac{dy'}{dt} + \hat{k}'\frac{dz'}{dt}\right) + \left(x'\frac{d\hat{i}'}{dt} + y'\frac{d\hat{j}'}{dt} + z'\frac{d\hat{k}'}{dt}\right) \quad \text{—(IV)}$$

Taking velocity and operator $\left(\frac{d}{dt}\right)$ in S frame of ref.

$$\frac{d}{dt}\vec{r} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\vec{v} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) \quad \text{—(V)}$$

let velocity & operator $\vec{v}'$ & $\frac{d'}{dt}$ in S' ref frame.

$$\frac{d'\vec{r}'}{dt} = \frac{d'}{dt}(x'\hat{i}' + y'\hat{j}' + z'\hat{k}')$$

$$\vec{v}' = \hat{i}'\frac{d'x'}{dt} + \hat{j}'\frac{d'y'}{dt} + \hat{k}'\frac{d'z'}{dt}$$

In initial state at $t=0$

$$\frac{d'}{dt} = \frac{d}{dt}$$

$$\vec{v}' = \hat{i}'\frac{dx'}{dt} + \hat{j}'\frac{dy'}{dt} + \hat{k}'\frac{dz'}{dt} \quad \text{—(VI)}$$

let a vector is $\vec{R}$ whose relation in Rotating frame of refrence given below

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$\because \quad \vec{V} = \vec{\omega} \times \vec{R}$

$\vec{\omega}$ = angular velocity

$\vec{V}$ = linear velocity

$$\frac{d\vec{R}}{dt} = \vec{\omega} \times \vec{R}$$

On putting $\hat{i}', \hat{j}', \hat{k}'$ @ place of $\vec{R}$

$$\frac{d\hat{i}'}{dt} = \vec{\omega} \times \hat{i}' \quad \text{—(VII)}$$

$$\frac{d\hat{j}'}{dt} = \vec{\omega} \times \hat{j}' \quad \text{—(VIII)}$$

$$\frac{d\hat{k}'}{dt} = \vec{\omega} \times \hat{k}' \quad \text{—(IX)}$$

by eq: (V) (VI) (VII) (VIII) & (IX) put eq^n in (IV)

$$\frac{dr}{dt} = \vec{v'} + \left[x'(\vec{\omega} \times \hat{i}') + y'(\vec{\omega} \times \hat{j}') + z'(\vec{\omega} \times \hat{k}')\right]$$

$$\vec{v} = \vec{v'} + \vec{\omega} \times (x'\hat{i}' + y'\hat{j}' + z'\hat{k}')$$

$$\vec{v} = \vec{v'} + \vec{\omega} \times \vec{r'}$$

$$\because \vec{r} = \vec{r'}$$

$$\boxed{\vec{v} = \vec{v'} + \vec{\omega} \times \vec{r}} \quad \text{—— (X)}$$

$\vec{v'}$ = Velocity in 'S'' F.O.R.

$\vec{v}$ = Velocity in 'S' F.O.R.

eq^n (X) represent Velocity transformation eq^n

* Acceleration Transformation eqⁿ :-

We know that,

$$\vec{v'} = \vec{v} - (\vec{\omega} \times \vec{r})$$

$$\vec{v} = \vec{v'} + (\vec{\omega} \times \vec{r})$$

$$\frac{dr}{dt} = \frac{d'r}{dt} + (\vec{\omega} \times \vec{r})$$

$$\frac{d}{dt} = \frac{d'}{dt} + \vec{\omega}\times \quad \text{— (A)} \quad @ \text{'t' time.}$$

$\because \frac{d}{dt}$ = operator in 'S'

$\frac{d'}{dt}$ = operator in S'

We know

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) \quad \text{— (B)}$$

by A and B

$$\vec{a} = \left(\frac{d'}{dt} + \vec{\omega}\times\right)\left[\frac{d'\vec{r}}{dt} + (\vec{\omega}\times\vec{r})\right]$$

$$\vec{a} = \frac{d'}{dt}\cdot\frac{d'r}{dt} + \frac{d'}{dt}(\vec{\omega}\times\vec{r}) + \vec{\omega}\times\frac{d'\vec{r}}{dt} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{a} = \frac{d'^2 r}{dt^2} + \vec{\omega}\times\frac{d'\vec{r}}{dt} + \vec{r}\times\frac{d'\vec{\omega}}{dt} + \vec{\omega}\times\frac{d'\vec{r}}{dt} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{a} = \frac{d'^2\vec{r}}{dt^2} + 2\vec{\omega}\times\frac{d'\vec{r}}{dt} + 0 + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{a} = \vec{a'} + 2(\vec{\omega}\times\vec{v'}) + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{a'} = \vec{a} - 2(\vec{\omega}\times\vec{v'}) - \vec{\omega}\times(\vec{\omega}\times\vec{r}) \quad \text{— (XI)}$$

eqⁿ (XI) is known as Acceleration Transformation eqⁿ

$a_c = -2(\vec{\omega}\times\vec{v'})$ = Coriolis ~~force~~ acceleration

$a_f = -\vec{\omega}\times(\vec{\omega}\times\vec{r})$ = Centrifugal ~~force~~ acceleration

* Force Transformation eq^n :-

multiply eq.(XI) by m

$$m\vec{a'} = m\vec{a} - 2m(\vec{\omega}\times\vec{v'}) - m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{F'} = \vec{F} - \vec{F_c} - \vec{F_f}$$

$$\vec{F_c} = -2m(\vec{\omega}\times\vec{v'}) = \text{coriolis force}$$

$$\vec{F_f} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}) = \text{Centrifugal force}$$

$$\vec{F_c} = -2m(\vec{\omega}\times\vec{v'})$$

$$|\vec{F_c}| = -2m|\vec{\omega}||\vec{v'}|\sin\theta\,\hat{n}$$

$\theta = 0$ (b/w $\vec{\omega}$ and $\vec{v'}$)

$$|\vec{F_c}| = -2m\omega v'(0)$$

$$\boxed{|\vec{F_c}| = 0}$$

If $\theta = 90°$

$$\boxed{|\vec{F_c}| = -2m\omega v'}$$

Effect of Centrifugal force :-

Note (i) Earth complete revolution in 24 hrs. about its axis.

(angular velocity) $\omega = \frac{2\pi}{T} = \frac{2\pi}{24\times60\times60} = 7.29\times10^{-5}$ rad./sec.

Earth rotates along the axis joining N to S pole from west to east. Angular velocity [$\omega = 7.29\times10^{-5}$ rad./sec.] of earth is very less.

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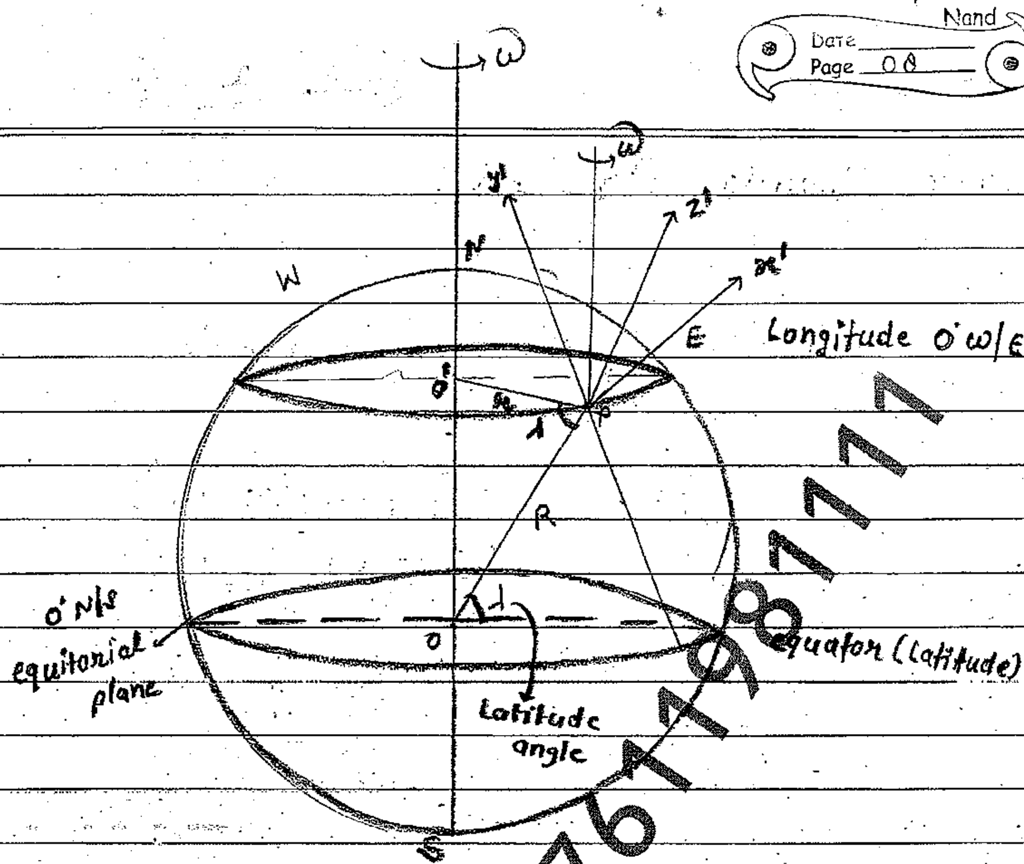
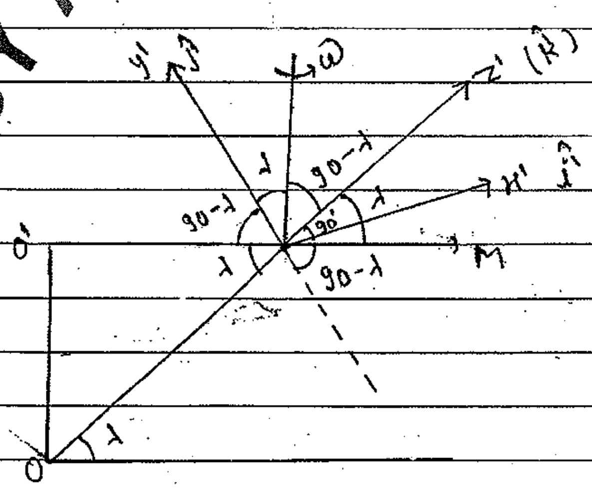
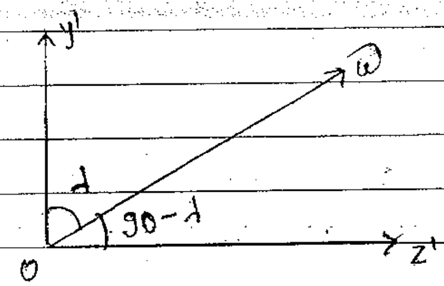


let's consider a particle situated on earth is stationary condition. this particle feels an imaginary force. This imaginary force is called Centrifugal force. Particle is situated at point 'P'.
To explain the position of particle, we imagine a refrence frame where x-axis is in east, y'-axis toward north and z-axis vertically upward.



Angular velocity (ω) makes λ, $90-\lambda$, with y' and z'-axis respectively.



