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Statistical Physics

CHAPTER-7

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UNIT-IV Quantum Statistics

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Postulates OF Quantum Statistics:

Acc^n to concept of Quantum statistics, Effect that occurs in nature are explained in Quantum mechanics. To explain the particle in classical mechanics we should have find the position x at time t. In this way, in Quantum statics, to explain the system (particle) we should have knowlege of state function. This state function is known as wave function that is denoted by $\psi(x,t)$. To calculate this wave function, in 1926 Schrondinger scientist define a differential equation that is known as Schrondinger differential equation.

⇒ Following properties should exists in wave function $\psi(x,t)$

1. Wave function should be single valued.

2. Wave function should be finite.

3. Wave function should be continuous.

4. It is not necessary that wave function is real. means it can also be complex.

5. Wave function should be zero at infinite.

6. First derivative of wave function should be continuous. If $V(x) = \infty$ then first derivative can also be

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discontineous.

⑦ Wave function should be squre integrable. It mean wave function should be Normalise.

$$\boxed{\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 d\tau = 1 \text{ (max)}}$$ Normalisation condition

8. Operator is define corresponding to every dynamic state.

<u>Operator</u> :-

Operator is a mathematical tool that convert a wave function into other after operation in second state is known as operator.

Ex $\boxed{\hat{A}\psi = \phi}$

$\hat{A}$ = Operator
ψ = function
ϕ = after operation function.

① Let $\psi = \sin x$

$\hat{A} = \frac{d}{dx}$

Now $\hat{A}\psi = \frac{d}{dx} \sin x$

$$\boxed{\hat{A}\psi = \cos x = \phi}$$

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(2) let $\psi = t^3$

$$\hat{A} = \frac{d^2}{dt^2}$$

Now $\hat{A}\psi = \frac{d^2}{dt^2}(t^3)$

$$\boxed{\hat{A}\psi = 6t = \phi}$$

8.

Physical Quantity	Dynamic operator	Corresponding oper
Position	x	$\hat{x}$
momentum	$\vec{P}$	$\hat{P} = -i\hbar\frac{\partial}{\partial x}$
Energy	E	$\hat{E} = i\hbar\frac{\partial}{\partial x}$
Angular momentum	$\vec{L} = \vec{r} \times \vec{P}$	$\hat{L} = \vec{r} \times (-i\hbar\vec{\nabla})$

9. Acc^n to Quantum statistics, Expectation value of Dynamic statistics is done / explain by following mathematical formula.

$$\langle A \rangle = \frac{\int \psi^*(x,t)\,\hat{A}\,\psi(x,t)}{\int |\psi(x,t)|^2\,dx}$$

$$\because \int |\psi(x,t)|^2 dx = \int \psi^*(x,t)\,\psi(x,t)\,dx = 1$$

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$$\boxed{\langle A \rangle = \int \psi^*(x,t)\, \hat{A}\, \psi(x,t)\, dx}$$

Ex: Expection of position

$$\langle x \rangle = \int \psi^*(x,t)\, \hat{x}\, \psi(x,t)\, dx$$

(ii) Expection of momentum

$$\langle P_x \rangle \int \psi^*(x,t)\, \hat{P}_x\, \psi(x,t)\, dx$$

(iii) Expection of energy

$$\langle E \rangle = \int \psi^*(x,t)\, \hat{E}_x\, \psi(x,t)\, dx$$

Ques Find the value of Normalization constant for following wave function?

$$\psi(x) = A \sin\left(\frac{\pi x}{a}\right) \quad ; \quad x : 0 \text{ to } a$$

Soln We know that
From Normalisation Condition

$$\int_{-\infty}^{+\infty} \psi^*(x)\, \psi(x)\, dx = \int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1$$

$$\because \quad |\psi(x) = A \sin\left(\frac{\pi x}{a}\right)$$

$$\because \quad \psi^*(x) = A^* \sin\left(\frac{\pi x}{a}\right)$$

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Now

$$\int_0^a A^* \sin\left(\frac{\pi x}{a}\right) \cdot A \sin\left(\frac{\pi x}{a}\right) dx = 1$$

$$\Rightarrow |A|^2 \int_0^a \sin^2\left(\frac{\pi x}{a}\right) = 1$$

$$\Rightarrow |A|^2 \int_0^a \frac{\left[1 - \cos\frac{2\pi x}{a}\right]}{2} dx = 1$$

$$\Rightarrow \frac{|A|^2}{2}\left[\int_0^a 1 \cdot dx - \int_0^a \cos\left(\frac{2\pi x}{a}\right) dx\right] = 1$$

$$\int_0^a \cos\left(\frac{2\pi x}{a}\right) dx = 0$$

$$\Rightarrow \frac{|A|^2}{2}\left[[x]_0^a - 0\right] = 1$$

$$\Rightarrow \frac{|A|^2}{2}(a-0) = 1$$

$$\Rightarrow |A| = \sqrt{\frac{2}{a}} = A$$

So, Normalise wave function

$$\boxed{\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)}$$ Ans

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Ques : Find the value of Normalization constant for following wave function :?

$$\psi = Ae^{ikx} \quad ; \; 0<x<a$$

Soln We know that from Normalization condition

$$\int_{-\infty}^{+\infty} |\psi|^2 d\tau = \int_{-\infty}^{+\infty} \psi^* \psi \, d\tau = 1$$

$$\because \psi = Ae^{ikx} \quad ; \; 0<x<a$$

$$\because \psi^* = A^* e^{-ikx}$$

$$\Rightarrow \int_0^a A^* e^{-ikx} \cdot Ae^{ikx} dx = 1$$

$$\because A^* A = |A|^2$$

$$\Rightarrow |A|^2 \int_0^a e^{-ikx+ikx} dx = 1$$

$$\Rightarrow |A|^2 \int_0^a dx = 1$$

$$\Rightarrow |A|^2 \cdot (a-0) = 1$$

$$\Rightarrow \boxed{|A| = \frac{1}{\sqrt{a}}}$$

$\therefore$ So, Normalization wave function:

$$\boxed{\psi = \frac{1}{\sqrt{a}} e^{ikx}} \quad ; \; 0<x<a$$

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Ques Calculate expection value for following wave function

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) \; ; \; 0<x<a$$

(i) Expection value of position. $\langle x \rangle$

(ii) $\langle x^2 \rangle$

(iii) $\langle P_x \rangle$

Soln We know that

From defination of expection value

$$\langle A \rangle = \int \psi^* \hat{A} \psi \, dx \quad \text{—— (1)}$$

$\because \psi =$ Normalized wave function

(i) **For position**

From eqn (1)

$\therefore A = x$

$$\langle x \rangle = \int \psi^* \hat{x} \psi \, dx$$

$$\because \psi = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) \; ; \; 0<x<a$$

$$\because \psi^* \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$$

$$\langle x \rangle = \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) \hat{x} \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) dx$$

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$$\langle x \rangle = \frac{2}{a}\int_0^a \hat{x}\, \sin^2\left(\frac{\pi x}{a}\right) dx$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\langle x \rangle = \frac{2}{a}\int_0^a \hat{x}\left[\frac{1-\cos\left(\frac{2\pi x}{a}\right)}{2}\right] dx$$

$$\langle x \rangle = \frac{2}{a}\cdot\frac{1}{2}\left[\int_0^a \hat{x}\, dx - \int_0^a \hat{x}\cos\left(\frac{2\pi x}{a}\right) dx\right]$$

$$\langle x \rangle = \frac{1}{a}\left\{\left[\frac{x^2}{2}\right]_0^a - \left[x\int \cos\left(\frac{2\pi x}{a}\right) - \int \frac{dx}{dx}\cdot\int \cos\frac{2\pi x}{a}\, dx\right] dx\right.$$

$$\langle x \rangle = \frac{1}{a}\left\{\frac{a^2}{2} - \left[x\sin\left(\frac{2\pi x}{a}\right)\cdot\left(\frac{2\pi}{a}\right) - \int \sin\left(\frac{2\pi x}{a}\right)\cdot\left(\frac{2\pi}{a}\right) dx\right.\right.$$

$$\langle x \rangle = \frac{1}{a}\left\{\frac{a^2}{2} - \left[\frac{2\pi x}{a}\sin\left(\frac{2\pi x}{a}\right) + \cos\left(\frac{2\pi x}{a}\right)\cdot\left(\frac{2\pi}{a}\right)\cdot\left(\frac{2\pi}{a}\right)\right.\right.$$

$$\langle x \rangle = \frac{1}{a}\left[\frac{a^2}{2} - \frac{2\pi x}{a}\sin\left(\frac{2\pi x}{a}\right) - \frac{4\pi^2}{a^2}\cos\left(\frac{2\pi x}{a}\right)\right]$$

$$\langle x \rangle = \frac{a}{2} - \frac{2\pi x}{a^2}\sin\left(\frac{2\pi x}{a}\right) - \frac{4\pi^2}{a^3}\cos\left(\frac{2\pi x}{a}\right)$$

$$\langle x \rangle = \frac{a}{2} - \frac{2\pi}{a^2}\left[x\sin\left(\frac{2\pi x}{a}\right) - \frac{2\pi}{a}\cos\left(\frac{2\pi x}{a}\right)\right]$$

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(ii) <u>Expection value of P_x</u>

$$\hat{A} = \hat{P}_x$$

$$\langle P_x \rangle = \int \psi^* \hat{P}_x \psi \, dx$$

$$\because \psi = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$$

$$\because \psi^* = \sqrt{\frac{2}{a}} \sin\frac{\pi x}{a}$$

$$\hat{P}_x = -i\hbar \frac{\partial}{\partial x}$$

$$\langle P_x \rangle = \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) \hat{P}_x \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) dx$$

$$\langle P_x \rangle = \frac{2}{a} \int_0^a \sin\left(\frac{\pi x}{a}\right) \hat{P}_x \sin\left(\frac{\pi x}{a}\right) dx$$

$$\langle P_x \rangle = \frac{2}{a} \int_0^a \sin\left(\frac{\pi x}{a}\right) \left[-i\hbar \frac{\partial}{\partial x} \sin\left(\frac{\pi x}{a}\right)\right] dx$$

$$\langle P_x \rangle = \frac{-2i\hbar}{a} \int_0^a \left[\sin\left(\frac{\pi x}{a}\right) \frac{\partial}{\partial x} \sin\left(\frac{\pi x}{a}\right)\right] dx$$

$$\langle P_x \rangle = -\frac{2i\hbar}{a} \int_0^a \sin\left(\frac{\pi x}{a}\right) \cdot \cos\left(\frac{\pi x}{a}\right) \frac{\pi}{a} \, dx$$

$$\langle P_x \rangle = \frac{-2i\hbar\pi}{a^2} \int_0^a \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi x}{a}\right) dx$$

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10. Eigen Value and Eigen function :-

When on any function operator is operate and after operation same function is define with constant Quantity, then this constant Quantity is known as Eigen value, and define function, is known as Eigen function.

So, from Eigen eqn.

$$\boxed{\hat{A}\psi = a\psi}$$

$\hat{A}$ = operator

ψ = Eigen function corresponding to $\hat{A}$ operator.

a = Eigen value corresponding to $\hat{A}$ operator.

Ex: let $\hat{A} = \frac{d^2}{dx^2}$

$\psi = \cos 3x$

Now $\hat{A}\psi = \frac{d^2}{dx^2}\cos 3x$

$\hat{A}\psi = -9\cos 3x$

$\because \psi = \cos 3x$

$\boxed{\hat{A}\psi = -9\psi} = a\psi$

Eigen value $a = -9$

Eigen function $\psi = \cos 3x$

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Ex-2 let $\hat{A} = \frac{d}{dt}$

$\psi = e^{2t}$

Now $\hat{A}\psi = \frac{d}{dt} e^{2t}$

$$\boxed{\hat{A}\psi = 2e^{2t} = a\psi}$$

$\because a = 2 =$ Eigen value

$\because \psi = e^{2t} =$ Eigen function.

(II) Degenerate Eigen Value :-

If an operator ~~is~~ operate on different ~~operator~~ wave function and operation eigen value is same for all wave function then this eigen value is said to be degenerate eigen value.

Ex

$$\hat{A}\psi_1 = a\psi_1$$
$$\hat{A}\psi_2 = a\psi_2$$
$$\hat{A}\psi_3 = a\psi_3$$
$$\vdots \qquad \vdots$$
$$\hat{A}\psi_n = a\psi_n$$

Here eigen value a is connected to wave function, so Rank of degeneracy will be $\underline{\underline{n}}$.

$$\boxed{\text{Rank} = n}$$

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This property of Quantum physics is known as Degeneracy.

12. Non-Degenerate Eigen Value:-

Acc^n to concept of Quantum mechanism if an operator is operate on different wavefunction and after operation value of eigen value finds different for all wave function then this eigen value is said to be Non-Degenerate Eigen value.

Ex:

$$\hat{A}\psi_1 = a\psi_1$$
$$\hat{A}\psi_2 = b\psi_2$$
$$\hat{A}\psi_3 = c\psi_3$$

Here a, b, c are called Non-Degenerate Eigen value.

→ Minimum Rank of Degeneracy is 2 (two)

→ Maximum Rank of Non-Degenerate eigen value is 1.

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13. Hemiltonian operator:-

In classical mechanics, total energy of particle is known as Hemiltonian energy and it is denoted by H. Operator connected to the energy is known as Hemiltonian operator in Quantum physics.

$$\hat{H} = \text{Total Energy}$$

$$H = K.E + P.E$$
$$H = \frac{1}{2}mv^2 + V(r)$$

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$$H = \frac{1}{2}\frac{m^2v^2}{m} + V(r) \qquad \because P = mv$$

$$H = \frac{P^2}{2m} + V(r)$$

in operator form

$$\boxed{\hat{H} = \frac{\hat{P}^2}{2m} + V(\hat{r})}$$

This is known as Hemiltonian operator.

(14) <u>Adjoint Operator</u> :-

If $\hat{A}$ and $\hat{B}$ operator are define as

$$\boxed{\int (\hat{A}\psi)^* \phi \, d\tau = \int \psi^* (\hat{B}\phi) \, d\tau}$$

Then $\hat{A}$ operator is said to be adjoint operator of $\hat{B}$ operator.

$$\boxed{\hat{A} = \hat{B}^\dagger} \qquad \dagger = \text{Pegar (Transpose of Conjugate)}$$

<u>Properties</u>

(i) $(\hat{A}^\dagger)^\dagger = A$

(ii) $(\hat{A} \pm \hat{B})^\dagger = \hat{A}^\dagger \pm \hat{B}^\dagger$

(iii) $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$

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(iv) $(\hat{A}\hat{B} \pm \hat{C}\hat{D})^{\dagger} = (\hat{A}\hat{B})^{\dagger} \pm (CD)^{\dagger}$

$$= \hat{B}^{\dagger}\hat{A}^{\dagger} \pm \hat{D}^{\dagger}\hat{C}^{\dagger}$$

(15) Hermition Operator :-

Acc^n to Quantum mechanics, def adjoint operator is known as hermition operator. If $\hat{A}$ operator is define as

$$\boxed{\int (\hat{A}\psi)^{*}\phi \, d\tau = \int \psi^{*}(\hat{A}\phi)\, d\tau}$$

Then $\hat{A}$ operator is is said to be Hermition operator.

$$\boxed{\hat{A}^{\dagger} = \hat{A}}$$

(16) Anti-Hermition operator :-

If $\hat{A}$ operator is such that

$$\boxed{\hat{A}^{\dagger} = -\hat{A}}$$

Then $\hat{A}$ Operator is said to be Anti-Hermition operator.

Ex. $\left(\frac{\partial}{\partial x}\right)^{\dagger} = \frac{-\partial}{\partial x}$

Ques Check Hermition operator :-

(i) $\hat{P}_x$ (ii) $\hat{E}$ (iii) $i(\hat{A} - \hat{A}^{\dagger})$ (iv) $i(\hat{A}^{\dagger} - \hat{A})$

(v) $(\hat{A} - i\hat{A}^{\dagger})$

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<u>solution</u> (i) $\hat{P}_x$

let $\hat{Y} = \hat{P}_x$

$\because \hat{P}_x = -i\hbar \frac{\partial}{\partial x}$ — ②

$\hat{Y} = -i\hbar \frac{\partial}{\partial x}$ — ①

On taking † to both sides.

$\hat{Y}^\dagger = \left(-i\hbar \frac{\partial}{\partial x}\right)^\dagger$ $\qquad i^\dagger = -i$

$\hat{Y}^\dagger = i\hbar \left(\frac{\partial}{\partial x}\right)^\dagger$

$\hat{Y}^\dagger = -i\hbar \frac{\partial}{\partial x} = \hat{P}_x$ from eqⁿ ②

so, $\boxed{\hat{Y} = \hat{Y}^\dagger}$

so, Momentum $\hat{P}_x$ is a Hermitian operator.

(ii) $\hat{E}$

<u>solⁿ</u> $\hat{E} = i\hbar \frac{\partial}{\partial t}$ — ①

on taking † to both sides.

$\hat{E}^\dagger = \left(i\hbar \frac{\partial}{\partial t}\right)^\dagger$

$\hat{E}^\dagger = -i\hbar \left(\frac{\partial}{\partial t}\right)^\dagger$

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$$\hat{E}^{\dagger} = i\hbar \frac{\partial}{\partial t}$$

from eq^n ①

$$\boxed{\hat{E}^{\dagger} = \hat{E}}$$

so, Energy $\hat{E}$ is called Hermition operator.

(iii) $i(\hat{A} - \hat{A}^{\dagger})$

Sol^n let $\hat{Y} = i(\hat{A} - \hat{A}^{\dagger})$ — ①

on taking † to both sides,

$$\hat{Y}^{\dagger} = [i(\hat{A} - \hat{A}^{\dagger})]^{\dagger} \qquad \because (\hat{A}^{\dagger})^{\dagger} = A$$

$$\hat{Y}^{\dagger} = -i(\hat{A}^{\dagger} - A)$$

$$\hat{Y}^{\dagger} = i(\hat{A} - \hat{A}^{\dagger})$$

so, $\boxed{\hat{Y} = \hat{Y}^{\dagger}}$ Hermition operator.

(iv) $(A - i\hat{A}^{\dagger})$

Sol^n let $\hat{Y} = (\hat{A} - i\hat{A}^{\dagger})$

on taking † to both sides,

$$\hat{Y}^{\dagger} = \hat{A}^{\dagger} - (iA^{\dagger})^{\dagger}$$

$$\boxed{\hat{Y}^{\dagger} = \hat{A}^{\dagger} + iA}$$

$\boxed{\hat{Y} \neq \hat{Y}^{\dagger}}$ Not Hermition operator

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(17)

Parity Operator :-

Eigen value of Parity operator define in Quantum statistics is $\boxed{d = \pm 1}$. If eigen value is +1 then define wave function will be symmetric and if eigen value is −1 then wave function will be Anti-symmetric.

(18) To calculate the state function of a particle, Schrondinger scientist gave a fundamental eqⁿ in 1926. that is of two types.

(a) Time dependent Schrongier differential eqⁿ :- If potential (b) energy of particle shows dependency on time then Schrondinger diffⁿ eqⁿ define for particle, is said to be time dependent Schrondinger diffⁿ eqⁿ.

$$\boxed{\frac{i\hbar \partial \psi}{\partial t} = \hat{H}\psi}$$

$\because \hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34}$ second

$\hbar$ = Dirac Constant/(Reduce plank.)

$h = 6.023 \times 10^{-34}$ J-S plank-constant

If P.E = $V(x,t)$

(b) Time independent Schrondinger diffⁿ eqⁿ :-

If potential energy of particle does not depend on time then the Schrondinger diffⁿ is known as Time independent Schrondinger diffⁿ eqⁿ.

