Chapter - Mathematical Concept

Chapter

1A

Mathematical Concepts and Computers

Logarithmic : In the filed of higher studies of technology and science the mathematical calculations become very compùter and lenghtly. Considering this difficulty, John Napier, in 1614 AD, discovered very important method for calculation i.e. logarithmically method. From the Greek word logs meaning "Property ration or word" and arithmos meaning "Number", which together makes "ratio number". This for calculation method is very important and useful. In sense logarithmic method, converts the complex mathematical operation into simple operation.

For example : Multiplication and division is converted in to simple addition and subtraction.

1.1 Definition and Notation

If a, x, N are three positive real number and

if $a^x = N$ (Exponential form)

then $\log_a N = x$ (Logarithmic form)

Definition :

If N, a and x are positive integers (While $a \neq 1$) and if $a^x = N$ these x is the logarithm of N with base a and is depicted as $\log_a N = x$

(i) $2^3 = 8 \rightarrow \log_2 8 = 3$

(ii) $2^4 = 16 \rightarrow \log_2 16 = 4$

Note :

(i) As 1^x will always be 1, for all value of x. It mean logarithms of number, having base 1 can not be calculated.

(ii) Because $a^0 = 1$ thus $\log_a 1 = 0$ (for any value of a and $a \neq 0$).

1.2 Systems of Logarithms

There are two types :

(i) Natural logarithms :

The natural logarithm has a base of e (approximate value of 2.718281828). It is represented by In (x). The natural logarithm of x(lnx) is the power to which e must be

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raised to obtain x.

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$
$$= 2.7182 \text{ (Approximate)}$$

(ii) Common logarithm : The common logarithm is the logarithm with base 10. It is also known as decadic logarithm and also known as the decimal logarithm. Represented by $\log_x$.

1.3 Laws of Logarithms

1. Multiplication law :

$$\text{Log}_a (MN) = \log_a M + \log_a N$$

Solution : Suppose $M = a^x \rightarrow \log_a M = x$ .....(i)

and $N = a^y \rightarrow \log_a N = y$ .....(ii)

Now $MN = a^x . y^x$

$MN = a^{x+y} \rightarrow \log_a MN = x + y$

From (i) & (ii) $\log_a MN = \log_a M + \log_a N$

2. Division Law :

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

Solution : Suppose $M = a^x \rightarrow \log_a M = x$ ....(i)

$N = a^y \rightarrow \log_a N = y$ ....(ii)

Now $M/N = \frac{a^x}{a^y} = a^{x-y}$

Thus $\log_a \frac{M}{N} = x - y$

From (i) & (ii) $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

3. Exponential law :

$$\log_a M^n = n \log_a M$$

Solution : Suppose $M = a^x \rightarrow \log_a M = x$ .....(i)

$M = a^x$

$(M)^n = (a^x)^n$

$M^n = a^{xn} = a^{nx}$

$\log_a (M)^n = nx$

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$$\log_a (M)^n = n \log_a M$$ ...from (i)

4. Change of base law :

$$\log_a M = \log_b M \times \log_a b$$

Solution : Suppose $\log_b M = x \rightarrow b^x = M$ .....(i)

$$\log_a b = y \rightarrow a^y = b$$ .....(ii)

Raising the equation to the power x

$$b^x = (a^y)^x \rightarrow b^x = a^{xy}$$

$\Rightarrow$ $M = a^{xy}$ (from (i))

$\Rightarrow$ $\log M = xy$

$\Rightarrow$ $\log_a M = \log_b (Mx) \log_a b$

Sub. Rule 1 : $\log_a a = 1$

We know that $a^1 = a \rightarrow \log_a a = 1$

Sub Rule 2 : $\log_a 1 = 0$

We know that $a^0 = 1 \rightarrow \log_a 1 = 0$

Sub Rule 3 : $\log_a b \times \log_b a = 1$

Suppose $\log_a b = x \rightarrow b = a^x$

$$a = b^{1/x}$$

Thus $\log_a b \times \log_b a = x\frac{1}{x} = 1$

Problem 1 : Write $5^3 = 125$ in logarithmic form

Solution : log of $5^3 = 125$ is $\log_5 125 = 3$

Problem 2 : Write $5^{-2} = \frac{1}{25}$ in log form

Solution : It is $\log_5 \left(\frac{1}{25}\right) = -2$

Problem 3 : (i) $\log_3 81 = 4$

Solution : $3^4 = 81$

(ii) $\log_2 256 = 8$

Solution : $2^8 = 256$

1.5.1 Slope of a straight line :

Step one : Identify two points on a line.

Step two : Select one to be (x_1, y_1) and the order to be (x_2, y_2)

Step three : Use the slope equation to calculate slope.

$$\boxed{m = \tan\theta}$$

1.5.2 Equation of a straight line :

(i) General form : $Ax + By + C = 0$

where A, B, C, are constands Slope = – A/B

(ii) Slope form : $Y = mx + C$

Where m slope of line and c is slope of straight line on the y-axis.

(iii) Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$

there a and b are straight line on x-axis and y-axis slope = – b/a

Problems 5 : $+x\sqrt{3} - y + \sqrt{7} = 0$ calculate the slope

Solution : $y = x\sqrt{3} + \sqrt{7}$

$\Rightarrow$ $m = \sqrt{3}$

Problem 6 : Determine the equation for a straight line, which cuts the y-axis at -5 and make an angle of 120° with the x-axis.