On dividing ω into components

$$\vec{\omega} = \omega \cos(90-\lambda)\hat{k} + \omega \cos\lambda\, \hat{j}'$$

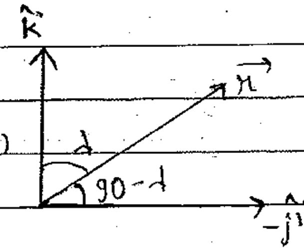
$$\vec{\omega} = \omega \cos\lambda\, \hat{j}' + \omega \sin\lambda\, \hat{k} \quad \text{---(i)}$$

On dividing $\vec{r}$ into component

$\vec{r}$; x-axis and y-axis makes $(90-\lambda)$ and z-axis $= \lambda$

$$\vec{r} = r\cos(90-\lambda)(-\hat{j}') + r\cos\lambda\, \hat{k}$$

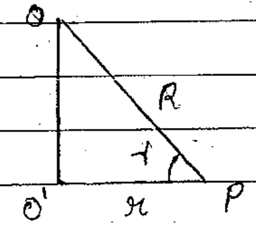
$$\vec{r} = -r\sin\lambda\, \hat{j}' + r\cos\lambda\, \hat{k} \quad \text{---(ii)}$$



by $\Delta OPO'$

$$\cos\lambda = \frac{r}{R}$$

$$r = R\cos\lambda \quad \text{---(iii)}$$



By acceleration transformation eq^n of rotating frame of ref.

$$\vec{a'} = \vec{a} - 2(\vec{\omega} \times \vec{v'}) - \vec{\omega} \times (\vec{\omega} \times \vec{r}) \quad \text{---(iv)}$$

$a_c = -2(\vec{\omega} \times \vec{v'})$ = Coriolis acceleration

$a_f = -\vec{\omega} \times (\vec{\omega} \times \vec{r'})$ = Centrifugal acceleration

Note:

→ Coriolis force act only on moving particle

→ Centrifugal force act on both stationary & moving particle,

so, $\vec{a_c} = -2(\vec{\omega} \times \vec{v}) = 0$ (P is stationary)

$a_f = -\vec{\omega} \times (\vec{\omega} \times \vec{r})$

by eq^n(IV)

$$\vec{a'} = \vec{a} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$\vec{a'} = \vec{g_1}$ (apparent acceleration due to gravity)

$\vec{a} = \vec{g}$ (real acceleration due to gravity toward centre)

$\vec{g} = g(-\hat{k})$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \omega\cos\lambda & \omega\sin\lambda \\ 0 & -r\sin\lambda & r\cos\lambda \end{vmatrix}$$

$\cancel{\vec{\omega} \times \vec{r} = \hat{i}(\omega r\cos^2\lambda) - \hat{j}(0) + \hat{k}(0)}$

$$\vec{\omega} \times \vec{r} = \hat{i}(\omega r\cos^2\lambda + \omega r\sin^2\lambda)$$

$$\vec{\omega} \times \vec{r} = \omega r\,\hat{i} \quad \text{---(vi)}$$

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$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \omega\cos\lambda & \omega\sin\lambda \\ \omega r & 0 & 0 \end{vmatrix}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \hat{i}(0) - \hat{j}(-\omega^2 r\sin\lambda) + \hat{k}(-\omega^2 r\cos\lambda)$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \omega^2 r\sin\lambda\,\hat{j} - \omega^2 r\cos\lambda\,\hat{k} \quad \text{—(VII)}$$

$$g_t = g(-\hat{k}) + \omega^2 r\cos\lambda\,\hat{k} - \omega^2 r\sin\lambda\,\hat{j}$$

$$g_t = (-\hat{k})(g - \omega^2 r\cos\lambda\,\hat{k}) - \omega^2 r\sin\lambda\,\hat{j}$$

by eq. (III)

$$r = R\cos\lambda$$

$$\vec{g_t} = (-\hat{k})(g - \omega^2 R\cos^2\lambda\,\hat{k}) - \omega^2 R\sin\lambda\cdot\cos\lambda\,\hat{j}$$

$$|\vec{g_t}| = \sqrt{(g - \omega^2 R\cos^2\lambda)^2 + (\omega^4 R^2\sin^2\lambda\cdot\cos\lambda^2)}$$

$\because$ $\omega = 7.29 \times 10^{-5}$ rad./sec.

$\omega^2 = (7.29)^2 \times 10^{-10}$ rad./sec.

$\omega^4 \approx 0\ (7.29)^2 \times 10^{-40}$ rad./sec. (very small)

$$|\vec{g_t}| = \sqrt{(g - \omega^2 R\cos^2\lambda)^2}$$

$$\boxed{|\vec{g_t}| = g - \omega^2 R\cos^2\lambda} \quad \text{—(VIII)}$$

eq^n (VIII) describe effect of rotation of earth on acceleration due to gravity.

* Direction of apparent acceleration :-

$$\vec{g_t} = (g - \omega^2 R\cos^2\lambda)(-\hat{k}) - \omega^2 R\sin\lambda\cdot\cos\lambda\,\hat{j}$$

$$\tan\theta = \frac{y}{x}$$

$$\tan\theta = \frac{-\omega^2 R \sin\lambda \cdot \cos\lambda}{-(g-\omega^2 R\cos^2\lambda)}$$

$g > \omega,\ g >> \omega^2$

$$\tan\theta = \frac{\omega^2 R \sin\lambda \cdot \cos\lambda}{g}$$

$$\tan\theta = \frac{\omega^2 R \cdot 2\sin\lambda \cdot \cos\lambda}{2g}$$

$$\tan\theta = \frac{\omega^2 R \sin 2\lambda}{2g}$$

R.U. Imp.

$$\boxed{\theta = \tan^{-1}\left(\frac{\omega^2 R \sin 2\lambda}{2g}\right)} \quad *** \quad \text{---(IX)}$$

Case I,

@ pole, $\cancel{\theta =}\ \lambda = 90^\circ$ (latitude angle)

by eq^n (VIII) & (IX)

$$g_\lambda = g - \omega^2 R\cos^2\lambda$$

$$= g - \omega^2 R\cos^2 90^\circ$$

$$\boxed{g_\lambda = g}$$

$$\theta = \tan^{-1}\left(\frac{\omega^2 R \sin 2\times 90^\circ}{2g}\right)$$

$$\theta = \tan^{-1}(0)$$

$$\boxed{\theta = 0^\circ}$$

No change in value of 'g'

@ pole,

apparent acceleration = real acceleration

Date
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Case II @ equator, $\lambda = 0°$

$$g_\lambda = g - \omega^2 R\cos^2 0°$$

$$\boxed{g_\lambda = g - \omega^2 R}$$

$$\theta = \tan^{-1}\left(\frac{\omega^2 R\sin 2\lambda}{2g}\right)$$

$$\theta = \tan^{-1}(0)$$

$$\boxed{\theta = 0°}$$

Case III @ $\lambda = 45°$

$$g_\lambda = g - \omega^2 R\cos^2 45°$$

$$\boxed{g_\lambda = g - \frac{\omega^2 R}{2}}$$

$$\theta = \tan^{-1}\left(\frac{\omega^2 R\sin 2\times 45°}{2g}\right)$$

$$\boxed{\theta = \tan^{-1}\left(\frac{\omega^2 R}{2g}\right)}$$

Effect of Coriolis force on a particle moving Horizontally on Earth :-

Let's consider a particle of mass 'm' moving horizontally on earth. Velocity of particle in x and y-axis is V'_x and V'_y respectively.

$$\vec{V'} = V'_x\hat{i} + V'_y\hat{j} \quad \text{—(i)}$$

$$\vec{\omega} = \omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k} \quad \text{—(ii)}$$

We know that

Coriolis force $\vec{F_c} = -2m(\vec{\omega'} \times \vec{v'})$

$$\vec{\omega'} \times \vec{v'} = \begin{vmatrix} \hat{i'} & \hat{j'} & \hat{k} \\ 0 & \omega\cos\lambda & \omega\sin\lambda \\ V'_x & V'_y & 0 \end{vmatrix}$$

$$\vec{\omega'} \times \vec{v'} = \hat{i'}(-V'_y\omega\sin\lambda) - \hat{j'}(0 - V'_x\omega\sin\lambda) + \hat{k}(-V'_x\omega\cos\lambda)$$

$$\vec{\omega'} \times \vec{v'} = -V'_y\omega\sin\lambda\,\hat{i'} + V'_x\omega\sin\lambda\,\hat{j'} - V'_x\omega\cos\lambda\,\hat{k}$$

$$\vec{F_{ch}} = 2m(V'_y\omega\sin\lambda\,\hat{i'} - V'_x\omega\sin\lambda\,\hat{j'} + V'_x\omega\cos\lambda\,\hat{k})$$

Horizontal component of coriolis force in $(x'-y')$

$$\vec{F_{ch}} = 2m(V'_y\omega\sin\lambda\,\hat{i'} - V'_x\omega\sin\lambda\,\hat{j'})$$

Vertical component of coriolis force

$$\vec{F_{ch}} = 2m\,V'_x\,\omega\cos\lambda\,\hat{k}$$

Magnitude of Horizontal component

$$\cancel{|\vec{F_{ch}}| = \sqrt{(2m)^2[(V'_y\omega\sin\lambda\,\hat{k}\,\hat{i})^2 - (V'_x\omega\sin\lambda)}}$$

$$|\vec{F_{ch}}| = \sqrt{(2m)^2((V'_y\omega\sin\lambda\,\hat{i'})^2 - (V'_x\omega\sin\lambda\,\hat{j'})^2)}$$

$$\boxed{|\vec{F_{ch}}| = 2m\omega\sin\lambda\sqrt{(V'_x)^2 + (V'_y)^2}}$$

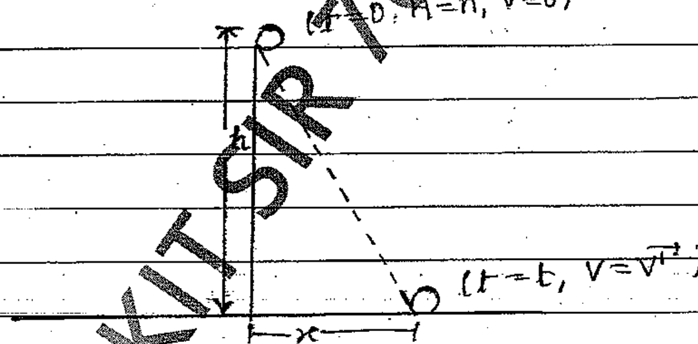
Thus horizontal component of coriolis force acts toward right in Northern Hemisphere and left in Southern Hemisphere.

The vertical component of force act always upward in both hemisphere.

Coriolis force describe natural phenomena.

Effect of coriolis force on body falling vertically downward on earth :-

let a body having mass 'm' falling downward from height 'h' due to gravity here velocity of body reach at surface $\vec{v}$ and time 't'.



Note :-
$\hat{i}$ = east dirxn
$\hat{j}$ = North
$\hat{k}$ = Vertically upward

We know that earth is a rotating frame of refrence in which a particle is moving hence coriolis force will act.

Coriolis force $\vec{F_c} = -2m(\vec{\omega} \times \vec{v'})$ —(i)

$\omega = \omega \cos\lambda \hat{j} + \omega \sin\lambda \hat{k}$ —(ii)

Velocity of particle (From 1st eqn of motion)

$$v = u + at \quad \text{—(iii)}$$

v = final velocity

u = Initial velocity

$\vec{a} = g(-\hat{k})$

$v = v'$

$u = 0$

$v' = 0 - gt\hat{k}$

$v' = -gt\hat{k}$ —(iv)

$$F_c = -2m(\vec{\omega} \times \vec{v'})$$

$$\vec{\omega} \times \vec{v'} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \omega\cos\lambda & \omega\sin\lambda \\ 0 & 0 & -gt \end{vmatrix}$$

$$\vec{\omega} \times \vec{v'} = \hat{i}(-gt\,\omega\cos\lambda) - \hat{j}(0) + \hat{k}(0)$$

$$\vec{\omega} \times \vec{v'} = -gt\,\omega\cos\lambda\,\hat{i}$$

$$\vec{F_c} = -2m(-gt\,\omega\cos\lambda\,\hat{i})$$

$$\vec{F_c} = 2mgt\,\omega\cos\lambda\,\hat{i} \quad \text{—— (v)}$$

When a particle fall from upward, then coriolis force acts in east dirxn in northern Hemisphere means particle move in east dirxn.

Acc. to Newton 2nd law

$$\vec{F} = m\frac{d^2x}{dt^2} \quad \text{—— (VI)}$$

by (V) & (VI)

$$m\frac{d^2x}{dt^2} = 2mgt\,\omega\cos\lambda$$

$$\frac{d^2x}{dt^2} = 2gt\,\omega\cos\lambda$$

Integerate above eqn.

$$\frac{dx}{dt} = 2g\frac{t^2}{2}\,\omega\cos\lambda + c$$

$$\frac{dx}{dt} = gt^2\,\omega\cos\lambda + c$$

@ $t = 0$

$$\frac{dx}{dt} = 0$$

$$0 = 0 + c$$

$$\boxed{c = 0}$$

$$\frac{dx}{dt} = gt^2\,\omega\cos\lambda \quad \text{—— (VII)}$$

again integerate eq. (VII)

$$x = \frac{1}{3}gt^3\,\omega\cos\lambda + c$$

@ $t = 0$, $x = 0$, $c = 0$

$$x = \frac{1}{3}gt^3\,\omega\cos\lambda \quad \text{—— (VIII)}$$

eqn (VIII) represent displacement of body falling downward.