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$$\boxed{\hat{H}\psi = E\psi}$$

E = Total energy = Eigen value of $\hat{H}$ operator

$\hat{H}$ = Hamiltonian operator,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(r)$$

on putting.

$$\left[\frac{\hat{p}^2}{2m} + V(\hat{r})\right]\psi = E\psi.$$

$$\frac{\hat{p}^2}{2m}\psi + V(\hat{r})\psi = E\psi$$

$\because \hat{p}$ = momentum operator
$\hat{p} = -i\hbar\nabla$

$$\boxed{\frac{-\hbar^2}{2m}\nabla^2\psi + V(\hat{r}) = E\psi}$$

∇^2 = laplacian diffⁿ Operator.

If $\boxed{P.E = V(x,0)}$

$\because t=0$

(19) Accⁿ to the concept of classical mechanics, particle and waves define seperate. but Accⁿ to Quantum mechanics every particle had a connected wave that is known as associate wave. This is called the Debroglie Dual nature Principle in Quantum mechanics.

(20) Accⁿ to Quantum mechanics, in only one moment in one dirⁿ, measurement is not possible for two physical Quantity means, Minimum error $\frac{h}{4\pi}$ exists.

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Let uncertainity in x-dirn in position is Δx and uncertainity in momentum is ΔP_x then from uncertainity principal.

$$\boxed{\Delta x \cdot \Delta P_x \geq \frac{h}{4\pi}}$$

$$\because \hbar = \frac{h}{2\pi}$$

$$\boxed{\Delta x \cdot \Delta P_x \geq \frac{\hbar}{2}}$$

$$\boxed{\Delta x \cdot \Delta P_x \approx \hbar}$$

$$\boxed{\Delta x \cdot \Delta P_x \approx h}$$

This is known as Higenberg's uncertainity principle in Quantum physics.

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Difference b/w Boson particle and Fermi particle:-

Difference b/w boson particle and Fermi particle explain as following:-

Boson Particle	Fermi Particle
1. That fundamental particle define in physics, in which value of spin Quantum no. is Integral known as Boson particle.	1. That elementary particle define in particle physics, in which value of spin Quantum no. is half odd Integer is known as Fermi particle.
Ex $S = 0, 1, 2, 3, 4 \ldots$	Ex $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

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2. Wave function for boson particle is $\psi(\vec{r},t)$. i.e symmetric.	2. Wave function for Fermi particle is $\psi(\vec{r},t)$ i.e skew-symmetric.
3. Boson particle follows that Quantum statistics is known as Bose-Einstein.	3. Fermi particle follows that Quantum statistics is known as Fermi-Dirac.
4. Bose-Einstein statistics is given in 1924.	4. Fermi-Dirac statistics is given in 1926.
5. Boson-particle doesn't follow the Pauli's exclusion law.	5. Fermi-particle follows the Pauli's exclusion law.
6. Photon (γ), Graviton, K^+ (कायोन), K^0, π^+, π^-, π^0, η^0, α-particle ($_2He^4$), etc.	6. e^-, p, n, positron (e^+), τ^-, μ^-, Σ^+, Σ^-, Σ^0, Ω^-, d^0, E^-, E^0 etc.

Black-Body :

That property of matter that absorb all the waves at low temperature (0 to ∞) and emits all the wave at high temperature (0 to ∞) is known as Black Body.

Example: Dust, sun,

Experimental Fact :-

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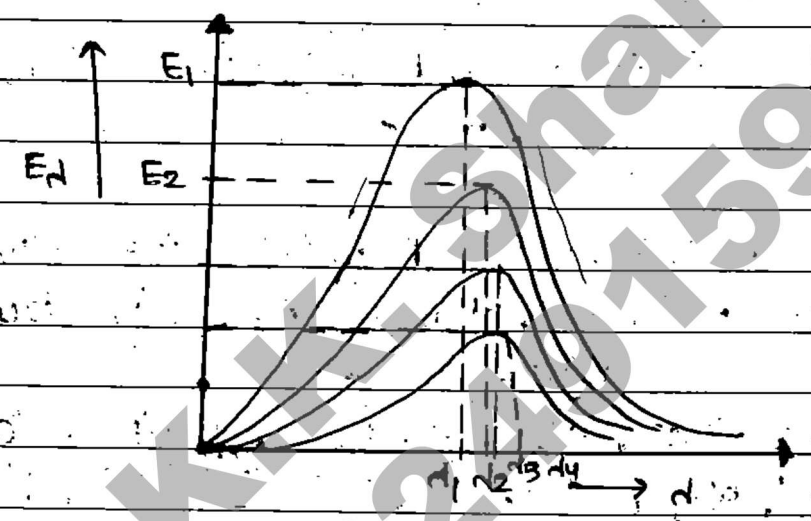
1. Energy Distribution curve of Black Body:-

For Black body, The curve drawn b/w Emission power (E_λ) and wavelength (λ) is known as Energy distribution curve of Black body.

∴ This curve is define as on the basis of experimental data:-



$T_1 > T_2 > T_3 > T_4$

$\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

From Black body Energy Distribution curve following Experimental curve are find:-

1. For any constant temp., Black Body Energy distribution curve is a Continuous curve.

2. In this curve, when the value of temperature increased then peak value of Energy also increased.

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3. In experimental form, The Conclusion find that Maximum Value of energy (Peak Value) is directly proportional to fifth power of Temperature.

MTR $\boxed{E_{max} \propto T^5}$

4. It is clear from Black Body distribution curve, that Value of energy increased on increasing wavelength firstly then ~~var~~ value of energy becomes maximum and after that on increasing value of wavelength, value of energy decreased.

*5. Wein's Displacement law:

In Black Body Energy distribution curve, when the value of temperature increased then wavelength corresponding to ~~energy~~ peak value of energy shifted to minimum value of wavelength, this is the wein's displacement law.

$$\lambda_m \propto \frac{1}{T}$$

MTR $\boxed{\lambda_m = \frac{b}{T}}$ b = wein's Constant
b = 2.98 m-k.

*6 Stefan-Boltzmann law:

Acc^n to Stefan Boltzmann law, spectrum Emission Capacity is directly proportional to four power of temperature.

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$E \propto T^4$

MTE $\boxed{E = \sigma T^4}$ ∵ σ = Stephen – Boltzmann Const.

$\boxed{E = \int_0^\infty U_\lambda d\lambda}$ E = Emissive power.
u = Energy density

Relation b/w Emmissive power and density.
(Albert Einstein) → $\boxed{E = \frac{cU}{4}}$ c = speed of light 3×10^8 m/s.

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**Failure OF Classical Physics:

1. We know that Acc^n to concept of classical physics, Energy distribution curve of Black body is explained by Weins and Rayleigh scientist. Weins Radiation formula explain the low wave length while Rayleigh Radiation formula only high wave length part. It means for high wavelength part weins Radiation formula is not valid, and Rayleigh formula is not valid for low wavelength part. In this way in classical physics, whole energy distribution curve is not define properly. This is the filure of classical physics.

In this an interacting conclusion is their of Quantum physics, that is successful in explaining the Black Body radiation. This mathematical formula is known as Plank's Radiation formula.

2. Acc^n to the concept of Classical physics, for matter, particle and wave nature are known seperate while Acc^n to
(माना)

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concept of Quantum physics, matter shows dual nature. means ~~fall~~ particle has associate wave. Particle and wave are both complimentary function. This is also a caused of failure which is explained by de-Broglie Hypothesis.

3. Acc^n to classical physics, Energy is considered Contineous. While acc^n to Quantum physics, Energy exists in discreate form. It means, Energy is a Quantisie Quantity, this is also a failure of classical physics. i.e is explained by Plank's Hypothesis. Acc^n to Plank's hypothesis, Oscillator are present in Black Body that are known as Plank's Oscillator

Value of Energy for plank's oscillator is:-

MTR

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0} \quad n = 0, 1, 2, 3, 4 ---$$

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ J-sec.}$$

for ground state

$n = 0$

$$\boxed{E_0 = \frac{1}{2}\hbar\omega_0}$$

in Quantum physics, this mathematical formula is known as zero point energy.

4. Acc^n to Uncertanity principle of Hygen Berg, Exact measurement of ~~two or more~~ than two physical Quantity in same dir^n, in same moment is not possible, While Acc^n

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to concept of classical physics, exact measurement is possible. This is also a failure of classical Physics.

$$\Delta x \cdot \Delta P_x \geq \frac{h}{4\pi}$$

$$\because \hbar = \frac{h}{2\pi}$$

MTR $\boxed{\Delta x \cdot \Delta P_x \geq \frac{\hbar}{2}}$

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Plank's Hypothesis :

Plank's hypothesis is a postulate of Quantum physics. Acc to this postulate, emission and Absorption of electromagnetic wave in black body is due to the presence of oscillator. This exchange of energy is in seperate form not in continuous. It means it is in in the minimum integer form. minimum value of energy is $h\nu$ that is known as Quanta of Energy. This Quanta of energy is known as Photon.

$$\boxed{E_n = nh\nu}$$

n = No of Oscillator

ν = frequency of oscillator

$h = 6.62 \times 10^{-34}$ J-sec.

This is the hypothesis of Plank's Constant.

NOTE :

① Acc to Quantum physics, energy is directly proportional to frequency.

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$E \propto \nu$

$$\boxed{E = h\nu}$$

$\because$ h = Plank's constant
$h = 6.62 \times 10^{-34}$ J-sec.

② Acc^n to the concept of cycladdical physics, energy is directly proportional to Temperature.

$E \propto T$

From (MBS) $\boxed{E = kT}$ $\because$ k = Boltzmann constant
$k = 1.38 \times 10^{-23}$ J/K.

③ Average energy of Plank's oscillator:-

$$\boxed{\langle E \rangle = \bar{E} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}}$$

④ Relation b/w energy density and emmisive power:-

$$\boxed{E = \frac{cu}{4}}$$ (By Albert Einstein)

⑤ Standard diff^n Integ.

$$\boxed{\int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}}$$

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Ques-1 Prove that Conclusion of Quantum physics satisfied Classical physics?

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Ques-2 What is Plank's Radiation Formula? Calculate in frequency form?

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Ques-10 Explain Bos-Einstein statistics?

Ques-11 Write short note on Ortho and Para Hydrogen?

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Ques-1 Prove that conclusion of Quantum physics satisfied classical physics?

Soln We know that Acc^n to Quantum physics, Average energy of Plank's oscillator, is:-

$$\langle E \rangle = \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1} \quad \text{--- (1)}$$

$$\because e^{x} = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$$

$$\text{let } x = \frac{h\nu}{KT}$$

$$e^{\frac{h\nu}{KT}} = 1 + \frac{h\nu}{KT} + \frac{1}{\lfloor 2}\left(\frac{h\nu}{KT}\right)^2 + \frac{1}{\lfloor 3}\left(\frac{h\nu}{KT}\right)^3 + \cdots$$

in Quantum physics

$$h = 6.62 \times 10^{-34} \text{ J-sec.}$$

Acc^n to classical physics

$$h \to 0$$

On neglecting the high power of h

$$h^2 = h^3 \approx 0$$

$$e^{\frac{h\nu}{KT}} = 1 + \frac{h\nu}{KT}$$

put the value in eq^n (1)

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$$\langle E \rangle = \frac{h\nu}{1 + \frac{h\nu}{KT} - 1}$$

$$\boxed{\langle E \rangle = KT}$$

$$\boxed{E \propto T}$$

This is the Result of classical physics. Hence it is clear that Quantum physics satisfies classical physics.

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Ques-2 What is Planks Radiation Formula? Calculate in Frequency Form?

Soln **Planks Radiation Formula:-**

Planks Radiation formula is an interacting fact of Quantum physics. The Radiation formula define in Quantum physics that define whole black body energy distribution curve is said to be Planks Radiation formula.

To explain the Planks Radiation formula in mathematical form, we use Maxwell's Boltzmann statistics.

Acc^n to Maxwell's Boltzmann statistics, No. of oscillator in wavelength range λ and $\lambda + d\lambda$ $= N_\lambda d\lambda$

$$N_\lambda d\lambda = \frac{8\pi}{\lambda^4} d\lambda \quad \text{—(1)}$$

Average energy of oscillator in Black body:-

$$\langle E \rangle = \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1} \quad \text{—(2)}$$

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$$\because \quad \nu = \frac{c}{\lambda}$$

from eq^n ②

$$\boxed{\langle E \rangle = \frac{hc}{\lambda\left[e^{\frac{hc}{\lambda kT}} - 1\right]}}$$

Energy Quantity present in unit volume is known as energy density. it is denoted by U.

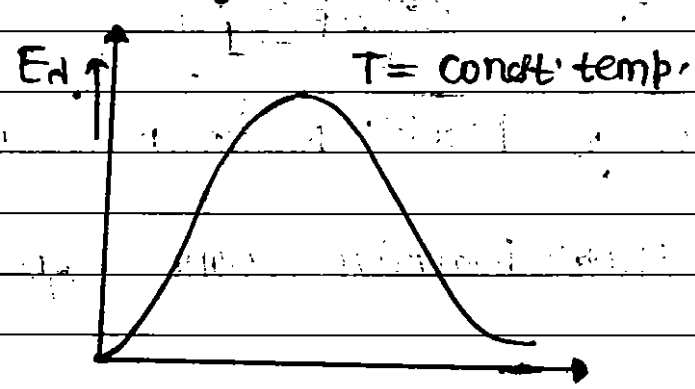
Energy density of Oscillator in wavelength range λ and $d\lambda$

$$= N_\lambda d\lambda \times \langle E \rangle$$

$$U_\lambda d\lambda = \frac{8\pi}{\lambda^4} d\lambda \cdot \frac{hc}{\lambda\left[e^{\frac{hc}{\lambda kT}} - 1\right]}$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left[e^{\frac{hc}{\lambda kT}} - 1\right]} d\lambda} \quad \text{—— ③}$$

eq^n ③ is known as planks Radiation formula in wave-length form. that explain energy distribution curve of whole Black Body.



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We know that

$$\nu = \frac{c}{\lambda}$$

$$\lambda = \frac{c}{\nu}$$

on diff with respect to ν

$$\frac{d\lambda}{d\nu} = c\left(\frac{-1}{\nu^2}\right)\cdot 1$$

$$d\lambda = \frac{-c}{\nu^2}\, d\nu$$

Here Negative sign is meaning less.

$$|d\lambda| = \frac{c}{\nu^2}|d\nu|$$

From eqn (3)

$$U_\nu d\nu = \frac{8\pi h c}{\left(\frac{c}{\nu}\right)^5 \cdot \left[e^{\frac{h\nu}{kT}} - 1\right]} \cdot \frac{c}{\nu^2}\, d\nu$$

$$\boxed{U_\nu d\nu = \frac{8\pi h \nu^3}{c^3\left[e^{\frac{h\nu}{kT}} - 1\right]}\, d\nu} \quad \text{--- (4)}$$

eqn (4) is known as Plank's Radiation formula in frequency form.

From Plank's Radiation formula, following Results are cleared.

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$$\boxed{U_\lambda \propto \lambda^{-5}}$$

$$\boxed{U_\nu \propto \nu^{3}}$$

Ques-3 Prove the Wein's Radiation formula by using Planks Radiation formula?

Ans Weind Radiation Formula:-

Weind Radiation formula is a result of claddical phydics. In classical phydics, that define low wavelength part of energy distribution curve define in mathematical, known as Wein's Radiation formula.

So, Weind Radiation formula,

$$U_\lambda d\lambda = 8\pi hc\lambda^{-5} e^{\frac{-hc}{\lambda kT}} d\lambda$$

$$\left.\begin{aligned} \text{let } A &= 8\pi hc \\ a &= \frac{hc}{k} \end{aligned}\right\} \text{Constant.}$$

$$\boxed{U_\lambda d\lambda = A\lambda^{-5} e^{\frac{-a}{\lambda T}} d\lambda}$$

proof Result of claddical phydics. explain in Planks Radiation formula explaind weind Radiation formula in mathematical formula.

We know that,

From Plank's Radiation formula.

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$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left[e^{\frac{hc}{\lambda KT}}-1\right]} d\lambda \quad \text{—— (1)}$$

Acc^n to Wein dientist:

$hc >>> \lambda KT$ (for low wavelength)

$$\frac{hc}{\lambda KT} >>> 1$$

$$e^{\frac{hc}{\lambda KT}} >>>> 1 \quad \text{—— (2)}$$

from eq^n (1) and (2)

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda KT}}} d\lambda$$

let $8\pi hc = A$, $\frac{hc}{K} = a$ } Constant.

$$U_\lambda d\lambda = \frac{A}{\lambda^5} \frac{1}{e^{\frac{a}{\lambda T}}} d\lambda$$

$$\boxed{U_\lambda d\lambda = A\lambda^{-5} e^{\frac{-a}{\lambda T}} d\lambda} \quad \text{—— (3)}$$

eq^n (3) is called Weind Radiation formula.

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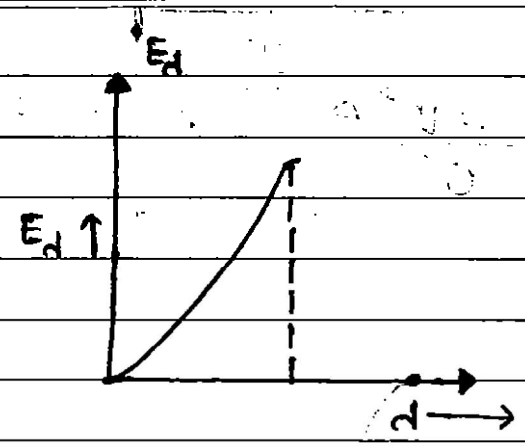

We know that

$$v = \frac{c}{d}$$

$$d = \frac{c}{v}$$

On diff^n w.r.t v

$$\frac{dd}{dv} = c\left(\frac{-1}{v^2}\right)$$

$$\boxed{dd = \frac{c}{v^2}\,dv}$$

On putting in eq^n (5)

$$U_d\,dd = \frac{A}{d^5} \cdot e^{\frac{-a}{dT}}\,dd$$

$$U_v\,dv = \frac{A}{\left(\frac{c}{v}\right)^5}\, e^{\frac{-a}{\frac{c}{v}T}} \cdot \frac{c}{v^2}\,dv$$

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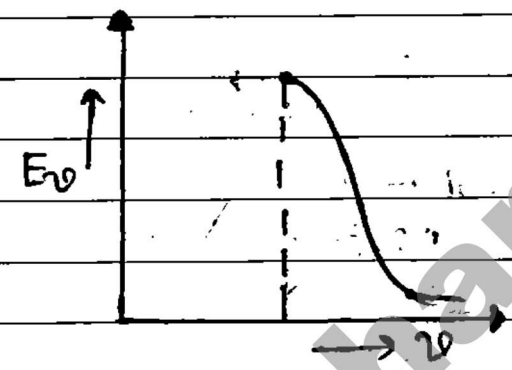
$$\boxed{U_\nu d\nu = \frac{A\nu^3}{c^4} e^{-\frac{a\nu}{cT}} d\nu} \quad \text{—④}$$



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eqⁿ ④ is known as Weins Radiation formula in frequency form.

Ques-3 What is the Meaning of Ultraviolet Catastrophe?

Ques-4 Prove the Rayle-jeans Radiation formula by using Planks Radiation formula?