Solution : Suppose equation of straight line is $y = mx + c$,

where $\theta = 120°$ and $c = -5$

Thus, $m = \tan 120° = -\sqrt{3}$

Thus, the equation. $y = -\sqrt{3x} - 5$.

Some theorems on differentiation :

(i) $$\frac{d}{dx}[kf(x)] = k\frac{d}{dx}f(x), \text{ there k is a}$$

example $$\frac{d}{dx}kx = k\frac{d}{dx}(x) = k.1 = k$$

(ii) $$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}(g(x))$$

Example $$\frac{d}{dx}(x^2 + \sin x) = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin x) = 2x + \cos x$$

(iii) $$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$$

(iv) $$\frac{d}{dx}[f(x).g(x)] = \frac{d}{dx}(uv) = U\frac{dv}{dx} + V\frac{du}{dx} [\therefore u = f(x), v = g(x)]$$

(v) $$\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}, v \neq 0$$

Problem : xe^x find the differential coefficient

Solution : $$\frac{d}{dx}(xe^x) = x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x)$$

$$= xe^x + e^x$$

$$= e^x(x + 1)$$

1.7 Maxima and Minima

Definition :

Any point on a graph where the slope falls down from a higher value to lower or lower to higher are sequentially known as maxima or minima. These points of maxima or minima are respectively denoted as 'a' or 'b' on the graph.

(ii) The value of function f(x) at x = a, f(a) is known as maxima if value of f(x) is smaller than f(a) for each value of x, in neigbourhood of x = a.

(iii) Similarly value of f(x) at x = a, is known as minimum value of f(a), if value of f(a) is higher than f(a) for every value of x in the neighborhood of x = a.

(iv) Increasing function : In a range (a, b) if value of f(x) increases with in crease in value of x, it is known as increasing function.

(v) Decreasing function : In a range (a, b) if value of function f(x) decrease with increasing value of x, it is known as decreasing function.

1.7.1 Important points :

(i) The maxima & minima points are also known as extreme point.

(ii) There exist at least one minima or maximum point between two equal values of f(x).

(iii) If at some point value o f(x) is maximum then at that point value of f(x) [f(x) ≠ 0] will be minimum and vice-versa.

(iv) If the value of f'(x) remains unchanged or both sides of a point then this point will neither be maxima nor a minima. Such point is point of inflexion.

(v) Maxima and minima occurs alternatively.

(vi) It is not necessary that value of f(x) is maximum at the point of maxima for a given interval and vice-versa.

1.7.2 Working method for determination of maxima & minima

(i) y = f(x) : Take the given function.

(ii) Determine the value of dy/dx or f'(x), equate it to zero and solve the equation to determine the roots a_1, a_2, a_3......

(iii) Determine d^2y/dx^2 or f''(x)

(iv) Substitute x = a_1, a_2, a_3...... respectively in d^2y/dx^2.

(v) If $\left(\frac{d^2y}{dx^2}\right)$ or f''(a_r), (where r = 1, 2, 3....) is negative then at x = a_r there will be maxima of f(x).

(vi) If the value of $\left(\frac{d^2y}{dx^2}\right)_{x=ar}$ of f''(ar); is +Ve (where r = 1, 2, 3....) then x = ar, will be minima of f(x).

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(vii) If $\left(\frac{d^2y}{dx^2}\right)_{x=ar} = 0$ or $f''(ar) = 0$ then differentiate f(x) further.

(viii) The point which is neither maxima, nor minima is known as point of inflexion.
Condition for point of inflexion : $y' = f'(a) = 0$

$$f''(a) = 0$$

$$f'''(a) \neq 0$$

1.7.3 Solved examples

Example 1 : Calculate the maximum value of r for the following curve.

$$\frac{C^4}{r^2} = \frac{a^2}{\sin^2\theta} + \frac{b^2}{\cos^2\theta}$$

Solution :

$$\frac{C^4}{r^2} = a^2 \operatorname{cosec}^2\theta + b^2 \sec^2\theta$$

$$= a^2 + a^2 \cot^2 + b^2 + b^2 \tan^2\theta$$

$$= (a^2 + b^2 + 2ab) + (a^2 \cot^2\theta + b^2\tan^2\theta - 2ab)$$

$$\frac{C^4}{r^2} = (a + b)^2 + (a \cot\theta - b \tan\theta)^2$$

$$\frac{C^4}{r^2} = (a + b)^2 + (a \cot\theta - b \tan\theta)^2$$

From above, We can note that value of C^4 will be minimum, if $(a \cot\theta - b \tan\theta) = 0$

or $\tan^2\theta = \frac{a}{b}$

In this situation minimum value of $\frac{C^4}{r^2} = (a + b)^2$

Thus maximum value $\frac{r^2}{C^4} = \frac{1}{(a+b)^2}$

Maximum value of $r = \frac{C^2}{a+b}$

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Example 2 : Prove that at x = 1, value of $x^5 - 5x^4 - 5x^3 - 10$ is at maxima and at x = 3, minima and also show that at x = 0, it is point of inflaxion.