Note :-

If coriolis force acting on a particle falling downward in southern hemisphere then force act in west dirxn, motion of body is in west dirxn

From second eqn. of motion,

$$s = ut + \frac{1}{2}at^2$$

s = displacement
u = initial velocity
t = time

$s = h, \quad u = 0, \quad a = g$

$$h = 0 + \frac{1}{2}gt^2$$

$$h = \frac{gt^2}{2}$$

$$t = \left(\frac{2h}{g}\right)^{1/2} \quad \text{--- (IX)}$$

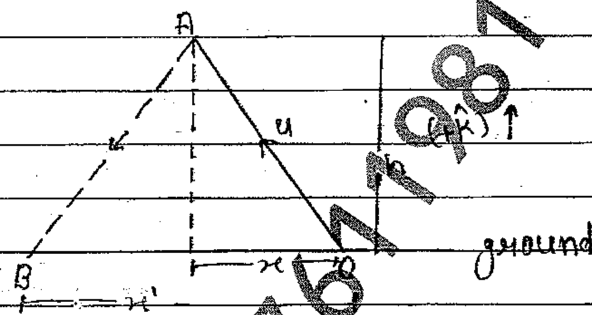
eqn. (IX) represent total time taken ~~displacement~~ of body falling downward

put value of t in eq. (VIII)

$$\boxed{x = \frac{1}{3} g\omega \left(\frac{2h}{g}\right)^{3/2} \cos\lambda} \quad \text{--- X}$$

Effect of coriolis force on Bodies thrown vertically upward from earth.

let a particle of mass 'm' is thrown vertically upward to a certain height of earth.



Earth is rotating hence coriolis force acts on particle

$$F_c = -2m(\vec{\omega} \times \vec{v'}) \quad \text{—(i)}$$

$$\vec{\omega} = \omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}$$

By 1st eqn of motion

$$V = u + at \quad \text{—(ii)}$$

$$V = V'$$

$$u = u(\hat{k})$$

$$a = -g\hat{k}$$

$$v' = u\hat{k} - g\hat{k}t$$

$$v' = (u - gt)\hat{k} \quad \text{—(iii)}$$

$$F_c = -2m(\vec{\omega} \times \vec{v'})$$

$$(\omega \times \vec{v'}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \omega\cos\lambda & \omega\sin\lambda \\ 0 & 0 & u-gt \end{vmatrix}$$

$$\vec{F_c} = -2m(\vec{\omega} \times \vec{v'}) = -2m(U-gt)\,\omega\cos\lambda\,\hat{i'}$$

$$\vec{F_c} = 2m(U-gt)\,\omega\cos\lambda\,(-\hat{i'}) \quad \text{---(iv)}$$

$\hat{i'}$ = east dirxn

$(-\hat{i'})$ = west dirxn

Hence coriolis force acting on particle thrown vertically upward in west dirxn in Northern Hemisphere.

By Newton's law

$$\vec{F} = m\frac{d^2x'}{dt^2} \quad \text{---(v)}$$

by (iv) and (v)

$$m\frac{d^2x'}{dt^2} = -2m(U-gt)\,\omega\cos\lambda\,\hat{i'}$$

$$\frac{d^2x'}{dt^2} = -2(U-gt)\,\omega\cos\lambda\,\hat{i'}$$

On integrating,

$$\frac{dx'}{dt} = -2\left(Ut - \frac{gt^2}{2}\right)\omega\cos\lambda\,\hat{i'} + c$$

@ $t = 0$, $\frac{dx'}{dt} = 0$

$c = 0$

$$\frac{dx'}{dt} = -2\left(Ut - \frac{gt^2}{2}\right)\omega\cos\lambda$$

$$\frac{dx'}{dt} = -2\omega g\left(\frac{U}{g}t - \frac{t^2}{2}\right)\cos\lambda \quad \text{---(vi)}$$

$\frac{dx'}{dt}$ = again integerate eq.(vi)

$$x' = -2\omega g\left(\frac{U}{g}\cdot\frac{t^2}{2} - \frac{t^3}{6}\right)\cos\lambda + c$$

$$x' = -\omega g\left(\frac{ut^2}{g} - \frac{t^3}{3}\right)\cos\lambda + c$$

@ $t=0$, $c=0$

$$x' = -\omega g\left(\frac{u}{g}t^2 - \frac{1}{3}t^3\right)\cos\lambda$$

by $v = u + at$

$a = g(-\hat{k})$

$t = t$, $v = 0$ (@ $\max^m$ height)

$0 = (u - gt)\hat{k}$

$u - gt = 0$

$$t = \frac{u}{g} \quad \text{— (VIII)}$$

by 3^{rd} eq^n of motion

$v^2 = u^2 + 2as$

$v = 0$ ($\max^m$ height)

$\vec{a} = g(-\hat{k})$

$0 = u^2 - 2gh$

$u = \sqrt{2gh}$ — (IX)

from eq (VI)

$$\frac{dx'}{dt} = -2\omega g\cos\lambda\left(\frac{u}{g}\cdot\frac{u}{g} - \frac{u^2}{2g^2}\right)$$

$$= -2\omega g\cos\lambda\left(\frac{u^2}{2g^2}\right)$$

$$= -\omega g\cos\lambda\left(\frac{2gh}{g^2}\right)$$

$$\frac{dx'}{dt} = -\omega g\cos\lambda\left(\frac{2h}{g}\right) \quad \text{— (X) at point A}$$

from eq (VII)

$$x' = -\omega g\cos\lambda\left(\frac{u}{g}t^2 - \frac{t^3}{3}\right)$$

$$x' = -\omega g \cos\lambda \left(\frac{v}{g} \cdot \frac{v^2}{g^2} - \frac{v^3}{3g^3} \right)$$

$$x' = -\omega g \cos\lambda \left(\frac{2}{3} \frac{v^3}{g^3} \right)$$

$$x' = -\omega g \cos\lambda \frac{2}{3} \left(\frac{(2gh)^{3/2}}{g^3} \right)$$

$$x' = -\frac{2}{3} \omega g \frac{\cos\lambda (2gh)^{3/2}}{g^3}$$

$$x' = -\frac{2}{3} \omega \frac{\cos\lambda (2gh)\sqrt{2gh}}{g^2}$$

$$x' = -\frac{2}{3} \omega g \cos \left(\frac{2h}{g} \right) \frac{\sqrt{2gh}}{g}$$

$$x' = -\frac{2}{3} \omega g \cos\lambda \left(\frac{2h}{g} \right) \left(\frac{2h}{g} \right)^{1/2}$$

$$x' = -\frac{2}{3} \omega g \cos\lambda \left(\frac{2h}{g} \right)^{3/2} \quad \text{—— (XI) at A}$$

eq (X) and (XI) represent horizontal displacement & velocity. at point A for thrown a particle vertically upward on earth.

Relation b/w Force when particle falling from a certain height.

$$\frac{m d^2 x'}{dt^2} = 2m\omega g t \cos\lambda$$

$$\frac{d^2 x'}{dt^2} = 2\omega g t \cos\lambda \quad \text{—— (XII)}$$

Integerate eq^n (XII)

$$\frac{dx'}{dt} = \omega g t^2 \cos\lambda + c$$

@ $t = 0$, ~~$c = 0$~~ $\frac{dx'}{dt} = -\omega g \cos\lambda \left(\frac{2h}{g}\right)$

again integerate

$x' =$ $\quad c = -\omega g \cos\lambda \left(\frac{2h}{g}\right)$

$$\frac{dx'}{dt} = \omega g t^2 \cos\lambda - \omega g \cos\lambda \left(\frac{2h}{g}\right)$$

Integerate again

$$x' = \omega g \frac{t^3}{3} \cos\lambda - \omega g \cos\lambda \left(\frac{2h}{g}\right) t + c'$$

@ $t = 0$, $c' = -\frac{2}{3} \omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2}$

$$x' = \omega g \frac{t^3}{3} \cos\lambda - \omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2} \cancel{t} - \frac{2}{3} \omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2}$$

$$t = \frac{v}{g} = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}}$$

$$x' = \frac{\omega g}{3} \left(\frac{2h}{g}\right)^{3/2} \cos\lambda - \omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2} - \frac{2}{3} \omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2}$$

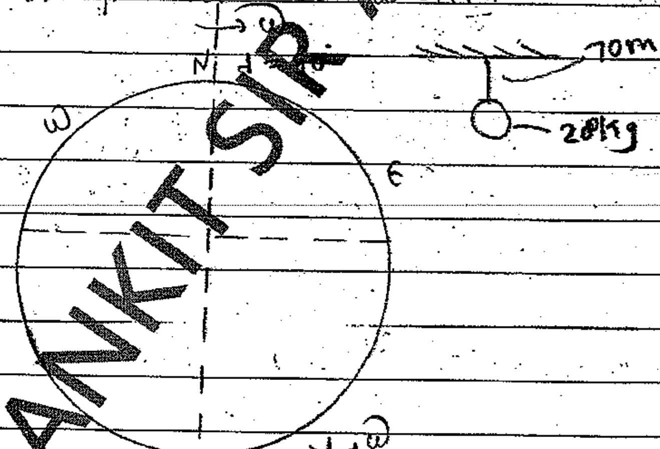
$$x' = \omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2} \left[\frac{1}{3} - 1 - \frac{2}{3}\right]$$

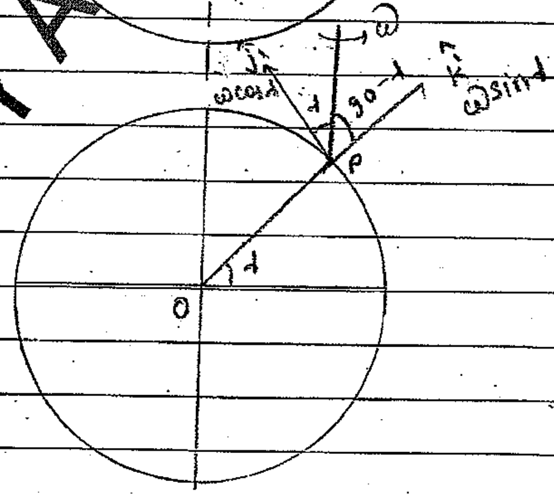
$$\boxed{x' = -\frac{4}{3} \omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2}}$$

Focault Pendulum / Effect of coriolis force on pendulum :-

Focault used 28 kg sphere tied with 70 m long thread. Time period was 17 sec. when focault pendulum carried to N-pole, coriolis force act prependicular to angular velocity of earth and velocity of pendulum. So both starts to rotate in opposite dirxn to rotation of earth in same manner.

So angular velocity of earth can be calculated using focault pendulum at latitude λ there be two component of coriolis force prependicular component be $\omega \sin\lambda$.





So angular velocity of earth with the help of focault pendulum.

If pendulum is carried at a point where latitudinal angle is λ, then there will be two component of angular velocity $-\omega \sin\lambda$ and $\omega \cos\lambda$. Hence coriolis force act only for gravitational force.

Hence Time Period, $T = \frac{2\pi}{-\omega}$

$\because$ (-) = opposite dirxn of pendulum to dirxn of earth.

$$T = \frac{2\pi}{\omega \sin\lambda}$$

angular velocity of earth, $\omega = \frac{2\pi}{T} = 7.29 \times 10^{-5}$ rad./sec.

$$\omega = \frac{2\pi}{24} \text{ rad./hr.}$$

$$T = \frac{2\pi}{\frac{2\pi}{24} \sin\lambda}$$

$$\boxed{T = \frac{24}{\sin\lambda}}$$

Case I If $\lambda = 90°$, point @ N-pole

$$T = \frac{24}{\sin 90°}$$

$$\boxed{T = 24 \text{ hr}}$$

Case II If $\lambda = 0°$

$$T = \frac{24}{0} = \infty$$

Rate of Rotation of pendulum :

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{24} \sin\lambda$$

$$\omega = \frac{2 \times 180}{24} \sin\lambda$$

$$\boxed{\omega = 15^\circ \sin\lambda} \text{ rad/sec}$$

Unit-I

ch-3

Special Theory of Relativity (STR)

Postulates of STR :-

1. The law of physics are same and can be state same in their simplest form in all inertial frame of refrence.

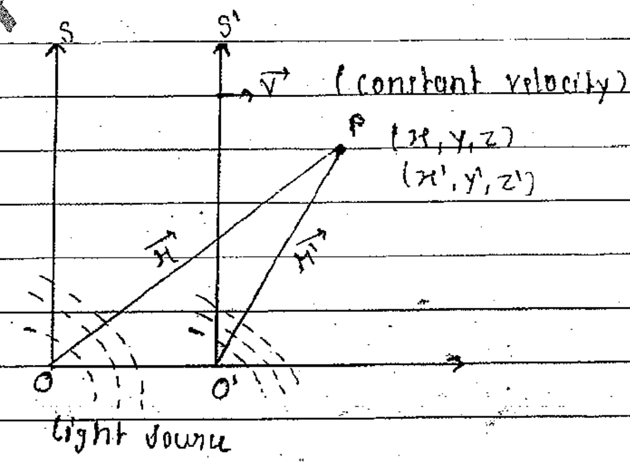
2. The speed of light in vacuum is same for all inertial observer regardless of motion of source or observer.

light velocity is a universal constant

3. $= 3 \times 10^8$ m/s

Lorentz Transformation :-

let two frame of ref. are S and S' S frame of ref. is in rest while S' is moving with constant velocity.