2019-4 and 2021-8 Radiation formula of Rayley-jeans :-

This Radiation formula is a result of classical physics. That Mathematical Radiation the formula which explain the high wavelength part of Black body energy distribution curve known as Radiation formula of Rayle-jeans. So, Radiation formula of Rayley-jeans.

$$\boxed{U_\lambda d\lambda = 8\pi KT\lambda^{-4} d\lambda}$$

Proof We know that, from Planck's Radiation formula.

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{\left[e^{\frac{hc}{\lambda KT}} - 1\right]} d\lambda \quad \text{—①}$$

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Acc^n to Rayleg-jeand

$hc <<< \lambda KT$ (for high wavelength)

$$\frac{hc}{\lambda KT} <<< 1$$

let $x = \frac{hc}{\lambda KT}$

$x <<< 1$

$$e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$$

$x <<<< 1$

$$e^x \approx 1 + x$$

$$e^{\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT} \quad \text{--- (2)}$$

from eq^n (1) and (2)

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{1 + \frac{hc}{\lambda KT} - 1} d\lambda$$

$$U_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda \quad \text{--- (3)}$$

$$\boxed{U_\lambda d\lambda = 8\pi KT \lambda^{-4} d\lambda} \quad \text{--- (4)}$$

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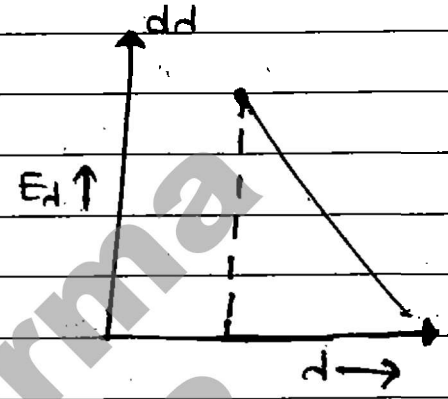
eq^n ④ is known as Rayley-jeand formula in frequency form.

Now We know that



$$\nu = \frac{c}{d}$$

$$d = \frac{c}{\nu}$$

on diff^n w.r to ν

$$\frac{dd}{d\nu} = c\left(\frac{-1}{\nu^2}\right)$$

$$\boxed{dd = \frac{c}{\nu^2}\, d\nu}$$

Here negative sign is negligible.

From eq^n ③

$$U_\nu d\nu = \frac{8\pi KT}{\left(\frac{c}{\nu}\right)^4} \frac{c}{\nu^2}\, d\nu$$

$$\boxed{U_\nu d\nu = \frac{8\pi KT\, \nu^2 d\nu}{c^3}} \quad \text{—⑤}$$

It is clear from eq^n ⑤ that energy density, is directly proportional to square of frequency.

$$\boxed{U_\nu \propto \nu^2}$$

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This conclusion of classical physics is known as Ultra-Violet Catastrophe.

Ques-5 Prove the Stephen Boltzmann law by using Planck's Radiation Formula?

Soln **Stephen Boltzmann law:-** "In Black Body energy distribution curve spectrum emission capacity (emission) curve is directly proportional to fourth power of temperature.

$$E \propto T^4$$

$$\boxed{E = \sigma T^4}$$

$$\sigma = \frac{2}{15}\frac{\pi^5 k^4}{h^3 c^2} = \text{const.}$$

σ = Stephen - Boltzmann constant.

Proof So, total energy density

$$U = \int_0^\infty U_\lambda d\lambda \quad \text{—— (1)}$$

We know that, from Planck's Radiation formula

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{\left[e^{\frac{hc}{\lambda kT}} - 1\right]} d\lambda \quad \text{—— (2)}$$

from eqn (1) and (2)

$$U = \int_0^\infty \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{\left[e^{\frac{hc}{\lambda kT}} - 1\right]} d\lambda \quad \text{—— (3)}$$

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let $x = \frac{hc}{\lambda KT}$

λ	0	∞
x	∞	0

$$\lambda = \frac{hc}{xKT}$$

on diff^n w.r.t to x

$$\frac{d\lambda}{dx} = \frac{hc}{KT}\left(\frac{-1}{x^2}\right)$$

$$d\lambda = \frac{-hc}{x^2 KT}\,dx$$

on putting the value in eq^n ③

$$U = -\int_{\infty}^{0} \frac{8\pi hc}{\left(\frac{hc}{xKT}\right)^5} \cdot \frac{1}{(e^x-1)} \cdot \frac{hc}{x^2 KT}\,dx$$

from limit property

$$U = \frac{8\pi (hc)^2 (KT)^5}{(hc)^5 KT} \int_0^{\infty} \frac{x^5}{x^2(e^x-1)}\,dx$$

$$U = \frac{8\pi K^4 T^4}{h^3 c^3} \int_0^{\infty} \frac{x^3}{(e^x-1)}\,dx$$

$$\int_0^{\infty} \frac{x^3}{(e^x-1)}\,dx = \frac{\pi^4}{15}$$

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$$U = \frac{8\pi \cdot k^4 T^4}{h^3 c^3} \cdot \frac{\pi^4}{15} \quad \text{—— (4)}$$

We know that, Relation b/w energy density and spectrum emission capacity

$$E = \frac{cu}{4} \quad \text{(Albert Einstein)}$$

From eqn (4)

$$E = \frac{c}{4} \cdot \frac{8\pi k^4 T^4}{h^3 c^3} \cdot \frac{\pi^4}{15}$$

$$E = \frac{2}{15} \frac{\pi^5 k^4}{h^3 c^2} \cdot T^4$$

$$\because \sigma = \frac{2}{15} \frac{\pi^5 k^4}{h^3 c^2} = \text{Const.}$$

$$\boxed{E = \sigma T^4}$$

This is the Stephen's Boltzmann law.

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Ques-6 Prove the Weind displacement law? by using Weind Radiation formula?

Ans Weind Displacement law:-

In Black Body energy distribution curve, on increasing the value of temp, wavelength connected to peak value of energy shifted to minimum value of wavelength, it means minimum value of wavelength is inversely proportional to temperature.

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$$\lambda_{min} \propto \frac{1}{T}$$

$$\boxed{\lambda_{min} = \frac{b}{T}} \quad \text{—— (1)}$$

b = Wein's constant.

We know that

$$\nu = \frac{c}{\lambda}$$

$$\lambda_{min} = \frac{c}{\nu_{max}} \quad \text{—— (2)}$$

from eqn (1) and (2)

$$\frac{c}{\nu_{max}} = \frac{b}{T}$$

$$\frac{\nu_{max}}{T} = \frac{c}{b} = \text{Constant.}$$

$$\boxed{\frac{\nu_{max}}{T} = \text{Constant}} \quad \text{—— (3)}$$

eqn (3) is known as Weins displacement law in frequency form.

Proof: We know that Weins Radiation formula is that result of classical physics that explain the low wavelength part of Black Body energy distribution curve.

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$$U_\lambda d\lambda = A\lambda^{-5} e^{\frac{-a}{\lambda T}} d\lambda$$

$$\because \left.\begin{aligned} A &= 8\pi hc \\ a &= \frac{hc}{k} \end{aligned}\right\} \text{Constant}$$

$$U_\lambda d\lambda = \frac{A}{\lambda^5} \cdot \frac{1}{e^{\frac{a}{\lambda T}}} d\lambda \quad \text{—④}$$

$$\because \nu = \frac{c}{\lambda}$$

$$\lambda = \frac{c}{\nu}$$

on diff. w.r.to ν

$$\left|\frac{d\lambda}{d\nu}\right| = \left|\frac{c}{-\nu^2}\right|$$

$$d\lambda = \frac{c}{\nu^2} d\nu$$

on putting value in eqn ④

$$U_\nu d\nu = \frac{A}{\left(\frac{c}{\nu}\right)^5} \cdot \frac{1}{e^{\frac{a\nu}{cT}}} \cdot \frac{c}{\nu^2} d\nu$$

$$\boxed{U_\nu d\nu = \frac{A\nu^3}{c^4 e^{\frac{a\nu}{cT}}} d\nu} \quad \text{—⑤}$$

eqn ⑤ is known as Weind Radiation formula in frequency form.

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From eqn (5)

$$U_v = \frac{A v^3}{c^4 e^{\frac{av}{cT}}}$$

$$\left[\frac{dU_v}{dv}\right]_{v=v_{max}} = 0$$

$$\Rightarrow \frac{A}{c^4}\left[\frac{d}{dv}\left(\frac{v^3}{e^{\frac{av}{cT}}}\right)\right]_{max} = 0$$

$$\Rightarrow \frac{A}{c^4}\left[\frac{3v^2 \cdot 1 \cdot e^{\frac{av}{cT}} - v^3 \frac{a}{cT} \cdot e^{\frac{av}{cT}}}{\left(e^{\frac{av}{cT}}\right)^2}\right]_{v=max} = 0$$

$$\Rightarrow \frac{A v_{max}^2}{c^4 e^{\frac{a v_{max}}{cT}}}\left[3 - \frac{a v_{max}}{cT}\right] = 0$$

$$\because \frac{A v_{max}^2}{c^4 e^{\frac{a v_{max}}{cT}}} \neq 0$$

$$3 - \frac{a v_{max}}{cT} = 0$$

$$\boxed{\frac{a v_{max}}{cT} = 3}$$

This is the Weind displacement law.

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$$\frac{\nu_{max}}{T} = \frac{3c}{a} = \text{Const.}$$

$$\boxed{\frac{\nu_{max}}{T} = \text{Constant}}$$

Ques-7 Prove the Weind Displacement law by using Plank's Radiation Formula?

Soln Weind Displacent Law:-

In Black Body energy distribution curve, on increasing the value of temperature wavelength connected to peak value of energy shifted to minimum value of wavelength. It means minimum value of wavelength is inversely proportional to temperature.

$$\lambda_{min} \propto \frac{1}{T}$$

$$\boxed{\lambda_{min} = \frac{b}{T}} \quad \text{—} \; (1)$$

b = Weind Constant

We know that

$$\nu = \frac{c}{\lambda}$$

$$\lambda_{min} = \frac{c}{\nu_{max}} \quad \text{—} \; (2)$$

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From eq^n ① and ②

$$\frac{c}{\nu_{max}} = \frac{b}{T}$$

$$\frac{\nu_{max}}{T} = \frac{c}{b} = \text{constant}$$

$$\boxed{\frac{\nu_{max}}{T} = \text{constant}} \quad \text{—③}$$

eq^n ③ is known as Weind' displacement law in frequency form.

We know that from Plank^{ns} Radiation formula.

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} d\lambda \quad \text{—④}$$

$$\because \nu = \frac{c}{\lambda}$$

$$\lambda = \frac{c}{\nu}$$

on diff^n w.r.t to ν

$$\frac{d\lambda}{d\nu} = \left| \frac{-c}{\nu^2} \right|$$

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$$d\lambda = \frac{c}{\nu^2} d\nu$$

From eqn (4)

$$U_\nu d\nu = \frac{8\pi h c}{\left(\frac{c}{\nu}\right)^5} \cdot \frac{1}{\left[e^{\frac{h\nu}{kT}} - 1\right]} \frac{c}{\nu^2} d\nu$$

$$\boxed{U_\nu d\nu = \frac{8\pi h \nu^3}{c^3\left[e^{\frac{h\nu}{kT}} - 1\right]} d\nu} \quad \text{--- (5)}$$

eqn (5) is known as plank's Radiation formula in frequency form that is the Conclusion of Quantum Mechanics.

Proof

We know that

$$U_\nu = \frac{8\pi h \nu^3}{c^3\left[e^{\frac{h\nu}{kT}} - 1\right]} \qquad \nu = \nu_{max}$$

on diffn w.r to ν,

$$\left[\frac{dU_\nu}{d\nu}\right]_{\nu_{max}} = 0$$

$$\Rightarrow \frac{8\pi h}{c^3} \frac{d}{d\nu}\left[\frac{\nu^3}{\left(e^{\frac{h\nu}{kT}} - 1\right)}\right]_{\nu = \nu_{max}} = 0$$

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$$\frac{8\pi h}{c^3}\left[\frac{3\nu_{max}^2}{\left(e^{h\nu_{max}}-1\right)} - \frac{\nu_{max}\cdot h}{kT}\frac{e^{h\nu_{max}}}{\left(e^{h\nu/kT}-1\right)}\right]=0$$

$$\frac{8\pi h\nu_{max}^2}{c^3\left(e^{h\nu_{max}}-1\right)}\left[3-\frac{h\nu_{max}}{kT}\frac{e^{h\nu_{max}/kT}}{\left(e^{h\nu_{max}/kT}-1\right)}\right]=0$$

$$\Rightarrow \frac{8\pi h}{c^3}\left[\frac{3\nu^2\left(e^{\frac{h\nu}{kT}}-1\right)^2-\nu^3\frac{h}{kT}\cdot e^{\frac{h\nu}{kT}}}{\left(e^{\frac{h\nu}{kT}}-1\right)^2}\right]_{\nu=\nu_{max}}=0$$

$$\Rightarrow \frac{8\pi h\nu_{max}^2}{c^3}\left[3-\frac{h\nu_{max}}{kT}\frac{e^{\frac{h\nu_{max}}{kT}}}{\left(e^{\frac{h\nu_{max}}{kT}}-1\right)}\right]=0$$

let $x=\frac{h\nu_{max}}{kT}$

$$\Rightarrow \frac{8\pi h\nu_{max}^2}{c^3\left(e^{h\nu_{max}/kT}-1\right)}\left[3-\frac{xe^x}{e^x-1}\right]=0$$

$$3-\frac{xe^x}{e^x-1}=0$$

$$3(e^x-1)-xe^x=0$$

$$(3-x)e^x=3$$

$$e^x=\frac{3}{3-x}$$

$$x=\log_e\left(\frac{3}{3-x}\right)$$

$$\boxed{x=\log_e 3-\log_e(3-x)} \quad \text{--- (6)}$$

eqn (6) is a transcendental eqn so, solution of this eqn. is done

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$$x = 2.98$$

$$\frac{h\nu_{max}}{kT} = 2.98$$

$$\boxed{\frac{\nu_{max}}{T} = \frac{2.98\,k}{h} = \text{constant}}$$

This is the Weind Displacement law.

NOTE :- Z = Atomic Number = No. of Proton.
A = Mass no. = No. of proton
X = Atom
Atom = $Z X^{A}$ = $^{A}_{Z}X$

Example

$_{8}O^{16}$, $_{7}N^{14}$

Question :- $_{8}O^{16}$, $_{6}C^{13}$, $_{7}N^{14}$, $_{8}O^{17}$

(i) $_{8}O^{16}$ = 8P + 8n + 8e⁻ = even (Boson)
(ii) $_{6}C^{13}$ = 6P + 6e⁻ + 7n = odd (Fermi)
(iii) $_{7}N^{14}$ = 7P + 7n + 7e⁻ = odd (Fermi)
(iv) $_{8}O^{17}$ = 8P + 9n + 8e⁻ = odd (Fermi)

Ques Identify Fermi and Boson Particle?
α-particle, H_2, e^+, β-particle, γ-particle, $_{3}Li^{6}$, $_{3}Li^{6+}$, $_{3}Li^{6++}$

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(i) $_3Li^6 = 3P + 3n + 3e^- = \text{odd}$ (Fermi)

(ii) α-particle = nucleus of He.

α-particle $_2He^4 = 2P + 2n = \text{even}$ (Boson)

(iii) H_2-Molecule = $2P + 2e^- = \text{even}$ (Boson)

(iv) e^+ (positron) = anti-particle of e^- = (Fermi)

(v) β-particle = $\beta^- = {}_{-1}e^0$, $\beta^+ = {}_{+1}e^0$ } Fermi

(vi) γ-particle = Photon (Boson)
spin Quantum no. is 0.

(vii) $_3Li^{6+} = 3P + 3n + 2e^- = \text{even}$ (Boson)

(viii) $_3Li^{6++} = 3P + 3n + 1e^- = \text{Odd}$ (Fermi)

25 Dec 2020
Ques **Given**

$T_1 = 600K$
$\lambda_{m_1} = 2\mu m$
$T_2 = 2000K$
$\lambda_{m_2} = ?$

Soln We know that, from Wein's displacement law.

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$$\lambda_m = \frac{b}{T}$$

$\lambda_m T = b =$ constant, $\because b = 2.89 \times 10^{-3}$ m-k

Acc^n to Question.

$$\lambda_{m_1} T_1 = \lambda_{m_2} T_2$$

$$\lambda_{m_2} = \frac{T_1 \lambda_{m_1}}{T_2}$$

$$\lambda_{m_2} = \frac{1600}{2000} \times 2 \times 10^{-6}$$

$$\lambda_{m_2} = \frac{4}{5} \times 2 \times 10^{-6}$$

$$\lambda_{m_2} = 1.6 \times 10^{-6}$$

$$\boxed{\lambda_{m_2} = 1.6\ \mu m}$$

Ques Compare the temp. of sun and moon given that the max. radiation emitted from moon is 4753 A° and from sun is 14 μm. ~~and~~

Sol^n given $\lambda_{m_{sun}} = 4753\ A^\circ$

$\lambda_{s_{moon}} = 14\ \mu m$

$$\frac{T_{sun}}{T_{moon}} = ?$$

We know that, from Weins displacement law.

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$$\lambda_m = \frac{b}{T}$$

$$T = \frac{b}{\lambda_m} \quad \text{--- (1)}$$

(i) for sun

$$T_{sun} = \frac{2.898 \times 10^{-3}\ mK}{4753 \times 10^{-10}}$$

$$T_{sun} = \frac{2.898 \times 10^{4}\ K}{4753}$$

$$\boxed{T_{sun} = 0.6 \times 10^{4}\ K}$$

(ii) for Moon

$$T_{moon} = \frac{2.898 \times 10^{-3}\ m\text{-}K}{14 \times 10^{-6}\ m}$$

$$\boxed{T_{moon} = 207\ K}$$

Now eqn (2) ÷ eqn (3)

$$\frac{T_{sun}}{T_{moon}} = \frac{6097}{207}$$

$$\boxed{\frac{T_{sun}}{T_{moon}} = 29.45}$$

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Ques given $T_{sun} = 6000\,K$

$\lambda_{max} = ?$

Soln We know that from Weind displacement law,

$$\lambda_m = \frac{b}{T}$$

$$\lambda_m = \frac{2.898 \times 10^{-3}\ m\text{-}K}{6000\ K}$$

$$\lambda_m = \frac{2898 \times 10^{-9}}{6}\ m$$

$$\boxed{\lambda_m = 483 \times 10^{-9}\ m}$$ **Ans**

$$\boxed{\lambda_m = 483\ nm}$$ **Ans**

Ques given

$\lambda_m = 5.76\ A^{\circ}$

$T = ?$

We know that from weind displacement law,

$$T = \frac{b}{\lambda_m}$$

$$T = \frac{2.898 \times 10^{-3}\ mK}{5.76 \times 10^{-10}\ m}$$

$$T = 5.03 \times 10^{6}$$

$$\boxed{T = 5 \times 10^{6}\ K}$$

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Ques given Wavelength of divided star,

$d_m = 240 nm \Rightarrow 240 \times 10^{-9} m$

$T = ?$

Soln We know that from wein's displacement law

$$d_m = \frac{b}{T}$$

$$T = \frac{b}{d_m}$$

$$T = \frac{2.898 \times 10^{-3} mK}{240 \times 10^{-9} m}$$

$$T = \frac{2.898 \times 10^{5}}{24}$$

$$T = 120.75 \times 10^{2} m$$

$$T = 1.2 \times 10^{4} m$$

$$\boxed{T \simeq 10^{4} m}$$ Ans

imp. Ques Write the Difference b/w Maxwell Boltzmann ~~statistics~~, Fermi Dirac ~~statistics~~ and Bose Einstein's ~~statistics~~?