Solution : Suppose

$$f(x) = x^5 - 5x^4 + 5x^3 - 10$$

$$f'(x) = 5x^4 - 20x^3 + 15x^2$$

$$= 5x^2 (x^2 - 4x + 3)$$

$$= 5x^2 (x - 3)(x - 1)$$

For maximum and minimum value

$$f'(x) = 0 = 5x^2(x - 3)(x - 1) = 0$$

$\Rightarrow$ $x = 0, 1, 3$

Thus from (i) $f''(x) = 20x^3 - 60x^2 + 30x$

$$= 10x (2x^2 - 6x + 3)$$

$\therefore$ at $x = 1, f''(x) = -10$ (negative)

Thus at $x = 1$, f(x) is maximum

$\therefore$ at $x = 3, f''(x) = 90$ (+Ve)

Thus, at $x = 3$, f(x) is minimum

$\therefore$ at $x = 0, f''(x) = 0$

Again by differentiating f''(x)

$$f'''(x) = 60x^2 - 120x + 30$$

at x = 0, f'''(x) is neither maximum nor minimum, it is point of inflexion.

Example 3 : Determine the points where value of θ for given equation $\sin^p\theta \cos^q\theta$ (p, θ > 0) gets maximum.

Solution :

$$y = \sin^p\theta \cos^q\theta \qquad \text{.....(i)}$$

If value of y will be maximum or minimum then value of log y will also be maximum or minimum.

Taking log on both side of equation (i) and equating to z-

$$z = \log y = p \log \sin\theta + q \log \cos\theta$$

$$\frac{dz}{d\theta} = p \cot\theta - q \tan\theta$$

$$\frac{d^2z}{d\theta^2} = -p \operatorname{cosec}^2\theta - q \sec^2\theta$$

For maximum and minimum value of z

$$\frac{dz}{d\theta} = 0 = p \cot\theta - q \tan\theta = 0 \text{ or } \tan^2\theta = \frac{p}{q}$$

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$$\Rightarrow \qquad \tan\theta = \sqrt{\frac{p}{q}} \text{ or } \theta = \tan^{-1}\left(\sqrt{\frac{p}{q}}\right)$$

Putting value of θ from (iv) in eq. (iii)

$$\frac{d^2z}{d\theta^2} = -p(1-\cot^2\theta) - q(1+\tan^2\theta)$$

$$= -p(1 + q/p(-q(1+p/q))$$

$$-p-q-q-p = -2(p+q) \text{ (negative)}$$

The value of $z = \log y$ is maximum at $\theta = \tan^{-1}\sqrt{p/q}$

Thus, y is maximum at $\theta = \tan^{-1}\sqrt{p/q}$

Example 4 : Calculate the value of r, for the curve

$$\mathbf{r^2 = a^2 \sec^2\theta + b^2 \operatorname{cosec}^2\theta}$$

Solution : Suppose $\qquad R = r^2 = a^2\sec^2\theta + b\operatorname{cosec}^2\theta$

For minimum value of r^2 (or R) $\dfrac{dR}{d\theta} = 0$

$$\frac{dR}{d\theta} = 2a^2\sec^2\theta\tan\theta - 2b^2\operatorname{cosec}^2\theta\cot\theta = 0$$

$$\Rightarrow \qquad a^2\tan^4\theta = b^2 \text{ or } \tan^2\theta = b/a$$

$$\Rightarrow \frac{d^2R}{d\theta^2} = 2a^2(\sec^4\theta + 2\sec^2\theta\tan^2\theta) + 2b^2(\operatorname{cosec}^4\theta + 2\operatorname{cosec}^2\theta\cot^2\theta)$$

Substituting value of $\tan^2\theta = \dfrac{b}{a}$ in $\dfrac{d^2R}{d\theta^2}$

$$\frac{d^2R}{d\theta^2} = 2a^2\left[\left(1+\frac{b}{a}\right)^2 + 2\left(1+\frac{b}{a}\right)\frac{b}{a}\right] + 2b^2\left[\left(1+\frac{a}{b}\right)^2 + 2\left(1+\frac{a}{b}\right)\frac{a}{b}\right]$$

Which is positive

Thus for $\tan^2\theta = \dfrac{b}{a}$, **R** is minimum or r^2 is minimum

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Examples :

Example 1 : For the given function

$$Z = 2x^2 + 3xy + y^3, \text{ calculate}$$

(a) $\frac{\partial Z}{\partial x}$ (b) $\frac{\partial Z}{\partial y}$

Solution : $Z = 2x^2 + 3xy + y^3$

(a) $\frac{\partial Z}{\partial x} = 4x + 3y$ [Differentiation, taking y as constant]

(b) Similarly $\frac{\partial Z}{\partial y} = 3x + 3y^2$ [Differentiation, taking x as constant]

Example 2 : Determine $\frac{\partial U}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial U}{\partial Z}$ **for the function**

$$u = ax^2 + by^2 + cz^2 + Z = yz + 2gzx + 2hxy$$

Solution : $u = ax^2 + by^2 + cz^2 + z = yz + 2gzx + 2hxy$

$$\frac{\partial u}{\partial x} = 2ax + 2gz + 2hy \text{ [taking y and z a constant]}$$

$$\frac{\partial u}{\partial y} = 2by + z + 2hx \text{ [taking x and z as constant]}$$

$$\frac{\partial y}{\partial z} = 2cz + y + 2gx \text{ [taking x and y as constant]}$$