In S and S' F.O.R., the position vector of particle at P are $\vec{r}$ and $\vec{r'}$ respectively. The co-ordinate of P are (x, y, z) and (x', y', z') respectively.

Time Taken by light source to travel from O & O' to P is t and t'.

Distance of P from O is $|\vec{r}|$

$$\because \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\Rightarrow t = \frac{|\vec{r}|}{c}$$

$$\Rightarrow |\vec{r}| = ct$$

$$\Rightarrow \sqrt{x^2+y^2+z^2} = ct$$

$$\Rightarrow x^2+y^2+z^2 = c^2t^2$$

$$\Rightarrow x^2+y^2+z^2-c^2t^2 = 0 \quad \text{---(I)}$$

Similarly in S'

$$x'^2+y'^2+z'^2 = c^2t'^2$$

$$\Rightarrow x'^2+y'^2+z'^2-c^2t'^2 = 0 \quad \text{---(II)}$$

let S' F.O.R. is moving along x-axis direction so by galilean transformation eq^n

$$\left.\begin{aligned} x' &= x - v_x t \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \text{(III)}$$

put value of eq.(III) in (II)

$$(x-v_x t)^2 + y^2 + z^2 - c^2t^2 = 0$$

$$x^2 - 2x\cdot v_x t + v_x^2 t^2 + y^2 + z^2 - c^2t^2 = 0$$

$$x^2+y^2+z^2 - 2xv_x t + v_x^2t^2 - c^2t^2 = 0 \quad \text{---(IV)}$$

eq^n (I) and (IV) both are not same means we can say that those frames in which speed of light is same, then there galilean transformation are not valid.

OR

The frames which have velocity like speed of light there galilean transformation are not valid.

Hence we required a new type of transformation eq^n, this is known as Lorentz tranformation eq^n

→ Lorentz Tranformation eq^n :-

$$\left.\begin{aligned} x' &= \alpha x + \beta t \quad &\text{—(V)} \\ y' &= y \quad &\text{—(VI)} \\ z' &= z \quad &\text{—(VII)} \\ t' &= \gamma t + \delta x \quad &\text{—(VIII)} \end{aligned}\right\}$$ frame moving in $+x$ dir^n

Note:-
Space → x, y, z
time → t
(x, y, z, t)
(x, y, z, it)
(x, y, z, ict)

Values of α, β, γ and δ can be found by using postulate of STR.

1. Event :-

The observer O' whose coordinate S' frame $x'=0$, moving along x-axis of frame S with velocity $\vec{v}$,

$$x'=0$$

$$\frac{dx}{dt} = \vec{v}$$

Lorentz transformation eqn

$$x' = \alpha x + \beta t$$

$$0 = \alpha x + \beta t$$

$$\beta t = -\alpha x \quad \text{—— (ix)}$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma t + \delta x$$

2 Event :

The observer O whose co-ordinate in S frame $x=0$, moving along (-ive) x-axis of frame S' with velocity $(-\vec{v})$

$$\frac{dx'}{dt} = -\vec{v}$$

$$t' = \gamma t + \delta x$$

$$t' = \gamma t + 0$$

$$t' = \gamma t \quad \text{—— (x)}$$

$$x' = \alpha x + \beta t$$

$$x' = 0 + \beta t$$

$$x' = \beta t \quad \text{—— (xi)}$$

by eqn (ix)

$$\beta t = -\alpha x$$

$$\beta = \frac{-\alpha x}{t}$$

$$\beta = -\alpha \vec{v} \quad \text{—— (xii)}$$

x and t are variable

$$eq^n (XI) \div eq^n (X)$$

$$\frac{x'}{t'} = \frac{\beta}{\alpha\gamma}$$

$$-\vec{v} = \frac{\beta}{\alpha\gamma}$$

$$\vec{\beta} = -\gamma\vec{v} \quad \text{—(XIII)}$$

by eq^n (XII) and (XIII)

$$-\alpha\vec{v} = -\gamma\vec{v}$$

$$\boxed{\alpha = \gamma} \quad \text{—(XIV)}$$

From Lorentz transformation eq^n

$$x' = \alpha x + \beta t$$

$$x' = \alpha x - \gamma\vec{v}t \quad (\text{by } eq^n (XIII)\ \beta = -\gamma v)$$

$$x' = \gamma x - \gamma v t \quad (\text{by (XIV)},\ \alpha = \gamma)$$

$$x' = \gamma(x - vt) \quad \text{—(XV)}$$

$$y' = y \quad \text{—(XVI)}$$

$$z' = z \quad \text{—(XVII)}$$

$$t' = \gamma t + \delta x \quad \text{—(XVIII)}$$

put value in eq^n (II)

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

$$[\gamma(x - vt)]^2 + y^2 + z^2 - c^2(\gamma t + \delta x)^2 = 0$$

$$\gamma^2(x^2 - 2xvt + v^2t^2) + y^2 + z^2 - c^2(\gamma^2t^2 + 2\gamma t\cdot\delta x + \delta^2x^2) = 0$$

$$\gamma^2x^2 - 2xvt\gamma^2 + v^2t^2\cdot\gamma^2 + y^2 + z^2 - c^2\gamma^2t^2 - 2c^2\gamma t\cdot\delta x - c^2\delta^2x^2 = 0$$

$$x^2(\gamma - c^2\delta^2) + y^2 + z^2 - t^2(-\gamma^2v^2 + c^2\gamma^2) + xt(-2v\gamma^2 - 2c^2\gamma\delta) = 0 \quad \text{—(XIX)}$$

on comparing eq^n (I) & (XIX)

$$\Rightarrow \quad \gamma^2 - c^2\delta^2 = 1$$

$$\Rightarrow \quad -\gamma^2 v^2 + c^2\gamma^2 = c^2$$

$$\Rightarrow \quad xt(-2\gamma\delta c^2 - 2\gamma^2 v) = 0$$

$$-\gamma^2 v^2 + c^2\gamma^2 = c^2$$

$$\Rightarrow \gamma^2(-v^2 + c^2) = c^2$$

$$\Rightarrow \gamma^2 = \frac{c^2}{-v^2 + c^2}$$

$$\Rightarrow \gamma = \frac{c}{\sqrt{c^2 - v^2}}$$

$$\Rightarrow \gamma = \frac{c}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad \text{--- (XX)}$$

$$xt(-2\gamma\delta c^2 - 2\gamma^2 v) = 0$$

$$2\gamma\delta c^2 + 2\gamma^2 v = 0$$

$$\delta\gamma c^2 + \gamma^2 v = 0$$

$$\delta c^2 + \gamma v = 0$$

$$\boxed{\delta = \frac{-\gamma v}{c^2}} \quad \text{--- (XXI)}$$

by eq^n (XV) (XVI) (XVII) & (XVIII)

$$x' = (x - vt)\gamma = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{--- (XXII)}$$

$$y' = y \quad \text{--- (XXIII)}$$

$$z' = z \quad \text{--- (XXIV)}$$

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$$t' = \gamma t + \delta x$$

$$t' = \gamma t - \frac{\gamma V x}{c^2}$$

$$t' = \gamma\left(t - \frac{Vx}{c^2}\right)$$

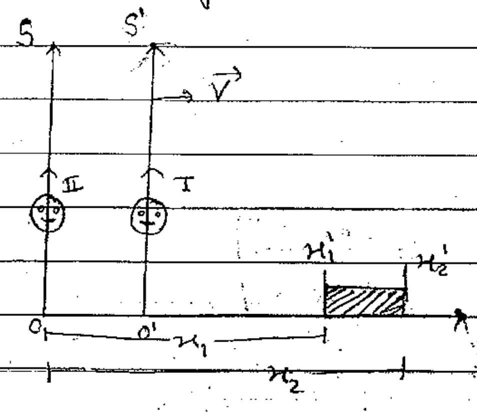
$$t' = \frac{t - Vx/c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{— (xxv)}$$

eq^n (22), (23), (24), (25) are known as Lorentz transformation eq^n in S' F.O.R. w.r.t. S

$$\left.\begin{aligned} x &= \frac{x' + Vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y &= y' \\ z &= z' \\ t &= \gamma\left(t + \frac{Vx}{c^2}\right) \end{aligned}\right\} \text{Lorentz transformation in S w.r.t. S' F.O.R.}$$

Lorentz Contraction OR Length Contraction

If a body moving along our length then contracts its length. This length is called length contraction.



length of rod for first observer is L_0 bcz S' is Inertial frame for 1st observer.
@ t' time, position of rod x_2' and x_1'

$$\Rightarrow \quad L_0 = x_2' - x_1' \quad \text{——— (i)}$$

length of rod for second observer is 't' at time.

$$\Rightarrow \quad L = x_2 - x_1 \quad \text{——— (ii)}$$

From lorentz transformation eq

$$\Rightarrow \quad x' = \gamma(x - vt)$$

$$\Rightarrow \quad x_1' = \gamma(x_1 - vt)$$

$$\Rightarrow \quad \frac{x_1'}{\gamma} = x_1 - vt$$

$$\Rightarrow \quad x_1 = \frac{x_1'}{\gamma} + vt \quad \text{——— (iii)}$$

Similarly,

$$\Rightarrow \quad x_2 = \frac{x_2'}{\gamma} + vt \quad \text{——— (iv)}$$

$$\Rightarrow \quad L = x_2 - x_1$$

$$\Rightarrow \quad L = \frac{x_2'}{\gamma} + vt - \frac{x_1'}{\gamma} - vt$$

$$\Rightarrow \quad \frac{1}{\gamma}(x_2' - x_1')$$

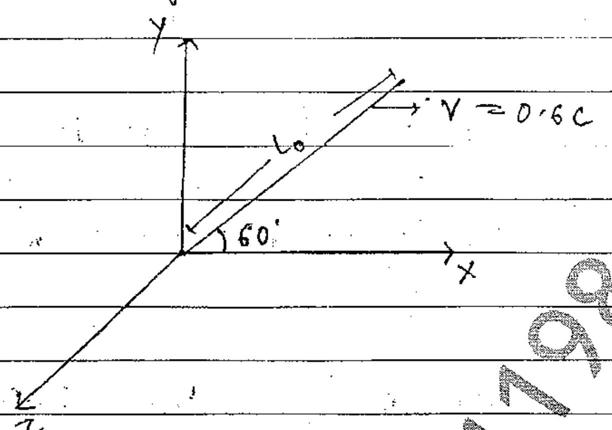
$$\Rightarrow \quad L = \frac{L_0}{\gamma}$$

$$\because \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \quad \boxed{L = L_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

Q. Calculate the % contraction of a rod moving with velocity 0.6 c in a dirxn inclined at 60° with its length.

Sol^n



When a rod is inclined at some angle then

$$L = L_0 \sqrt{1 - \frac{V^2}{C^2} \cos^2\theta}$$

$$\Rightarrow \quad L = L_0 \sqrt{1 - \frac{(0.6c)^2}{c^2} \cos^2 60°}$$

$$\Rightarrow \quad L = L_0 \sqrt{1 - \left(\frac{6}{10}\right)^2 \cdot \left(\frac{1}{2}\right)^2}$$

$$L = L_0 \sqrt{1 - \frac{\cancel{36}^{9}}{100} \times \frac{1}{\cancel{4}}}$$

$$L = L_0 \sqrt{1 - \frac{9}{100}}$$

$$L = L_0 \sqrt{1 - 0.09}$$

$$L = L_0 \sqrt{0.}$$

Time dilation :-

An important conclusion obtained by lorentz transformation eq^n that there is no existence of absolute time.