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28 Dec 2020

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Maxwell-Boltzmann statistics	Bose-Einstein statistics	Fermi-Dirac statistics
1. Maxwell Boltzmann statistical is a classical statistics in which energy of particle is explain by classical physics/concept. $E = KT$	1. Bose-Einstein statistical is a Quantum statistics in which energy of particle is explain by Planck hypothesis. $E_n = nh\nu$, $n = 1, 2, 3 \ldots$ n = No. of Photon.	1. Fermi-Dirac statistics is also a Quantum statistics in which energy of particle is explain by Planck hypothesis. $E_n = nh\nu$, $n = 1, 2, 3 \ldots$ n = No. of Photon.
2. In phase space volume of phase shell is not fixed	2. In phase space, volume of phase shell is of h^3 order. h = Plank's constant $h = 6.623 \times 10^{-34}$ Js	2. In phase space, volume of phase shell is h^3 order. h = Plank's constant $h = 6.62 \times 10^{-34}$ J-sec.
3. Distribution law for Maxwell Boltzmann statistics $n_i = \frac{g_i}{A \exp\left(\frac{E_i}{KT}\right)}$ $A = e^{\alpha}$ = const. n = No. of phase point. g = No. of shell.	Distribution curve for Bose Einstein statistics. $n_i = \frac{g_i}{A \exp\left(\frac{E_i}{KT}\right) - 1}$ $A = e^{\alpha}$ = const n = No. of phase point. g = No. of shell.	Distribution law for Fermi-Dirac statistics. $n_i = \frac{g_i}{A \exp\left(\frac{E_i}{KT}\right) + 1}$ $A = e^{\alpha}$ = const. n = No. of phase point. g = No. of shell.
4. Pauli's exclusion law is not followed in maxwell-Boltzmann statistics.	4. Pauli's exclusion law is not followed in Bose-Einstein statistics.	4. Pauli's exclusion law is ~~not~~ followed in Fermi Dirac statistics.

⑤ At zero kelvin temperature, value of particle can be zero is possible.	⑤ At zero kelvin temperature value of particle can be zero is possible.	⑤ In Fermi-Dirac statistics at absolute zero kelvin temperature, value of particle can't be zero is possible.
⑥ In Maxwell Boltzmann statistics, arrangement on particle, there is no condition.	⑥ In Bose Einstein statistics, there is no boundation on arrangement of particle.	⑥ In Fermi Dirac statistics only one particle can be exists in one Quantum state.
⑦ Maxwell Boltzmann statistics is valid for different particles.	⑦ Bose Einstein statistics, ~~Quantum~~ is valid for different particles. but in this Pauli's exclusion law is not followed.	⑦ Fermi dirac statistics is valid for different particle but in this pauli's exclusion law is followed.
⑧ In Maxwell Boltzmann statistics wavefunction is not define for state of particle. Ex H_2-gas, Li-gas, He-gas.	⑧ In Bose Einstein statistics, wavefunction $\psi(\vec{r},t)$ is define symmetric for state of particle, and this statistics is valid for Boson particle. ↓ Ex Photon, Phenon, gravition, K^+, K^0, etc.	⑧ In Fermi-Dirac statistics wavefunction $\psi(\vec{r},t)$ is define anti-symmetric and this statistics is valid for Fermi-particle. Ex Fermi-particle Proton, e^-, n, p-particle, Σ^+ Σ^-, Σ^0, τ^-, μ^- etc.

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2019
V.imp.

Ques Explain Bose Einstein statistics and Derive the distribution law?

$$n_i = \frac{g_i}{A\exp\left(\frac{E_i}{KT}\right)-1}$$

29 Dec 2020

NOTE :

① We know that meadument of disorder of system is done by Entropy that is a energy of system. This physical Quantity is denoted by S.

And Probability thermodynamic is denoted by W or Ω. Acc^n to Maxwell-Boltzmann statistics, Entropy of particle, is directly proportional to logrithim of thermodynamic probability.

$$S \propto \log_e W$$

$$\boxed{S = k\log_e W}$$

k = Boltzmann const.
$k = 1.38 \times 10^{-23}$ J/k

② Starling Formula :-

$$\boxed{\log_e N! = N\log_e N - N}$$

$$\underline{|N} = N! = N(N-1)(N-2)(N-3)\ldots 3.2.1$$

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31 Dec 2020

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Bose-Einstein statistics :-

In 1924, S.N Bose and Albert Einstein gave a statistics. This statistics is based on principle of Quantum physics. Means this statistics is valid for Micro particle. In this statistics, Wave function of particle is explain by $\psi(x,t)$. This wave function explain the state of particle so this is also known as state function. In this statistics this state function is symmetric.

Acc^n to particle physics, That statistics that is valid for Boson particle, is said to be Bose Einstein statistics. Boson particle are that elementary particle for which spin Quantum no is integer.

$$S = 0, 1, 2, 3 - - -$$

Ex: Photon (γ), Phonon, graviton, kion's, Pion's etc.

In Bose Einstein statistics, Pauli's exclusion law is invalid for arrangement of particle. It means No. of particle in a shell can be possible two or more than two but this statistics is valid for ~~different~~ same individual particles. To explain the Bose Einstein statistics, Phase space is used. Phase space is a six dimensional frame of reference in which 3 co-ordinate of position and 3-co ordinate of momentum (P_x, P_y, P_z). In phase space volume of any particle is represent as.

$$\boxed{H = dx\,dy\,dz\,dP_x\,dP_y\,dP_z}$$

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Let, No. of orbitals/shell in system is g and no. of phase point is n.

Mathematical Analysis:

Mathematical analysis of Bose Einstein is done by thermodynamic probability function. Thermodynamic probability for i^{th} phase point is following

$$W_i = \frac{g_i \lfloor g_i + n_i - 1}{\lfloor g_i \, \lfloor n_i} \quad \text{—— (1)}$$

For whole system

$$n_1 + n_2 + n_3 + \cdots$$

$$W = \prod_i W_i$$

From eqn (1)

$$W = \prod_i \frac{g_i \lfloor g_i + n_i - 1}{\lfloor g_i \, \lfloor n_i}$$

$$\lfloor n = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$

$$\lfloor n = n \lfloor n-1$$

$$W = \prod_i \frac{g_i \lfloor g_i + n_i - 1}{g_i \lfloor g_i - 1 \, \lfloor n_i}$$

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$$W = \prod_i \frac{\lfloor g_i + n_i - 1}{\lfloor g_i - 1 \, \lfloor n_i} \quad \text{--- (2)}$$

1 Jan 2021

We know that, logarithm function of thermodynamic probability is said to be entropy of system. So,

$$S = K \log_e W$$

$$\because K = \text{const} = 1.38 \times 10^{-23} \, J/K$$

$$S \propto \log_e W$$

eqn (2), taking log

$$\log W = \sum \log_e \left[\frac{\lfloor g_i + n_i - 1}{\lfloor g_i - 1 \, \lfloor n_i} \right]$$

$$\because \log_e \left(\frac{A}{BC} \right) = \log_e A - (\log_e B + \log_e C)$$

$$\log_e W = \sum_i \left[\log_e \lfloor g_i + n_i - 1 - \log_e \lfloor g_i - 1 - \log_e \lfloor n_i \right]$$

For Macrosystem

$$\log_e W = \sum_i \log_e \lfloor g_i + n_i - \log_e \lfloor g_i - \log_e \lfloor n_i \Big]$$

From stirling formula.

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$$\log_e A! = A\log_e A - A$$

$$\log_e W = \sum\left[(g_i+n_i)\log(g_i+n_i) - (g_i+n_i) - g_i\log_e g_i + g_i - n_i\log_e n_i + n_i\right]$$

$$\log_e W = \sum_i\left[(g_i+n_i)\log_e(g_i+n_i) - g_i\log_e g_i - n_i\log_e n_i\right] \quad -③$$

in eqⁿ ③, it define the entropy of particle and we know that in equilibrium system, value of entropy is maximum (constant) so, differential form of $\log_e W$ finds zero. Here g_i is no. of shell, so this is define as constant quantity that's differential form defined zero.

on writing eqⁿ ③ in diffⁿ form.

$$\Rightarrow \quad \delta(\log_e W) = 0 = \sum_i\left[(0+\delta n_i)\log_e(g_i+n_i) + \frac{(g_i+n_i)\delta n_i}{(g_i+n_i)} - 0 - \left\{\delta n_i\log_e n_i + n_i\cdot\frac{1}{n_i}\delta n_i\right\}\right]$$

$$\Rightarrow \quad 0 = \sum_i\left[\delta n_i\log_e(g_i+n_i) + \delta n_i - \delta n_i\log_e n_i - \delta n_i\right]$$

$$\Rightarrow \quad 0 = \sum_i\left[\log_e(g_i+n_i) - \log_e n_i\right]\cdot\delta n_i$$

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$$\boxed{0 = \sum_i \log_e \left(\frac{g_i + n_i}{n_i}\right) \delta n_i} \quad \text{—— (4)}$$

let total no. of phase point present in system is N. and total energy is U. that is a constant Quantity. so their differential will also be zero.

$$N = \sum_i n_i = n_1 + n_2 + n_3 + n_4 + \cdots$$

$$\because N = \text{Constant}$$

$$\delta N = 0 = \sum \delta n_i$$

$$\boxed{\sum \delta n_i = 0} \quad \text{—— (5)}$$

so, total Energy

$$U = \sum_i n_i E_i = \text{Constant}$$

in differential form.

$$\delta U = 0 = \sum_i \delta n_i E_i$$

$$\boxed{\sum_i \delta n_i E_i = 0} \quad \text{—— (6)}$$

eq^n (4), (5) and (6) is a first order diff^n eq^n. To do the sol^n of these diff^n eq^n langrange method is used.

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From langrange method.

$$eq^n\ ④ + eq^n\ ⑤ \times(-\alpha) + eq^n\ ⑥ \times(-\beta)$$

$$\Rightarrow \sum_i \left[\log_e\left(\frac{g_i+n_i}{n_i}\right)\right]\delta n_i + (-\alpha)\sum_i \delta n_i + (-\beta)\sum_i \delta n_i E_i = 0$$

$$\Rightarrow \sum_i \left[\log\left(\frac{g_i+n_i}{n_i}\right) - \alpha - \beta E_i\right]\delta n_i = 0$$

Here $\delta n_i \neq 0$, becoz no. of phase points are free variable.

NOW

$$\log_e\left(\frac{g_i+n_i}{n_i}\right) - \alpha - \beta E_i = 0$$

$$\log_e\left(\frac{g_i+n_i}{n_i}\right) = \alpha + \beta E_i$$

$$\frac{g_i+n_i}{n_i} = e^{\alpha+\beta E_i}$$

$$\frac{g_i}{n_i} + 1 = e^{\alpha+\beta E_i}$$

$$\frac{g_i}{n_i} = e^{\alpha+\beta E_i} - 1$$

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$$\frac{n_i}{g_i} = \frac{1}{e^{\alpha+\beta E_i}-1}$$

$$n_i = \frac{g_i}{e^{\alpha}\cdot e^{\beta E_i}-1}$$

$$\because \quad e^{\alpha} = A \text{ (let) } = \text{constant}$$

$$n_i = \frac{g_i}{Ae^{\beta E_i}-1}$$

$$\because \quad \beta = \frac{1}{KT}$$

MTR

$$\boxed{n_i = \frac{g_i}{A\exp\left(\frac{E_i}{KT}\right)-1}} \quad \text{——— (7)}$$

This is the mathematical formula of Bose Einstein statistics. (distribution law).

20JAN2021

Ques Explain Fermi-Dirac statistics and derive the distribution law?

$$n_i = \frac{g_i}{A\exp\left(\frac{E_i}{KT}\right)+1}$$

Ans Fermi Dirac statistics:-

In 1926 Fermi scientist and Dirac scientist gave a Quantum statistics. In this Quantum

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Statistic particle is explain by wave function $\Psi(r,t)$. This wave function define the state of particle. So this wave is also known as state function. In fermi-dirac statistic, This wave function $\Psi(r,t)$ is Anti-symmetric. means,

$$\boxed{\Psi_{12} = -\Psi_{21}}$$

Acc^n to Concept of ~~Quantum~~ particle physics, This Quantum statistics is valid for fermi particles. Fermi-particle are that particle for which spin Quantum no. is half odd integer.

$$\boxed{S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \text{-----}}$$

Ex: β-particle, e^-, Proton, neutron, Baryon.

These fermi-particle follows Pauli's exclusion law, means in one Quantum state only one particle can be present. and in one orbital maximum two particle can be exists. If first particle is present in up state then second will present in down state. Fermi dirac statistics is explain by phase space. In phase space, Volume of phase sheet is:-

$$\boxed{H = dx\,dy\,dz\,dP_x\,dP_y\,dP_z}$$

To do the mathematical analysis of Fermi-Dirac statistical, we use the thermodynamic probability function.

Thermodynamic probability for i^{th} particle is W_i

$$W_i = \frac{g_i!}{n_i!\,(g_i - n_i)!} \quad \text{——} (1)$$

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Thermodynamic probability for whole system

$$W = \prod_i W_i$$

From eqⁿ ①

$$W = \prod_i \frac{\lfloor g_i}{\lfloor n_i \, \lfloor g_i - n_i} \quad \text{—— ②}$$

on taking log to both sides,

$$\log W = \sum_i \log_e \frac{\lfloor g_i}{\lfloor n_i \, \lfloor g_i - n_i}$$

$$\log_e W = \sum_i \left[\log_e \lfloor g_i - \log_e \lfloor n_i - \log_e \lfloor g_i - n_i \right]$$

From stirling formula

$$\log_e \lfloor N = N \log_e N - N$$

$$\Rightarrow \log_e W = \sum_i \left[g_i \log_e g_i - g_i - n_i \log_e n_i + n_i - (g_i - n_i) \log_e (g_i - n_i) + g_i - n_i \right]$$

$$\Rightarrow \log_e W = \sum_i \left[g_i \log_e g_i - n_i \log_e n_i - (g_i - n_i) \log_e (g_i - n_i) \right] \quad \text{—— ③}$$

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In eqn (3) It define the entropy of particle and we know in equilibrium system, Value of entropy is maximum. (Constant) so, differential form of $\log_e W$ finded zero. Here g_i is no. of shell. so this is define as constant Quantity that's differential form defined zero.

$$0 = \sum_i \left[0 - \delta n_i \log_e n_i - n_i \frac{1}{n_i} \delta n_i - (0 - \delta n_i) \cdot \log_e (g_i - n_i) - \frac{(g_i - n_i)}{(g_i - n_i)} \cdot (0 - \delta n_i) \right]$$

$$\Rightarrow \quad 0 = \sum_i \left[-\delta n_i \log_e n_i - \delta n_i + \delta n_i \log_e (g_i - n_i) + \delta n_i \right]$$

$$\Rightarrow \quad 0 = \sum_i \left[-\log_e n_i + \log (g_i - n_i) \right] \delta n_i$$

$$\Rightarrow \quad \boxed{\sum_i \left[\log_e \left(\frac{g_i - n_i}{n_i} \right) \right] \delta n_i = 0} \quad \text{--- (4)}$$

Let Total no. of phase point present in system is N. and total energy is U. that is a Constant Quantity. so their differential is <u>zero</u>.

$$N = \sum_i n_i = \text{const.}$$

$$\delta N = 0 = \sum_i \delta n_i$$

$$\boxed{\sum_i \delta n_i = 0} \quad \text{--- (5)}$$

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So, Total energy

$$U = \sum_i n_i E_i = \text{Constant}$$

$$\delta U = 0 = \sum_i \delta n_i E_i$$

$$\boxed{\sum_i \delta n_i E_i = 0} \quad \text{—} (6)$$

eq^n (4), (5) and (6) is a first order diff^n eq^n. To do the det^n of these diff^n eq^n langrange method is used, So, From langrange method:

$$\text{eq}^n (4) + \text{eq}^n (5) \times (-\alpha) + \text{eq}^n (6) \times (-\beta)$$

$$\sum_i \left[\log_e \left(\frac{g_i - n_i}{n_i} \right) \right] \delta n_i + (-\alpha) \sum_i \delta n_i + (-\beta) \sum_i \delta n_i E_i = 0$$

$$\sum_i \left[\log_e \left(\frac{g_i - n_i}{n_i} \right) - \alpha - \beta E_i \right] \delta n_i = 0$$

$$\delta n_i \neq 0$$

Now

$$\log \left(\frac{g_i - n_i}{g_i} \right) - \alpha - \beta E_i = 0$$

$$\log_e \left(\frac{g_i - n_i}{n_i} \right) = \alpha + \beta E_i$$

$$\frac{g_i - n_i}{n_i} = e^{\alpha + \beta E_i}$$

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$$\frac{g_i}{n_i} - \frac{n_i}{n_i} = e^{\alpha} \cdot e^{\beta E_i}$$

Let $A = e^{\alpha} = \text{const.}$

$$\frac{g_i}{n_i} - 1 = A e^{\beta E_i}$$

$$\frac{g_i}{n_i} = A e^{\beta E_i} + 1$$

$$\frac{n_i}{g_i} = \frac{1}{A e^{\beta E_i} + 1}$$

$$n_i = \frac{g_i}{A e^{\beta E_i} + 1}$$

$$\beta = \frac{1}{KT}$$

$$\boxed{n_i = \frac{g_i}{A \exp\left(\frac{E_i}{KT}\right) + 1}} \quad \text{—— (7)}$$

eqn (7) is the ~~differential~~ distribution law for Fermi-Dirac statistics.

A Jan 2021

Fermi Energy :-

At Absolute zero kelvin temp., ~~Fermion~~ Maximum Energy

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of Fermion particle present in system is known as Fermi-Energy of that system. It is denoted by 'Ef. So, Fermi-energy of system.

$$\boxed{E_F = -\alpha kT}$$

$$\alpha = -\frac{E_f}{kT}$$

Now

$$A = e^{\alpha}$$
$$A = e^{\frac{-E_f}{kT}}$$

From eqn (7)

$$n_i = \frac{g_i}{e^{-E_f/kT} \cdot e^{+E_i/kT} + 1}$$

$$n_i = \frac{g_i}{e^{\frac{E_i - E_f}{kT}} + 1}$$

$$\boxed{n_i = \frac{g_i}{\exp\left(\frac{E_i - E_f}{kT}\right) + 1}} \quad \text{— (8)}$$

eqn (8) is said to be distribution law of Fermi Dirac statistics in form of Fermi Energy.

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Case I If $T = 0K$ and $E_i < E_f$

then,

$$e^{\frac{E_i - E_f}{KT}} = e^{\frac{-(E_f - E_i)}{0}}$$

$$e^{\frac{E_i - E_f}{KT}} = e^{-\infty} = 0$$

From eqn (8)

$$n_i = \frac{g_i}{0+1}$$

$$\boxed{n_i = g_i}$$

Occupation of Probability (f) $= \frac{n_i}{g_i}$

$$\boxed{f_i = \frac{n_i}{g_i} = 1}$$

Case-II If at $T = 0K$ and $E_i > E_f$

then,

$$e^{\frac{E_i - E_f}{KT}} = e^{\frac{+(E_i - E_f)}{0}}$$

$$e^{\frac{E_i - E_f}{KT}} = e^{\infty} = \infty$$

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from eqn (8)

$$n_i = \frac{g_i}{\infty + 1}$$

$$\frac{n_i}{g_i} = \frac{1}{\infty}$$

$$\boxed{\frac{n_i}{g_i} = 0}$$

<u>Case III</u> If $T > 0K$ and $E_i = E_f$

then

$$e^{\frac{E_i - E_f}{kT}} = e^0$$

$$e^{\frac{E_i - E_f}{kT}} = 1 \qquad \because e^0 = 1$$

from eqn (8)

$$n_i = \frac{g_i}{1+1}$$

$$\boxed{n_i = \frac{g_i}{2}}$$

$$\boxed{f_i = \frac{n_i}{g_i} = \frac{1}{2}}$$

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where value of energy level is same (degeneracy)

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Nuclear spin statistics:-

That statistics define in nuclear physics that define the spin state of nuclear / nucleus is known as nuclear spin statistics.