Example 3 : If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = \frac{3}{x+y+z}$$

Solution : $u = \log(x^3 + y^3 + z^3 - 3xys)$

taking partial diffrentiation w.r.t x, y, z

$$\frac{\partial u}{\partial x} = \frac{3x^2 - 3yz}{x^3+y^3+z^3-3xyz} = \frac{3(x^2-yz)}{x^2+y^3+z^3-3xyz}$$

$$111_y \frac{\partial u}{\partial y} = \frac{3(y^2-xz)}{x^3+y^2+z^3-3xyz}$$

$$\frac{\partial u}{\partial z} = \frac{3(z^2-xz)}{x^3+y^3+z^2-3xyz}$$

Example 4 : Proove, $\frac{\partial^2 U}{\partial x \partial y} = \frac{\partial^2 U}{\partial y \partial x}$ **when**

$$u = ax^3 + 3bx^2y + 3cxy^2 + dy^3$$

Solution : $u = ax^3 + 3bx^2y + 3cxy^2 + dy^3$

$$\frac{\partial u}{\partial x} = 3ax^2 + 6bxy + 3cy^2, \quad \frac{\partial u}{\partial y} = 3bx^2 + 6cxy + 3dy^2$$

$$\frac{\partial^2 u}{\partial y \partial x} = 6bx + 6cy,$$

$$\frac{\partial^2 u}{\partial x \partial y} = 6bx + 6cy$$

$$\therefore \quad \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} \quad \text{.....Hence proved}$$

1.9 Integration

Integration was introduced for summation of infinite series so as to calculate the area of planes. To determine variable of function $y = f(x)$, whose differential coefficient is dy/dx. Thus it is a process by which a function f(x) is determined by the given

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function f(x) whose differential $\frac{d[f(x)]}{dx}$ is again the same function f(x), is known as integration.

From above it is understood that process of integration is just inverse process to the differentiation and is also known as antiderivatives.

1.9.1 Definition :

For the given function f(x), the integration is denoted as F(x) w.r.t to x.

Such that

$$\frac{d}{dx}[F(x)] = f(x) \qquad \text{[Given function]}$$

and integration of function f(x) is represented as

$$\int (x)dx = F(x)$$

Here '$\int$' sign is use to depict integration and dx, shows the variable x, with those reference, the function is integrated and the function to be integrated is known as integrand.

1.9.2 The constant of Integration :

We know that differential coefficient of a constant is zero, thus $\frac{d(c)}{dx} = 0$ where c is constant.

Suppose $$\frac{d}{dx}[F(x)] = f(x)$$

$$\Rightarrow \quad \frac{d}{dx}[F(x) + c] = \frac{d}{dx}F(x) + \frac{d}{dx}(c)$$

$$= \frac{d}{dx}F(x) - 0 = \frac{d}{dx}F(x) = f(x)$$

upon integrating both side w.r.t x,

$$\int f(x)dx = \left[\frac{d}{dx}[F(x)+c]\right]dx$$

$$\int f(x)dx = F(x) + c \text{ [According to definition]}$$

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where c is constant whose value is independent of x. By taking different value of c, we get different values after integration.

1.9.3 Definite integral :

Integration done within the given limits are known as definite integration

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

where a is lower limit and b is upper limit

1.9.4 Standard formulae of integration :

We can derive the standard formulae of integration easily by using, the above stated definition of integration in the standard formulae of their corresponding differentiation.

For example :

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c_1 \ (n \neq -1)$$

as we know $\frac{d}{dx}(x^n) = nx^{n-1}$

By integrating both sides

$$\int \left[\frac{d}{dx}(x^n)\right] dx = \int nx^{n-1} dx.$$

$$\Rightarrow \quad n\int x^{n-1} dx = x^n$$

Replacing n, with n + 1, we get

$$(n+1)\int x^n dx = x^{n+1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$$

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Table of general integration

Formulae for differentiation		**Formula of integration**
1. $\frac{d}{dx}(\log x) = \frac{1}{x}$	$\Rightarrow$	$\int \frac{1}{x} dx = \log x + C$
2. $\frac{d}{dx}(e^x) = e^x$	$\Rightarrow$	$\int e^x dx = e^x + c$
3. $\frac{d}{dx}(a^x) = a^x \log_e a$	$\Rightarrow$	$\int a^x dx = \frac{a^x}{\log_e a} + c$
4. $\frac{d}{dx}(x^{n+1}) = (n+1)\, x^n$	$\Rightarrow$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
5. $\frac{d}{dx}(\sin x) = \cos x$	$\Rightarrow$	$\int \cos x dx = \sin x + c$
6. $\frac{d}{dx}(\cos x) = -\sin x$	$\Rightarrow$	$\int \sin x dx = -\cos x + c$
7. $\frac{d}{dx}(\tan x) = \sec^2 x$	$\Rightarrow$	$\int \sec^2 x dx = \tan x + c$
8. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$	$\Rightarrow$	$\int \operatorname{cosec}^2 x dx = -\cot x + c$
9. $\frac{d}{dx}(\sec x) = \sec x \tan x$	$\Rightarrow$	$\int \sec x \tan x dx = \sec x + c$
10. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$	$\Rightarrow$	$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$

1.9.5 Theorems of Integration :

Theorem 1 : Integration of a constant and a function is equal to product of that constant and integration function.

$$\boxed{\int af(x)dx = a\int f(x)dx + c}$$

Theorem 2 : Integration of algebric sum of function is equal to the sum/substraction of integrated function.