A absolute time means its value is same for all observer.

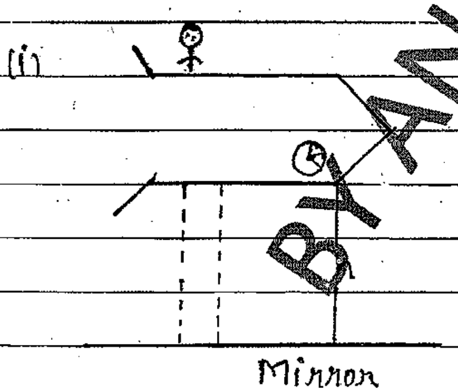
Time is relative and depend on frame of refrence.

" For a moving observer, time be dilated relative to stationary observer, in Stationary watch.

Daily life Example :-

To understand time dilation concept. we assume a example of daily life.

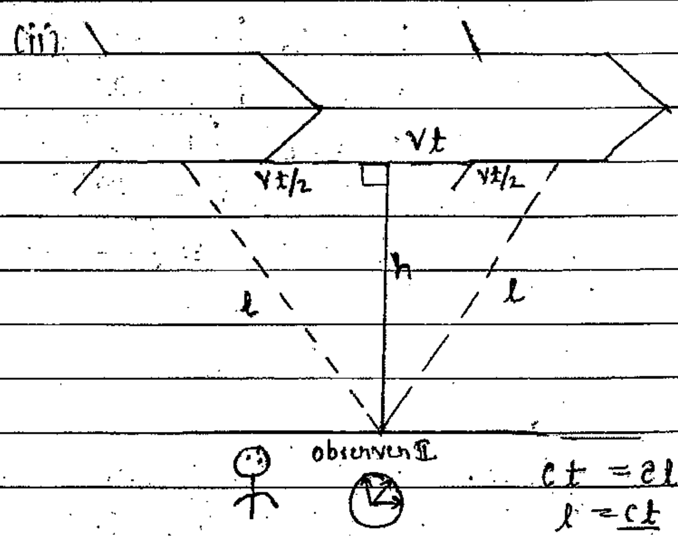
(i)



$ct' = 2h$

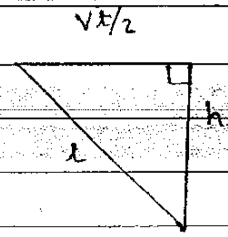
$t' = \frac{2h}{c}$ —(1)

(ii)



$ct = 2l$

$l = \frac{ct}{2}$ —(II)



Using pythogoreaus theorim

$$\Rightarrow \quad l^2 = h^2 + \left(\frac{vt}{2}\right)^2$$

$$\Rightarrow \quad l^2 = h^2 + \frac{v^2t^2}{4}$$

by eq^n (II)

$$\Rightarrow \quad \frac{c^2t^2}{4} = h^2 + \frac{v^2t^2}{4}$$

$$h^2 = \frac{v^2t^2}{4} - c$$

$$\Rightarrow \quad h^2 = \frac{c^2t^2}{4} - \frac{v^2t^2}{4}$$

$$\Rightarrow \quad h^2 = \frac{t^2}{4}(c^2 - v^2)$$

$$\Rightarrow \quad \frac{t^2}{4} = \frac{h^2}{c^2 - v^2}$$

$$\Rightarrow \quad t^2 = \frac{4h^2}{c^2 - v^2}$$

$$\Rightarrow \quad t = \frac{2h}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

by eq. (I) $t' = \frac{2h}{c}$

$$\Rightarrow \quad t = \frac{t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\because \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t = \gamma t'} \quad \text{———} \quad (iii)$$

when $\gamma = 1$, means $v = 0$, means object is stationary.

where

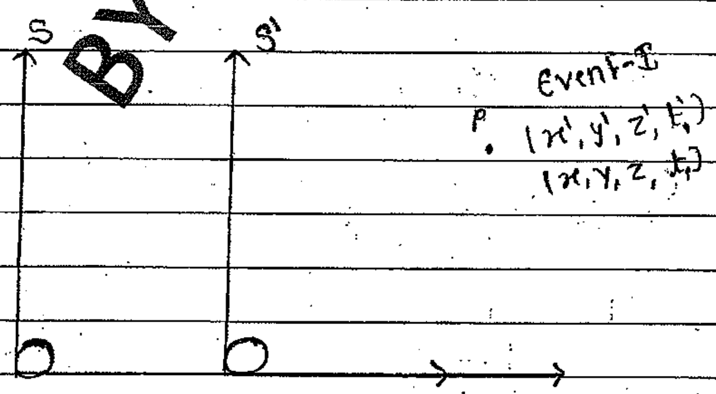
t = Time for observer II

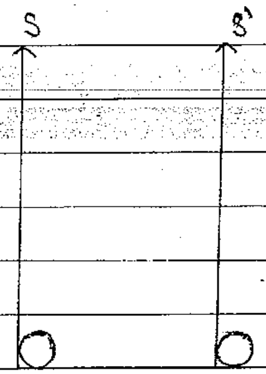
t' = Time for observer I

From eq. (iii), it is clear that time dilate for stationary observer.

If an object is moving then value of γ will be greater than 1 we can say that for moving object, time dilate happen in stationary watch.

By Lorentz transformation :-





Event-II
(x', y', z', t_2')
(x, y, z, t_2)

By Lorentz transformation eq^n

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \quad \text{—— (I)}$$

$$t_1' = \gamma\left(t_1 - \frac{vx}{c^2}\right)$$

$$t_1 = \gamma\left(t_1' + \frac{vx_1'}{c^2}\right) \quad \text{—— (II)}$$

$$t_2 = \gamma\left(t_2' + \frac{vx_2'}{c^2}\right) \quad \text{—— (III)}$$

$$t_2 - t_1 = \gamma(t_2' - t_1') \quad \text{—— (iv)}$$

$$t_2 - t_1 = \Delta t = \tau \quad (\text{Toe})$$

$$t_2' - t_1' = \Delta t' = \tau_0$$

$$\tau = \gamma \tau_0$$

$$\because \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{\tau = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Note:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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Unit I

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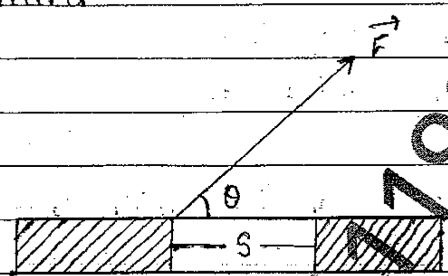
Energy Conservation

Work :-

Work is a scalar product of force and displacement.



$$W = \vec{F} \cdot \vec{S}$$

$$W = |\vec{F}||\vec{S}| \cos\theta \quad \text{Joule}$$

Unit → S.I. → N-m and Joule

Dimension → $W = F \cdot S$

$= ma \cdot s$

$= m \cdot \frac{d^2x}{dt^2} \cdot s$

$= kg \cdot \frac{m^2}{t^2} \cdot m$

$= kg \cdot m^3 \cdot t^{-2}$

$= [M^1 L^3 T^{-2}]$

Case I If force and displacement are in same dirxn

$\theta = 0°$

$W = F \cdot S \cdot \cos 0$

$\boxed{W = F \cdot S}$ N-m

Case II. If force and displacement are in opposite dirxn.

$\theta = \pi$

$W = F \cdot S \cdot \cos\pi$

$\boxed{W = -F \cdot S}$ N-m

Case III. If force and displacement are $\perp$ to each other.

$\theta = 90'$

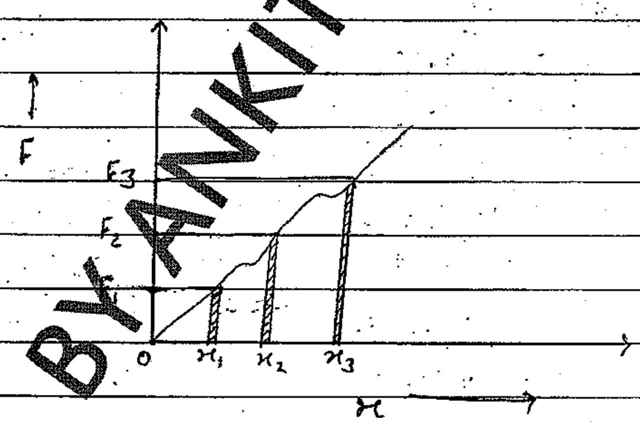
$W = F \cdot S \cdot \cos 90'$

$\boxed{W = 0}$ N-m

$W = |F||S| \cos\theta$ valid only when force is constant

Integeral form of Work :-

If force is variable then work is represented in Integeral form.



$W = F_1 dx_1 + F_2 dx_2 + F_3 dx_3 + \ldots$

$$\boxed{W = \int_{x_1}^{x_2} \vec{F} \cdot dx}$$

Work Energy Theorem :-

According to this theorem, work done by all forces on a body is equal to change in k.E. of body.

(Conservative, Non conservative, inertial, external)

Mathematical Statement :-

$$W_c + W_N + W_i + W_e + \ldots = \Delta \cdot E_K$$

$$W_{all\ forces} = (E_K)_f - (E_K)_i$$

F = Final , I = initial

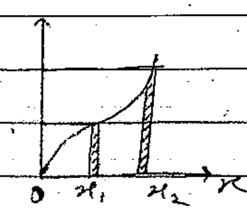
Differential form

$$(dw)_{all\ forces} = d(E_K)$$

proof

By definition of Work done

$$W = \int_{x_1}^{x_2} F \cdot dx$$



$\because F = ma$

$$m\frac{dv}{dt} = m\frac{dv}{dt} \times \frac{dx}{dx} = m\frac{dv}{dx} \cdot \frac{dx}{dt} = m \cdot v \cdot \frac{dv}{dx}$$

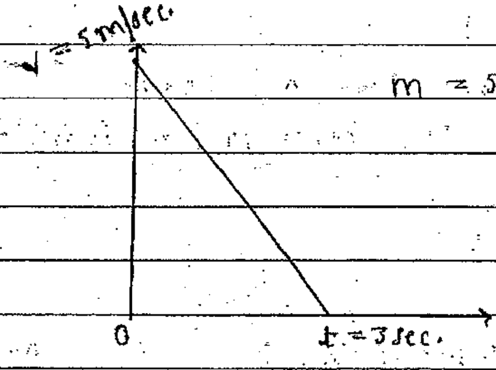
$$W = \int_{x_1}^{x_2} m \cdot v \frac{dv}{dx} \cdot dx$$

$$W = \int_{x_1}^{x_2} m v \, dv$$

$$W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\boxed{W = (K.E)_f - (K.E)_i}$$ H.P.

e.g.



$m = 5kg$

@ $t = 0$ sec., $v = 5$ m/sec.
@ $t = 3$ sec., $v = 0$ m/sec.

$$W = \vec{F}\cdot\vec{S} = \Delta K$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

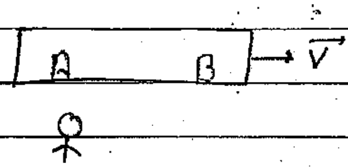
$$= \frac{1}{2}m(0)^2 - \frac{1}{2}m(5)^2$$

$$= -\frac{25\times 5}{2}$$

$$\boxed{W = -\frac{125}{2}}$$

Note :

Kinetic energy depend on frame of ref.



K.E. w.r.t. B of A $= \frac{1}{2}m(0) = 0$
K.E. of B w.r.t. A $= \frac{1}{2}mv^2$

Restoring force :-

If force act on opposite to displacement is called restoring force.