We know that electron does orbital motion as well as spin motion due to which spin angular momentum define. similarly as a electron, nucleon particle proton and neutron also does spin motion due to which spin angular momentum define. spin angular momentum is denoted by I or J. It is a Quantised Quantity. whose value is $\sqrt{I(I+1)}\,\hbar$. and value of degeneracy is $(2I+1)$.

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By Nuclear angular statistics, we can explain the ortho and para state.

* → That Rotational state of Hydrogen atom for which value of Quantum no. defined odd. is said to be ortho state of Hydrogen.

$J = 1, 3, 5, 7 \ldots$

For ortho state, Rotational wavefunction is Anti-symmetric. and for this state, value of Probability in equilibrium state is high. spin state is symmetric for ortho hydrogen. There are the special features of ortho-Hydrogen.

* → That Rotational state of Hydrogen atom for which value of Quantum no. defined even is said to be Para state of Hydrogen.

$J = 2, 4, 6, 8 \ldots$

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For Para state of Hydrogen, Rotational wave-function is symmetric. And for this state, Probability in equilibrium state is low. Spin state of Para hydrogen is Anti-symmetric. At Normal temperature, 75% ortho exists and 25% part para state exists in Normal Hydrogen.

Chemical properties of ortho and para Hydrogen is approximately equal. But physical properties are different. For example:

Nuclear magnetic moment differ different for both state.

According to experimental knowledge, This conclusion has been clear that temperature of Hydrogen atom, on decreasing the temp. of hydrogen by Normal temperature then ortho state of hydrogen is change in Para state. Means Probability of Para state of hydrogen increased. This Postulate can also be define as Ortho state change in Para state becoz of emission of energy in hydrogen. But Para state is more stable. In this way Physical properties are different of ortho and para hydrogen.

According to Concept of molecular Physics, molecules are design by atom. Two Hydrogen atom are present in hydrogen molecule. If spin state of Nuclear of both hydrogen atom is same then it is called ortho state of hydrogen, and if Nuclear spin state is different then it is called Para state of Hydrogen. In this way on the basis of nuclear spin state, Ortho and Para state of Hydrogen is explain.

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UNIT I–III Classical statistics (MBS)

Lesson-6

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→ Introduction of classical statistics
→ Degree of Freedom.
→ Heat Capacity
→ Relation b/w Entropy and Thermodynamic Probability
→ Ideal gas Equation
→ Barometric Equation
→

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Introduction OF Classical statistics:-

That branch of Physical science that is based on classical concepts, known as classical physics. Statistics related to classical physics is known as classical statistics. Maxwell and Boltzmann scientist, to study the macrosystem they gave a statistics based on classical concept. i.e. known as Maxwell Boltzmann statistics. ~~Maxwell~~ To explain the particle in Maxwell Boltzmann statistics Phase space (6-dim^n) is used.

Degree OF Freedom:-

To ~~def~~ decide the Configuration of particle or system, independent variable are necessary. No. of independent variable are known as Degree of Freedom.

To explain any particle position vector is required in Frame of Reference. Position vector of particle is explain by co-ordinates. It means Position vector of particle is a function of co-ordinate. These co-ordinates does not depends on each other it means these are independent variable. No. of independent variable is known as Degree of freedom of particle. Degree of freedom is denoted by f.

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* If any particle does Rectilinear Motion then to explain the motion of that particle, one no. of free variable is required. Therefore Degree of freedom in one dim^n motion is One.

$$\boxed{f_{1-D} = 1}$$

* If any particle does two dim^n motion then to explain the motion of that particle, two free variable are required. So in this condition Degree of freedom is two.

$$\boxed{f_{2-D} = 2}$$

* If any particle does 3-dim^n motion then to explain the motion of that particle, Three free variable are required. So in this condition Degree of freedom is Three.

$$\boxed{f_{3-D} = 3}$$

* We know that monoatomic molecule does translation motion. and for translational motion, Maximum Degree of freedom is 3.

$$\boxed{f = 3}$$

* In classical physics / statistics, Degree of freedom at normal temperature is define by following mathematical

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formula.

$$\boxed{f = 3N - K}$$

$\because$ f = Degree of freedom.
$\because$ N = No. of atoms
$\because$ K = No. of constraint / bond.

ⓐ **For Monoatomic Molecule:-**

We know that

$$f = 3N - K$$

$\because$ $N = 1$, $K = 0$

$$f = 3 \times 1 - 0$$

$$\boxed{f = 3}$$

ⓑ **For Diatomic Molecule:-**

We know that

$$f = 3N - K$$

$\because$ $N = 2$, $K = 1$

$$f = 3 \times 2 - 1$$

$$\boxed{f = 5}$$

ⓒ **For Triatomic / Polyatomic Molecule:-**

Degree of freedom for Triatomic molecule depends on structure.

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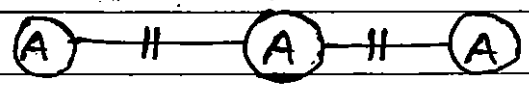
① **For linear structure:-**



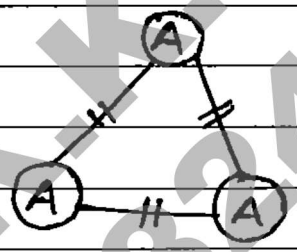
$k=2$, $N=3$

$f = 3N-k$

$f = 3\times3-2$

$\boxed{f=7}$

② **For cyclic / Non-linear structure:-**



$N=3$, $k=3$

$f = 3\times3-3$

$\boxed{f=6}$

NOTE! In classical Physics, these degree of freedom are define at Normal temperature. It means, on increase in the temperature of molecule, degree of freedom also increase.

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Classical Theory of Heat Capacity :-

According to concept of classical physics, Capacity of Heat absorbed by any system is known as heat capacity or specific Heat capacity. In Mathematical form, change in heat with respect to temperature is known as Heat capacity.
It is denoted by C.

$$\boxed{C = \frac{dQ}{dT}}$$

Heat capacity is of two type.

1. Heat capacity at constant Pressure

$$C_P = \left(\frac{dQ}{dT}\right)_P \quad \text{--- (1)}$$

2. Heat capacity at Constant Volume

$$\boxed{C_V = \left(\frac{dQ}{dT}\right)_V} \quad \text{--- (2)}$$

From 1st law of Thermodynamics

$$dQ = dU + dW$$

$$\because W = PdV$$

$$dQ = dU + PdV \quad \text{--- (3)}$$

If V = const. then dV = 0

From eqn (3)

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$$\boxed{dQ = dU}$$

From eqn ②

$$\boxed{C_v = \left(\frac{dU}{dT}\right)_v} \quad \text{—— ④}$$

We know that,
from ideal gas eqn

$$PV = RT$$

on diffn w.r.t to Temp, P = Constant

$$\frac{PdV}{dT} = R$$

$$\boxed{PdV = RdT} \quad \text{—— ⑤}$$

from eqn ③

$$dQ = dU + RdT$$

on dividing by dT

$$\left(\frac{dQ}{dT}\right)_P = \left(\frac{dU}{dT}\right)_v + R\left(\frac{dT}{dT}\right)$$

$$C_p = C_v + R$$

$$\boxed{C_p - C_v = R} \quad \text{—— ⑥ Mayer's Relation}$$

eqn ⑥ is known as "Mayer's Relation".

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1. Internal energy of system:-

Acc^n to concept of equipartition principle of energy, Energy related to every degree of freedom is $\frac{1}{2}KT$. If for any system degree of freedom is f, then internal energy of system is following:-

$$\boxed{U = \frac{1}{2}KT \cdot f}$$

k = Boltzmann constant.
$k = 1.38 \times 10^{-23}$ J/K

2. Heat capacity at constant volume:-

$$C_v = \left(\frac{dU}{dT}\right)_V$$

$$\because U = f \cdot \frac{1}{2}KT$$

$$C_v = \frac{d}{dT}\left(f \cdot \frac{1}{2}KT\right)$$

$$C_v = f \cdot \frac{1}{2}K$$

If N no. of atoms are present in a system.

$$C_v = f \cdot \frac{1}{2}NK$$

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$\because Nk = nR$

$$C_V = f \cdot \frac{1}{2} nR$$

$$\boxed{C_V = \frac{1}{2} fR}$$

3. Heat capacity at constant Pressure :-

We know that

$$C_P - C_V = R$$
$$C_P = C_V + R$$

$$C_P = \frac{1}{2} fR + R$$

$$\boxed{C_P = \frac{R}{2}(f+2)}$$

4. Ratio of Heat Capacity :-

$$\gamma = \frac{C_P}{C_V}$$

$$\gamma = \frac{R/2\,(f+2)}{1/2\, fR}$$

$$\gamma = \frac{(f+2)}{f}$$

$$\boxed{\gamma = 1 + \frac{2}{f}}$$

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I. For Monoatomic molecule:-

We know that Degree of freedom for monoatomic molecule is 3.

$$f = 3N - k$$

$$\because N = 1$$

$$k = 0$$

$$f = 3 \times 1 - 0$$

$$\boxed{f = 3}$$

(a) Internal Energy:-

$$U = f \cdot \frac{1}{2} KT$$

$$\because f = 3$$

$$\boxed{U = \frac{3}{2} KT} \quad \because Nk = nR$$

$$\boxed{U = \frac{3}{2} RT}$$

(b) Heat Capacity at constant volume:-

We know that

$$C_v = f \cdot \frac{1}{2} R$$

$$\because f = 3$$

$$\boxed{C_v = \frac{3}{2} R}$$

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(c) Heat Capacity at constant Pressure:-

We know that

$$C_P = \frac{R}{2}(f+2)$$

$$C_P = \frac{R}{2}(3+2)$$

$$\boxed{C_P = \frac{5}{2}R}$$

(d) Ratio of Heat Capacity:-

We know that

$$\gamma = 1 + \frac{2}{f}$$

$$\because f = 3$$

$$\gamma = 1 + \frac{2}{3}$$

$$\gamma = \frac{5}{3}$$

$$\boxed{\gamma = 1.6}$$

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For Diatomic Molecule:-

We know that, degree of freedom for diatomic molecule is 5.

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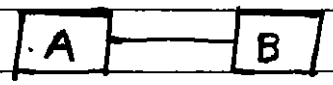
$$f = 3N - K$$



$$N = 2, \quad K = 1$$

$$f = 3 \times 2 - 1$$

$$\boxed{f = 5}$$

ⓐ **Internal Energy :-**

We know that

$$U = \frac{1}{2} KT \cdot f$$

$$\because f = 5$$

$$\boxed{U = \frac{5}{2} KT}$$

ⓑ **Heat capacity at constant volume :-**

We know that

$$C_V = \frac{1}{2} f \cdot R$$

$$\because f = 5$$

$$\boxed{C_V = \frac{5}{2} R}$$

ⓒ **Heat capacity at constant Pressure :-**

$$C_P = \frac{R}{2}(f+2)$$

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$$C_p = \frac{R}{2}(5+2)$$

$$\boxed{C_p = \frac{7R}{2}}$$

ⓓ **Ratio of Heat capacity:-**

We know that

$$\gamma = 1 + \frac{2}{f}$$

$$\because f = 5$$

$$\gamma = 1 + \frac{2}{5}$$

$$\gamma = \frac{7}{5}$$

$$\boxed{\gamma = 1.4}$$

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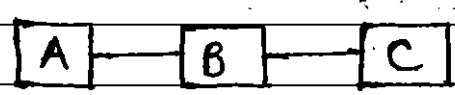
III For Triatomic Molecule:-

We know that Degree of freedom for triatomic molecule is depends on its geometrical structure.

a (i) For linear structure:-



$$f = 3N - K$$

$\because N = 3$

$K = 2$

$$f = 3 \times 3 - 2$$

$$\boxed{f = 7}$$

It is clear that degree of freedom for linear structure is 7.

ⓐ <u>Internal Energy:-</u>

We know that

$$f = 3N - K$$

$\because N = 3, K = 2$

$$U = \frac{1}{2} f KT$$

$$\boxed{U = \frac{7}{2} KT}$$

ⓑ <u>Heat capacity at Constant Pressure:-</u>

$$C_p = \frac{R}{2}(f+2)$$

$$C_p = \frac{R}{2}(7+2)$$

$$\boxed{C_p = \frac{9R}{2}}$$

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(c) Heat capacity at Constant Volume:-

We know that

$$C_V = \frac{1}{2} f \cdot R$$

$$\boxed{C_V = \frac{7}{2} R}$$

(d) Ratio of Heat capacity:-

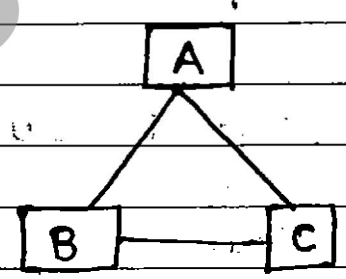
$$\gamma = \cancel{C_p} \; 1 + \frac{2}{f}$$

$$\gamma = 1 + \frac{2}{7}$$

$$\gamma = \frac{9}{7}$$

$$\boxed{\gamma = 1.28}$$

(ii) For Cyclic structure:-



$$f = 3N - K$$

$\because N = 3, \; K = 3$

$$f = 3 \times 3 - 3$$

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$$\boxed{f=6}$$

So, Degree of freedom for cyclic structure is 6.

ⓐ **Internal Energy:-**

We know that

$$U = \frac{1}{2} f \cdot KT$$

$$U = \frac{6}{2} KT$$

$$\boxed{U = 3KT}$$

ⓑ **Heat capacity at constant Volume:-**

$$C_V = \frac{1}{2} f R$$

$$C_V = \frac{6}{2} R$$

$$\boxed{C_V = 3R}$$

ⓒ **Heat capacity at constant Pressure:-**

$$C_P = \frac{R}{2}(f+2)$$

$$C_P = \frac{R}{2}(6+2)$$

$$\boxed{C_P = 4R}$$

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(d) Ratio of Heat capacity:

$$\gamma = 1 + \frac{2}{f}$$

$$\gamma = 1 + \frac{2}{6}$$

$$\boxed{\gamma = \frac{4}{3}}$$

IV Heat capacity for Diatomic Molecule:-

We know that In diatomic molecule, two atoms of finite distance are present. These atoms are connected by a bond due to which structure of diatomic molecule is as a Dumble. Acc^n to concept of Thermodynamic physics, degree of freedom ~~of~~ for diatomic molecule is 5. three degree of freedom are define as translational motion and two degree of freedom are define as Rotational motion. We know that degree of freedom depends on temperature. If Heat energy is given to the diatomic molecule then degree of freedom define is 7. For every ~~one~~ degree of freedom related to energy is $\frac{1}{2}KT$ so energy related to translational motion is $\frac{3}{2}KT$ and energy related to Rotational motions is KT and energy related to vibrational motion is KT. so total internal energy of diatomic molecule.

$$U = U_T + U_R + U_V$$

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$\because U_T = \frac{3}{2}KT$

$\because U_R = KT$

$\because U_V = KT$

So,

$U = \frac{3}{2}KT + KT + KT$

$\boxed{U = \frac{7}{2}KT}$ or $\boxed{U = \frac{7}{2}RT}$ $\because Nk = nR$

So, Heat Capacity of system:-

$C_V = (C_V)_T + (C_V)_R + (C_V)_V$

$C_V = \frac{3}{2}R + R + R$

$\boxed{C_V = \frac{7}{2}R}$

So, it is clear that at High temperature, Heat capacity of Diatomic molecule is $\frac{7}{2}R$. That is a Conclusion of classical physics. By this Conclusion, Conclusion of Quantum physics doesn't satisfy. it means this result doesn't satisfy the curve.

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Explanation by Quantum Theory:-

We know that, acc to Quantum physics, Value of energy for system is following:-

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$$\boxed{E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0} \qquad n = 0, 1, 2, 3 ----$$

$$E_n = \left(n+\frac{1}{2}\right)h\nu$$

$$E_n = \left(n+\frac{1}{2}\right)h\nu \cdot \frac{2\pi}{2\pi}$$

$$\because \omega_0 = 2\pi\nu$$

$$\because \hbar = \frac{h}{2\pi}$$

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0 \quad \text{--- (1)}$$

n = quantum state

Energy in ground state.

(i) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0$

$$\because n = 0$$

$$\boxed{E_0 = \frac{1}{2}\hbar\omega_0}$$

(ii) Energy in first state

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0$$

$$n = 1$$

$$E_1 = \left(1+\frac{1}{2}\right)\hbar\omega_0$$

$$\boxed{E_1 = \frac{3}{2}\hbar\omega_0}$$

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We know that from defination of partition function

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n}$$

$$Z = \sum_{n=0}^{\infty} \exp(-\beta E_n)$$

$$\beta = \frac{1}{KT}$$

$$Z = \sum_{n=0}^{\infty} e^{-\frac{E_n}{KT}}$$

$$\because E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0$$

$$Z = \sum_{n=0}^{\infty} e^{-\left(n+\frac{1}{2}\right)\frac{\hbar\omega_0}{KT}}$$

let $\theta_v = \frac{\hbar\omega_0}{K}$ (characteristic temp. of Vibrational state)

$$Z = \sum_{n=0}^{\infty} e^{-\left(n+\frac{1}{2}\right)\frac{\theta_v}{T}}$$

$$Z = \sum_{n=0}^{\infty} e^{-\frac{n\theta_v}{T}} \cdot e^{-\frac{\theta_v}{2T}}$$

$$Z = e^{-\frac{\theta_v}{2T}} \sum_{n=0}^{\infty} e^{-\frac{n\theta_v}{T}}$$

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$$\left(e^{-\frac{\theta_v}{2T}}\right) e^{+\frac{\theta_v}{T}} = e^{\frac{\theta_v}{T} - \frac{\theta_v}{2T}}$$

$$e^{(\theta_v/2T)}$$

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$$z = e^{\frac{-\theta_v}{2T}} \left[1 + e^{\frac{-\theta_v}{T}} + e^{\frac{-2\theta_v}{T}} + e^{\frac{-3\theta_v}{T}} + \cdots\right]$$

$$\because e^{x} + e^{2x} + e^{3x} = (e^{x})^1 + (e^{x})^2 + (e^{x})^3$$

$$z = e^{\frac{-\theta_v}{2T}} \left[1 + \left(e^{\frac{-\theta_v}{T}}\right)^1 + \left(e^{\frac{-\theta_v}{T}}\right)^2 + \left(e^{\frac{-\theta_v}{T}}\right)^3 + \cdots\right]$$

$$S_\infty = \frac{a}{1-r}$$

$$a = 1, \quad r = e^{\frac{-\theta_v}{T}}$$

$$\boxed{z = \frac{e^{\frac{-\theta_v}{2T}} \cdot 1}{1 - e^{-\theta_v/T}}} \quad \text{—} (2)$$

$$z = \frac{e^{\frac{-\theta_v}{2T}}}{e^{-\theta_v/T}\left[\frac{1}{e^{-\theta_v/T}} - 1\right]}$$

$$\boxed{z = \frac{e^{+\frac{\theta_v}{2T}}}{(e^{\theta_v/T} - 1)}}$$

eq^n (2) is the mathematical formula of partition function for diamtomic molecule.