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$$\left[\int f_1(x)dx \pm \int f_2(x)dx + \ldots\ldots\right] = \int f_1(x)dx \pm \int f_2(x)dx \ldots\ldots$$

1.9.6 Examples :

Example 1 : $\int\left(\sin x + 3x^2 + \frac{1}{\sqrt{1-x^2}}\right)dx$

Solution : $\int\left(\sin x + 3x^2 + \frac{1}{\sqrt{1-x^2}}\right) dx$

$$= \int \sin x dx + \int 3x^2 dx + \int \frac{1}{\sqrt{1-x^2}} dx.$$

$$= -\cos x + \frac{3x^3}{3} + \sin^{-1} x + c$$

$\Rightarrow$ $-\cos x + x^3 + \sin^{-1} x + c$

Example 2: $\int 6x^4 dx$

Solution : $\int 6x^4 dx$

$\Rightarrow$ $6\left(\frac{x^{4+1}}{4+1}\right) + c$

$\Rightarrow$ $\frac{6x^5}{5} + c$

Example 3 : $\int_1^2 x^2 dx$

Solution : $\int_1^2 x^2 dx$

$\Rightarrow$ $\left[\frac{x^{2+1}}{3}\right]_1^2 + C$

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$$\Rightarrow \left[\frac{x^3}{3}\right]_1^2 + C$$

$$\Rightarrow \frac{(2)^3}{3} - \frac{(1)^3}{3} + C$$

$$\Rightarrow \frac{8-1}{3} + C$$

$$\Rightarrow \frac{7}{3} + C$$

Example 4 : $\int \tan x \sec x dx$

Solution : $\int \tan x \sec x dx$

Example 5 : $\int \frac{ax^2 + bx + c}{x} dx.$

$$I = \int \left(ax + b + \frac{c}{x} \right) dx$$

$$= a\int x dx + b\int dx + c\int \frac{1}{x} dx$$

$$= \frac{ax^2}{2} + bx + c \log x + d$$

where d is an integration constant

1.10 Factorials

Let there be a natural integer denoted by 'n'. the product of multiplication of n and all the numbers before n. is known as ordered multiple n.

It is represented by $\underline{|n}$ and n!

$\Rightarrow$ $\underline{|n} = n! = n (n-1) (n - 2)........3.2.1.$

Example : $\underline{|5} = 5! = 5(4)(3)(2)(1) = 120$

$\underline{|3} = 3! = 3(2) = 6$

Notes : N = natural number = [1, 2, 3.......]

$\Rightarrow$ $|\underline{0} = 1$

$\Rightarrow$ If n is negative then $|\underline{n}$ or n! does not exist

$\Rightarrow$ $n! = n\,(n-1)!$

$\Rightarrow$ $\ln N! = \ln N - N$

1.11 Permutation and combination

1.11.1 Permutation :

Permutation are the set of different arrangements for the given objects which may or may not include all the given object. These arrangements are all unique and non repeated.

For example : Different arrangement for the given set of alphabets i.e. A, B and C taking two objects at a time are :

AB, BC, CA, BA, CB, AC.

Since there are 6 arrangement for the given set hence permutation for given set is 6.

(ii) Consider another example : We are given with 3 nos. 1,2,3

$\therefore$ Permutation for the given no. will be

123, 321, 231, 132, 213, 312.

1.11.2 From n numbers of object, compute the permutation of r.

Anything can be chosen in the first place, $\therefore$ We are having 'n' options for the first place. Now for the second place, one has to choose from the rest $(n-1)$ objects left. Similarly for r^{th} object one can have $(n-r+1)$ methods or ways. Thus from different n objects, the number of permutation for all the objects r at a time are :

No. of permutation $\quad {}^nP_r = n \times (n-1) \times (n-2) \times ..... \times (n-r+1)$

Rule 1 : $\quad {}^nP_n = n(n-1).....3 \times 2 \times 1 = n!$ or $|\underline{n}$

$\Rightarrow$ $\quad {}^nP_n = |\underline{n}$

Rule 2 : $\quad {}^nP_r = n(n-1)(n-2)....(n-r+1)$

$$= \frac{n(n-1).....(n-r+1)(n-r)......3.2.1.}{(n-r)....3 \times 2 \times 1} = \frac{n!}{(n-r)!}$$

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

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Rule 3 : $$\lfloor 0 = 1$$

$$^nP_r = \frac{\lfloor n}{\lfloor (n-r)}$$

$$\Rightarrow \quad ^nP_r = \frac{n!}{\lfloor (n-n)} = \frac{\lfloor n}{\lfloor 0}$$

$$\Rightarrow \quad \lfloor n = \frac{\lfloor n}{\lfloor 0}$$

$$\Rightarrow \quad \lfloor 0 = 1$$

Rule 4 : $$^nP_r = n^{n-1}P_{r-1}$$

Rule 5 : $$^nP_0 = 1$$

$$^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$$\Rightarrow \quad ^nP_0 = \frac{\lfloor n}{\lfloor n} = 1$$

1.11.3 Conditional permutation :

In some cases we might be give a condition to use some particular things/object at particular places. These types of permutation are known as conditional permutation.