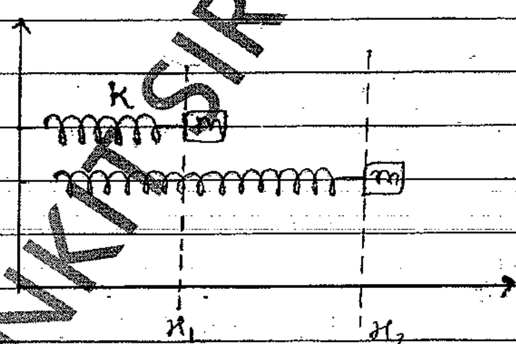
$$F = -kx$$

k = Spring constant

x = displacement

Q. Calculate work done by spring at 't' time.

Let a block of mass 'm' which is connected to spring and spring constant 'k' and A force act opposite to displacement. Restoring force $F = -kx$



By definition of work,

$$W = \int_{x_1}^{x_2} F \cdot dx$$

$$\because F = -kx$$

$$W = -k \int_{x_1}^{x_2} x \, dx$$

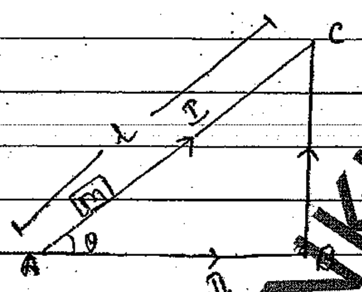
$$W = -\frac{1}{2} k \left[x^2 \right]_{x_1}^{x_2}$$

Initial $x_1 = 0$
final $x_2 = x$

$$W = -\frac{1}{2} K x^2$$

Here (-)ive sign shows work done in opposite dirxn.

Conservative force :-
Work done by such force which does not depend on path are called conservative force. depends only and only on initial and final position eg.



By path I
$W_I = |F||S| \cos\theta$
$= |F||S| \cos 90+\theta$
$\boxed{W_I = -mgl\sin\theta}$

By path II
$W_{AB} = |F||S| \cos 90' = 0$
$W_{BC} = |F||S| \cos 180'$
$= -mgl\sin\theta$
$W_{II} = W_{AB} + W_{BC} = -mgl\sin\theta$

Potential energy :-

Energy of body due to its position or change in shape is known as potential energy.

"Change in potential energy = ~~(-)~~ workdone by conservative force"

e.g. of conservative force

gravity → gravitational potential energy

elastic → elastic potential energy

electrostatic → electrostatic energy

Mathematical Statement :-

Let a body moves from A to B under conservative force, then work done is equal to change in potential energy.

Let's assume displacement is $d\vec{r}$ then workdone is defined by

$$W_c = \int \vec{F} \cdot d\vec{r} \quad \text{(Integeral form)}$$

$$\Delta U = -\int_A^B \vec{F} \cdot d\vec{r}$$

Special cond^n

If point A is @ infinite then corresponding potential energy at this point will be zero.

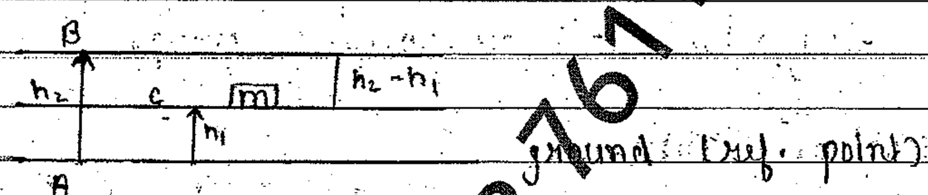
$$U_A \xrightarrow{@\infty} 0$$

$$U_A - U_B = -\int_A^B \vec{F}\cdot d\vec{r}$$

$$\boxed{U_B = -\int_\infty^B \vec{F}\cdot d\vec{r}}$$

Note, above refrence point, potential energy $= +mgh$
below refrence point, potential energy $= -mgh$

e.g.



potential energy $\Delta U = U_f - U_i$

$$= U_{AB} - U_{CB}$$
$$= mgh_2 - mgh_1$$
$$\Delta U = mg(h_2 - h_1) \quad \text{--- (I)}$$

By definition of workdone,

$$W = |F||S|\cos\theta$$
$$W = -mg(h_2 - h_1) \quad \text{--- (II)}$$

by (I) and (II)

$$\cancel{W} = \boxed{\Delta U = -W_c} \quad H.P.$$

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Q. check whether given forces are conservative & non conservative.

(i) $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$
(ii) $\vec{F} = x^2y\hat{i} + y\hat{j} + z^2\hat{k}$
(iii) $\vec{F} = -k\vec{r}$

proof.
(i) $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$
for conservative force, $\nabla \times \vec{F} = 0$
for non conservative force, $\nabla \times \vec{F} \neq 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) - \hat{j}\left(\frac{\partial z}{\partial x} - \frac{\partial}{\partial y}x\right) + \hat{k}\left(\frac{\partial}{\partial x}\cdot y - \frac{\partial}{\partial y}x\right)$$

$$= 0 - 0 + 0$$

$$\boxed{\nabla \times \vec{F} = 0}$$

(ii) $\vec{F} = x^2y\hat{i} + y\hat{j} + z^2\hat{k}$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & y & z^2 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}\left(\frac{\partial}{\partial x}\cdot z^2 - \frac{\partial}{\partial z}\cdot x^2y\right) + \hat{k}\left(\frac{\partial}{\partial x}y - \frac{\partial}{\partial y}\cdot x^2y\right)$$

$$\nabla \times \vec{F} \neq 0$$

(iii) $\vec{F} = -k\vec{r}$ $(\vec{r} = x\hat{i} + y\hat{j} + z\hat{k})$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -kx & -ky & -kz \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k}(0)$$

$$\boxed{\nabla \times \vec{F} = 0}$$

Q. Prove that potential energy is equal to (−) U

Q. Prove that conservative force is equal to (−) gradient of potential energy.

Note :- ∇ = del operator
$\nabla \cdot$ = divergence
$\nabla \times$ = curl
∇ = gradient

proof.

We know that

$$U = -W_c$$

$$= -\int \vec{F} \cdot d\vec{r} \quad \text{—(1)} \quad \text{(By definition of potential energy)}$$

$$\left.\begin{aligned} \vec{F} &= F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \\ d\vec{r} &= dx\hat{i} + dy\hat{j} + dz\hat{k} \end{aligned}\right\} \text{(II)}$$

by (I) and (II)

$$U = -\int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$U = -\int (F_x dx + F_y dy + F_z dz) \quad \text{—(III)}$$

potential energy is a function of position

$$U = U(x, y, z) \quad \text{—(IV)}$$

doing partial differentiation

$$dU = \frac{\partial U}{\partial x} \cdot dx + \frac{\partial U}{\partial y} \cdot dy + \frac{\partial U}{\partial z} \cdot dz \quad \text{—(V)}$$

doing differentiation of eq^n. (III)

$$dU = -d\int (F_x dx + F_y dy + F_z dz)$$

$$dU = -(F_x dx + F_y dy + F_z dz) \quad \text{—(VI)}$$

by (5) and (6)

$$-F_x = \frac{\partial U}{\partial x}$$

$$\left.\begin{aligned} F_x &= -\frac{\partial U}{\partial x} \\ F_y &= -\frac{\partial U}{\partial y} \\ F_z &= -\frac{\partial U}{\partial z} \end{aligned}\right\} \text{—(VII)}$$

by eq. (II)

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

$$\vec{F} = -\left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right] U$$

$$\boxed{\vec{F} = -\nabla U}$$ H.P.

Q.3. Prove that curl of conservative force is equal to '0'

$$\nabla \times \vec{F_c} = 0$$

proof,

We know,

$$\because \vec{F_c} = -\nabla U \quad \text{---(i)}$$

Taking curl of eq. (i)

$$\nabla \times \vec{F_c} = -\nabla \times \nabla U$$

$$\nabla \times \vec{F_c} = -\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial U/\partial x & \partial U/\partial y & \partial U/\partial z \end{vmatrix}$$

$$\nabla \times \vec{F_c} = -\left[\hat{i}\left(\frac{\partial}{\partial y}\cdot\frac{\partial U}{\partial z} - \frac{\partial}{\partial z}\cdot\frac{\partial U}{\partial y}\right) - \hat{j}\left(\frac{\partial}{\partial x}\cdot\frac{\partial U}{\partial z} - \frac{\partial}{\partial z}\cdot\frac{\partial U}{\partial x}\right)\right.$$
$$\left. + \hat{k}\left(\frac{\partial}{\partial x}\cdot\frac{\partial U}{\partial y} - \frac{\partial}{\partial y}\cdot\frac{\partial U}{\partial x}\right)\right]$$

$$\boxed{\nabla \times \vec{F_c} = 0}$$ H.P.

Mechanical energy :- (Conservation law)

Initially a object is moving then K.E. increased and potential energy decrease and sometime velocity of object will be decreased means K.E. decreased and potential energy is increased so we can say that if K.E. increases then potential energy decrease or vice versa, that is mechanical energy conservation law.

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$$\boxed{K.E + P.E = E = U}$$

Where,

K.E = kinetic energy
P.E = potential energy
E = Total energy

Q.4. If the mechanical energy of a system is conserved then prove that force act on system is conservative force.

proof,

By mechanical energy conservation law

$$E = K.E + P.E$$

$$E = \frac{1}{2}mv^2 + U \quad \text{——(i)}$$

D. eq.(i) w.r.t. 't'

$$\frac{dE}{dt} = \frac{1}{2}m.2v\frac{dv}{dt} + \frac{dU}{dt}$$

E = constant

$$0 = mv\frac{dv}{dt} + \frac{dU}{dt} \quad \text{——(ii)}$$

We know, potential energy is function of position

$$U = U(x, y, z)$$

doing partial derivative

$$dU = \frac{\partial U}{\partial x}\cdot dx + \frac{\partial U}{\partial y}\cdot dy + \frac{\partial U}{\partial z}\cdot dz$$

on dividing by dt

$$\frac{dU}{dt} = \frac{\partial U}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial U}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial U}{\partial z}\cdot\frac{dz}{dt}$$

$$\frac{dU}{dt} = \left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)\cdot\left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}\right)$$

$$\frac{dU}{dt} = \nabla U \cdot \left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}\right)$$

$$\because \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\frac{dU}{dt} = \nabla U \cdot \vec{v} \quad \text{---(iv)}$$

by eq. (iii) & eq. (iv)

$$m\vec{v}\cdot\frac{d\vec{v}}{dt} + \nabla U \cdot \vec{v} = 0$$

$$m\vec{v}\cdot\vec{a} + \nabla U \cdot \vec{v} = 0$$

$$\boxed{\vec{v} \neq 0}$$

$$m\vec{a} + \nabla U = 0$$

$$m\vec{a} = -\nabla U$$

$$\boxed{\vec{F} = -\nabla U}$$

So we can say that F is conservative force.

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Note :- Some important result which are conservative

1. $\vec{F} = F(x)\hat{i} + F(y)\hat{j} + F(z)\hat{k}$
2. $\vec{F} = \frac{K}{r^n}(\pm\hat{r})$

Q. Prove that under conservative force, motion of a particle is straight line. (Linear)

OR

Rectilinear Motion

Proof,

let a particle of mass 'm' is moving in x-dirsn under conservative force $\vec{F}$

We know that under conservative force, mechanical energy is also conserved.

$$E = K + U(x)$$

Here,

E = Total energy

K = Kinetic energy

U = potential energy

$$E = \frac{1}{2}mV_x^2 + U(x)$$

$$\because \vec{V_x} = \frac{d\vec{x}}{dt}$$

$$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + U_{(x)}$$

$$E - U(x) = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$$

$$\frac{dx}{dt} = \sqrt{\frac{2}{m}(E - U_x)}$$

$$\frac{dx}{dt} = \sqrt{\frac{2}{m}} (E - U_x)^{1/2} \quad \text{—— (I)}$$

By definition of conservative force

$$\vec{F} = -\nabla U$$

If particle move along x-axis

$$F_x = -\frac{\partial U}{\partial x} \quad \text{—— (II)}$$

$$F_x \partial x = -\partial U$$

Integrate both side

$$\int F_x \partial x = -\int \partial U$$

$$F_x x = -U + C$$

at $x = 0$, $U(x) = 0$, $C = 0$

$$F_x x = -U_{(x)} \quad \text{—— (III)}$$

by (I) and (III)

$$\frac{dx}{dt} = \sqrt{\frac{2}{m}} (E + F_x \cdot x)^{1/2}$$

by variable separation

$$\frac{dx}{(E + F_x \cdot x)^{1/2}} = \sqrt{\frac{2}{m}}\, dt$$

Integerating;

$$\int_0^x \frac{dx}{(E + F_x \cdot x)^{1/2}} = \sqrt{\frac{2}{m}} \int_0^t dt$$

$$\left[\frac{2(E + F_x \cdot x)^{1/2}}{F_x} \right]_0^x = \sqrt{\frac{2}{m}}\, t$$

$$\frac{2}{F_x} \left[(E + F_x \cdot x)^{1/2} - E^{1/2} \right] = \sqrt{\frac{2}{m}}\, t$$

$$(E + x \cdot F_x)^{1/2} = \frac{F_x}{2} \sqrt{\frac{2}{m}}\, t + E^{1/2}$$

Squaring both side

$$(\cancel{E} + \cancel{F_x} \cdot x) = \frac{F_x^{\cancel{2}}}{4} \cdot \frac{2}{m} \cdot t^2 + \cancel{E} + 2 \cdot \frac{\cancel{F_x}}{2} \sqrt{\frac{2}{m}}\, t \cdot E^{1/2}$$

$$x = \frac{F_x}{2m} t^2 + \sqrt{\frac{2}{m}}\, t \cdot E^{1/2}$$

$$F_x = m a_x$$

$$x = \frac{1}{2} a_x t^2 + \sqrt{\frac{2E}{m}}\, t$$

let initial velocity of particle is V_0

$$E = \frac{1}{2} m V_0^2$$

$$V_0 = \sqrt{\frac{2E}{m}}$$

$$x = \frac{1}{2} a_x t^2 + V_0 t$$

$$\therefore \boxed{x = V_0 t + \frac{1}{2} a_x t^2} \quad \text{(IV)}$$

eq. (IV) represent rectilinear motion.

$$S = ut + \frac{1}{2}at^2$$

Hence on basis of eq. (IV), we can say that motion of particle under conservative force is linear.