Again we know that from mathematical formula of probability distribution function.

$$P = \frac{e^{-\beta E_n}}{Z} \quad \text{—} (3)$$

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No. of molecule for n-Quantum state:

$$N_n = NP$$

$$P = \frac{e^{-\beta E_n}}{Z}$$

$$N_n = \frac{Ne^{-\beta E_n}}{Z}$$

$$Z = e^{-\theta_v/2T} \cdot \frac{1}{1-e^{-\theta_v/T}}$$

$$N_n = \frac{N e^{-\beta E_n}\left(1-e^{-\frac{\theta_v}{T}}\right)}{e^{-\theta_v/2T}}$$

$$\because E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0$$

$$\beta = \frac{1}{KT}$$

$$N_n = \frac{N\left(1-e^{-\theta_v/T}\right)e^{-\left(n+\frac{1}{2}\right)\frac{\hbar\omega_0}{KT}}}{e^{-\frac{\theta_v}{2T}}}$$

$$\because \theta_v = \frac{\hbar\omega_0}{K}$$

$$N_n = \frac{N\left(1-e^{-\frac{\theta_v}{T}}\right)e^{-\left(n+\frac{1}{2}\right)\frac{\theta_v}{T}}}{e^{-\theta_v/2T}}$$

$$N_n = \frac{N\left(1-e^{-\frac{\theta_v}{T}}\right)e^{-\frac{n\theta_v}{T}} \cdot e^{-\frac{\theta_v}{2T}}}{e^{-\theta_v/2T}}$$

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$$\boxed{N_n = N e^{-\frac{n\theta_D}{T}}\left(1-e^{-\frac{\theta_D}{T}}\right)} \quad \text{--- (4)}$$

$n = 0, 1, 2, 3 ----$

<u>For Ground State</u>

$n = 0$

From eqn (4)

$$N_0 = N e^{-0}\left(1-e^{-\frac{\theta_D}{T}}\right)$$

$\because e^{-0} = e^{0} = 1$

$$N_0 = N\left(1-e^{-\frac{\theta_D}{T}}\right)$$

At absolute zero kelvin temp.

$$N_0 = N\left(1-e^{-\infty}\right)$$

$\because e^{-\infty} = 0$

$$\boxed{N_0 = N}$$

So, Average energy of system

We know that

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E_n e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}} \quad \text{--- (6)}$$

$$\because E_n = \left(n+\frac{1}{2}\right)\frac{\hbar\omega_D}{KT}$$

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$$\because \beta = \frac{1}{KT}$$

$$\bar{E} = \frac{\sum_{n=0}^{\infty} \left(n+\frac{1}{2}\right)\hbar\omega_0 e^{-\left(n+\frac{1}{2}\right)\frac{\hbar\omega_0}{KT}}}{\sum_{n=0}^{\infty} e^{-\left(n+1/2\right)\frac{\hbar\omega_0}{KT}}}$$

let $x = \frac{\hbar\omega_0}{KT}$

$$\bar{E} = \frac{\hbar\omega_0 \sum_{n=0}^{\infty} \left(n+\frac{1}{2}\right) e^{-\left(n+\frac{1}{2}\right)x}}{\sum_{n=0}^{\infty} e^{-\left(n+1/2\right)x}}$$

$$\bar{E} = \frac{\hbar\omega_0 \left[\frac{1}{2} e^{-\frac{1}{2}x} + \frac{3}{2} e^{-\frac{3}{2}x} + \frac{5}{2} e^{-\frac{5}{2}x} + \cdots\right]}{e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots}$$

$$\because -\frac{d}{dx} \log_e \left[e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots\right] = \frac{1/2\, e^{-1/2x} + 3/2\, e^{-3/2x} + \cdots}{e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots}$$

$$\Rightarrow \bar{E} = -\hbar\omega_0 \frac{d}{dx} \log_e \left[e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots\right]$$

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$$\Rightarrow \bar{E} = -\hbar\omega_0 \frac{d}{dx} \log_e \left[e^{-1/2x} (1 + e^{-x} + e^{-2x} + e^{-3x} + \cdots \right]$$

$$\because \quad S_\infty = \frac{a}{1-r} \; ; \; a = 1, \; r = e^{-x}$$

$$\bar{E} = -\hbar\omega_0 \frac{d}{dx} \left[\log_e \left\{ e^{-1/2x} \cdot \frac{1}{1-e^{-x}} \right\} \right]$$

$$\bar{E} = -\hbar\omega_0 \frac{d}{dx} \left[\frac{-1}{2}x + \log_e \frac{1}{1-e^{-x}} \right]$$

$$\bar{E} = -\hbar\omega_0 \left[\frac{-1}{2} + \frac{(1-e^{-x})(-1)\cdot(0+e^{-x})}{(1-e^{-x})^2} \right]$$

$$\bar{E} = \hbar\omega_0 \left[\frac{1}{2} + \frac{e^{-x}}{1-e^{-x}} \right]$$

$$\bar{E} = \hbar\omega_0 \left[\frac{1}{2} + \frac{e^{-x}}{e^{-x}\left(\frac{1}{e^{x}} - 1\right)} \right]$$

$$\bar{E} = \hbar\omega_0 \left[\frac{1}{2} + \frac{1}{e^{x} - 1} \right]$$

$$x = \frac{\hbar\omega_0}{KT}$$

$$\boxed{\bar{E} = \hbar\omega_0 \left[\frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega_0}{KT}} - 1} \right]} \quad \text{—(8)}$$

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If N molecule are present in system then Average energy of system.

$$\bar{E} = N\hbar\omega_0\left[\frac{1}{2} + \frac{1}{(e^{\hbar\omega_0/KT}-1)}\right]$$

$$dE = dQ = dU +$$

We know that if Volume of any system is constant then total energy is equal internal energy. Internal energy is denoted by U.

$$U = N\hbar\omega_0\left[\frac{1}{2} + \frac{1}{(e^{\hbar\omega_0/KT}-1)}\right] \quad \text{--- (9)}$$

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We know that

$$C_V = \left(\frac{dU}{dT}\right)_V$$

from eqn (9)

$$C_V = \frac{d}{dT}\left[N\hbar\omega_0\left\{\frac{1}{2} + \frac{1}{(e^{\hbar\omega_0/KT}-1)}\right\}\right]$$

$$C_V = N\hbar\omega_0\left[0 + \frac{(-1)}{(e^{\hbar\omega_0/KT}-1)^2}\left\{e^{\frac{\hbar\omega_0}{KT}} \cdot \frac{\hbar\omega_0}{K}\left(\frac{-1}{T^2}\right) - 0\right\}\right]$$

$$C_V = N\hbar\omega_0 \frac{\frac{\hbar\omega_0}{KT^2} e^{\frac{\hbar\omega_0}{KT}}}{(e^{\hbar\omega_0/KT}-1)^2}$$

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$$C_V = Nk\left(\frac{\hbar\omega_0}{kT}\right)^2 \frac{e^{\frac{\hbar\omega_0}{kT}}}{\left(e^{\hbar\omega_0/kT}-1\right)^2}$$

let $\theta_V = \frac{\hbar\omega_0}{k}$

$Nk = nR$, $n=1$

$$\boxed{C_V = R\left(\frac{\theta_V^2}{T^2}\right)\frac{e^{\frac{\theta_V}{T}}}{\left(e^{\theta_V/T}-1\right)^2}} \quad (10)$$

eqn (10) defined the Heat capacity for diatomic molecule ie based on Quantum Theory.

Code-I At High Temperature

$$T >>> \theta_V$$

$$\frac{T}{T} >>> \frac{\theta_V}{T}$$

$$e^x = 1 + x + \frac{x^2}{\lfloor 2} + \cdots$$

$$x = \frac{\theta_V}{T} \text{ (let)}$$

$$e^{\frac{\theta_V}{T}} = 1 + \frac{\theta_V}{T} + \frac{1}{\lfloor 2}\frac{\theta_V^2}{T^2} + \cdots$$

$\frac{\theta_V}{T} <<< 1$ on neglect the high power of θ_V

$$e^{\theta_V/T} \approx 1 + \frac{\theta_V}{T}$$

from eqn (10)

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from eqn (10)

$$C_v = R\left(\frac{\theta_v}{T}\right)^2 \frac{\left(1+\frac{\theta_v}{T}\right)}{\left(1+\frac{\theta_v}{T}-1\right)^2}$$

$$C_v = R\left(1+\frac{\theta_v}{T}\right)$$

$$1 >>> \frac{\theta_v}{T}$$

$\boxed{C_v = R = \text{const.}}$ At high temp value of Heat Capacity is constant.

<u>Case – II</u>

At low Temperature,

$$T <<< \theta_v$$

$$1 <<< \frac{\theta_v}{T}$$

$$e^{\frac{\theta_v}{T}} >>> 1$$ on Neglecting 1

from eqn (10)

$$C_v = R\left(\frac{\theta_v}{T}\right)^2 \frac{e^{\frac{\theta_v}{T}}}{(e^{\theta_v/T})^2}$$

$$C_v = R\left(\frac{\theta_v}{T}\right)^2 \frac{1}{e^{\theta_v/T}}$$

$$\boxed{C_v = R\left(\frac{\theta_v}{T}\right)^2 e^{-\frac{\theta_v}{T}}}$$

it is clear that at low temp, Heat capacity decreases in exponential form.

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very Imp. 2019

Heat Capacity OF solid Matter:

To explain the Heat capacity of of solid matter, equipartition principle of energy is used. Accn to equipartition principle of energy, ~~every degree~~ energy connected to every degree of freedom is $\frac{1}{2}KT$. If in solid matter vibration of molecule is 1-dimn then define energy will be KT. and if atom/molecule present in solid matter Heat energy is provide then molecule does 3-dimn vibration, for which define energy is 3KT.

$$U = 3KT$$

If n-molecule are present in solid matter then energy.

$$U = 3NKT$$

$$\boxed{U = 3RT} \qquad \because NK = R$$

We know that

$$C_V = \left(\frac{dU}{dT}\right)_V$$

$$C_V = \frac{d}{dT}(3RT)$$

$$\boxed{C_V = 3R}$$

This Result of classical physics is a Concept of Dulong petit model. That is define the Heat capacity at high temp for solid matter, For low temperature this

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Dulong petit model is invalid ~~bef~~ because this does not satisfy the experimental curve.

① Einstein Model

To define the Heat Capacity of solid matter, A model based on Quantum theory is define, this model is known as Einstein model that explain the limitations present in Dulong Petit. following Hypothesis of this model are:-

ⓐ Acc^n to Einstein model, molecule present in solid matter does simple harmonic vibration means behave as a simple Harmonic Oscillator

ⓑ Behave of these simple Harmonic oscillator is as free oscillator.

ⓒ Distribution of energy is in Discreate form not in continuous form means energy is a Quantized Quantity. So, define energy:-

$$\boxed{E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0}$$

$n = 0, 1, 2, 3 \ldots$

For ground state:

$n = 0$

$$\boxed{E_n = \frac{1}{2}\hbar\omega_0}$$

This is known as Zero Point Energy.

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$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ J-sec.}$$

Planck's Reduced Constant.

(a) Molecule present in solid matter had vibration in 3-dim^n.

<u>Mathematical Analysis:</u>

We know that

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E_n e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}} \quad \text{--- (1)}$$

$$\beta = \frac{1}{kT}$$

$$E_n = \left(n+\frac{1}{2}\right)\frac{\hbar\omega_0}{kT}$$

$$\bar{E} = \frac{\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)\hbar\omega_0 e^{-\left(n+\frac{1}{2}\right)\frac{\hbar\omega_0}{kT}}}{\sum_{n=0}^{\infty} e^{-(n+1/2)\hbar\omega_0/kT}}$$

Let $x = \frac{\hbar\omega_0}{kT}$

$$\bar{E} = \frac{\hbar\omega_0 \sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right) e^{-(n+1/2)x}}{\sum_{n=0}^{\infty} e^{-(n+1/2)x}}$$

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$$\bar{E} = \frac{\hbar\omega_0\left[\frac{1}{2}e^{-\frac{1}{2}x} + \frac{3}{2}e^{-\frac{3}{2}x} + \frac{5}{2}e^{-\frac{5}{2}x} + \cdots\right]}{e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots}$$

$$\because \frac{-d}{dx}\log_e\left[e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots\right] = \frac{1/2\, e^{-1/2x} + 3/2\, e^{-3/2x} + \cdots}{e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots}$$

$$\Rightarrow \quad \bar{E} = -\hbar\omega_0 \frac{d}{dx}\log_e\left[e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots\right]$$

$$\Rightarrow \quad \bar{E} = -\hbar\omega_0 \frac{d}{dx}\log_e\left[e^{-1/2x}(1 + e^{-x} + e^{-2x} + e^{-3x} + \cdots)\right]$$

$$\because S_\infty = \frac{a}{1-r} \; ; \; a = 1, \; r = e^{-x}$$

$$\bar{E} = -\hbar\omega_0 \frac{d}{dx}\left[\log_e\left\{e^{-1/2x} \cdot \frac{1}{1-e^{-x}}\right\}\right]$$

$$\bar{E} = -\hbar\omega_0 \frac{d}{dx}\left[\frac{-1}{2}x + \log_e \frac{1}{1-e^{-x}}\right]$$

$$\bar{E} = -\hbar\omega_0\left[\frac{-1}{2} + \frac{(1-e^{-x})(-1)\cdot(0+e^{-x})}{(1-e^{-x})^2}\right]$$

$$\bar{E} = \hbar\omega_0\left[\frac{1}{2} + \frac{e^{-x}}{1-e^{-x}}\right]$$

$$\bar{E} = \hbar\omega_0\left[\frac{1}{2} + \frac{e^{-x}}{e^{-x}\left(\frac{1}{e^{-x}} - 1\right)}\right]$$

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$$\bar{E} = \hbar\omega_0 \left[\frac{1}{2} + \frac{1}{e^{x}-1} \right]$$

$$x = \frac{\hbar\omega_0}{kT}$$

$$\boxed{\bar{E} = \hbar\omega_0 \left[\frac{1}{2} + \frac{1}{e^{\hbar\omega_0/kT}-1} \right]} \quad \text{—(8)}$$

If N molecule are present in system then Average energy of system.

$$\bar{E} = N\hbar\omega_0 \left[\frac{1}{2} + \frac{1}{e^{\hbar\omega_0/kT}-1} \right]$$

We know that if Volume of any system is constant then total energy is equal to internal energy. Internal energy is denoted by U.

$$U = N\hbar\omega_0 \left[\frac{1}{2} + \frac{1}{e^{\hbar\omega_0/kT}-1} \right] \quad \text{—(9)}$$

We know that

$$C_V = \left(\frac{dU}{dT}\right)_V$$

From eqn (9)

$$C_V = \frac{d}{dT}\left[N\hbar\omega_0 \left\{ \frac{1}{2} + \frac{1}{e^{\hbar\omega_0/kT}-1} \right\} \right]$$

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$$C_V = N\hbar\omega_0 \left[0 + \frac{-1}{(e^{\hbar\omega_0/KT}-1)^2} \left\{ e^{\frac{\hbar\omega_0}{KT}} \cdot \frac{\hbar\omega_0}{K}\left(\frac{-1}{T^2}\right) - 0 \right\} \right]$$

$$C_V = \frac{N\hbar\omega_0 \, \frac{\hbar\omega_0 \cdot e^{\frac{\hbar\omega_0}{KT}}}{KT^2}}{\left(e^{\frac{\hbar\omega_0}{KT}}-1\right)^2}$$

$$C_V = NK\left(\frac{\hbar\omega_0}{KT}\right)^2 \frac{e^{\frac{\hbar\omega_0}{KT}}}{\left(e^{\hbar\omega_0/KT}-1\right)^2}$$

Let $\theta_E = \frac{\hbar\omega_0}{K}$

$$\boxed{C_V = R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{\left(e^{\theta_E/T}-1\right)^2}} \quad \text{—— (5)}$$

Acc^n to Albert Einstein, Molecule present in solid matter do 3-dim^n vibration so Heat Capacity.

for 3-dim^n $$\boxed{C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{\left(e^{\theta_E/T}-1\right)^2}} \quad \text{—— (6)}$$

eq^n (6) is the result of Albert Einstein model.

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Case I st. At high temperature

$$T >>> \theta_E$$

$$1 >>> \frac{\theta_E}{T}$$

$$\because \quad e^{x} = 1 + x + \frac{x^2}{\lfloor 2} + \cdots$$

let $x = \frac{\theta_E}{T}$

$$e^{\frac{\theta_E}{T}} = 1 + \frac{\theta_E}{T} + \left(\frac{\theta_E}{T}\right)^2 \cdot \frac{1}{\lfloor 2}$$

$$1 >>> \frac{\theta_E}{T}$$

on neglecting high power of $\frac{\theta_E}{T}$

$$e^{\frac{\theta_E}{T}} \approx 1 + \frac{\theta_E}{T} \quad \text{—(7)}$$

from eqn (6) and (7)

$$C_V = 3R \cdot \left(\frac{\theta_E}{T}\right)^2 \frac{1 + \theta_E/T}{\left(1 + \frac{\theta_E}{T} - 1\right)^2}$$

$$\therefore \quad C_V = 3R\left(1 + \frac{\theta_E}{T}\right)$$

$$1 >>> \frac{\theta_E}{T}$$

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$$\boxed{C_V = 3R} \quad \text{—— (8)}$$

It is clear from eqn (8) that Einstein model satisfy the Dulong petit model. It means this result of Quantum physics satisfy the classical concept.

Case IInd

At low temperature.

$$T <<< \theta_E$$

$$1 <<< \frac{\theta_E}{T}$$

$$1 <<< e^{\theta_E/T} \quad \text{—— (9)}$$

from eqn (6) and (9)

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \cdot \frac{e^{\theta_E/T}}{(e^{\theta_E/T})^2}$$

$$\boxed{C_V = 3R\left(\frac{\theta_E}{T}\right)^2 e^{-\theta_E/T}} \quad \text{—— (10)}$$

It is clear from eqn (10) that Heat Capacity of solid matter decreases in exponential form at low temperature acc to Einstein model. But acc to experimental result Heat capacity of solid matter follows the T^3 Rule. ($C_V \propto T^3$). so at low temperature, Einstein model is invalid. Explanation of Heat capacity of solid matter is further explain by Dibay model.

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Qued find the

Given $h = 6.62 \times 10^{-34}$ J-sec.