Example : How many permutation can be formed out of the word 'UDA1PUR" when :

(i) In every permutation P, U, R comes in the same order and together

(ii) P,U,R comes together

(iii) Every permutation starts with P.

Solution : As PUR, should come in same order always this is considered to one letter group. Thus now the no. of letters are U, D, A, I, PUR i.e. 5.

Thus No. of permutation = $^5P_5$

$\Rightarrow$ $5 \times 4 \times 3 \times 2 \times 1 = 120$

(ii) Taking PUR as one letter group, the no. of permutations = 120, as above.

Now, by changing the places of P, U, R new permutation can be formed $^3P_3 = 6$

Hence total permutation = $120 \times 6 = 720$

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(iii) As first letter should be P. position of P is fixed thus only 6 letters are to be arranged.

Thus No. of permutation = ${}^6P_6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

1.11.4 Permutation of things, where some of them are similar :

Suppose there are n things, out of which 'P' are of one type 'a' are of second time and 'r', of third type and remaining things are of different types.

Suppose number of required permutation is x,

Thus $$x = \frac{\lfloor n}{\lfloor P \lfloor q \lfloor r}$$

Example : Determine the number of words formed by taking letters from word

'MATHEMATICS'

Solution : There are 11 letters from which M,A,T are repeated 2 times

$$\therefore \qquad \text{Permutation} = \frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2} = 4989600$$

1.11.5 Circular permutation :

The concept of arrange in n no. of various objects around a circle is known as circular permutation.

In circular arrangement place of one object is decided and then the other objects are arranged with reference to that object.

Suppose n things are to be arranged in a circle, out of these n things, place of one object at conglent place remaining (n – 1) things are arranged w.r.t this, whose total no. of way are $\lfloor n-1$.

Thus for n things the number of circular permutation is (n – 1). If clock wise and anti-clock wise, permutation are taken as same.

Thus $\frac{1}{2}$ (n – 1)! will be the permutation.

Example : In how many ways can a garland with 20 flowers can be made ?

Solution : $\frac{1}{2}(n-1)!$

$$\Rightarrow \qquad \frac{1}{2}(20-1)! \Rightarrow \frac{\lfloor 19}{2}$$

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Example 2 : In how many ways 9 people can be seated around a round table ?

Solution : $(n-1)! \Rightarrow (9-1)! = 8! = 40320$ ans.

1.11.6 Solved example :

Example : If $^{12}P_r = 11889$ calculate value of r

Solution : $\therefore$

$$^{12}p_r = 11880$$

$$^{12}P_r = 12 \times 11 \times 10 \times 9$$

$$\Rightarrow \quad \frac{12 \times 11 \times 10 \times 9}{4} = 11880$$

2nd method :

$$^{12}P_r = \frac{|\underline{12}}{|\underline{12-r}} = 11880$$

$$\Rightarrow \quad \frac{12|\underline{11}}{|\underline{12-r}} = 11880$$

$$\frac{11|\underline{10}}{|\underline{12-r}} = 990$$

$$\text{or} \quad \frac{10|\underline{9}}{|\underline{12-r}} = 90 \text{ or } \frac{10|\underline{8}}{|\underline{12-r}} = 9$$

$$\text{or} \quad \frac{|\underline{8}}{|\underline{12-r}} = 1 \text{ or } |\underline{8} = |\underline{12-r}$$

$$\Rightarrow \quad 8 = 12 - r$$

$$r = 4 \text{ Ans.}$$

Example 2 : From the given numbers 1,2,3,4,5 how many numbers can be formed without repeating the numbers ?

Solution : The required number or essential

$$= {}^5P_3 \text{ (No. of permutation)}$$

$$= 5 \times 4 \times 3$$

$$= 60$$

Example 3 : With the word 'Jaipur' how many permutation can be formed

(i) If all the letters are taken

(ii) If only four letters are taken

Solution : There are 6 deferent letters in JAIPUR

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(i) If 6 letters are taken

No. of permutation $= {}^6P_6$

$\Rightarrow \quad \underline{|6} = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

(ii) By taking four letter

No. of permutation $= {}^6P_4$

$\Rightarrow \quad = 6 \times 5 \times 4 \times 3$

$= 360$

1.11.7 Combination :

In a given no. of objects, grouping those objects in such a way that they are non-repeated and have unique pattern such grouping is known as combination.

This number is denoted by ${}^nC_r$

$$\Rightarrow \quad {}^nC_r = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}$$

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

Rule 1 : ${}^nC_r = \underline{|r} \times {}^nP_r$

Rule 2 : ${}^nC_r = {}^nC_{n-r}$

Solution :

$$ {}^nC_{n-r} = \frac{\underline{|n}}{\underline{|(n-r)}\,\underline{|(n-n+r)}}$$

$$= \frac{\underline{|n}}{\underline{|(n-r)}\,\underline{|n}} = {}^nC_r = \text{L.H.S}$$

Rule 3 : ${}^nC_r = \frac{n-r+1}{r} . {}^nC_{r-1}$

Solution :

$$ {}^nC_r = \frac{n!}{(r-1)!}\ (n-r+1)!$$

$$= \frac{r \times n!}{r \times (r-1)!(n-r+1)!(n-r)!} \quad (\therefore n! = n(n-1)$$