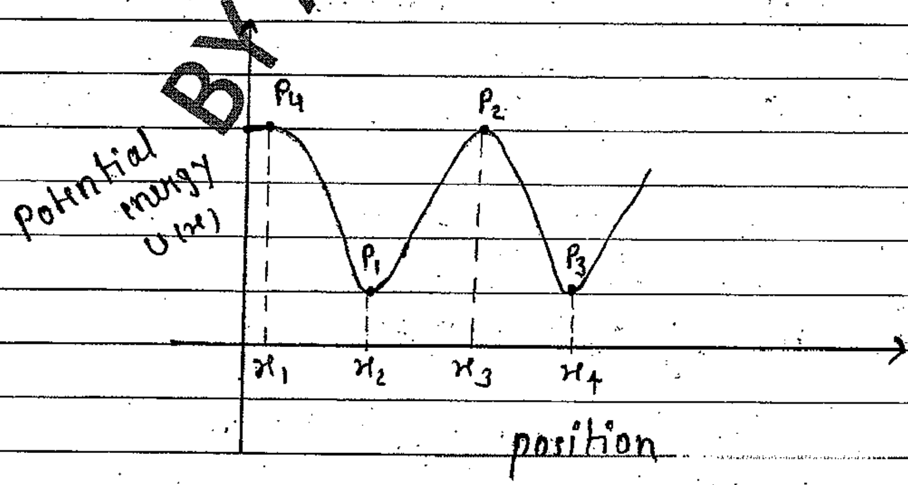
Potential Energy Curve and motion of particle :-

let's consider a particle of mass 'm' moves in x - dirxn under conservative force.

Conservative force, $F_c = -\nabla U$

$$\boxed{F_x = -\frac{\partial U}{\partial x}}$$

Curve drawn ~~b/w~~ potential energy and position is known as potential energy curve.



1. If we increase value of position, value of potential energy $U(x)$ also increase, slope of potential Energy $\frac{\partial U}{\partial x}$ is (+)ive so force work in (-)ive x-dirxn.

2. If we increase value of position, value of potential energy $U(x)$ decrease and $\frac{\partial U}{\partial x}$ (slope) will be (-)ive means force will act in (+)ive x-dirxn.

1. Balance State :-
In this curve P_1 and P_2 represent minimum and maximum potential energy. potential energy slope at this point will be zero. So no force act on that particle so we can say that point is balance point state.

2. Stable equilibrium State :-
In curve point P_1 and P_3 are known stable equilibrium point. In this state we apply a slide force on particle then particle come back from initial condition so we can say, P_1 and P_3 are stable equill^m point.

Nand

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3. Unstable equil^m point :-

In given curve point P_2 is known as unstable equil^m point. Bez when we apply a slide force on P_2 then it never come back from final to initial cond^n so we can say P_2 is unstable equil^m point.

4. Neutral equil^m state :-

point F is a neutral equil^m point in this state we apply a force then particle get a new equil^m state. so we can say F is neutral equil^m point.

Completed Unit-E

Numerical :-

Q.1. Prove that force $\vec{F} = (2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}$ is conservative force.

(ii) Determine potential energy function of this force.

(iii) Calculate workdone in displacing the particle from a point (0,1,2) to point (5,2,7)

Solⁿ

Given,

$$\vec{F} = (2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}$$

prove,

$\vec{F}$ is conservative force

proof,

for conservative force, $\nabla \times \vec{F} = 0$

non conservative force, $\nabla \times \vec{F} \neq 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy+z^2 & x^2 & 2xz \end{vmatrix}$$

$$\nabla \times \vec{F} = \hat{i}\left(\frac{\partial}{\partial y} 2xz - \frac{\partial}{\partial z} x^2\right) - \hat{j}\left[\frac{\partial}{\partial x} 2xz - \frac{\partial}{\partial z}(2xy + z^2)\right] + \hat{k}\left[\frac{\partial}{\partial x} x^2 - \frac{\partial}{\partial y}(2xy + z^2)\right]$$

$$= 0 - \hat{j}(2z - 2z) + \hat{k}(2x - 2x)$$

$$\boxed{\nabla \times \vec{F} = 0}$$

means force is conservative force.

(i) Potential Energy :-

We know
$$U = -\int \vec{F}\cdot d\vec{r}$$

$$U = -\int\{(2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}\}\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$U = -\int (2xy + z^2)\,dx + x^2 dy + 2xz\,dz$$

$$U = -\left[\frac{2x^2}{2}y + z^2x\right] - x^2y - \frac{2xz^2}{2}$$

$$U = -x^2y - z^2x - x^2y - z^2x$$

$$\boxed{U = -(2)(x^2y + z^2x)} \quad \text{---(1)}$$

(ii) Work done in displacing particle from (0,1,2) to (5,2,7)

$$\because W = \int_{x_1}^{x_2} \vec{F}\cdot d\vec{x}$$

let $x_1 = 0,1,2$ and $x_2 = 5,2,7$

$$W = \int_{x_1}^{x_2} F\cdot dx$$

$$W = 2\left[x^2y + z^2x\right]_{x_1}^{x_2}$$

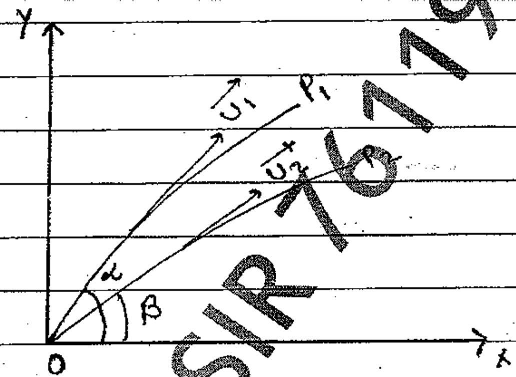
$$W = 2\{(5)^2(2) + (7)^2(5) - (0)\}$$
$$= 2(50 + 49\times5)$$
$$= 2(50 + 245)$$
$$= 2\times295$$
$$= 590$$

Imp.

Q.2. Prove that one projectile motion relative to another projectile motion is linear.

Soln

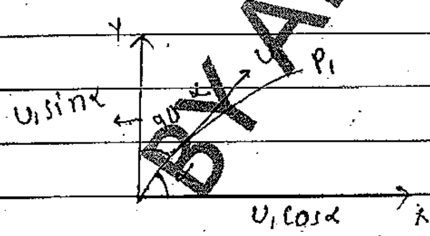
Let us consider, two particle whose initial velocity are U_1 and U_2 respectively. These are projected at an angle α and β then these particle do projectile motion.



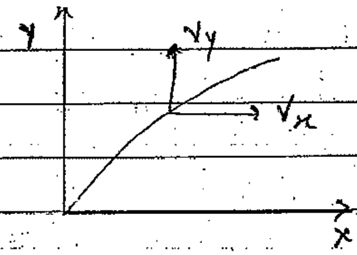
Component of U_1 velocity in x, y dirxn are U_x and U_y.

$$U_x = U_1 \cos\alpha \quad \text{——(I)}$$

$$U_y = U_1 \sin\alpha \quad \text{——(II)}$$



after 't' time velocity of particle V_x, V_y



from 1st eqn of motion

$v = u + at$

v = final velocity

u = initial velocity

a = acceleration

Particle is moving with constant acceleration in x-dirxn

$a_x = 0$

$v_x = u_x + a_x t$

$v_x = u_x$

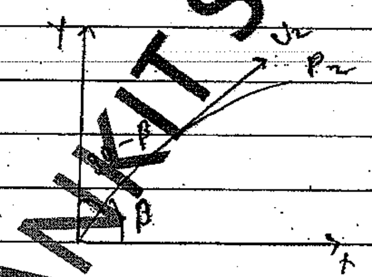
$v_x = u_1 \cos\alpha$ —— (III)

similarly

$v_y = u_y + a_y t$

$v_y = u_1 \sin\alpha - gt$ —— (IV)

similarly for 2nd object



$v_x' = u_2 \cos\beta$ —— (V)

$v_y' = u_2 \sin\beta - gt$ —— (VI)

let velocity of object are v_A and v_B

- Velocity of B w.r.t A $= v_B - v_A$
- " " A w.r.t B $= v_A - v_B$

let relative to 1^{st} object, second object have have vellocity V_x'' and V_y'' along x-y axis.

$$V_x'' = V_x' - V_x$$

by eq. (III) and (IV)

$$V_x'' = U_2\cos\beta - U_1\cos\alpha$$

$$\frac{dx''}{dt} = U_2\cos\beta - U_1\cos\alpha \quad \text{——(VII)}$$

$$dx'' = (U_2\cos\beta - U_1\cos\alpha)\,dt$$

Integerate

$$\int dx'' = \int (U_2\cos\beta - U_1\cos\alpha)\,dt$$

$$x'' = (U_2\cos\beta - U_1\cos\alpha)\,t + C$$

@ $t=0$, Initial condⁿ

$$x'' = 0$$

$$C = 0$$

$$x'' = (U_2\cos\beta - U_1\cos\alpha)\,t \quad \text{——(VIII)}$$

similarly

$$V_y'' = V_y' - V_y$$

$$V_y'' = U_2\sin\beta - gt - U_1\sin\alpha + gt$$

$$V_y'' = U_2\sin\beta - U_1\sin\alpha$$

$$\frac{dy''}{dt} = U_2\sin\beta - U_1\sin\alpha \quad \text{——(IX)}$$

$$dy'' = (U_2\sin\beta - U_1\sin\alpha)\,dt$$

Integerate

$$\int dy'' = \int (U_2\sin\beta - U_1\sin\alpha)\,dt$$

$$y'' = (U_2 \sin\beta - U_1 \sin\alpha)t + c$$

@ Initial, $t = 0$

$y'' = 0$

$c = 0$

$$y'' = U_2 \sin\beta - U_1 \sin\alpha \quad \text{--- (X)}$$

eq. (X) $\div$ eq. (VIII)

$$\frac{y''}{x''} = \frac{U_2 \sin\beta - U_1 \sin\alpha}{U_2 \cos\beta - U_1 \cos\alpha}$$

$$y'' = \left(\frac{U_2 \sin\beta - U_1 \sin\alpha}{U_2 \cos\beta - U_1 \cos\alpha}\right) x'' \quad \text{--- (XI)}$$

$$\because m = \frac{U_2 \sin\beta - U_1 \sin\alpha}{U_2 \cos\beta - U_1 \cos\alpha} = \text{slope}$$

$$\boxed{y'' = m x''}$$

from eq. (XI), it is clear that one projectile motion relative to another be linear.

Q.3. Position vector of any particle in ref. S and S' are $\vec{r} = (8t^2 - 3t)\hat{i} + 2\hat{j} + 4t^3\hat{k}$ and $\vec{r'} = (8t^2 - 5t)\hat{i} + 9\hat{j} + 4t^3\hat{k}$

then find.

(I) Velocity of S' relative to S

(II) prove acceleration is same for both F.O.R.