$\theta_E = 240K$

$k_B = 1.38 \times 10^{-23}$ J/K

$\omega_0 = ?$

Soln

$$\theta_E = \frac{\hbar\omega_0}{k}$$

$$\omega_0 = \frac{\theta_E k}{\hbar}$$

$$\omega_0 = \frac{240 \times 1.38 \times 10^{-23}}{1.054 \times 10^{-34}}$$

$$\omega_0 = \frac{240 \times 1.38 \times 10^{-23+34}}{1.054}$$

$$\omega_0 = \frac{240 \times 1.38 \times 10^{11}}{1.054}$$

$$\omega_0 = \omega_E = 3.14 \times 10^{13}$$

$$\omega = 2\pi\nu.$$

$$\nu = \frac{\omega}{2\pi} = \frac{3.14 \times 10^{13}}{2 \times 3.14}$$

$$\boxed{\nu = 5 \times 10^{12} \, sec^{-1}}$$

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Question Given

$$\nu = 3\times10^{13}\ Hz$$
$$\theta_E = ?$$

Soln

$$\theta_E = \frac{\hbar\omega_0}{k}$$

$$\theta_E = \frac{h}{2\pi}\cdot\frac{2\pi\nu}{k}$$

$$\theta_E = \frac{h\nu}{k}$$

$$\nu = \frac{\theta_E k}{h}$$

$$\theta_E = \frac{6.62\times10^{-34}\times3\times10^{13}}{1.38\times10^{-23}}$$

$$\boxed{\theta_E = 1439K}$$

Ques Given

$$\theta_E = 300K$$
$$\nu_E = ?$$

Soln

$$\theta_E = \frac{\hbar\omega_0}{k}$$

$$\theta_E = \frac{h}{2\pi}\cdot\frac{2\pi\nu}{k}$$

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$$\theta_E = \frac{h\nu}{k}$$

$$\nu_E = \frac{\theta_E k}{h}$$

$$\nu_E = \frac{300 \times 1.38 \times 10^{-23}}{6.62 \times 10^{-34}}$$

$$\nu_E =$$

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② Debye Model:-

Debye model is based on Quantum Principle / theorem. means it is a modifiy form of Einstein model To explain the heat capacity of solid matter Debye scientist gave some Hypothesis that are given below.

① Accn to Einstein model, atom present in solid matter behaved as a free oscillator. While Accn to Hypothesis of Debye model, interatomic bond is present b/w atoms and due to this bond, atoms present in solid matter behaved as a Coupled oscillator.

② Accn to Einstein model, distribution of energy for all atoms present in solid matter is in Contineous form. Means atoms which do vibration motion have same energy While Accn to Debye model, all atoms does ~~m~~ vibration motion with different frequency. It means value of energy for all atoms is different becoz pairing energy is define.

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Mathematical Analysis:

We know that

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E_n e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}} \quad \text{—— (1)}$$

$$\because \beta = \frac{1}{KT}$$

$$\because E_n = \left(n+\frac{1}{2}\right)\frac{\hbar\omega_0}{KT}$$

$$\bar{E} = \frac{\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)\hbar\omega_0 e^{-\left(n+\frac{1}{2}\right)\frac{\hbar\omega_0}{KT}}}{\sum_{n=0}^{\infty} e^{-(n+1/2)\hbar\omega_0/KT}}$$

Let $x = \frac{\hbar\omega_0}{KT}$

$$\bar{E} = \frac{\hbar\omega_0 \sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right) e^{-\left(n+\frac{1}{2}\right)x}}{\sum_{n=0}^{\infty} e^{-(n+1/2)x}}$$

$$\bar{E} = \frac{\hbar\omega_0\left[\frac{1}{2}e^{-\frac{1}{2}x} + \frac{3}{2}e^{-\frac{3}{2}x} + \frac{5}{2}e^{-\frac{5}{2}x} + \cdots\right]}{e^{-1/2x} + e^{-3/2x} + e^{-5/2x} + \cdots}$$

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$$\because \frac{-d}{dx}\log_e\left[e^{-1/2x}+e^{-3/2x}+e^{-5/2x}+\cdots\right]=\frac{1/2\,e^{-1/2x}+3/2\,e^{-3/2x}+\cdots}{e^{-1/2x}+e^{-3/2x}+e^{-5/2x}+\cdots}$$

$$\Rightarrow \quad \bar{E} = -\hbar\omega_0\frac{d}{dx}\log_e\left[e^{-1/2x}+e^{-3/2x}+e^{-5/2x}+\cdots\right]$$

$$\Rightarrow \quad \bar{E} = -\hbar\omega_0\frac{d}{dx}\log_e\left[e^{-1/2x}\left(1+e^{-x}+e^{-2x}+e^{-3x}+\cdots\right)\right]$$

$$\because S_\infty = \frac{a}{1-r}\ ;\ a=1,\ r=e^{-x}$$

$$\bar{E} = -\hbar\omega_0\frac{d}{dx}\left[\log_e\left\{e^{-1/2x}\cdot\frac{1}{1-e^{-x}}\right\}\right]$$

$$\bar{E} = -\hbar\omega_0\frac{d}{dx}\left[\frac{-1}{2}x+\log_e\frac{1}{1-e^{-x}}\right]$$

$$\bar{E} = -\hbar\omega_0\left[-\frac{1}{2}+\frac{(1-e^{-x})}{(1-e^{-x})^2}(-1)\cdot(0+e^{-x})\right]$$

$$\bar{E} = \hbar\omega_0\left[\frac{1}{2}+\frac{e^{-x}}{1-e^{-x}}\right]$$

$$\bar{E} = \hbar\omega_0\left[\frac{1}{2}+\frac{e^{-x}}{e^{-x}\left(\frac{1}{e^{-x}}-1\right)}\right]$$

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$$\bar{E} = \hbar\omega_0\left[\frac{1}{2} + \frac{1}{e^{x}-1}\right]$$

$$x = \frac{\hbar\omega_0}{KT}$$

$$\boxed{\bar{E} = \hbar\omega_0\left[\frac{1}{2} + \frac{1}{e^{\hbar\omega_0/KT}-1}\right]} \quad \text{--- (8)}$$

If N molecules are present in system then Average energy of system.

$$\bar{E} = N\hbar\omega_0\left[\frac{1}{2} + \frac{1}{e^{\hbar\omega_0/KT}-1}\right]$$

We know that if volume of any system is Constant then total energy is equal to internal energy. Internal energy is denoted by U.

$$U = 3N\hbar\omega_0\left[\frac{1}{2} + \frac{1}{e^{\hbar\omega_0/KT}-1}\right]$$

To calculate the Heat Capacity of solid matter, Debye gave a standard mathematical integral formula, that is given by as:-

$$\boxed{C_v = 3R\cdot\frac{3}{x_0^3}\int_0^{x_0}\frac{x^4 e^{x}}{(e^{x}-1)^2}\,dx} \quad \text{--- (9)}$$

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Here, $x = \frac{h\nu}{KT}$, $x_0 = \frac{h\nu_{max}}{KT}$

$$\theta_D = \frac{h\nu_{max}}{K} = \frac{\hbar\omega_D}{K}$$

Case-I At High temperature

$$h\nu <<< KT$$

$$\frac{h\nu}{KT} <<< 1$$

$$\because x = \frac{h\nu}{KT}$$

$$x <<< 1$$

$$\because e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$$

For high value of x is negligible

$$e^x \simeq 1 + x \quad \text{—— (b)}$$

from eqn (a) and (b)

$$C_V = 3R \cdot \frac{3}{x_0^3} \int_0^{x_0} \frac{x^4 (1+x)}{(1+x-1)^2} dx$$

$$C_V = 3R \cdot \frac{3}{x_0^3} \int_0^{x_0} (1+x) x^2 dx$$

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$\because x << 1$

$$C_V = 3R \cdot \frac{3}{x_0^3} \int_0^{x_0} x^2 dx$$

$$C_V = 3R \cdot \frac{3}{x_0^3} \left[\frac{x^3}{3}\right]_0^{x_0}$$

$$C_V = 3R \cdot \frac{3}{x_0^3} \cdot \frac{x_0^3}{3}$$

$$C_V = 3R$$

It is clear from eqn (c) that Debye model divides the Dulong Petit model at High Temperature.

<u>Case-II</u> At Low Temperature

$$h\nu >>> KT$$

$$\frac{h\nu}{KT} >>> 1$$

$$\because x = \frac{h\nu}{KT}$$

$$x >>> 1$$

$$e^x >>> 1 \quad \text{—— (d)}$$

from eqn (a) and (d)

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$$C_V = 3R \cdot \frac{3}{x_0^3} \int_0^{x_0} \frac{x^4 e^x}{(e^x)^2} dx$$

$$C_V = 3R \cdot \frac{3}{x_0^3} \int_0^{x_0} \frac{x^4}{e^x} dx \qquad \because \int_0^{x_0} \frac{x^4}{e^x} dx = \frac{4}{15}\pi^4$$

$$C_V = \frac{9R}{x_0^3} \cdot \frac{4}{15} \pi^4$$

$$C_V = \frac{12 \pi^4 R}{5 x_0^3} \qquad \because x_0 = \frac{h\nu_{max}}{kT}$$

$$C_V = \left(\frac{12 k^3}{5 h^3 \nu_{max}^3}\right) T^3$$

$$\boxed{C_V \propto T^3}$$

It is clear that at low temperature Debye model satisfy the T^3 law.

Ques Given

$\theta_D = 300K$

$\nu_D = ?$ (cut off frequency)

Soln

We know that

$$\theta_D = \frac{\hbar \omega_D}{k}$$

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$$\hbar = \frac{h}{2\pi}, \quad \omega_D = 2\pi\nu_D$$

$$\theta_D = \frac{h \cdot 2\pi\nu_D}{2\pi k}$$

$$\theta_D = \frac{h\nu_D}{k}$$

$$\nu_D = \frac{\theta_D k}{h}$$

$$\nu_D = \frac{300 \times 1.38 \times 10^{-23}\ J/k}{6.62 \times 10^{-34}\ J\text{-}sec}$$

$$\boxed{\nu_D = \cancel{300}\ 2.08 \times 10^{12}\ Hz}$$

Ques Given

$$\omega_D = 5.65 \times 10^{12}\ Hz$$
$$\theta_D = ?$$

Ans

We know that

$$\theta_D = \frac{\hbar\omega_D}{k}$$

$$\theta_D = \frac{1.054 \times 10^{-34} \times 5.65 \times 10^{-12}}{1.38 \times 10^{-23}}$$

$$\boxed{\theta_D = 43.15\ K}$$

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Ques Given

V

$T = 10K$

$C_V = ?$

$\nu_D = 6.55 \times 10^{12} Hz$

$R = 8.31 \times 10^3$

Soln We know that

$$\theta_D = \frac{h\nu_D}{k}$$

$$\theta_D = \frac{6.62 \times 10^{-34} \times 6.55 \times 10^{12}}{1.38 \times 10^{-23}}$$

$$\boxed{\theta_D = 314.2K}$$

from Debye Model

$$C_V = \frac{12}{5}\pi^4 R \left(\frac{T}{\theta_D}\right)^3$$

$$C_V = \frac{12}{5} \times (3.14)^4 \times 8.31 \times 10^3 \times \left(\frac{10}{314.2}\right)^3$$

$$\boxed{C_V = 62.3}$$

Ques Given

$C_V = 4.6 \times 10^{-2} T^3$

$N = 6.02 \times 10^{26}$

$K = 1.38 \times 10^{-23}$

$\theta_D = ?$

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Soln We know that from Debye model

$$C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$$

$$\because \quad Nk = R$$

$$C_V = \frac{12}{5}\pi^4 Nk\left(\frac{T}{\theta_D}\right)^3$$

$$\theta_D^3 = \frac{12}{5}\frac{Nk\,T^3\,\pi^4}{C_V}$$

$$\theta_D^3 = \left[\frac{\frac{12}{5}(3.14)\pi^4 \times 6.02\times10^{26}\times1.38\times10^{-23}\,T^3}{4.6\times10^{-2}\,T^3}\right]$$

$$\theta_D = \frac{1.936\times10^{6}}{4.6\times10^{-2}}$$

$$\boxed{\theta_D = 348\,K}$$

Ques Given

$C_V = 152$

$\theta_D = 348\,K$

$T = ?$

Soln $C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$ from Debye model

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$$C_V = \frac{12}{5} \times (3.14)^4 \times NK \times \frac{T^3}{\theta_D^3}$$

$$T^3 = \frac{C_V \times 5 \times \theta_D^3}{12 \times \pi^4 \times NK}$$

$$T^3 = \frac{152 \times 5 \times (348)^3}{12 \times (3.14)^4 \times 6.02 \times 10^{26} \times 1.38 \times 10^{-23}}$$

$$T^3 = 225 \times 15$$

$$\boxed{T = 15K}$$

Phase space :-

Acc^n to Concept of classical physics "To explain ~~the~~ molecule of any gas in easy way, a frame of reference is necessary. In this frame of reference to define the Configuration of particle, position as well as momentum co-ordinate is necessary in the form of free Variable. In this way define six dimensional frame of reference is known as Phase space." small Volume is define by H in phase space.

$$H = dx\,dy\,dz\,dP_x\,dP_y\,dP_z \quad \text{(momentum space)}$$

or

$$H = dx\,dy\,dz\,dV_x\,dV_y\,dV_z \quad \text{(Position-Velocity space)}$$

Micro and Macro state :-

Acc^n to Concept of thermodynamic physics, If many particles are present in any system

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then the system is known as macro system with respect to these particle and exists particle are known as micro state.

Example Let an atom of uranium then e^-, proton and neutron are known as micro ~~sy~~ state and these atom is known as Macro state.

V.V mat

Relation b/w Thermodynamic probability and Entropy :-

Acc^n to the concept of thermodynamic physics, That physical Quantity by which measurement of disorder of system is known as entropy of system. Entropy is a energy of system. that is denoted by S.

That Microstate that follow the law of Macro system known as Acceciable state of system. Acceciable state of system is known as Thermodynamic probability. It is denoted by W or $\Omega(E)$.

Mathematical Analysis :-

Acc^n to Boltzmann statics, Entropy of any system is the function of thermodynamic probability. It means,

$$S = f(W) \quad \text{——} (1)$$

Let In Phase space, two macro system are present. For 1st system, Value of entropy is S_1 and for second entropy is S_2. Thermodynamic probability for 1st is W_1 and

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Thermodynamic probability for IInd is W_2,

(i) For 1st system

$$S_1 = f(W_1) \quad \text{--- (2)}$$

(ii) For 2nd system

$$S_2 = f(W_2) \quad \text{--- (3)}$$

Total Entropy of system.

so, $S = S_1 + S_2$

$$\boxed{f(W) = f(W_1) + f(W_2)} \quad \text{--- (4)}$$

These two system do interaction due to which a Combined system is ~~define~~ generate, so, Thermodynamic probability of Combined system.

$$W = W_1 W_2$$

so, Thermodynamic probability in mathematical function form.

$$f(W) = f(W_1 W_2) \quad \text{--- (5)}$$

from eqn (4) and (5)

$$\Rightarrow \boxed{f(W_1 W_2) = f(W_1) + f(W_2)} \quad \text{--- (6)}$$

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Thermodynamic probability W_1 and W_2 are free variable present in eqn (6) so on differentiate with respect to each other will be zero.

(i) $f(W_1 W_2) = f(W_1) + f(W_2)$

on diffn w.r.t to W_1; W_2 = const.

$$\Rightarrow \frac{\partial f(W_1, W_2)}{\partial W_1} = \frac{\partial f(W_1)}{\partial W_1} + 0$$

$$\Rightarrow \frac{W_2}{W_2} \frac{\partial f(W_1 W_2)}{\partial W_1} = \frac{\partial f(W_1)}{\partial W_1}$$

from property of differential operator.

$$\Rightarrow W_2 \frac{\partial f(W_1 W_2)}{\partial (W_1 W_2)} = \frac{\partial f(W_1)}{\partial W_1}$$

$$\Rightarrow \frac{\partial f(W_1 W_2)}{\partial (W_1 W_2)} = \frac{1}{W_2} \frac{\partial f(W_1)}{\partial W_1} \quad \text{--- (7)}$$

(ii) from eqn (6)

$$f(W_1 W_2) = f(W_1) + f(W_2)$$

on diffn w.r.t to W_2; W_1 = const.

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$$\frac{\partial f(\omega_1\omega_2)}{\partial \omega_2} = 0 + \frac{\partial f(\omega_2)}{\partial \omega_2}$$

$$\frac{\omega_1}{\omega_1}\frac{\partial f(\omega_1\omega_2)}{\partial \omega_2} = \frac{\partial f(\omega_2)}{\partial \omega_2}$$

From property of diff^n operator.

$$\omega_1 \frac{\partial f(\omega_1\omega_2)}{\partial(\omega_1\omega_2)} = \frac{\partial f(\omega_2)}{\partial \omega_2}$$

$$\frac{\partial f(\omega_1\omega_2)}{\partial(\omega_1\omega_2)} = \frac{1}{\omega_1}\frac{\partial f(\omega_2)}{\partial \omega_2} \quad \text{—— (8)}$$

From eq^n (7) and (8)

$$\frac{1}{\omega_2}\frac{\partial f(\omega_1)}{\partial \omega_1} = \frac{1}{\omega_1}\frac{\partial f(\omega_2)}{\partial \omega_2}$$

$$\Rightarrow \omega_1 \frac{\partial f(\omega_1)}{\partial \omega_1} = \omega_2 \frac{\partial f(\omega_2)}{\partial \omega_2} = \text{Constant}$$

Acc^n to maxwell's Boltzmann distribution in general form.

$$\omega \frac{\partial f(\omega)}{\partial \omega} = \text{Constant} = K$$

K = Boltzmann Constant $= 1.38 \times 10^{-23}$ J/K

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$$\Rightarrow \quad \omega \frac{\partial f(\omega)}{\partial \omega} = K$$

$$\Rightarrow \quad \frac{\partial f(\omega)}{\partial \omega} = \frac{K}{\omega}$$

on integrate

$$\Rightarrow \quad \int \partial f(\omega) = \int \frac{K}{\omega} \partial \omega$$

$$\Rightarrow \quad f(\omega) = K \log_e W$$

$S = f(\omega)$

from eqn (1)

MTR $\boxed{S = K \log_e \omega}$

$$S \propto \log_e \omega$$

Hence It is clear that Acc^n to Maxwell's Boltzmann statistics entropy of system is logarithm function of thermodynamic probability.

Ques 1 Prove that partition function for monoatomic ideal gas is following:

$$Z = \frac{V}{H} (2\pi mKT)^{3/2}$$

if $H = h^3$

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$$Z = \frac{V}{h^3}(2\pi mKT)^{3/2}$$

Soln Partation function for monoatomic ideal gas:-

To do the calculation of Partion function for monoatomic ideal gas we image a ~~sp~~ phase space. let in this Phase space a system is present in which molecule of monoatomic ideal gas is present, mass of everey molecule is m. and velocity is v, so total energy of gas molecule is equal to kinetic energy and potential energy.

$$E = K.E + P.E$$

$$E = \frac{1}{2}mv^2 + mgh$$

$$\boxed{E = \frac{P^2}{2m} + mgh} \quad \text{--- (1)} \qquad \because P = mv$$

To do the mathematical analysis of Partion function for monoatomic gas, let effect of gravitational field is negligible. means $g \to 0$

from eqn (1)

$$E = \frac{P^2}{2m}$$

Energy for i^{th} shell

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$$\boxed{E_i = \frac{P_i^2}{2m}}$$

We know that, from defination of partition function.

$$Z = \sum_i e^{-\beta E_i}$$

Partition function in phase space.

$$Z = \sum_i \frac{e^{-\beta E_i} H}{H}$$

$$\beta = \frac{1}{KT}$$

$$E_i = \frac{P_i^2}{2m}$$

$$Z = \frac{1}{H} \sum_i \frac{P_i^2}{e^{2mKT} H}$$

So, Volume in phase space,

$$H = dx\,dy\,dz\,dP_x\,dP_y\,dP_z$$

Acc^n to Maxwell's Boltzmann statistics, Distribution of gas molecule in system is Continuous. So, summation is shifted to integration.