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$$\Rightarrow \qquad {}^nC_r = \frac{n-r+1}{r}\ {}^nC_{r-1}$$

Rule 4 : $\quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Rule 5 : $\quad {}^nC_n = {}^nC_0 = 1$

$${}^nC_n = \frac{\lfloor n}{\lfloor n \lfloor (n-n)!}$$

$$\Rightarrow \qquad {}^nC_n = {}^nC_0 = 1$$

Example : If ${}^{25}C_r = {}^{25}C_{r+5}$ then calculate

Value of r and ${}^{10}C_3$

Solution : $\quad {}^{25}C_r = {}^{25}C_{r+5}$

$$\Rightarrow \qquad {}^{25}C_{25-r} = {}^{25}C_{r+5}$$

$$\Rightarrow \qquad 25 - r = r + 5$$

$$2r = 20$$

$$\therefore \qquad r = 10$$

Again $\quad {}^{10}C_3 = \frac{10\times 9\times 8}{3\times 2\times 1} = 120$

1.11.8 Examples :

Example 1 : Compute ${}^{10}C_3$

Solution : $\quad {}^{10}C_3 = \frac{10\times 9\times 8}{3\times 2\times 1} = 120$

Example 2 : If $\quad {}^nC_{15} = {}^nC_{13}$

$${}^nC15 = n - 13 \Rightarrow n = 28$$

Example 3 : Out of 16 players how many teams of 11 players could be made

Solution : $\quad {}^{16}C_{11} = {}^{16}C_{16-11}$

$$= {}^{16}C_3 = \frac{16\times 15\times 14\times 13\times 12}{5\times 4\times 3\times 2\times 1}$$

$$= 4368$$

Example 4 : From deferent n things, determine the number of combination by taking r things together if.

(i) The particular things p are always taken

(ii) The particular things p are not taken

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Solution : (i) If things p are always taken in every group to form group of r things only r-p thing have to be chosen from the remaining n-p thing.

$$\text{Required number} = {}^{n-p}C_{r-p}$$

(ii) If p things are never chosen than r things must be taken from remaining n-p things.

$$\text{Required number} = {}^{n-p}C_{r}$$

1.12 Probability

Introduction : There are many day to day events about which we know the probable result but we are not sure about them. For example, when a coin is tossed, we are sure about the result whether tails or head but we are sure enough that the result is either going to be tails or head. This situation when we know about the result but are not sure enough is known as probability.

Probability theorem : It is the mathematical explaination, for the chances of any act or event to take place, out of many possible events.

1.12.1 Terminologies and their definitions related to probability

(i) Trials and events : Any experiment in which out of many possible results one is sure, is known as trial and possible events are known as events.

Example (i) To toss a coin is a trial but the result head or tail is and event.

(ii) To throw a dice is trial and the outcome whether 1,2,3,4,5,6, is event.

(ii) Simple event : If in a trial, there is only one event at a time it is known as simple event

Example : There are some whits some yellow balls in a bag. To take out one ball is a simple event.

(iii) Compound event : If two or more events take place, at the same time or take place with reference to each other, then the event is known as compound event.

Example : Two bag contain different number of 50 paise and one rupee coin to select one bag and take out one coin is a compound event.

(iv) Independent and Dependent event : (a) Independents event : Two or more such events where result on event does not effect the result of other events are known as independent events.

Example : Two coins are tossed together one has tail as outcome and other head as outcome.

(b) Dependent events : If two or more events takes place in such a way that one affects another they are known as dependent events.

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Example : If there are some blue and some green balls in a bag. If one bag is taken out and the other ball is taken out without mixing the previous ball.

(v) Mutually exclusive or disjoint events : If two or more events could not take place together or if one event in taking place, other events do not take place such events are mutually exclusive

Example : On throwing a dice only 1,2,3,4,5,6 number can appear. This is mutually exclusive events.

(vi) Equally likely events : In an experiment if possibilities of all the results or events are equal.

Example : In tossing a coin the possibilities of head and tail are equal.

(vii) Favorable events : The favorable conditions for a particular result or event in an experiment are known as favorable events.

(viii) Exhaustive events : All possible results of an experiment or event are known as exhaustive event.

(ix) Sample space : Set of all the possible outcome of an event is termed as sample space.

1.12.2 Mathematical definition of probability :

If there are n possible outcomes for an experiment which are equally likely mutually exclusive or exhaustive and out of these, m possibilities are favourable to a particular event A.

Then probability of A i.e. P(A) can be given by a ratio m/n thus

$$p(A) = \frac{\text{Favorable condition for event A}}{\text{Exhaustive events for the event}}$$

If probability of not happening the event A, is expressed by $P(\bar{A})$ then

$$P(\bar{A}) = \frac{n-m}{n} = \frac{\text{non favorable condition for event A}}{\text{exhaustive condition for that event}}$$

$$\text{Clearly, } P(A) + P(\bar{A}) = 1 = p + q = 1$$

$$\Rightarrow \quad P(\bar{A}) = 1 - p(A)$$

probability of an event is not more than one. If an event is impossible

$$P(A) = 0$$

$$0 \leq P(A) \leq 1$$

1.12.7 Examples :

Example 1 : Determine the probability of getting less than 3 in playing a dice.