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Sol^n

given $\vec{r} = (8t^2 - 3t)\hat{i} + 2\hat{j} + 4t^3\hat{k}$

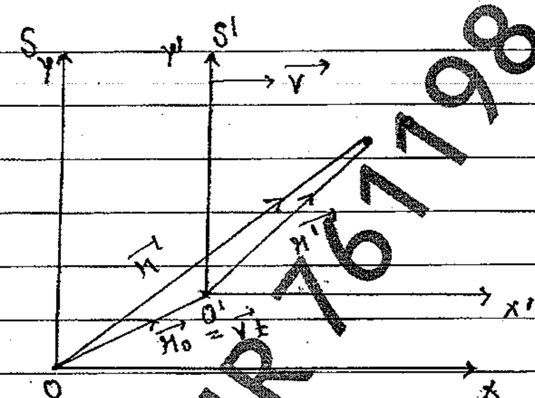
$\vec{r'} = (8t^2 - 5t)\hat{i} + 9\hat{j} + 4t^3\hat{k}$

To find.

(I) Velocity of S' w.r.t. S $(\vec{V}) = ?$

(II) $\vec{a'} = \vec{a}$

Sol^n



So (i)

We know, By Δ law of vector addition in $\Delta OO'P$

$\vec{OP} = \vec{OO'} + \vec{O'P}$

$\vec{r} = \vec{r_0} + \vec{r'}$

$\vec{r'} = \vec{r} - \vec{r_0}$

$\vec{r'} = \vec{r} - \vec{v}t$

$\vec{v}t = \vec{r} - \vec{r'}$

$\vec{v}t = (8t^2\hat{i} - 3t\hat{i} + 2\hat{j} + 4t^3\hat{k}) - (8t^2\hat{i} - 5t\hat{i} + 9\hat{j} + 4t^3\hat{k})$

$\vec{v}t = 2t\hat{i} - 7\hat{j}$

on comparing coefficient

$\vec{v} = 2\hat{i}$

OR

$\boxed{|\vec{v}| = 2\ m/s}$

(II) $\vec{a'} = \vec{a}$

Sol^n: $\vec{r} = (8t^2 - 3t)\hat{i} + 2\hat{j} + 4t^3\hat{k}$

$\vec{a} = \frac{d^2\vec{r}}{dt^2}$

$= \frac{d\vec{r}}{dt} = (16t - 3)\hat{i} + 12t^2\hat{k}$

$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 16\hat{i} + 24t\hat{k}$ —(I)

Similarly

$\vec{r'} = (8t^2 - 5t)\hat{i} + 9\hat{j} + 4t^3\hat{k}$

$\frac{d\vec{r'}}{dt} = (16t - 5)\hat{i} + 12t^2\hat{k}$

$\vec{a'} = \frac{d^2\vec{r'}}{dt^2} = 16\hat{i} + 24t\hat{k}$ —(II)

by (I) and (II)

$\boxed{\vec{a} = \vec{a'}}$

Q.4. If in frame of ref. at any time 't' position of particle is $\vec{r} = 3t^2\hat{i} - 5t\hat{j} + 10\hat{k}$ the find velocity and acceleration at $t = 1$ sec.

Sol^n: $\vec{r} = 3t^2\hat{i} - 5t\hat{j} + 10\hat{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = 6t\hat{i} - 5\hat{j}$

@ $t = 1$ sec.

$\vec{v} = 6\hat{i} - 5\hat{j}$

$|\vec{v}| = \sqrt{36 + 25} = \sqrt{61}$ m/s

$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 6\hat{i}$

$|\vec{a}| = 6\ m/s^2$

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5. At any instant, position of two particles are $(4\hat{i} - 4\hat{j} + 7\hat{k})$ and $(2\hat{i} + 2\hat{j} + 5\hat{k})$ m. If velocity of first particle is $0.4(\hat{i} - \hat{j} + \hat{k})$ m/s then find velocity of 2^{nd} particle due to which both particle can collide after $t = 10$ sec.

Soln for collision

$$(\vec{r_2} - \vec{r_1}) = (\vec{v_1} - \vec{v_2})t$$

let, given

$$\vec{r_1} = (4\hat{i} - 4\hat{j} + 7\hat{k}) \text{ m}$$

$$\vec{r_2} = (2\hat{i} + 2\hat{j} + 5\hat{k}) \text{ m}$$

$$\vec{v_1} = 0.4(\hat{i} - \hat{j} + \hat{k}) \text{ m}$$

$$t = 10 \text{ sec.}$$

To find

$$\vec{v_2} = ?$$

Soln

$$\vec{r_2} - \vec{r_1} = -2\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\vec{v_2}t = (\vec{r_2} - \vec{r_1}) - \vec{v_1}t$$

$$= (-2\hat{i} + 6\hat{j} - 2\hat{k}) - (\hat{i} - \hat{j} + \hat{k})0.4 \times 10$$

$$\vec{v_2}t = -2\hat{i} + 6\hat{j} - 2\hat{k} - 4\hat{i} + 4\hat{j} - 4\hat{k}$$

$$= -6\hat{i} + 10\hat{j} - 6\hat{k}$$

$$\vec{v_2} = 0.1(-6\hat{i} + 10\hat{j} - 6\hat{k})$$

or if

$$(\vec{r_1} - \vec{r_2}) = (\vec{v_2} - \vec{v_1})t$$

then

$$\vec{v_2} = 0.1(6\hat{i} - 10\hat{j} + 6\hat{k})$$

Imp.

6. A meter scale is moving with a velocity $\frac{c}{2}$. Calculate apparent length.

Sol^n Given: length of scale $L_0 = 1m$
Velocity $v = \frac{c}{2}$

apparent length, $L = ?$

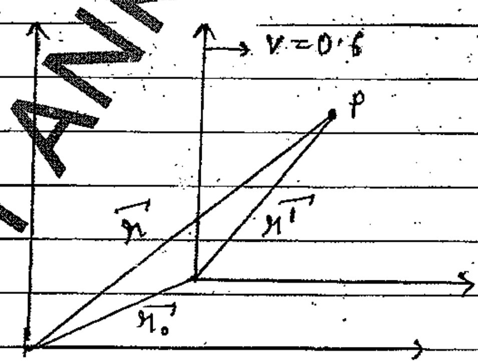
∴ We know

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = 1 \sqrt{1 - \frac{c^2}{4c^2}}$$

$$\boxed{L = \frac{\sqrt{3}}{2}\, m}$$

7. A vector is represented by $(4\hat{i} + 3\hat{j})$ in frame S. If another ref. frame 'S'' moving along +x direction with velocity 0.6c then find vector in S'.



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$\vec{r} = (4\hat{i} + 3\hat{j})\,m.$

$v = 0.6c$

$\vec{r'} = ?$

We know

length contradiction in x dirⁿ

$$L_x = L_{0x}\sqrt{1 - \frac{v^2}{c^2}}$$

$$L_x = 4\sqrt{1 - (0.6)^2}$$

$$\boxed{L_x = 3.2}\ m$$

'S'' is not moving in y-dirⁿ.

so, L_y = constant

$$\boxed{L = 3.2\hat{i} + 3\hat{j}}\ m$$

Q.8. At what speed should a rod moves so that its length contracts by 60% along dirⁿ of motion

Solⁿ

let, length of rod $= l_0$

length contraction $= \frac{60}{100} \times l_0 = 0.6\,l_0$

$v = ?$

$$\because\ L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{L}{L_0} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\left(\frac{L}{L_0}\right)^2 = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\left(\frac{L}{L_0}\right)^2$$

$$\frac{v^2}{c^2} = 1-\left(\frac{0.6\,L_0}{L_0}\right)^2$$

$$\frac{v^2}{c^2} = 1-\frac{36}{100}$$

$$\frac{v^2}{c^2} = \frac{64}{100}$$

$$v^2 = 0.64\,c^2$$

$$\boxed{v = 0.8\,c} \text{ m/s}$$

Q.9. Calculate velocity of particle at which its mass become thrice of its rest mass.

Soln

let rest mass = m_0

m = mass contraction = $3m_0$

$$\Rightarrow \quad m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \quad 1-\frac{v^2}{c^2} = \left(\frac{m_0}{m}\right)^2$$

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$1 - \frac{1}{9} = \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = \frac{8}{9}$$

$$v^2 = \frac{8}{9} c^2$$

$$\boxed{v = \frac{2\sqrt{2}}{3} c} \text{ m/s}$$

Imp. RU

Q.10. A clock keeps correct time with what speed should it travel relative to an observer so that it appears 1 min. slow in 24 hr.

Sol^n We know

$$t = \frac{t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left(\frac{t'}{t}\right)^2 = 1 - \frac{v^2}{c^2}$$

given

One day = 24 hrs. = 24×60 min.

$\tau_0 = 24 \times 60$ min.

dilate time $\tau = 24 \times 60 - 1$

$$\tau = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$1 - \frac{v^2}{c^2} = \left[\frac{\tau_0}{\tau}\right]^2$$

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$$1-\frac{v^2}{c^2} = \left(\frac{24\times 60}{24\times 60-1}\right)^2$$

$$\boxed{v = 0.35\times 10^{8}\ cm/s}$$

Q. 11. A rod of length 3m is moving parallel to its length what will be apparent length when its apparent mass become $\frac{3}{2}$ of its rest mass.

Soln given,

length of rod, $l_0 = 3m$

let apparent mass $= m$

rest mass $= m_0$

$m = \frac{3}{2} m_0$

To find, apparent length $l = ?$

We know,

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

$$1-\frac{v^2}{c^2} = \left(\frac{m_0}{m}\right)^2$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0}{\frac{3}{2}m_0}$$

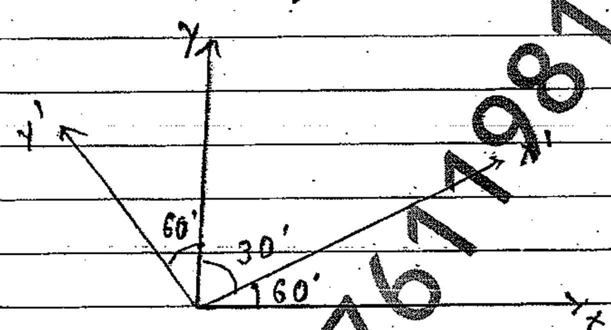
$$\sqrt{1-\frac{v^2}{c^2}} = \frac{2}{3}$$

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$L = 3 \times \frac{2}{3}$

$\boxed{L = 2m}$

Q. Find the co-ordinate of P in S' f.O.R. when co-ordinate of P in S f.O.R. (6, 8)



Sol^n We know

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

$$x' = 6\cos 60' + 8\sin 60'$$
$$x' = 6 \cdot \frac{1}{2} + 8 \cdot \frac{\sqrt{3}}{2}$$

$$x' = 3 + 4\sqrt{3}$$

$$y' = -6\sin 60' + 8\cos 60'$$
$$y' = -6 \cdot \frac{\sqrt{3}}{2} + 8 \cdot \frac{1}{2}$$

$$y' = -3\sqrt{3} + 4$$

$$(x', y') = (3 + 4\sqrt{3}, -3\sqrt{3} + 4)$$

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Note $E = mc^2$ (total energy)

$E_0 = m_0c^2$ (rest mass energy)

Q.

Sol^n given,

$E = 2E_0$

To find,

$v = ?$

Sol^n

$$E = 2E_0$$

$$mc^2 = 2m_0c^2$$

$$m = 2m_0$$

$$\because m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \left(\frac{m_0}{m}\right)^2$$

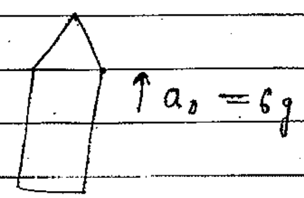
$$1-\frac{v^2}{c^2} = \frac{1}{4}$$

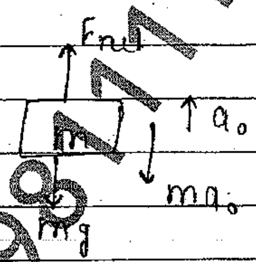
$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$\boxed{v = \frac{\sqrt{3}}{2}c}$$

Q Determine effective weight of an astronaut of mass 60 kg when it's rocket is moving vertically upward from ground with acceleration 6g.

Sol^n





$\because a_0 = 6g$

$$F_{net} = m(g + a_0)$$

$$F_{net} = 60(g + 6g)$$

$$F_{net} = 60 \times 7g$$

$$\boxed{F_{net} = 420\, g}$$

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[illegible]