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Here, Derivative term are independent in 6 order, so,
Partition function,

$$Z = \frac{1}{H}\iiiint\!\!\int e^{\frac{-p^2}{2mKT}} dx\,dy\,dz\,dP_x\,dP_y\,dP_z.$$

$$Z = \frac{1}{H}\iiint dx\,dy\,dz \iiint e^{\frac{-p_i^2}{2mKT}} dP_x\,dP_y\,dP_z$$

so, Volume in general space,

$$V = \iiint dx\,dy\,dz$$

$$Z = \frac{V}{H}\iiint e^{\frac{-p_i^2}{2mKT}} dP_x\,dP_y\,dP_z$$

$$\because \vec{p} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$$

$$|\vec{p}| = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

$$p^2 = P_x^2 + P_y^2 + P_z^2$$

$$Z = \frac{V}{H}\iiint e^{\frac{-(P_x^2+P_y^2+P_z^2)}{2mKT}} dP_x\,dP_y\,dP_z$$

$$Z = \frac{V}{H}\int e^{\frac{-P_x^2}{2mKT}} dP_x \int e^{\frac{-P_y^2}{2mKT}} dy \int e^{\frac{-P_z^2}{2mKT}} dz$$

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Acc^n to Maxwell molecular velocity law, every value of velocity can be possible, so Here limit of momentum will be $-\infty$ to $+\infty$, so,

$$Z = \int_{-\infty}^{+\infty} e^{\frac{-P_x^2}{2mKT}} dP_x \int_{-\infty}^{+\infty} e^{\frac{-P_y^2}{2mKT}} dP_y \int_{-\infty}^{+\infty} e^{\frac{-P_z^2}{2mKT}} dP_z$$

$$\because \int_{-\infty}^{+\infty} e^{\frac{-P_x^2}{2mKT}} dP_x = (2\pi mKT)^{1/2}$$

$$\int_{-\infty}^{+\infty} e^{\frac{-P_y^2}{2mKT}} dP_y = (2\pi mKT)^{1/2}$$

$$\int_{-\infty}^{+\infty} e^{\frac{-P_z^2}{2mKT}} dP_z = (2\pi mKT)^{1/2}$$

$$\boxed{Z = \frac{V}{H} (2\pi mKT)^{3/2}} \quad \text{—— (2)}$$

if $H = h^3$ then

$$\boxed{Z = \frac{V}{h^3} (2\pi mKT)^{3/2}}$$

eq^n (2) is called the mathematical formula for Partation function for monoatomic ideal gas.

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(i) Velocity Distribution function:

We know that from Probability Distribution function.

$$P = \frac{e^{-\beta E}}{Z}$$

Let no. of molecules in a system is N so, no. of small molecule in i^{th} shell. (phase space).

$$d^6N_i = NP$$

$$d^6N_i = \frac{Ne^{-\beta E_i}}{Z}$$

From eqn (2)

$$d^6N_i = \frac{Ne^{-\beta E_i} H}{V(2\pi mKT)^{3/2}}$$

$$\because H = dx\,dy\,dz\,dP_x\,dP_y\,dP_z$$

$$d^6N_i = \frac{N e^{-\beta E_i}}{V(2\pi mKT)^{3/2}} dP_x\,dP_y\,dP_z\,dx\,dy\,dz \quad \text{—(3)}$$

eqn (3) is the 6 order differential eqn. in this diffn eqn, on integrate three ~~tyt~~ times with respect to momentum.

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$$d^3N_i = \frac{N\,dx\,dy\,dz}{V(2\pi mKT)^{3/2}} \iiint e^{-\beta E_i}\,dP_x\,dP_y\,dP_z$$

$$\frac{d^3N_i}{dx\,dy\,dz} = \frac{N}{V(2\pi mKT)^{3/2}} \iiint e^{-\beta E_i}\,dP_x\,dP_y\,dP_z$$

$$\because \quad \beta = \frac{1}{KT}$$

$$\because \quad E_i = \frac{P_i^2}{2m} = \frac{P_x^2 + P_y^2 + P_z^2}{2m}$$

$$\frac{d^3N_i}{dx\,dy\,dz} = \frac{N}{V(2\pi mKT)^{3/2}} \iiint e^{-\frac{(P_x^2+P_y^2+P_z^2)}{2mKT}}\,dP_x\,dP_y\,dP_z$$

$$\frac{d^3N_i}{dx\,dy\,dz} = \frac{N}{V(2\pi mKT)^{3/2}} \int_{-\infty}^{+\infty} e^{\frac{-P_x^2}{2mKT}}\,dx \int_{-\infty}^{+\infty} e^{\frac{-P_x^2}{2mKT}}\,dP_y \int_{-\infty}^{+\infty} e^{\frac{-P_z^2}{2mKT}}\,dP_z$$

$$\frac{d^3N_i}{dx\,dy\,dz} = \frac{N}{V(2\pi mKT)^{3/2}} (2\pi mKT)^{3/2}$$

$$\boxed{\frac{d^3N_i}{dx\,dy\,dz} = \frac{N}{V}} \quad \text{—— (4)}$$

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From eq^n (3)

$$d^6N_i = \frac{N e^{-\beta E_i}}{V(2\pi mKT)^{3/2}} dx\,dy\,dz\,dP_x\,dP_y\,dP_z$$

On ~~diff~~ integrate three times with respect to position.

$$d^3N_i = \frac{Ne^{-\beta E_i}}{V(2\pi mKT)^{3/2}} dP_x\,dP_y\,dP_z \iiint dx\,dy\,dz$$

$\because \iiint dx\,dy\,dz$ Volume in general space

$$d^3N_i = \frac{Ne^{-\beta E_i}}{V(2\pi mKT)^{3/2}} dP_x\,dP_y\,dP_z \cdot V$$

$\because P = mv = dP_x = m\,dV_x$; m = constant

$$d^3N_i = \frac{Ne^{-\beta E_i}}{(2\pi mKT)^{3/2}} m\,dV_x \cdot m\,dV_y \cdot m\,dV_z$$

$$\because \beta = \frac{1}{KT}$$

$$\because E_i = \frac{P_i^2}{2m} = \frac{1}{2}mv^2$$

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$$d^3 N_i = \frac{N m^3}{(2\pi KT)^{3/2} \cdot m^{3/2}} e^{-\frac{1}{2}\frac{mV^2}{KT}} dV_x dV_y dV_z$$

$$\because V^2 = V_x^2 + V_y^2 + V_z^2$$

$$\boxed{d^3 N_i = N \left[\frac{m}{2\pi KT}\right]^{3/2} \cdot e^{\frac{-mV_x^2}{2KT}} \cdot e^{\frac{-mV_y^2}{2KT}} \cdot e^{\frac{-mV_z^2}{2KT}} dV_x dV_y dV_z} \quad \text{—(5)}$$

eqn (5) is known as Distribution function of velocity for monoatomic ideal gas.

Topic

(ii) Ideal gas equation :-

We know that from mathematical formula of ~~Hail~~ Helmholtz energy.

$$F = U - ST$$

on writing in differential form.

$$dF = dU - d(ST)$$

$$dF = dU - SdT - TdS \quad \text{—(6)}$$

From first law of Thermodynamics.

$$dQ = dU + dw$$

$$\because dw = PdV$$

$$dQ = dU + PdV \quad \text{—(7)}$$

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From second law of Thermodynamics,

$$ds = \frac{dQ}{T}$$

$$dQ = Tds \quad \text{--- (8)}$$

From eqn (7) and (8)

$$Tds = dU + PdV \quad \text{--- (9)}$$

putting eqn (9) in eqn (6)

$$dF = dU - SdT - (dU + PdV)$$

$$dF = dU - SdT - dU - PdV$$

$$\boxed{dF = -SdT - PdV}$$

For isothermal system

$T =$ constant

$dT = 0$

so,

$$dF = 0 - PdV$$

$$\left(\frac{\partial F}{\partial V}\right)_T = -P \quad \text{--- (10)}$$

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We know that, value of Helmholtz energy in the form of partition form is :-

$$F = -NKT \log_e Z$$

$$\because \quad Z = \frac{V}{H}(2\pi mKT)^{3/2}$$

$$F = -NKT \log_e\left[\frac{V}{H}(2\pi mKT)^{3/2}\right]$$

$$F = -NKT\left[\log_e V - \log_e H + \frac{3}{2}\log_e(2\pi mKT)\right]$$

On differentiate with respect to V, T = const.

$$\left(\frac{\partial F}{\partial V}\right)_T = -NKT\left[\frac{1}{V} - 0 + 0\right]$$

$$\left(\frac{\partial F}{\partial V}\right)_T = \frac{-NKT}{V} \quad \text{——— (11)}$$

From eqn (10) and (11)

$$-P = \frac{-NKT}{V}$$

$$PV = NKT \qquad \because NK = R$$

$$\boxed{PV = RT}$$

So, This is ideal gas eqn.

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(iii) Internal Energy of system:

We know that

$$U = NKT^2 \frac{\partial}{\partial T} \log_e Z \quad \text{—} (12)$$

$$\because \quad Z = \frac{V}{H} (2\pi mKT)^{3/2}$$

$$Z = \frac{V}{H} (2\pi mK)^{3/2} \cdot T^{3/2}$$

$$\because \frac{V}{H} (2\pi mK)^{3/2} = A = \text{Const.}$$

$$Z = AT^{3/2}$$

From eqn (12)

$$U = NKT^2 \frac{\partial}{\partial T} \log_e (AT^{3/2})$$

$$U = NKT^2 \frac{\partial}{\partial T} \left[\log_e A + \frac{3}{2} \log_e T \right]$$

$$U = NKT^2 \left[0 + \frac{3}{2} \cdot \frac{1}{T} \cdot 1 \right]$$

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$$U = NkT^2 \cdot \frac{3}{2T}$$

$$U = \frac{3}{2} NkT \qquad \because Nk = R$$

$$\boxed{U = \frac{3}{2} RT}$$

(iv) <u>Heat Capacitance</u> :-

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V$$

$$U = \frac{3}{2} RT$$

$$\boxed{C_v = \frac{3}{2} R}$$

$$\because C_p - C_v = R$$

$$C_p = C_v + R$$

$$C_p = \frac{3}{2} R + R$$

$$\boxed{C_p = \frac{5}{2} R}$$

<u>NOTE</u>

$$\gamma = \frac{C_p}{C_v}$$

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$$\gamma = \frac{\frac{5}{2}R}{3/2R}$$

$$\boxed{\gamma = \frac{5}{3}}$$

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Barometric Equation:-

To study the Barometric eqn, we image an air column in Phase space. In this air column air particles are ~~fif~~ filled. Let mass of every gas molecule is m and velocity is v. So total energy for gas particle.

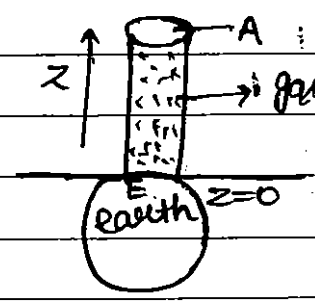
$$E = E_k + E_p$$

$$E = \frac{1}{2}mv^2 + mgh.$$

Let air column is present in z-dirn.

$$\boxed{E = \frac{1}{2}mv^2 + mgz} \quad \text{—— (1)} \quad (\because h = z)$$

To do the mathematical explanation of Barometric eqn, effect of gravitational force is also included.



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So, cross section area of gas column:

$$A = \iint dx\,dy \quad \text{—②}$$

We know that from defination of partition fun.

$$Z = \sum_i e^{-\beta E_i}$$

$$Z = \frac{1}{H} \sum_i e^{-\beta E_i} H$$

$$\because \beta = \frac{1}{KT}, \quad E = \frac{1}{2} mV^2 + mgz$$

$$Z = \frac{1}{H} \sum_i e^{-\left(\frac{1}{2}mV^2 + mgz\right)\cdot\frac{1}{KT}} \cdot H$$

$$Z = \frac{1}{H} \sum_i e^{\frac{-mV^2}{2KT}} \cdot e^{\frac{-mgz}{KT}} H$$

So, Volume in velocity space

$$H = dx\,dy\,dz\,dV_x\,dV_y\,dV_z$$

If distribution of gas molecule is continuous then summation is shifted to integration.

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$$Z = \frac{1}{H}\iiiint\!\!\int e^{\frac{-mv^2}{2KT}} \cdot e^{\frac{-mgz}{KT}}\, dx\, dy\, dz\, dV_x\, dV_y\, dV_z$$

Acc^n to maxwell molecular distribution law, velocity of every gas molecule can be possible.

$$Z = \frac{1}{H}\iiint e^{\frac{-mgz}{KT}}\, dx\, dy\, dz \int_{-\infty}^{+\infty}\!\!\int_{-\infty}^{+\infty}\!\!\int_{-\infty}^{+\infty} e^{\frac{-mv^2}{2KT}}\, dV_x\, dV_y\, dV_z$$

$$\because \iint dx\, dy = A \quad , z = 0 \text{ to } \infty$$

$$\Rightarrow \quad Z = \frac{A}{H}\int_0^{\infty} e^{\frac{-mgz}{KT}} \int_{-\infty}^{+\infty} e^{\frac{-mv_x^2}{2KT}}\, dV_x \int_{-\infty}^{+\infty} e^{\frac{-mv_y^2}{2KT}}\, dV_y \int_{-\infty}^{+\infty} e^{\frac{-mv_z^2}{2KT}}\, dV_z$$

$$\Rightarrow \quad \int_{-\infty}^{+\infty} e^{\frac{-mv_x^2}{2KT}}\, dV_x = \left(\frac{2\pi KT}{m}\right)^{1/2}$$

$$\int_{-\infty}^{+\infty} e^{\frac{-mv_y^2}{2KT}}\, dV_y = \left(\frac{2\pi KT}{m}\right)^{1/2}$$

$$\int_{-\infty}^{+\infty} e^{\frac{-mv_z^2}{2KT}}\, dV_z = \left(\frac{2\pi KT}{m}\right)^{1/2}$$

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$$\int_0^\infty e^{\frac{-mgz}{KT}} dz = \frac{KT}{mg}$$

$$\boxed{Z = \frac{AKT}{Hmg}\left(\frac{2\pi KT}{m}\right)^{3/2}} \quad \text{—— (3)}$$

No. of molecules in i^{th} shell,

$$d^6N_i = -NP$$

$$\because P = \frac{e^{-\beta E_i}}{Z}$$ (Probability distribution function)

$$d^6N_i = \frac{Ne^{-\beta E_i}}{Z}$$

from eqn (3)

$$d^6N_i = \frac{NHmg}{AKT}\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\beta E_i}$$

$$\because \beta = \frac{1}{KT}, \quad E = \frac{1}{2}mv^2 + mgz$$

$$d^6N_i = \frac{NmgH}{AKT}\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\left(\frac{1}{2}mv^2 + mgz\right)\frac{1}{KT}}$$

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$$d^6N_i = \frac{NmgH}{AKT}\left(\frac{m}{2\pi KT}\right)^{3/2} e^{\frac{-mv^2}{2KT}} \cdot e^{\frac{-mgz}{KT}}$$

$$\because H = dx\,dy\,dz\,dV_x\,dV_y\,dV_z$$

on integration three times with respect to velocity

$$\Rightarrow d^3N_i = \frac{Nmg}{AKT}\left(\frac{m}{2\pi KT}\right)^{3/2} e^{\frac{-mgz}{KT}}\left[\int_{-\infty}^{+\infty} e^{\frac{-mv_x^2}{2KT}}dV_x \int_{-\infty}^{+\infty} e^{\frac{-mv_y^2}{2KT}}dV_y \int_{-\infty}^{+\infty} e^{\frac{-mv_z^2}{2KT}}dV_z\right] dx\,dy\,dz$$

$$\Rightarrow \frac{d^3N_i}{dx\,dy\,dz} = \frac{Nmg}{AKT}\left(\frac{m}{2\pi KT}\right)^{3/2} \cdot e^{\frac{-mgz}{KT}}\left(\frac{2\pi KT}{m}\right)^{3/2}$$

$$\because \frac{d^3N_i}{dx\,dy\,dz} = \frac{N}{V}$$

$$\frac{N}{V} = \frac{Nmg}{AKT} e^{\frac{-mgz}{KT}} \quad \text{—— (4)}$$

From ideal gas equation

$$PV = NKT$$

$$P = \frac{N}{V}KT \quad \text{—— (5)}$$

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from eqn (4) and (5)

$$P = \frac{Nmg}{AKT} e^{\frac{-mgz}{KT}} \cdot KT$$

$$P = \frac{Nmg}{A} e^{\frac{-mgz}{KT}} \quad \text{--- (6)}$$

At origin point

$z = 0$; $P = P_0$

from eqn (6)

$$P_0 = \frac{Nmg}{A} e^{-0} \qquad \because e^{-0} = e^{0} = 1$$

$$P_0 = \frac{Nmg}{A}$$

Put the value in eqn (6)

$$\boxed{P = P_0 e^{\frac{-mgz}{KT}}} \quad \text{--- (7)}$$

(i) if $z = z_1$ then $P = P_1$

from eqn (7)

$$P_1 = P_0 e^{\frac{-mgz_1}{KT}} \quad \text{--- (8)}$$

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(iii) If $z = z_2$ then $P = P_2$

from eqn (7)

$$P_2 = P_0 e^{\frac{-mgz_2}{KT}} \quad \text{--- (9)}$$

on dividing eqn (8) and (9)

$$\frac{P_1}{P_2} = \frac{P_0 e^{\frac{-mgz_1}{KT}}}{P_0 e^{\frac{-mgz_2}{KT}}}$$

$$\frac{P_1}{P_2} = e^{\frac{-mg}{KT}(z_1 - z_2)}$$

on taking log to both sides,

$$\log\left(\frac{P_1}{P_2}\right) = \frac{-mg}{KT}(z_1 - z_2)$$

$$\boxed{(z_2 - z_1) = \frac{KT}{mg} \log_e\left(\frac{P_1}{P_2}\right)} \quad \text{--- (10)}$$

This is the mathematical explanation of Barometric equation.

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Validity of classical approximation:-

Acc^n to the concept of classical physics, explanation of macro system is done by Maxwell Boltzmann statics. If size of particle is in micro order then wavelength of particle is dominant on the size of particle, and explanation of that particle is done by Wave Nature, that is a postulate of Quantum Physics.

Ex. Photoelectric effect, Compton effect, Pair Production, Black Body Radation.

And when size of particle is dominant on wavelength of particle then explaination of that particle is done by classical physics.

let size of any particle is s and wavelength is λ then Validity of classical approximation.

$$\boxed{s >>> \lambda}$$

let a particle is present in Phase space, that does simple Harmonic motion, Total energy of that particle is equal to kinetic energy and potential energy and frequency of that particle is

$$\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}} \quad \text{--- (1)}$$

$$E = E_k + E_p = \text{Constant}$$

$$E = \frac{1}{2} m V_x^2 + \frac{1}{2} k x^2 \quad \text{in } x\text{-dirn}$$

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$$\Rightarrow \frac{p_x^2}{2mE} + \frac{1}{2E} k x^2 = 1$$

$$\Rightarrow \frac{p_x^2}{2mE} + \frac{x^2}{2E/k} = 1$$

$$\boxed{\frac{p_x^2}{(\sqrt{2mE})^2} + \frac{x^2}{\left(\sqrt{\frac{2E}{k}}\right)^2} = 1} \quad \text{—— (2)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

eqn (2) is the eqn of ellipse in Phase space.

$$a = \sqrt{2mE}, \quad b = \sqrt{\frac{2E}{k}}$$

So, Area of ellipse

$$A = \pi ab$$

$$A = \pi \sqrt{2mE}, \sqrt{\frac{2E}{mk}} \qquad \because \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$A = \frac{E}{\nu}$$

on diffn with respect to E

$$\boxed{dA = \frac{dE}{\nu}}$$

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Anubhuti Jain

$$\boxed{\frac{dA}{h} = \frac{dE}{h\nu}}$$

This is the validity of classical Approximation.

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