Solution : In dice one can get any number from 1 to 6, thus total no. of cases or exhausters events are 6.

Favorable events for less than 3 are (1 and 2) i.e.2. Thus probability = $\frac{2}{6} = \frac{1}{3}$

Example 2 : In throwing two dice, determine the probability of getting 6 in at least one dice.

Solution : Total exhaustive events = $6 \times 6 = 36$

Favorable events for getting 6, in at least one dice

(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6) (6, 1) (6, 2) (6,3) (6, 4) (6, 5)

Total favorable solution = 11

$\therefore$ Required probability = 11/36

Problem 1 : On throwing two dice together determine the probability of getting the number whose total is 6 or 7.

Solution : $P(A + B) = P(A) + P(B)$

Here for two dice exhaustive events = 36

Favorable events for event A;

(5, 1), (4, 2), (3, 3), (2, 4) (1,5) = 5

$$\therefore \quad P(A) = \frac{5}{36}.$$

Favorable events of B, (6, 1), (5, 2), (4, 3) (3, 4) (2, 5) (1, 6) = 6

$$\therefore \quad P(B) = \frac{6}{36}$$

Required probability $P(A + B) = \frac{5}{3} + \frac{6}{36} = \frac{11}{36}$

Exercise

(a) Very Short Answer Type Questions

1. Find the value of $\log_{10} 1000$. [Ans. 3.0]
2. Find expanded form of $\log_e (a \times b)$. [Ans. 2.303 (log a + log b)]
3. Write $a^{5/2} = 243$ in logarithmic form. [Ans. $\log_a 243 = 5/2$]
4. If $\log_{10}{}^{2} = 0.3010$ and $\log_{10}{}^{3} = 0.4771$, then find $\log_{10}6$. [Ans. 0.7781]
5. Write a short note on logarithms.
6. Find slope of equation $x - 2y + 3 = 0$ [Ans. 1/2]
7. Find length of intercepts on axes cut by straight line $x - 3y = 3$. [Ans. 3, – 1]
8. Define coordinates.
9. Write a note on differentiation.
10. Defferentiate $y = e^x + x^2$ with respect to x. [Ans. $e^x + 2x$]
11. Write differential coefficient of x^6. [Ans. $6x^5$]
12. Find the differential coefficient of any constant. [Ans. 0 (zero)]
13. What is the extrame points.
14. What is point of inflexion ? Explain.
15. Write any two properties of maxima and minima.
16. Find the points where function $x^5 - 5x^4 + 5x^3$ has maximum and minimum.
[Ans. maxima x = 2, minima x = 1]
17. Write reciprocal relation for partial differentiation.
18. If $f(x, y) = x^3 - 3y^2 - 3$ then find $\frac{\partial^2 f}{\partial x^2}$. [Ans. 6x]
19. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ when $u = 2x^2 - 3xy + 4y^2$. [Ans. 4x – 3y, – 3x + 8y]
20. State Euler's Theorem for homogeneous function.
21. Write commutative property of partial differentiation.
22. Determine the value of $\int (2x^2 - 5x + 3)dx$. [Ans. $\frac{2x^3}{3} - \frac{5x^2}{2} + 3x + c$]
23. Write any four standard formula of integration.
24. Evaluate $\int (x+3)^2 dx$. [Ans. $\frac{(x+3)^3}{3} + c$]

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25. Write a short note on Integration.

26. Write the main difference between permutation and combination.

27. Define factorial and find the value of $\underline{|5}$. [Ans. 120]

28. How many 3-digit numbers can be formed by digit 1, 2, 3, 4, 5. [Ans. 10]

29. Evaluate $^{15}C_{11}$. [Ans. 1365]

30. Explain mathematical definition of probability.

31. Define simple event.

32. State addition theorem of probability.

33. What is the probability of same number appears on three dice in any throw of three dice.

[Ans. 1/36]

34. How many words forms by the word BANANA. [Ans. 60]

35. Evaluate $\int_0^1 x^2 dx$. [Ans. 1/3]

36. Explain permutation and combination. [राज. 2012]

37. Expand the following. $\log_e (a \times b)$

38. In an equation $y = mx + c$ what will be the slope ? [अजमेर, 2011]

39. Find the slope of the line $x + 3y = 6$ [अजमेर, 2012]

(b) Short Answer Type Questions

Prove that :

1. $\log\frac{12}{7} + \log\frac{9}{4} - \log\frac{27}{7} = 0$

2. $\log_5 3 \times \log_3 4 \times \log_2 5 = 5$

3. $\log_b a \times \log_c b \times \log_a c = 1$

4. $2\log\frac{8}{45} + 3\log\frac{25}{8} - 4\log\frac{5}{6} = 2$

5. Sketch the graph of equation $x - 2y + 3 = 0$.

6. Draw the graph of equation $y = 3x + 2$.

7. Find equation of straight line which cuts at -4 on y-axis and makes an angle 30° with x-axis.

[Ans. $y = \frac{1}{\sqrt{3}}x - 4$]

8. Find slope of a line which passes through the points (1, 2) and (−1, 4). [Ans. 1]

9. Find differential coefficient of $y = \sin x + \cos x$.

[Ans. $\cos x - \sin x$]

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Chapter -
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